Viscous and Thermal Boundary Layers in Turbulent Rayleigh-Bénard Convection

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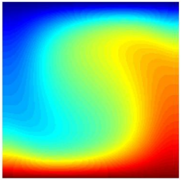
Questions to answer

- Are thermal and viscous boundary layers laminar or turbulent?

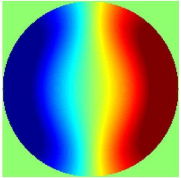
- Does it matter how x, y, t are averaged and how z is scaled?

Rayleigh-Bénard Convection ($\sigma = 0.4, \Gamma = 1$)

Steady-State $R = 2 \times 10^4$:

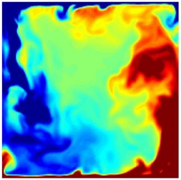


Y-const

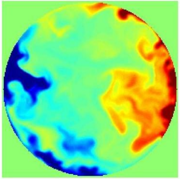


Z-const

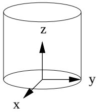
Turbulence $R = 1 \times 10^8$:



Y-const



Z-const



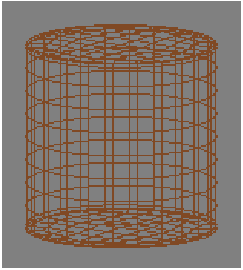
Dimensionless Boussinesq equations

$$\sigma^{-1}\left(\partial_t + \vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\vec{\nabla}P + \nabla^2\vec{v} + \theta\hat{z},$$
$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}\right)\theta = \nabla^2\theta + Rw,$$
$$\vec{\nabla}\cdot\vec{v} = 0,$$

- $\vec{v}$ = (u, v, w) = velocity field, θ = temperature deviation field, P = pressure field
- ν= kinematic viscosity, κ= therm diffusivity, α = therm exp coeff
- R=Rayleigh number=$\alpha g \Delta T d^3/\kappa\nu$, σ=Prandtl number=ν/κ
- Γ = diameter/depth for cylinders

Numerical Simulations—Nekton

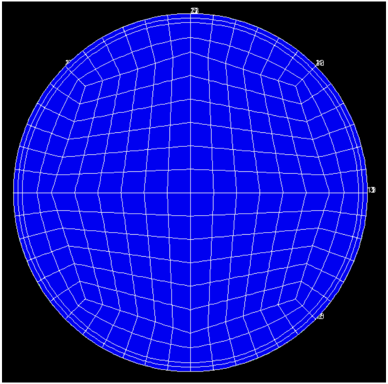
Numerically solves the Boussinesq equations

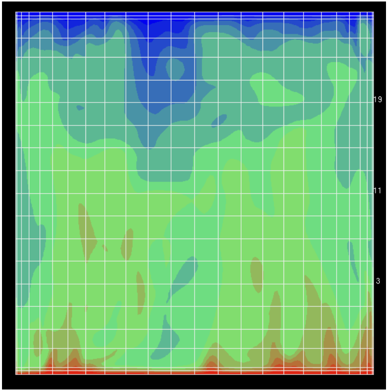


- P.F. Fischer—J. Comp. Phys **133**, 84 (1997)
  - Parallel, spectral element method
  - Gauss-Lobatto-Legendre grid
  - Semi-implicit: Treat off-diagonal terms explicitly:
    - Convective terms, Buoyancy term
- Experimentally realistic boundary conditions
  - No-slip velocity boundary conditions
  - Thermal boundary conditions can be conducting or insulating

Need sufficient resolution

$\sigma = 0.7, \Gamma = 1, R = 1 \times 10^8$





Need sufficient resolution

$\sigma = 0.7, \Gamma = 1$

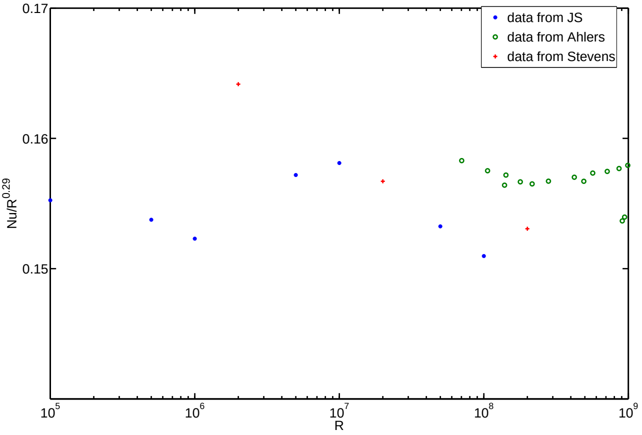
R	Nu	N_{bl}	l/η
1×10^5	4.38	20	.28
5×10^5	6.91	16	.47
1×10^6	8.37	14	.58
5×10^6	13.78	13	.99
1×10^7	16.94	13	1.24
5×10^7	26.19	12	2.07
1×10^8	31.54	20	1.27

$$\text{Kolmorgorov scale } \eta = d \left(\frac{\sigma^2}{RNu} \right)^{1/4}$$

$$l = \text{max grid scale}$$

Comparison of Nusselt numbers with other results

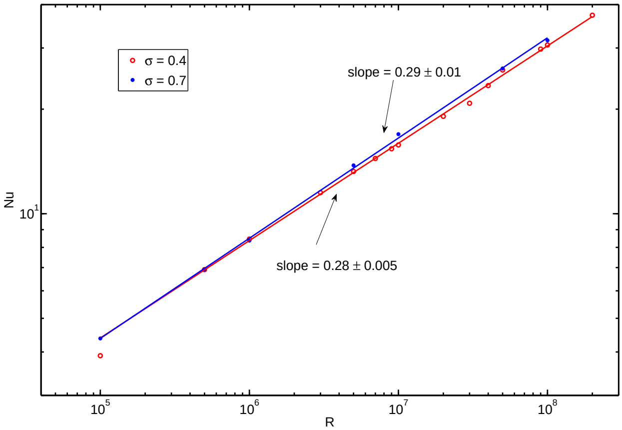
$\sigma = 0.7$, JS: $\Gamma = 1$, Others: $\Gamma = 1/2$



Ahlers et.al. JFM 2009, Stevens et. al. JFM 2010

Nusselt number scaling

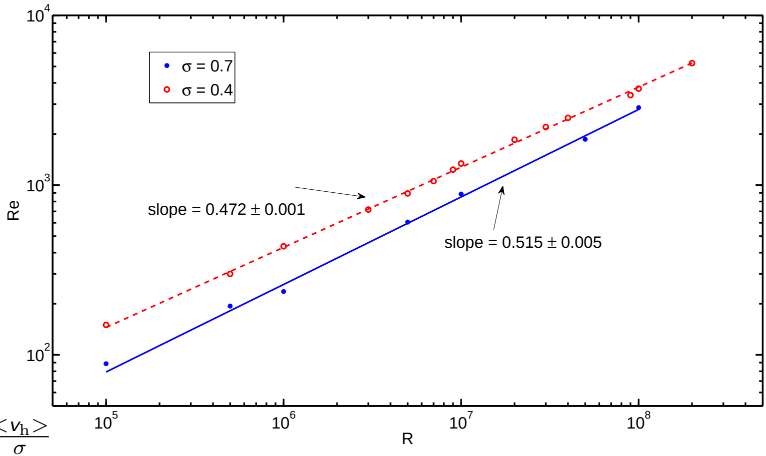
$Nu = Nu_o R^{\gamma}, \Gamma = 1$



$$\mathrm{Nu} = \frac{\int \vec{\nabla} T \cdot d\vec{A}}{\int \hat{n} \cdot d\vec{A}}$$

Reynolds number scaling

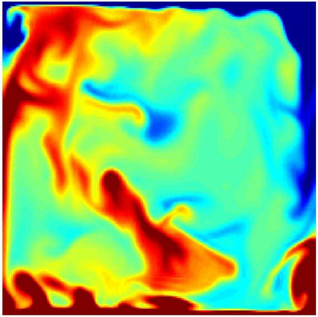
$Re = Re_o R^{\gamma}, \Gamma = 1$



$$\mathrm{Re} = \frac{<v_h>}{\sigma}$$

Movie

$R = 5 \times 10^7, \sigma = 0.7, \Gamma = 1$



Temperature plot, with Y held constant.
Time runs from 0-44 turnover times.

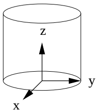
Boundary Layer Profiles

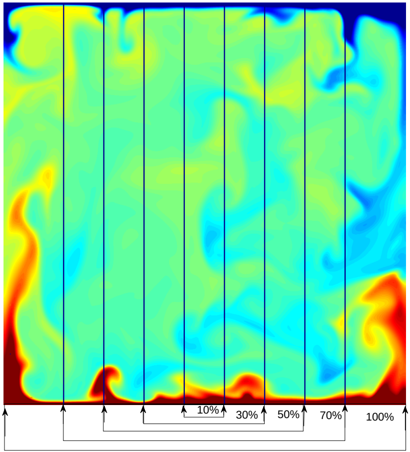
$R = 1 \times 10^8, \sigma = 0.7$, X-const

Want plots of

- temperature θ versus z
- rms horizontal velocity v versus z

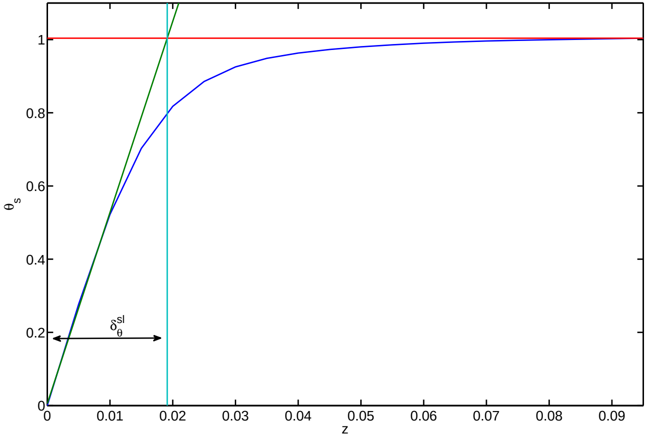
Need to average over x, y, t





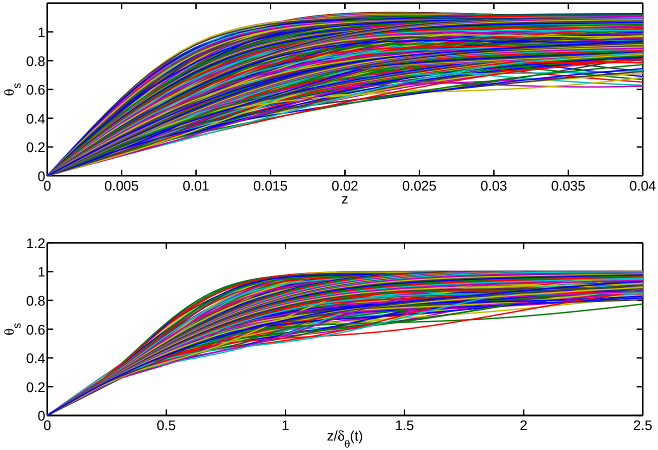
Measuring boundary layer thickness

$R = 5 \times 10^7, \sigma = 0.7$, X-const, cut $= 10\%$, $\theta_s = 2 - 2 < \theta >$



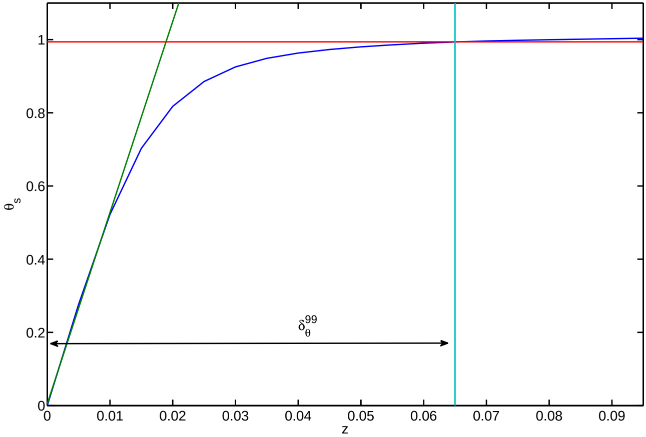
Time average profiles before or after rescaling z

$\sigma = 0.7$, $R = 1 \times 10^8$, X-Slice, cut $= 10\%$, $\theta_s = 2 - 2 < \theta >$



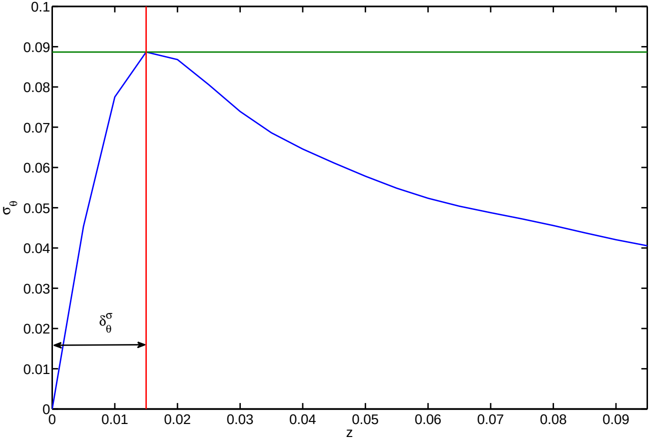
Different methods of measuring boundary layers

$R = 5 \times 10^7, \sigma = 0.7$, X-const, cut $= 10\%$, $\theta_s = 2 - 2 < \theta >$



Different methods of measuring boundary layers

$R = 5 \times 10^7, \sigma = 0.7$, X-const, cut $= 10\%$, $\sigma_{theta} = \sqrt{\Delta < \theta >^2}$

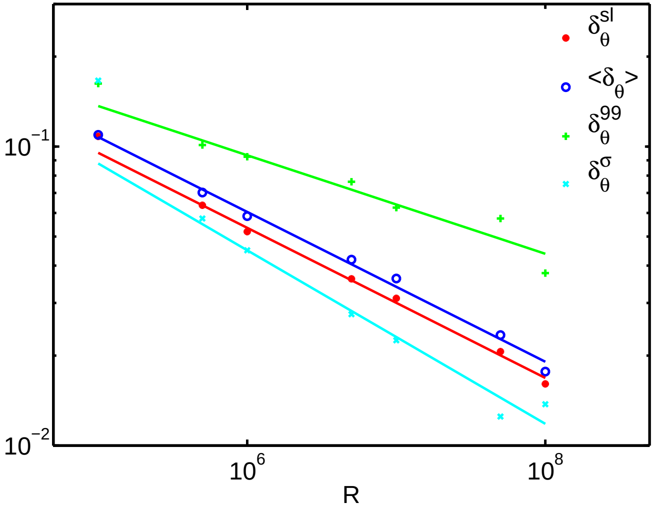


Dependence of boundary layer thickness on Rayleigh no.

$\delta_\theta = \delta_0 R^\beta, \sigma = 0.7, \Gamma = 1$, cut $= 10\%$

δ_θ^{sl}, slope $=-0.26$
$<\delta_\theta>$, slope $=-0.26$
δ_θ^{99}, slope $=-0.10$
δ_θ^{σ}, slope $=-0.29$

Expect $\delta_\theta \propto \frac{1}{Nu}$

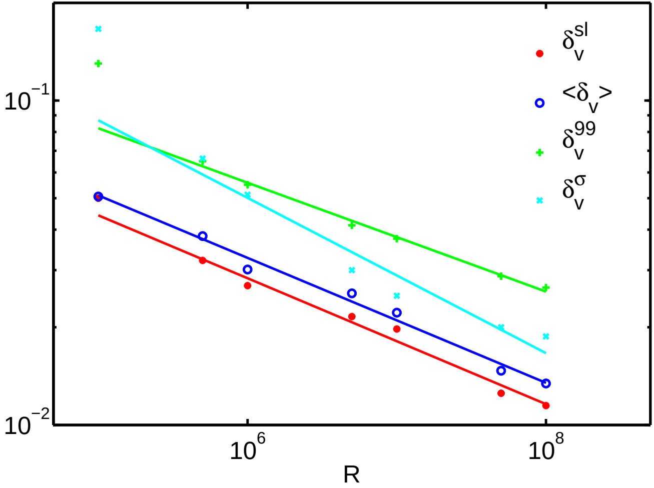


Dependence of boundary layer thickness on Rayleigh no.

$\delta_v = \delta_0 R^{\beta}, \sigma = 0.7, \Gamma = 1$, cut = 10%

δ_v^{sl}, slope $= -0.19$
$< \delta_v >$, slope $= -0.19$
δ_v^{99}, slope $= -0.17$
δ_v^{σ}, slope $= -0.24$

Expect $\delta_v \propto \frac{1}{\sqrt{Re}}$



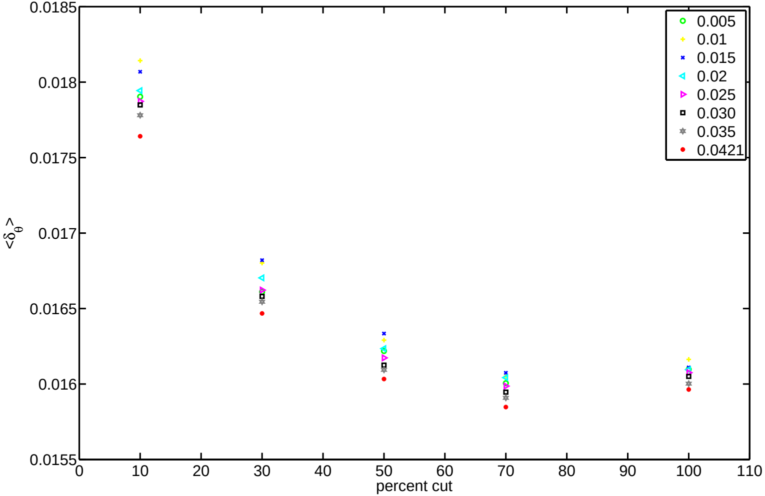
Scaling Results

$\sigma = 0.7, \Gamma = 1$

Coeff	100%	70%	50%	30%	10%
δ_θ^{sl}	-0.26	-0.25	-0.25	-0.25	-0.25
$< \delta_\theta >$	-0.26	-0.25	-0.25	-0.26	-0.25
δ_θ^{99}	-0.11	-0.19	-0.18	-0.16	-0.16
δ_θ^{σ}	-0.29	-0.28	-0.29	-0.29	-0.29
δ_v^{sl}	-0.20	-0.19	-0.19	-0.19	-0.19
$< \delta_v >$	-0.20	-0.19	-0.19	-0.20	-0.19
δ_v^{99}	-0.16	-0.17	-0.16	-0.16	-0.17
δ_v^{σ}	-0.20	-0.20	-0.18	-0.18	-0.24

Does it matter how long you time average?

$R = 2 \times 10^8, \sigma = 0.4, \Gamma = 1$, turnover time $= 0.0005$



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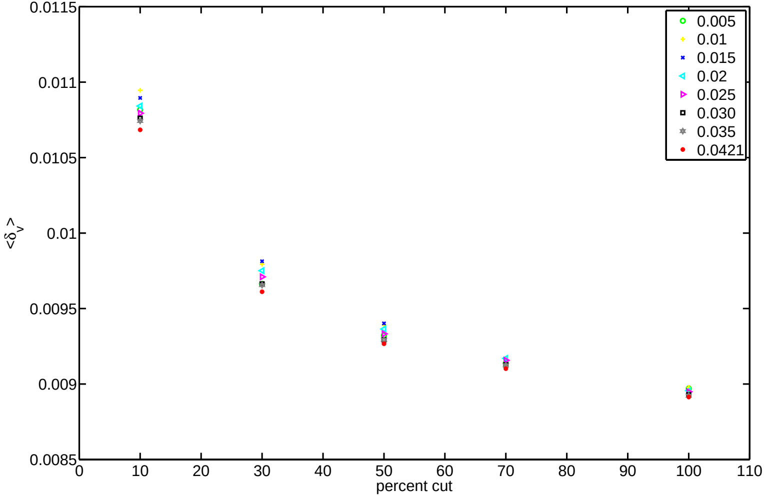
$R = 2 \times 10^8, \sigma = 0.4, \Gamma = 1,$ turnover time $= 0.0005$



Theoretical Predictions—Prandtl-Blausius

Laminar theory

Solutions to

$$\begin{aligned} \frac{d^3\Psi}{d\xi^3} + 0.5\Psi\frac{d^2\Psi}{d\xi^2} &= 0 \\ \frac{d^2\theta}{d\xi^2} + 0.5\sigma\Psi\frac{d\theta}{d\xi} &= 0 \end{aligned}$$

Initial Conditions:

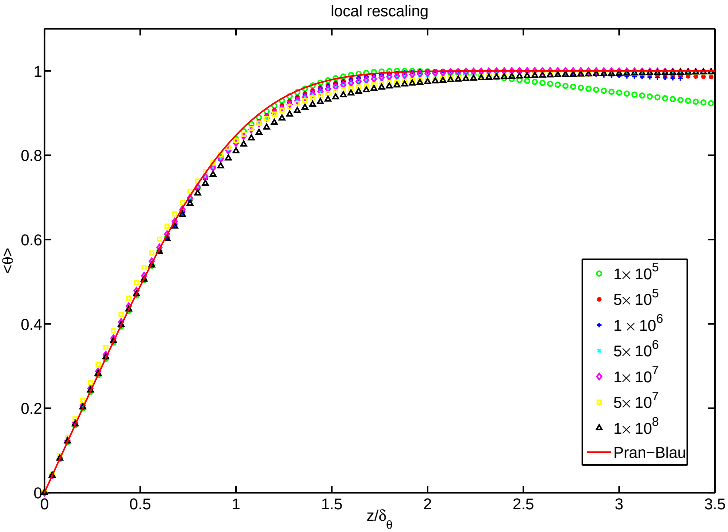
$$\Psi(0) = 0, \frac{d\Psi}{d\xi}(0) = 0, \frac{d\Psi}{d\xi}(\infty) = 1$$

$$\theta(0) = 0, \theta(\infty) = 1$$

Horizontal velocity, $v_h = \frac{d\Psi}{d\xi}$
$\xi = \frac{z}{l}$, l is the length scale = boundary layer thickness.

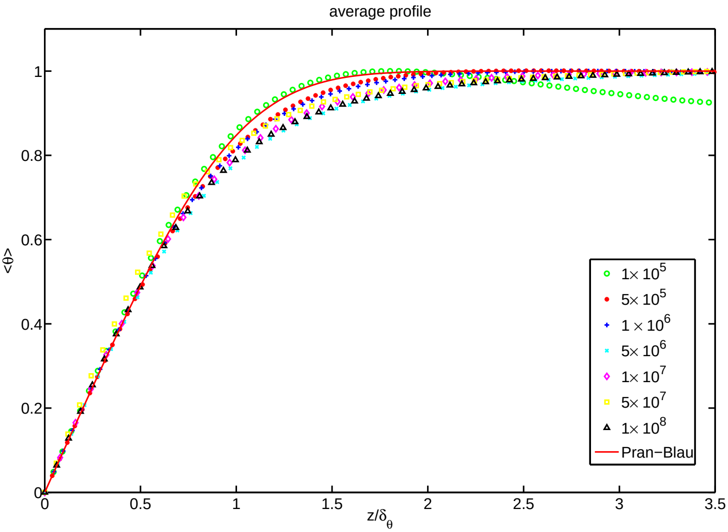
Thermal Boundary Layer Profiles

$\sigma = 0.7, \Gamma = 1$



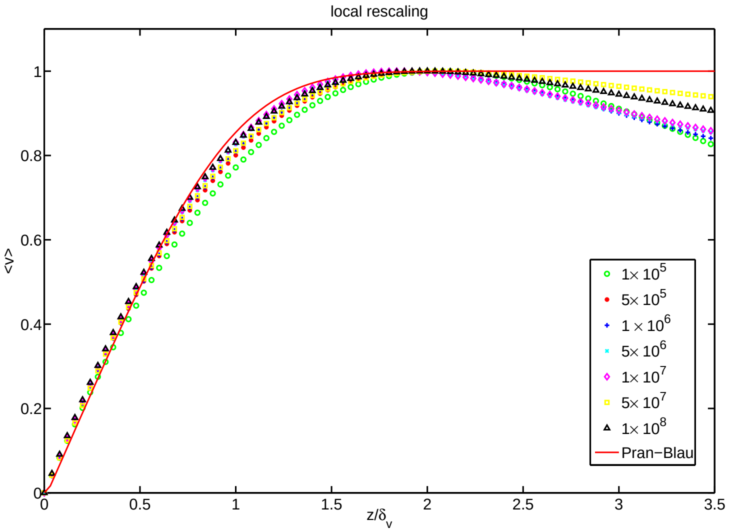
Thermal Boundary Layer Profiles

Time average before rescaling



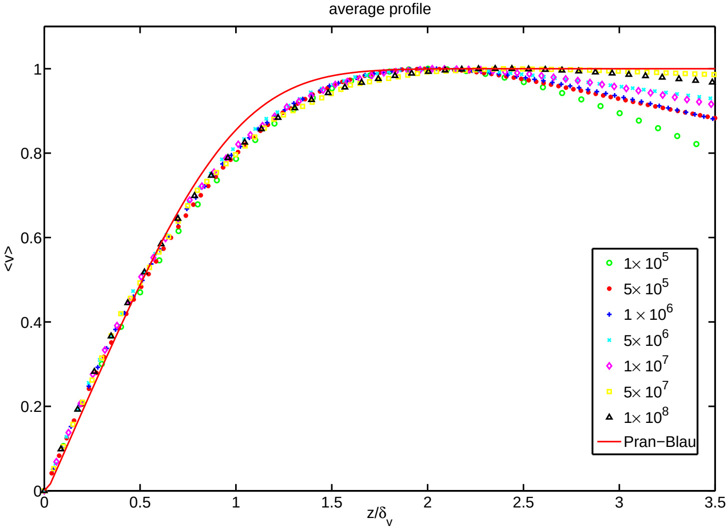
Viscous Boundary Layer Profiles

Time average after rescaling



Viscous Boundary Layer Profiles

Time average before rescaling



Shape Factors

$\sigma = 0.7, \Gamma = 1$

Displacement thickness:

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy \tag{1}$$

Momentum thickness:

$$\delta_2 = \int_0^\infty \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy \tag{2}$$

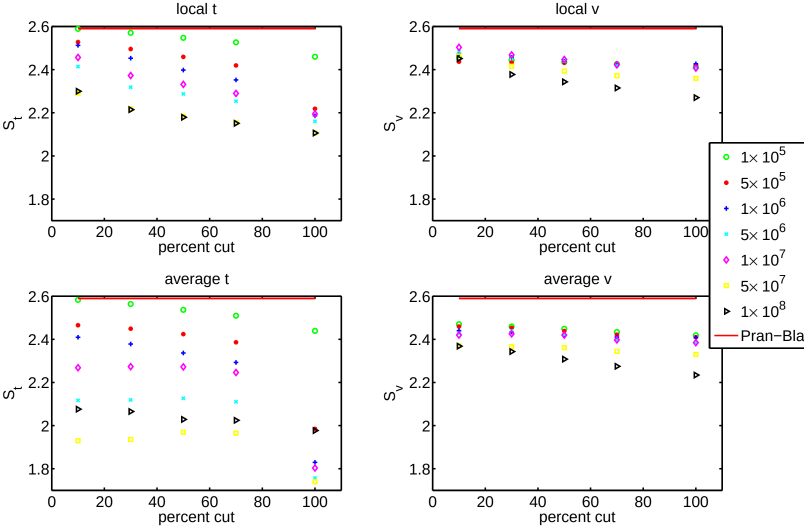
Shape Factor:

$$S_v = \frac{\delta_1}{\delta_2} \tag{3}$$

$S_v = 2.59$ for Prandtl-Blausius

Shape Factors

$\sigma = 0.7, \Gamma = 1$



Conclusions

- Scaling exponents agree fairly well with experimental and theoretical results, and they are generally robust with respect to averaging.
- Local rescaling generally gives better agreement with laminar Prandtl-Blausius Theory (this implies that the fluctuations don't simply average out), but there is still some disagreement which grows with R.
- Averaging near the center of the cell gives better agreement with laminar Prandtl-Blausius Theory.
- Running sims for long times is not necessary as long as you have at least 10 turnover times (more is always better of course).

Future Work

- Look at boundary layer profiles for $\sigma = 0.4$ data
- Use top boundary layer data for better statistics
- Go to higher Rayleigh numbers
- What other quantities are interesting to investigate?

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$+$ undergraduate students Elissa Kim and Katelyn White