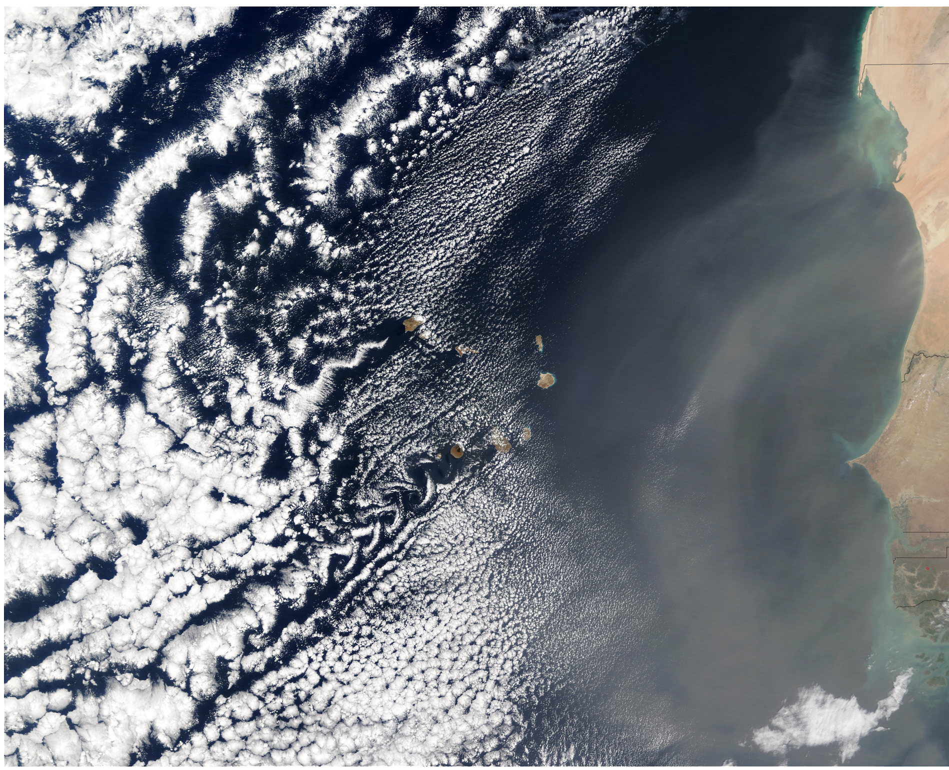
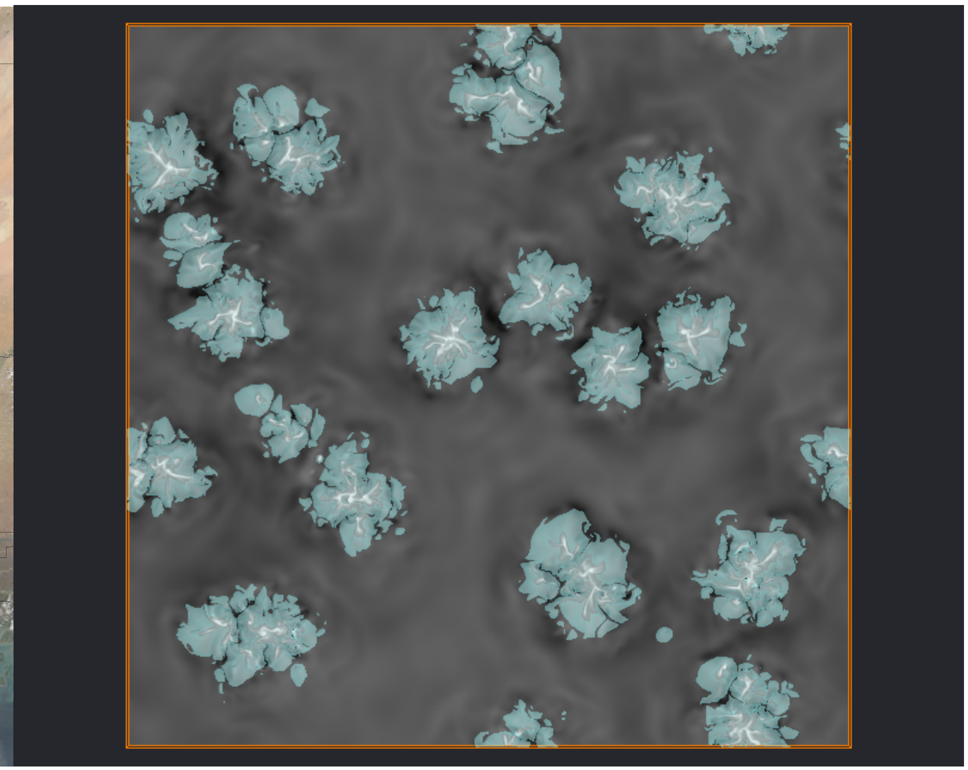
Conditionally unstable moist convection

Thomas Weidauer[1], Olivier Pauluis[2] & Jörg Schumacher[1]

[1]Institute of Thermodynamics and Fluid Mechanics, TU Ilmenau, Germany

[2]Center for Atmospheric and Ocean Science, Courant Institute for Mathematical Sciences, NYU, USA

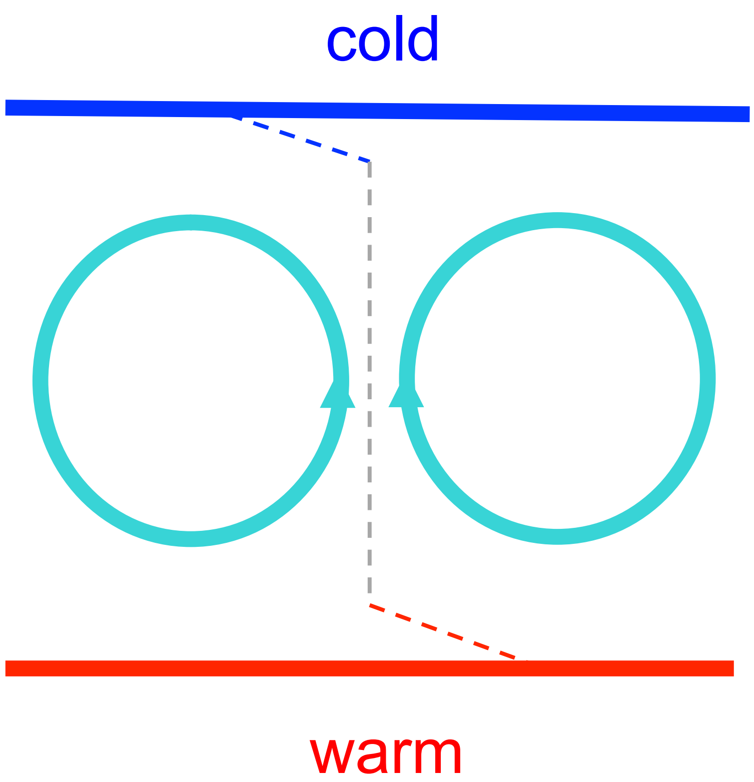


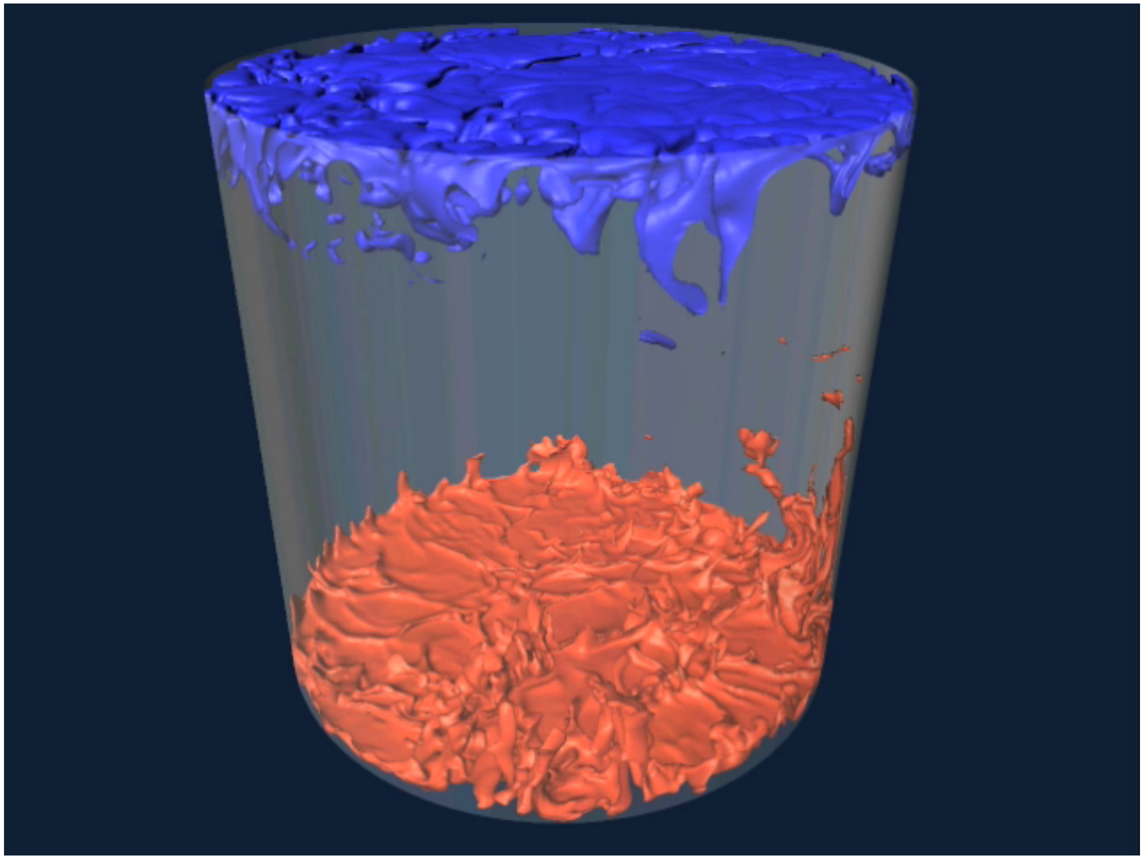


Outline

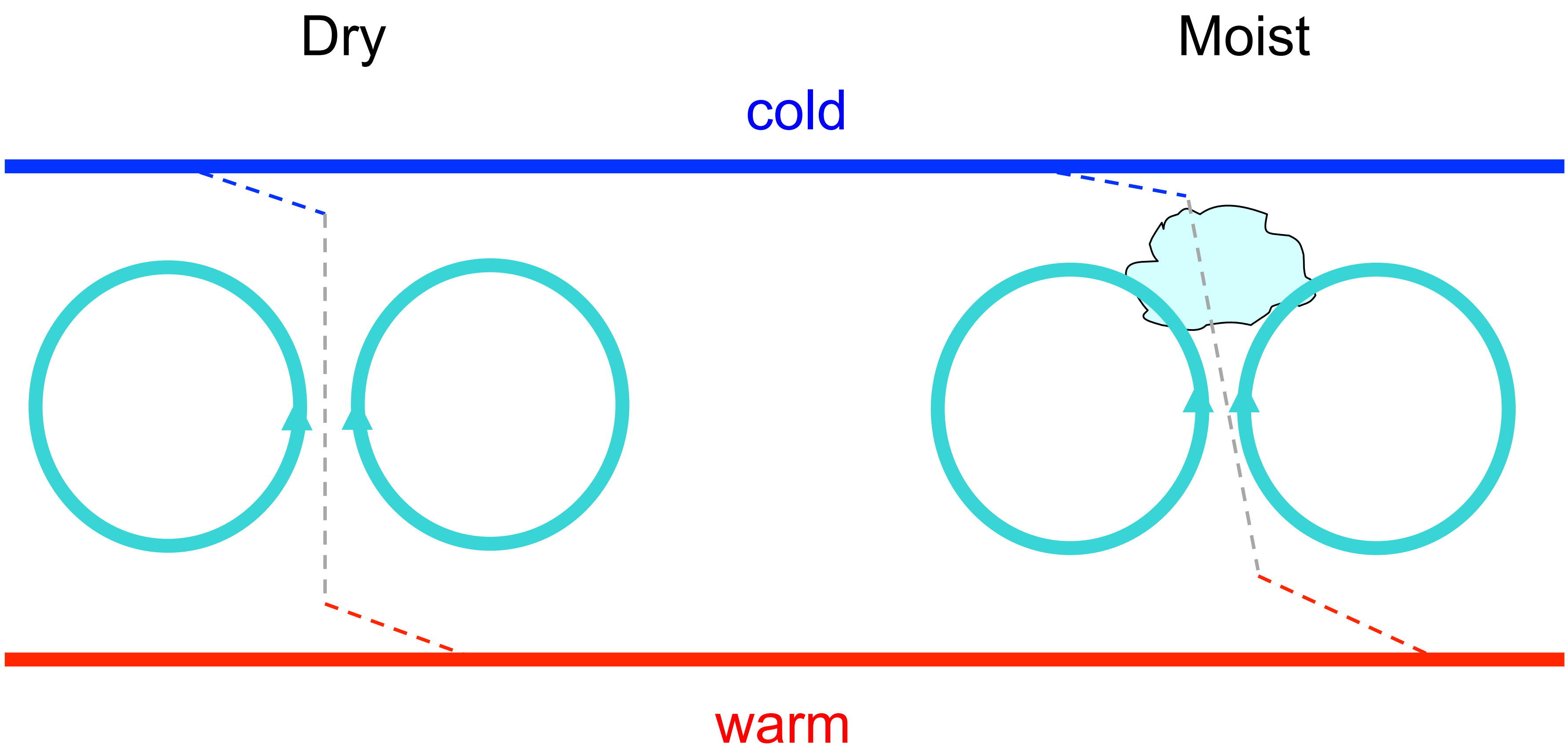
- Motivation
- Moist convection model
- Linearly unstable case: Buoyancy flux and cloud patterns
- Conditionally unstable case: Transition & aggregation
- Summary & outlook

Rayleigh-Bénard convection





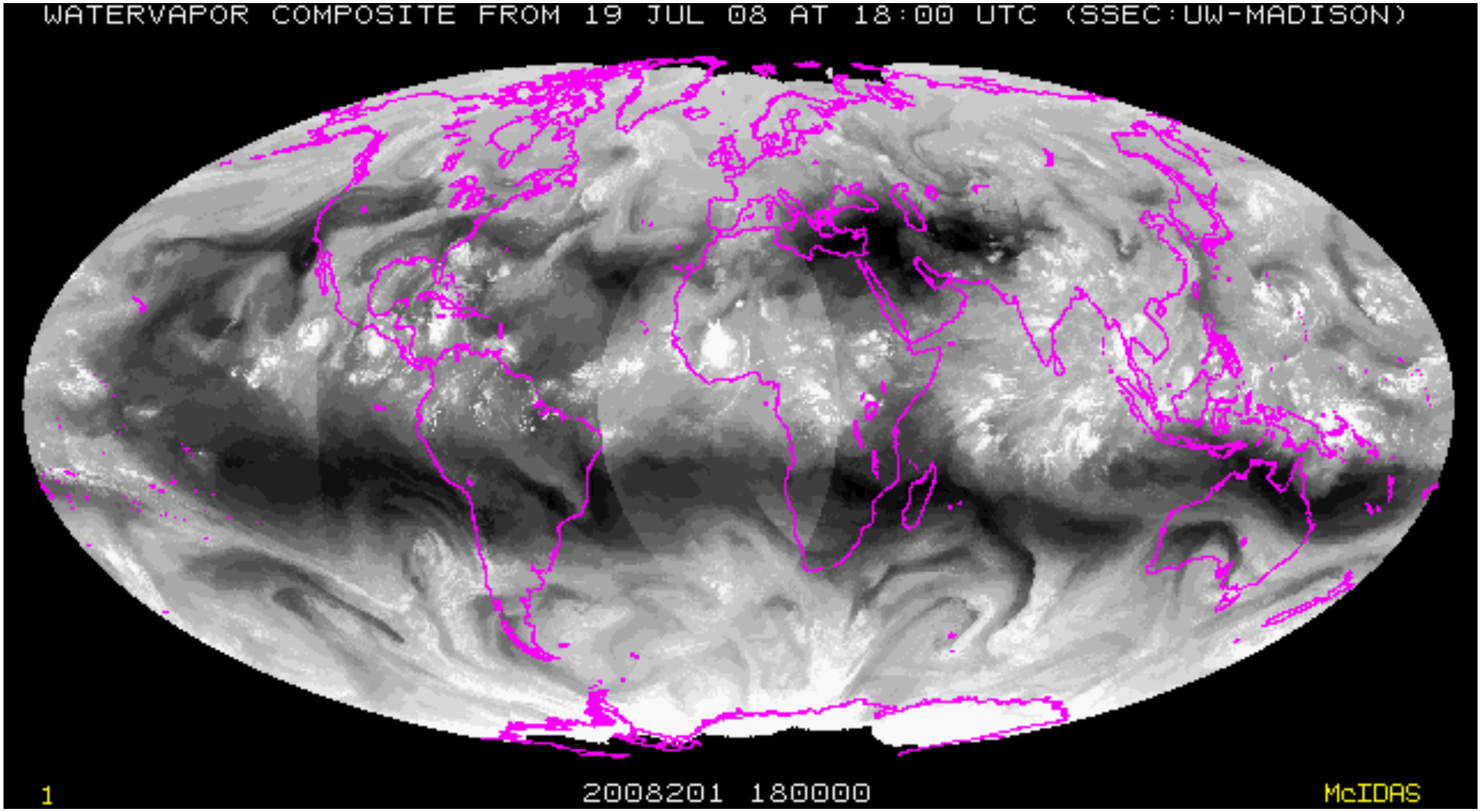
Dry and moist convection



Warm air rises, expands and its temperature drops. The amount of vapour contained in an air parcel decreases with temperature.

Atmospheric turbulence

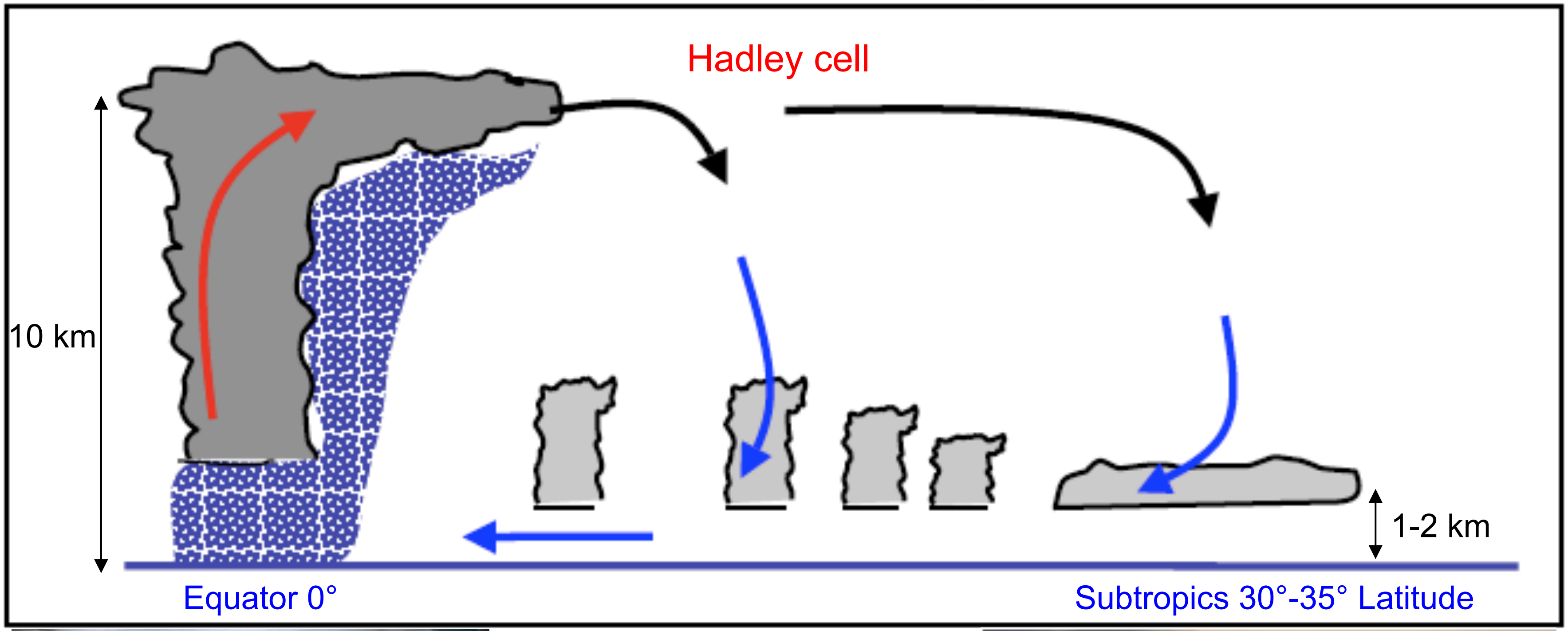
IPCC Report 2007

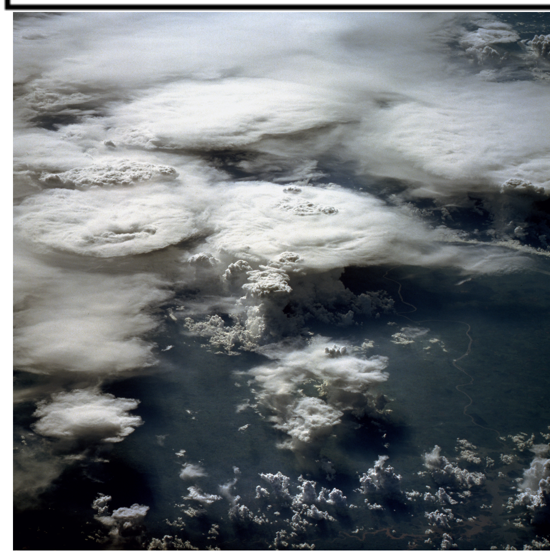


Clouds add the biggest uncertainties to more reliable prognoses of climate change

Fast (<1 hour) & Small-scale (<1 km)

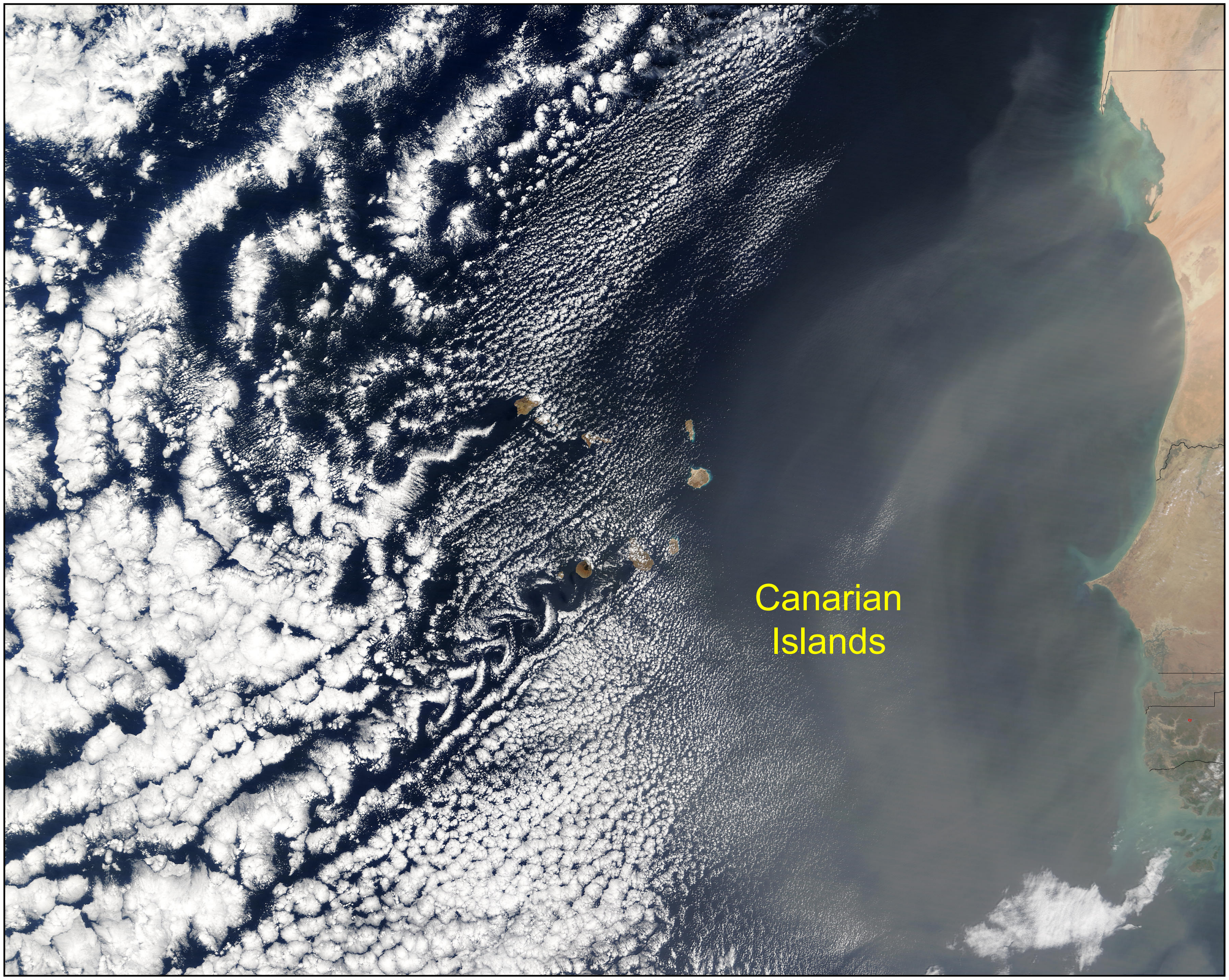
Deep and shallow convection





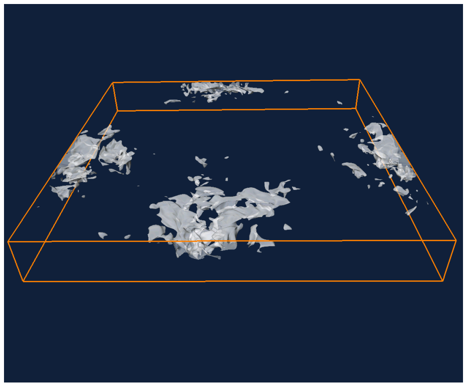
Shallow maritime clouds in the subtropics

Stevens & Brenguier, in Clouds in the perturbed climate system, MIT Press 2009

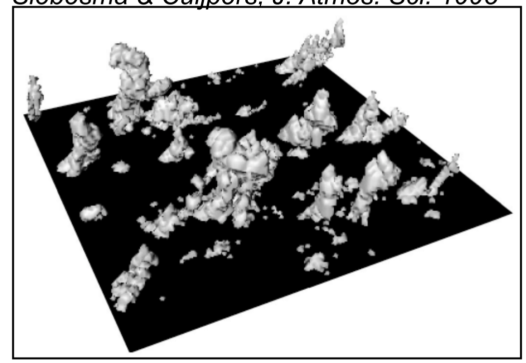


- Low clouds up to heights H < 2km
- No rain with n_d=75-200 cm^{-3}
- Warm low clouds can emit more IR than cold high ones
- 2-3% increase of cloud fraction results in 1 Wm^{-2} change in net solar radiation
- Total incoming solar radiation is 342 Wm^{-2}

How to study cloud formation?



Siebesma & Cuijpers, J. Atmos. Sci. 1995



Reduced complexity

No turbulence modeling

All scales resolved

Method: DNS

$Ra \sim 10^8$ $A \leq 64$

Full complexity

Turbulence modeling, cloud parametrizations

Fine scales not resolved

Methods: LES, RANS

$Ra \sim 10^{15}$ $A = 10\text{-}10^3$

Study moist Rayleigh-Bénard convection in shallow layers

Approximations

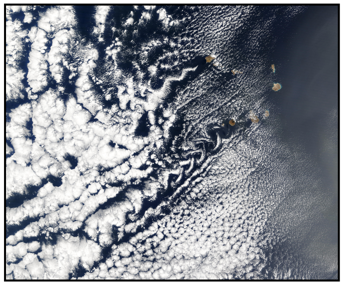
Bretherton, J. Atmos. Sci. 1987; Pauluis & JS, Comm. Math. Sci. 2010

$$\rho(S, p, q_v, q_l, q_i)\,\vec{g}$$

Warm clouds $q_i = 0$ (maritime low clouds)

Thermodynamic equilibrium (no rain)

→Total liquid water mixing ratio $q_T = q_v + q_l$



$$\rho(S, p, q_T)\,\vec{g}$$

Boussinesq approximation (shallow convection, H<2 km)

Buoyancy $B = -g(\rho - \rho_0)/\rho_0$

$$B(S, q_T, z)\,\vec{e}_z$$

Piecewise linear thermodynamics on both sides of the phase boundary: $(q_T, S) \to (D, M)$

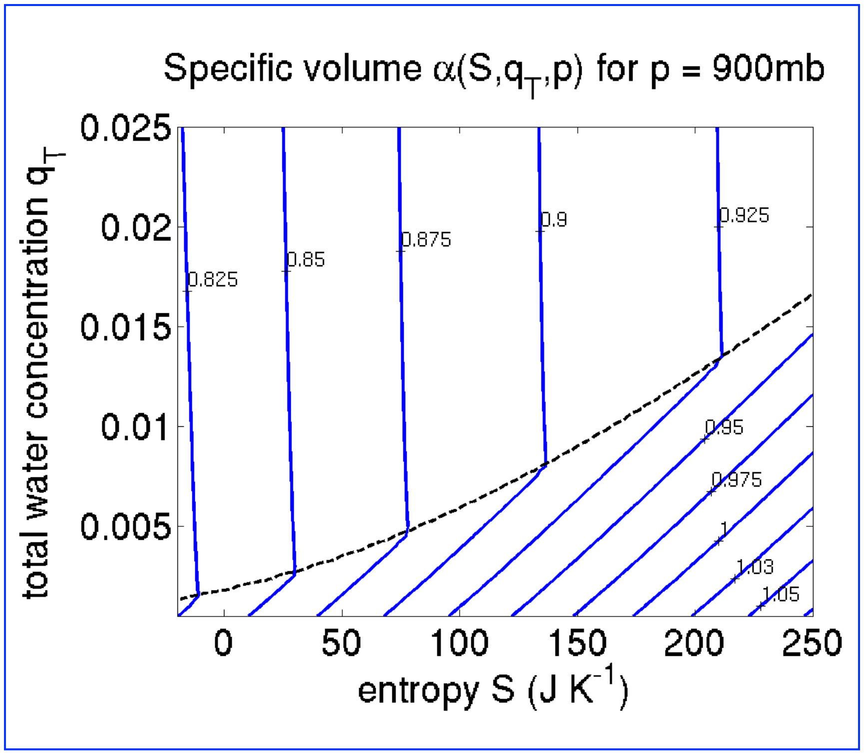
No supersaturation (heterogeneous nucleation)

$$B(D, M, z)\,\vec{e}_z$$

Piecewise linear thermodynamics

Pauluis & JS, Comm. Math. Sci. 2010

$$B(S, q_T, z)\,\vec{e}_z$$



Linear expansion on both sides of phase boundary

$$\left.\frac{\partial B}{\partial S}\right|_{q_T,z} = \begin{cases} B_{S,u} & \text{if } q_T \le q_{sat}(S,z) \\ B_{S,s} & \text{if } q_T > q_{sat}(S,z) \end{cases}$$

$$\left.\frac{\partial B}{\partial q_T}\right|_{S,z} = \begin{cases} B_{q_T,u} & \text{if } q_T \le q_{sat}(S,z) \\ B_{q_T,s} & \text{if } q_T > q_{sat}(S,z)\,. \end{cases}$$

Linear combination to new variables

$$D = B_{S,u}(S - S_{ref}) + B_{q_T,u}(q_T - q_{T,ref})$$

$$M = B_{S,s}(S - S_{ref}) + B_{q_T,s}(q_T - q_{T,ref})$$

Shallow moist RB convection

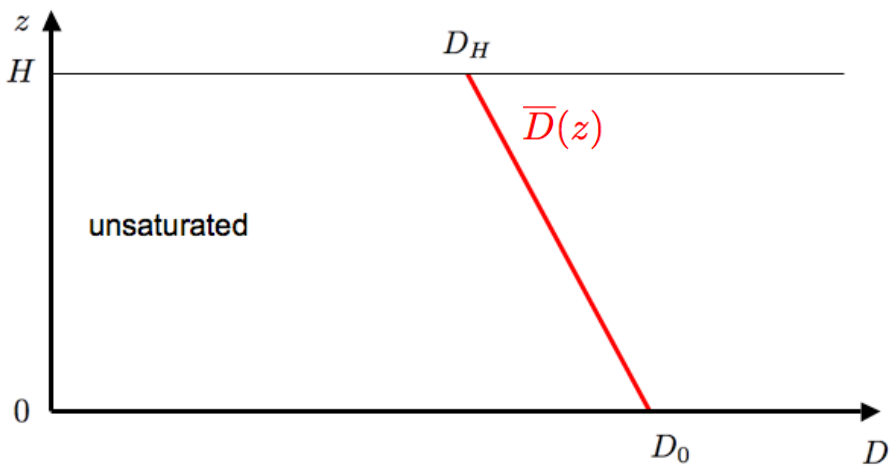
$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla p + \nu \nabla^2 \vec{u} + B(D, M, z)\vec{e}_z$$

Dry RB convection

$$\frac{\partial D'}{\partial t} + (\vec{u} \cdot \nabla) D' = \kappa \nabla^2 D' + \frac{D_0 - D_H}{H} u_z$$

$$B(D) = D$$



Shallow moist RB convection

$$\nabla \cdot \vec{u} = 0$$

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RB convection

$$\frac{\partial D'}{\partial t} + (\vec{u} \cdot \nabla) D' = \kappa \nabla^2 D' + \frac{D_0 - D_H}{H} u_z$$

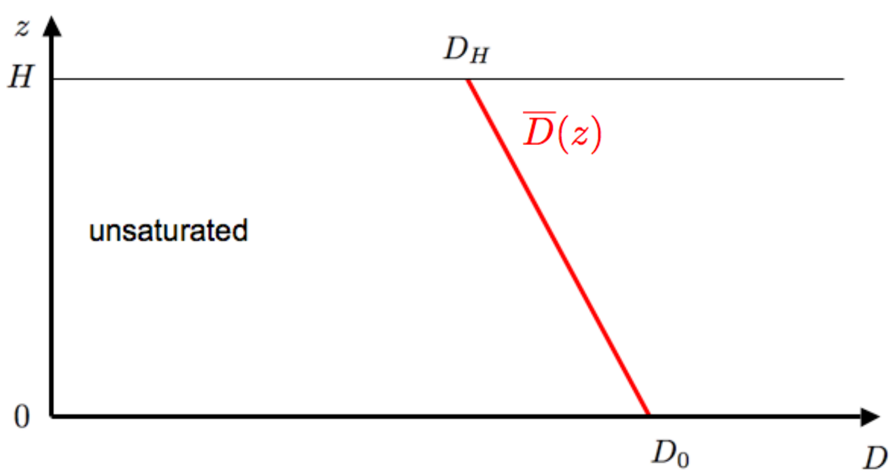
$$B(D) = D$$

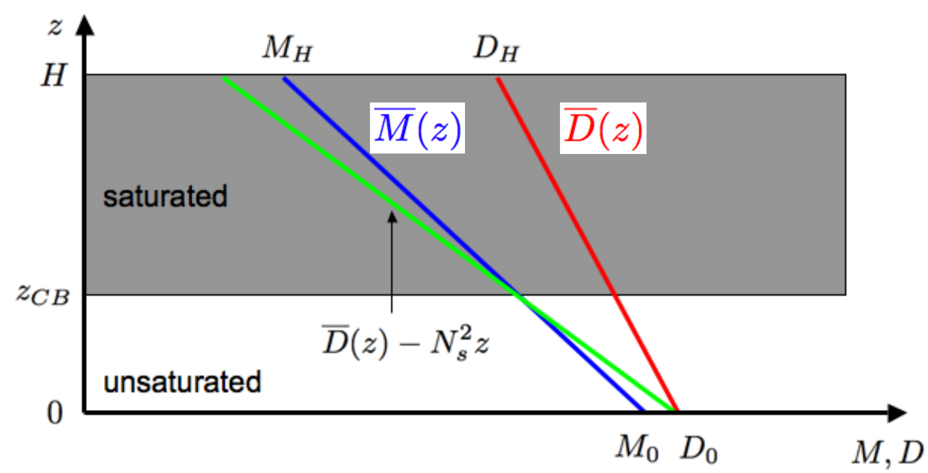
Moist RB convection

$$\frac{\partial D'}{\partial t} + (\vec{u} \cdot \nabla) D' = \kappa \nabla^2 D' + \frac{D_0 - D_H}{H} u_z$$

$$\frac{\partial M'}{\partial t} + (\vec{u} \cdot \nabla) M' = \kappa \nabla^2 M' + \frac{M_0 - M_H}{H} u_z$$

$$B(D, M, z) = \max(M, D - N_s^2 z)$$

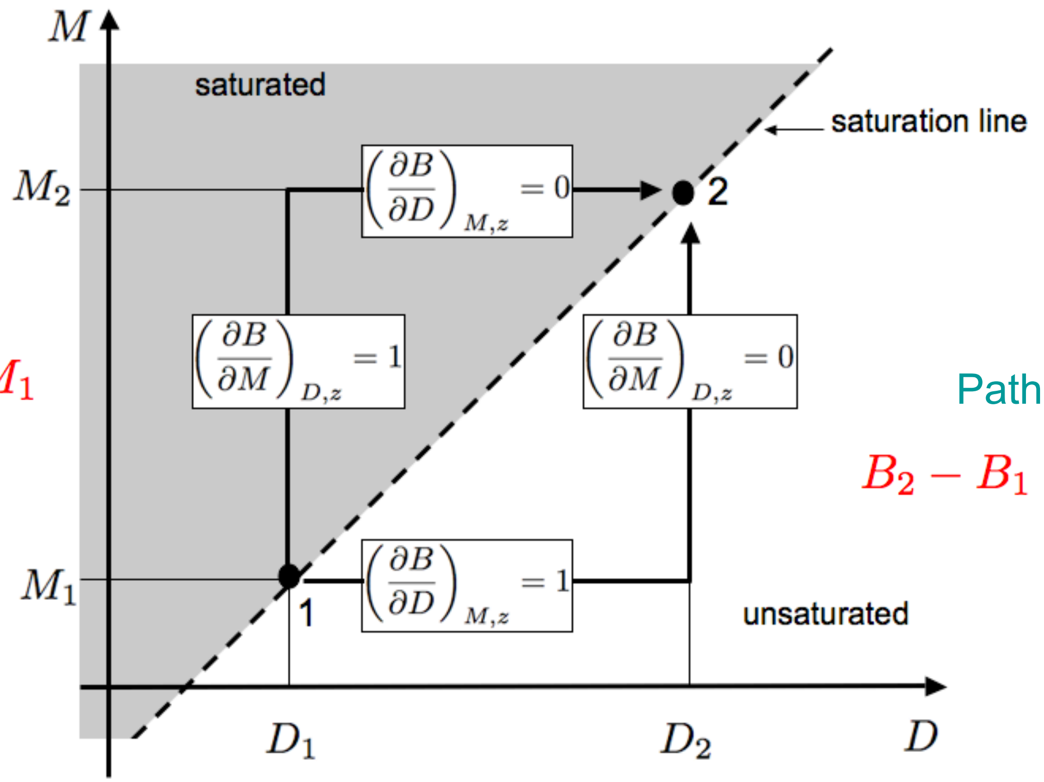




Explicit saturation condition

Path 1

$$B_2 - B_1 = M_2 - M_1$$



Path 2

$$B_2 - B_1 = D_2 - D_1$$

Saturation condition $F(M, D, z) \geq 0$

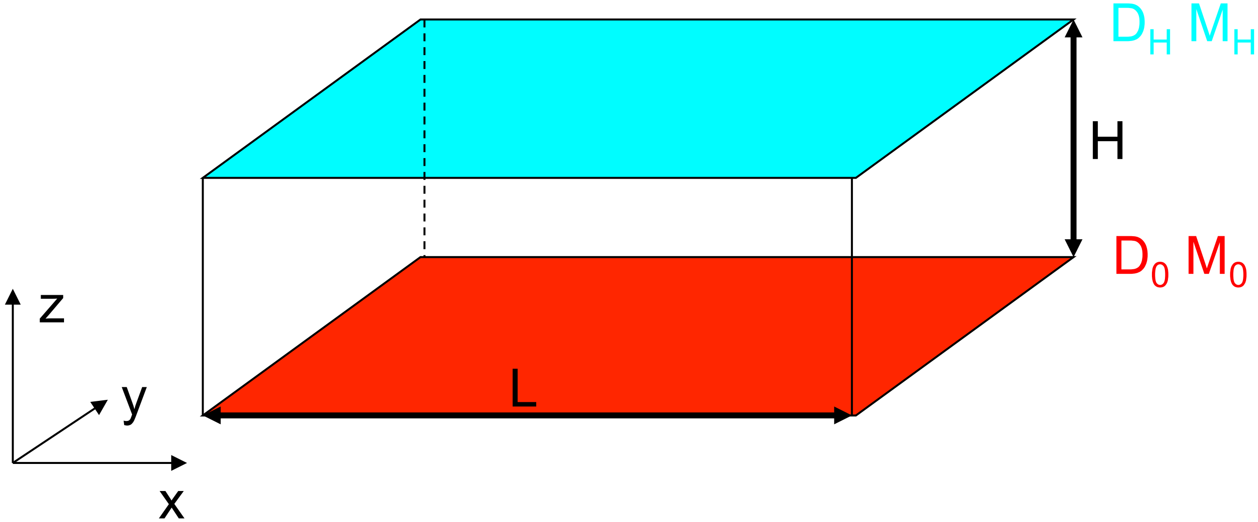
$$\rightarrow \quad M_2 - M_1 = D_2 - D_1$$

$$\rightarrow \quad M - D = f(z) \quad \text{with} \quad f(z) = -N_s^2 z$$

N_s is a Brunt-Vaisala fequency given by $N_s^2 = \frac{g}{T_{ref}}(\Gamma_d - \Gamma_s)$

Numerical method

Kuo, Tellus 1965; Bretherton, J. Atmos. Sci. 1987, 1988



$$A = \frac{L}{H} \geq 4$$

x,y periodic boundary conditions
z free-slip boundary condition $z = 0, H: \quad u_z = \frac{\partial u_x}{\partial z} = \frac{\partial u_y}{\partial z} = 0 \quad \& \quad D' = M' = 0$

Pseudospectral method which uses volumetric Fast Fourier Transformations

Two-dimensional parallelization

Simulations with up to 8192 MPI tasks for grids with up to $4096^2 \times 513$ points

Five-dimensional parameter space

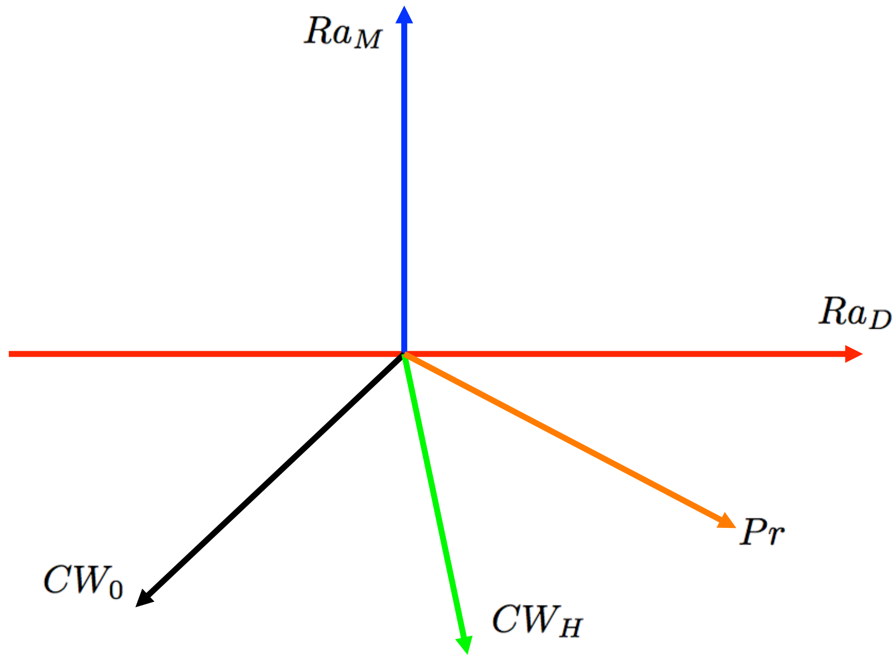
dry and moist Rayleigh number

Prandtl number

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = \frac{\nu}{\kappa}$$



$$CW_0 = \frac{M_0 - D_0}{N_s^2 H}$$

$$CW_H = \frac{M_H - D_H}{N_s^2 H} + 1$$

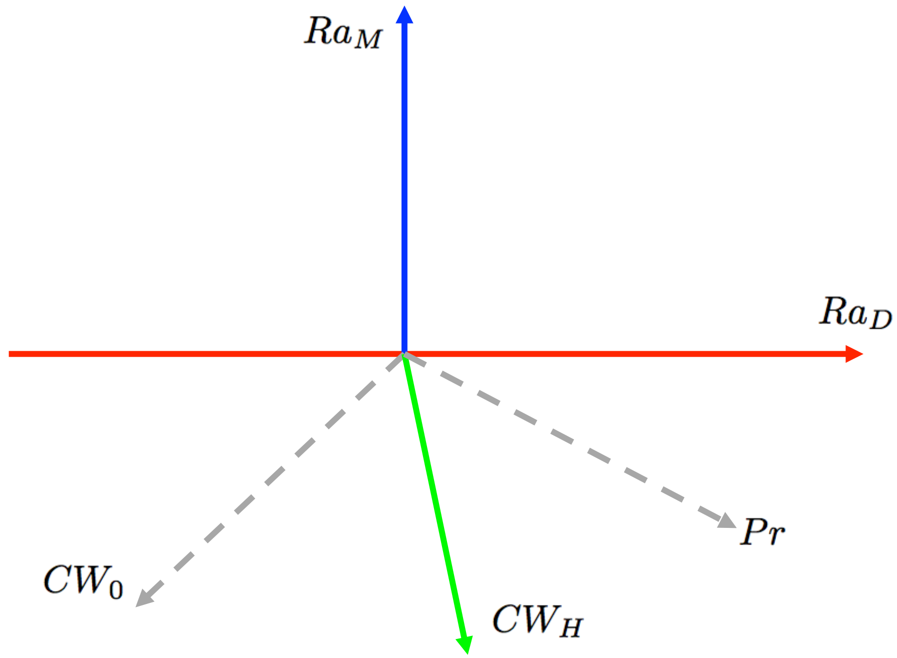
amount of cloud water at the bottom (z=0) and top (z=H)

Convection in air

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$



$$CW_0 = 0$$

$$CW_H = \frac{M_H - D_H}{N_s^2 H} + 1$$

Convectively available potential energy

Prescribed amplitudes of D and M at top and bottom determine the amount of potential energy that can be converted into fluid motion

$$\mathrm{CAPE} = -\int_0^H [B(\overline{D}(z), \overline{M}(z), z) - B(D_0, M_0, z)]$$

Potential energy of the background equilibrium

Potential energy of an air parcel that starts at the bottom of the layer

$$D_0 = M_0 = 0, M_H < 0$$

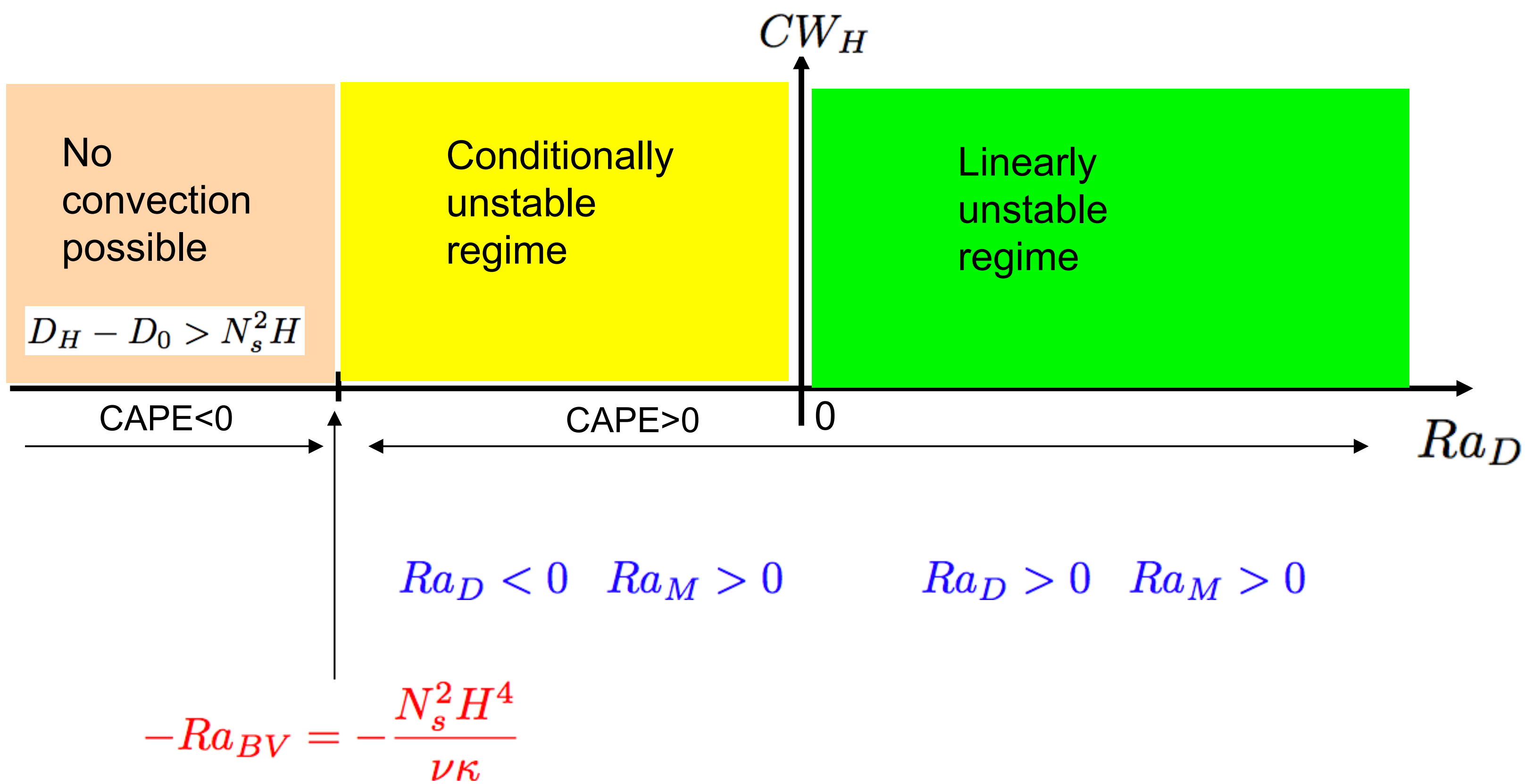
Case 1: Equilibrium is fully subsaturated $\overline{M}(z) < \overline{D}(z) - N_s^2 z$

$$\mathrm{CAPE} > 0 \Leftrightarrow D_H < N_s^2 H$$

Case 2: Equilibrium is fully saturated $\overline{M}(z) > \overline{D}(z) - N_s^2 z$

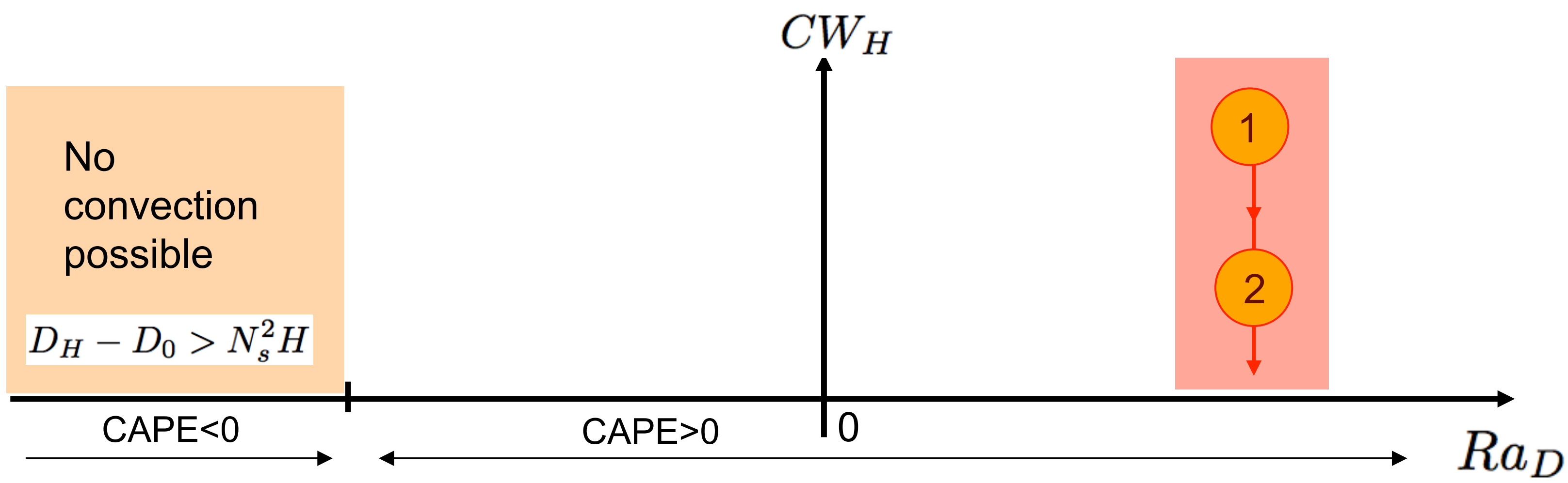
$$\mathrm{CAPE} > 0 \Leftrightarrow M_H < 0$$

Linearly & conditionally unstable equilibrium

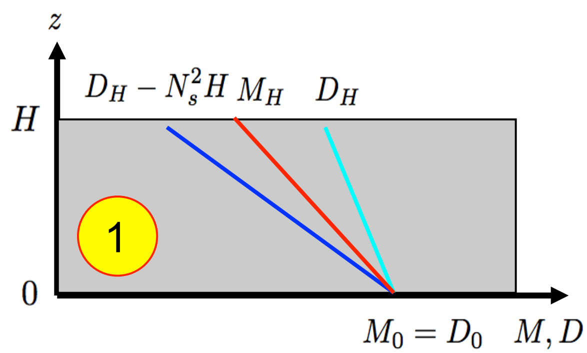


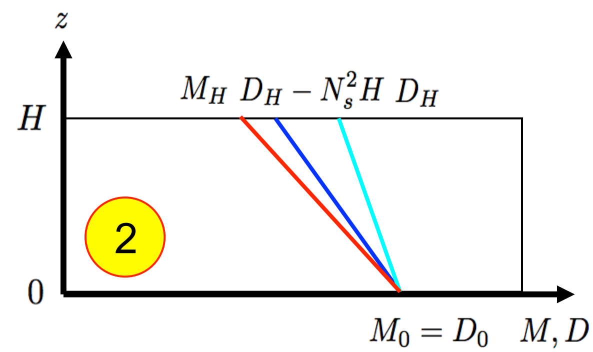
In both regimes one can set up a fully saturated and a fully unsaturated intial equilibrium state!

Linearly unstable equilibrium



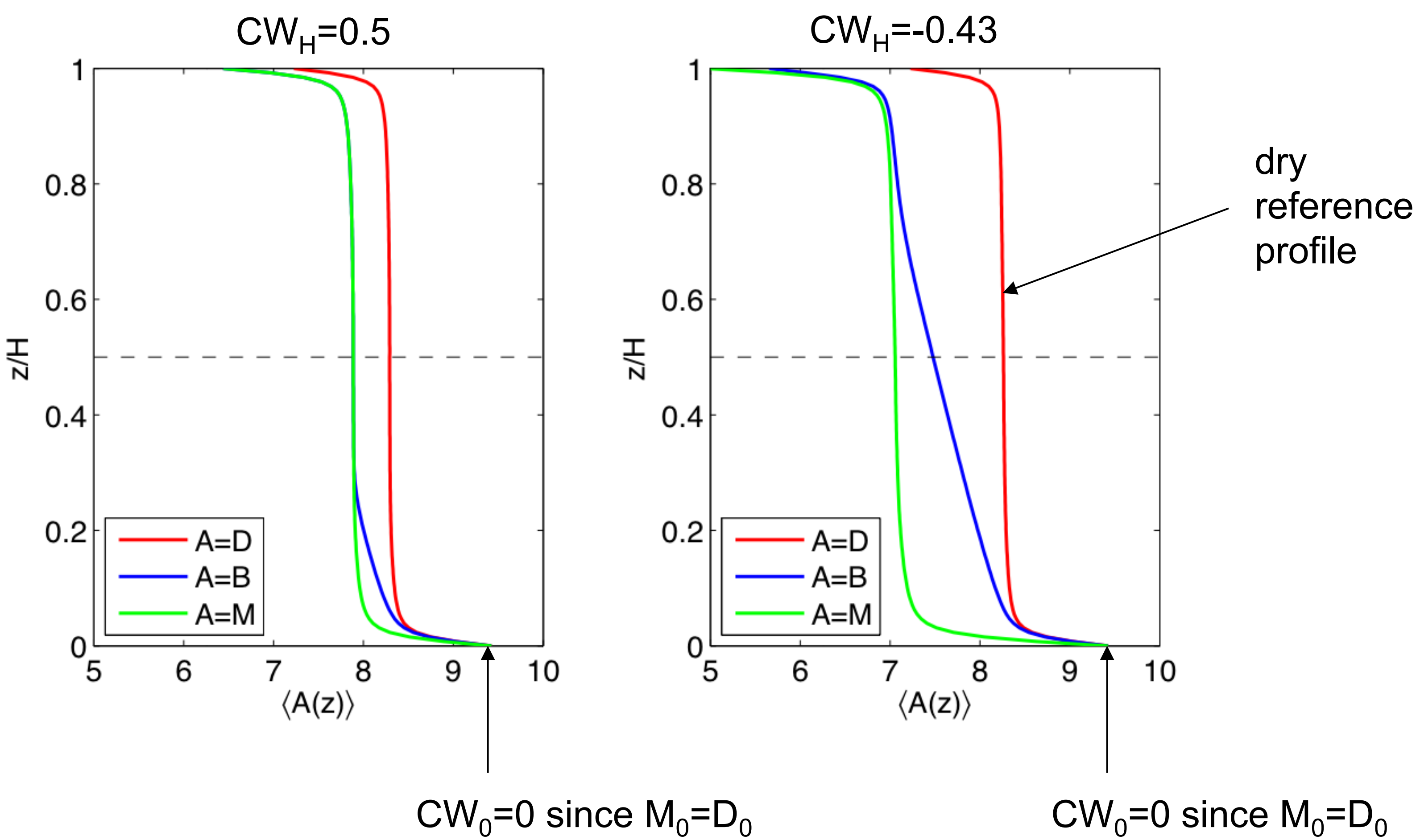
D,M unstably stratified!





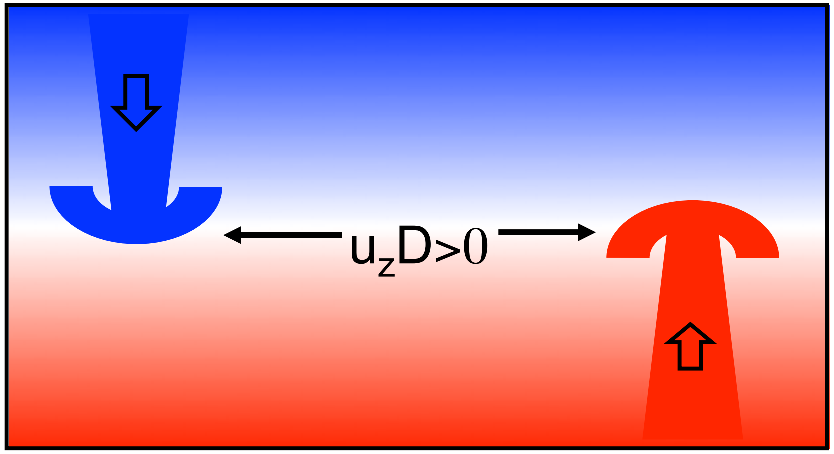
Increase of Ra_M = Decrease of CW_H

Mean buoyancy profiles

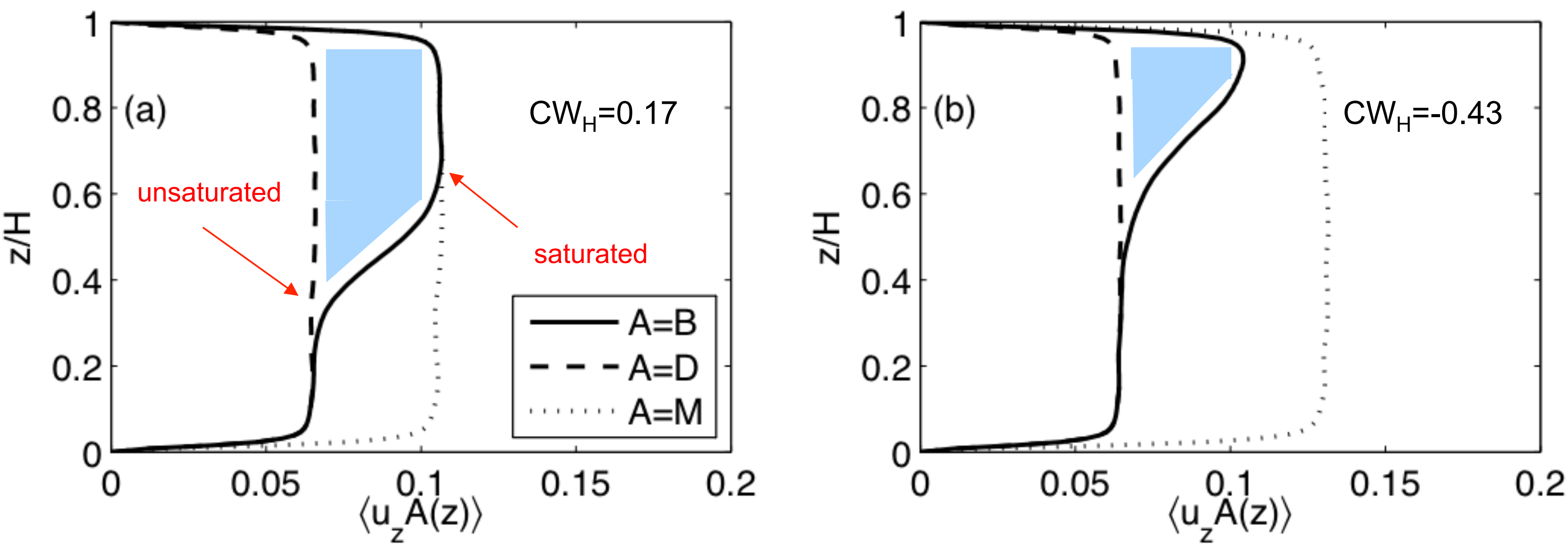


In contrast to dry convection: linear mean profile

Convective buoyancy flux

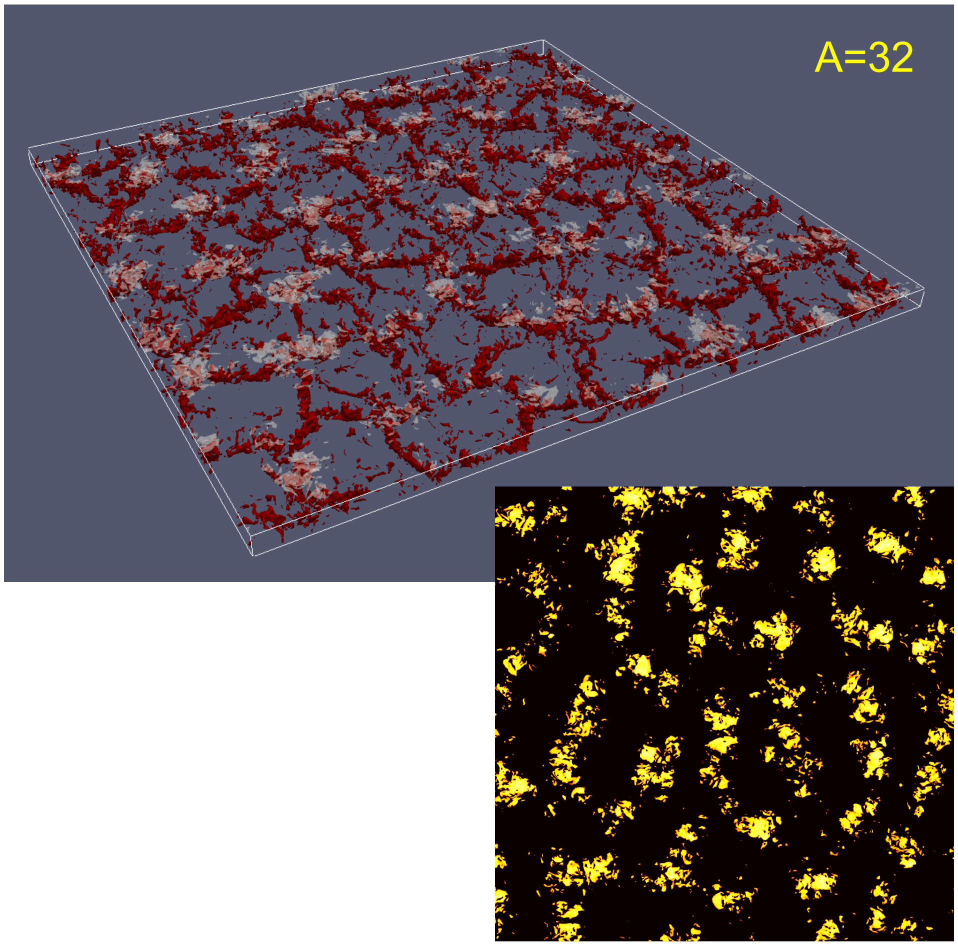


$$\langle u_z D(z)\rangle_{A,t} \leq \langle u_z B(z)\rangle_{A,t} \leq \langle u_z M(z)\rangle_{A,t}$$

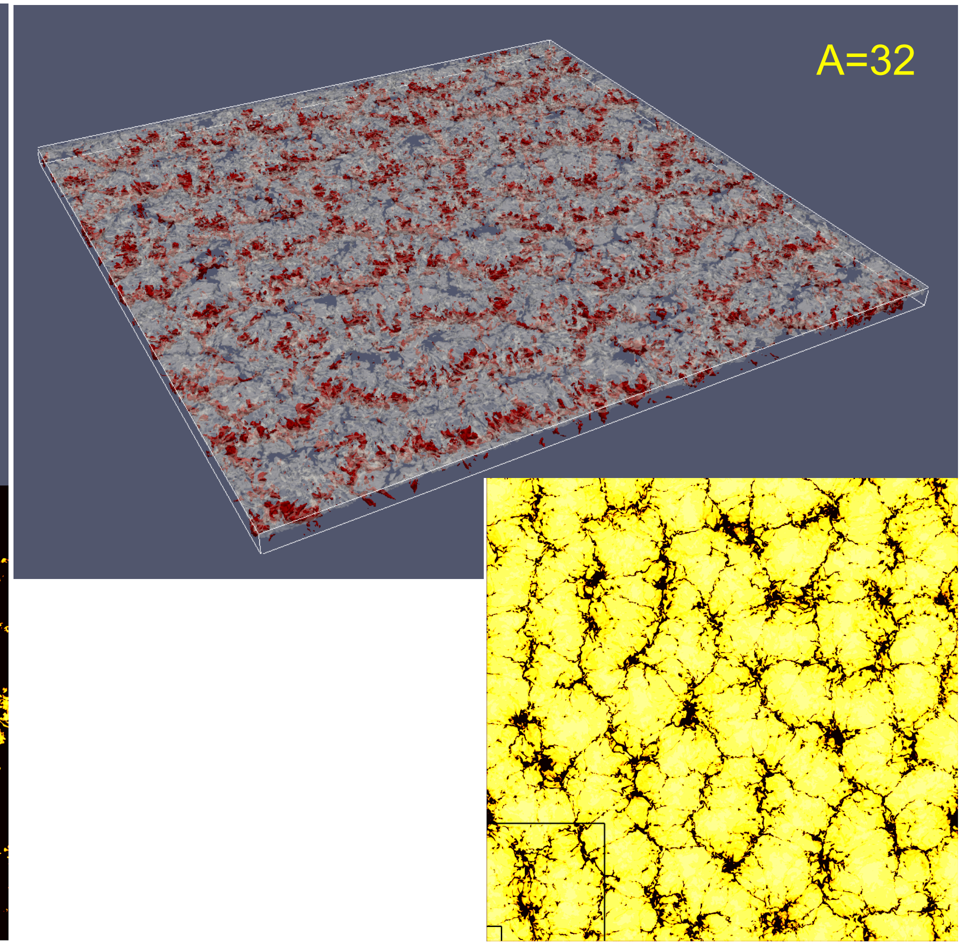


Very large aspect ratio systems

Isolated clouds
CW_H=-1.43

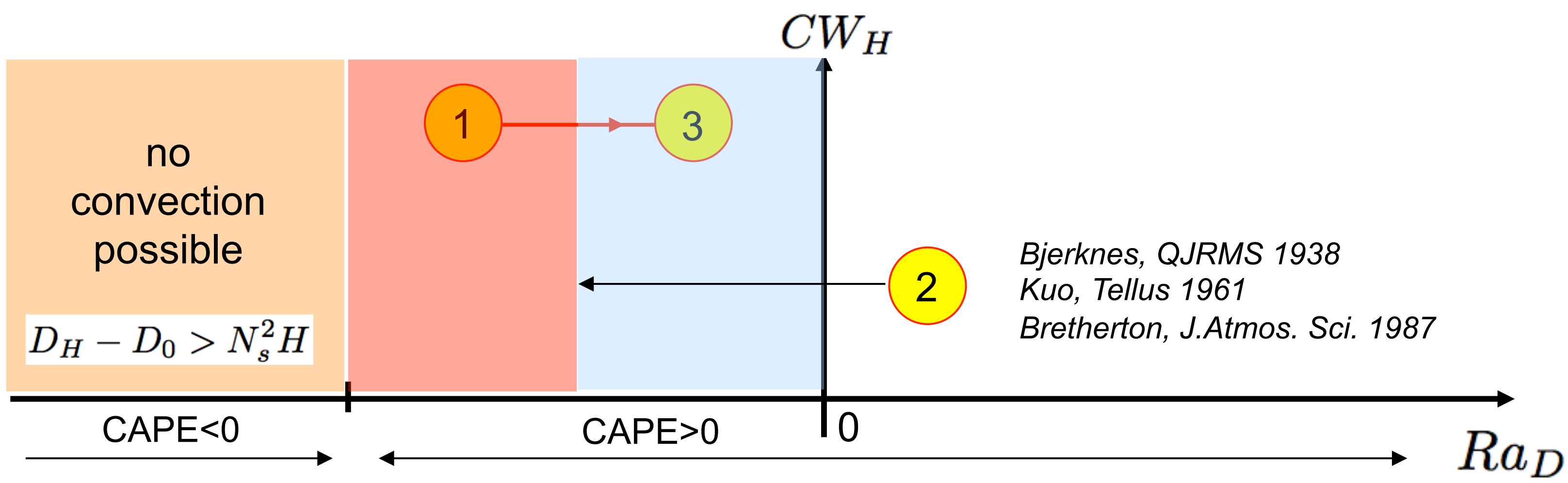


Almost closed cloud layer
CW_H=-0.43

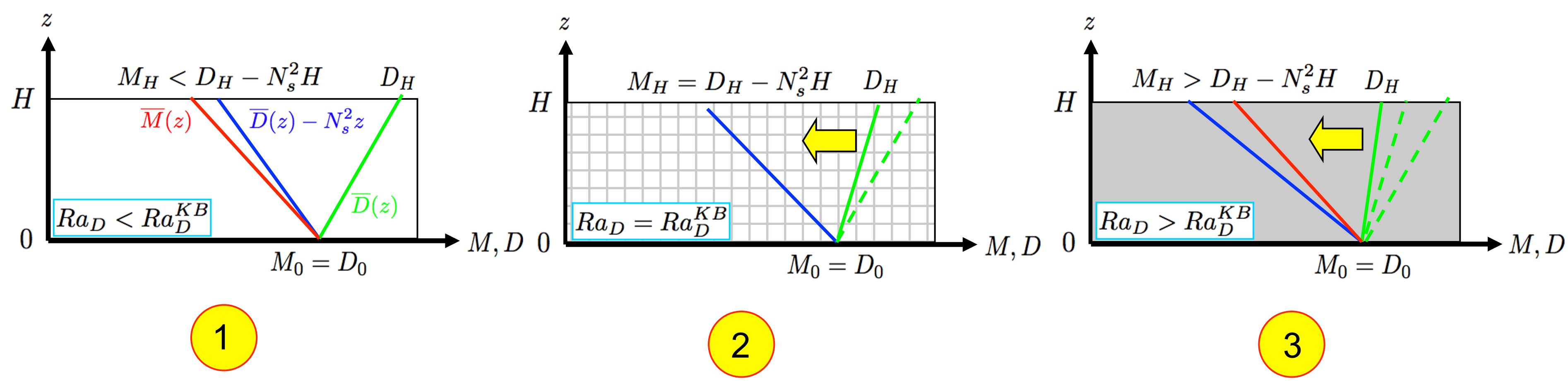


Vertically averaged liquid water content and buoyancy flux

Conditionally unstable equilibrium



D stably and M unstably stratified!



Conditional instability

Bjerknes, Quat. J. Royal Meteor. Soc. 1938

SATURATED-ADIABATIC ASCENT OF AIR THROUGH DRY-ADIABATICALLY DESCENDING ENVIRONMENT

By Professor J. BJERKNES, Bergen, Norway

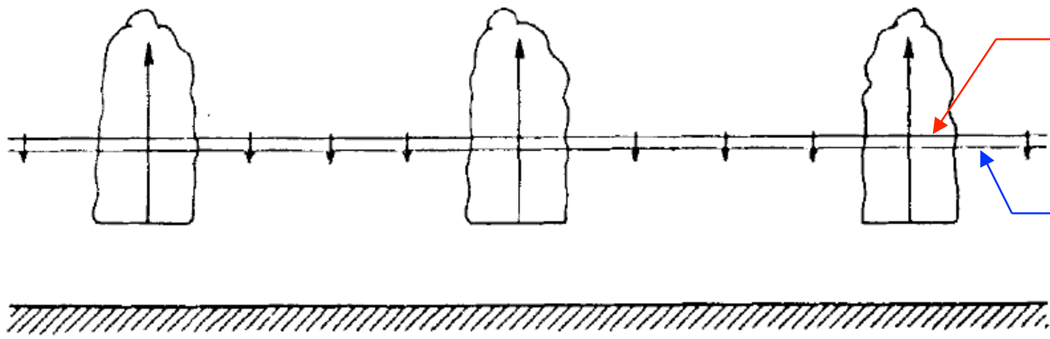
Summary

This paper gives a theoretical treatment of the cumulus convection in an atmosphere with a temperature lapse-rate between the saturated-adiabatic and the dry-adiabatic (conditional instability). The dry-adiabatic downward motion of the air between the clouds is taken into account. The result is that the cumulus convection does not convert heat into kinetic energy unless the ratio between the width of the cumulus towers and that of the cloudless intervals is below a certain critical value. Quite generally, the present treatment shows that the rate of production of kinetic energy through convection must be far below the estimates based on the assumption of the "infinitely small particle rising through an environment at rest."

Conditionally unstable = saturated air parcels unstable & unsaturated air parcels stable

Bjerknes model in short

Bjerknes, Quat. J. Royal Meteor. Soc. 1938



$$\rho' u'_z A' = \rho u_z A \;\Rightarrow\; \frac{m'}{m} = \frac{u_z}{u'_z}$$

Fig. 1.

$$\partial_t T = u_z(\Gamma_u - \partial_z T)$$

$$\partial_t T' = u'_z(\Gamma_s - \partial_z T)$$

$$\int_{z_1}^{z_2} c_p \left(m \frac{\partial T}{\partial t} + m' \frac{\partial T'}{\partial t} \right) \mathrm{d}z \;\simeq\; c_p m' u'_z \left(1 + \frac{m'}{m}\right) \int_{z_1}^{z_2} (\partial_z T - \Gamma_u)\mathrm{d}z + c_p m' u'_z \left[\int_{z_1}^{z_2} (\Gamma_s - \partial_z T)\mathrm{d}z - \frac{m'}{m} \int_{z_1}^{z_2} (\partial_z T - \Gamma_u)\mathrm{d}z \right]$$

Accumulation of heat per second set free by condensation

Uniform Heating

Surplus heating by clouds convertible in convective motion if in total positive

Bjerknes model in short

Bjerknes, Quat. J. Royal Meteor. Soc. 1938

$$\int_{z_1}^{z_2} c_p \left(m\frac{\partial T}{\partial t} + m'\frac{\partial T'}{\partial t} \right) \mathrm{d}z \simeq c_p m' u'_z \left(1 + \frac{m'}{m}\right) \int_{z_1}^{z_2} (\partial_z T - \Gamma_u)\mathrm{d}z$$

$$+ \; c_p m' u'_z \Bigg[\underbrace{\int_{z_1}^{z_2} (\Gamma_s - \partial_z T)\mathrm{d}z}_{=b(>0)} - \frac{m'}{m} \underbrace{\int_{z_1}^{z_2} (\partial_z T - \Gamma_u)\mathrm{d}z}_{=a(>0)} \Bigg]$$

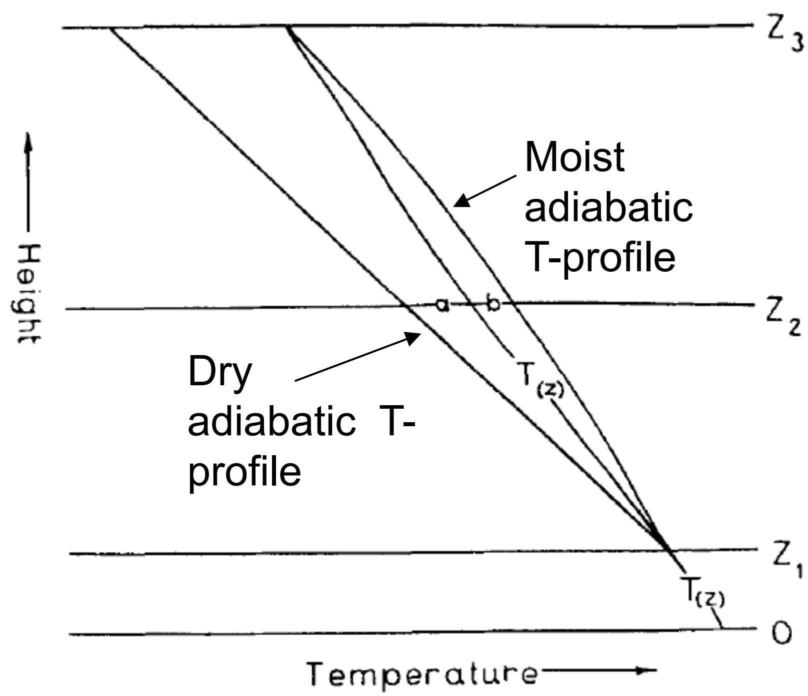


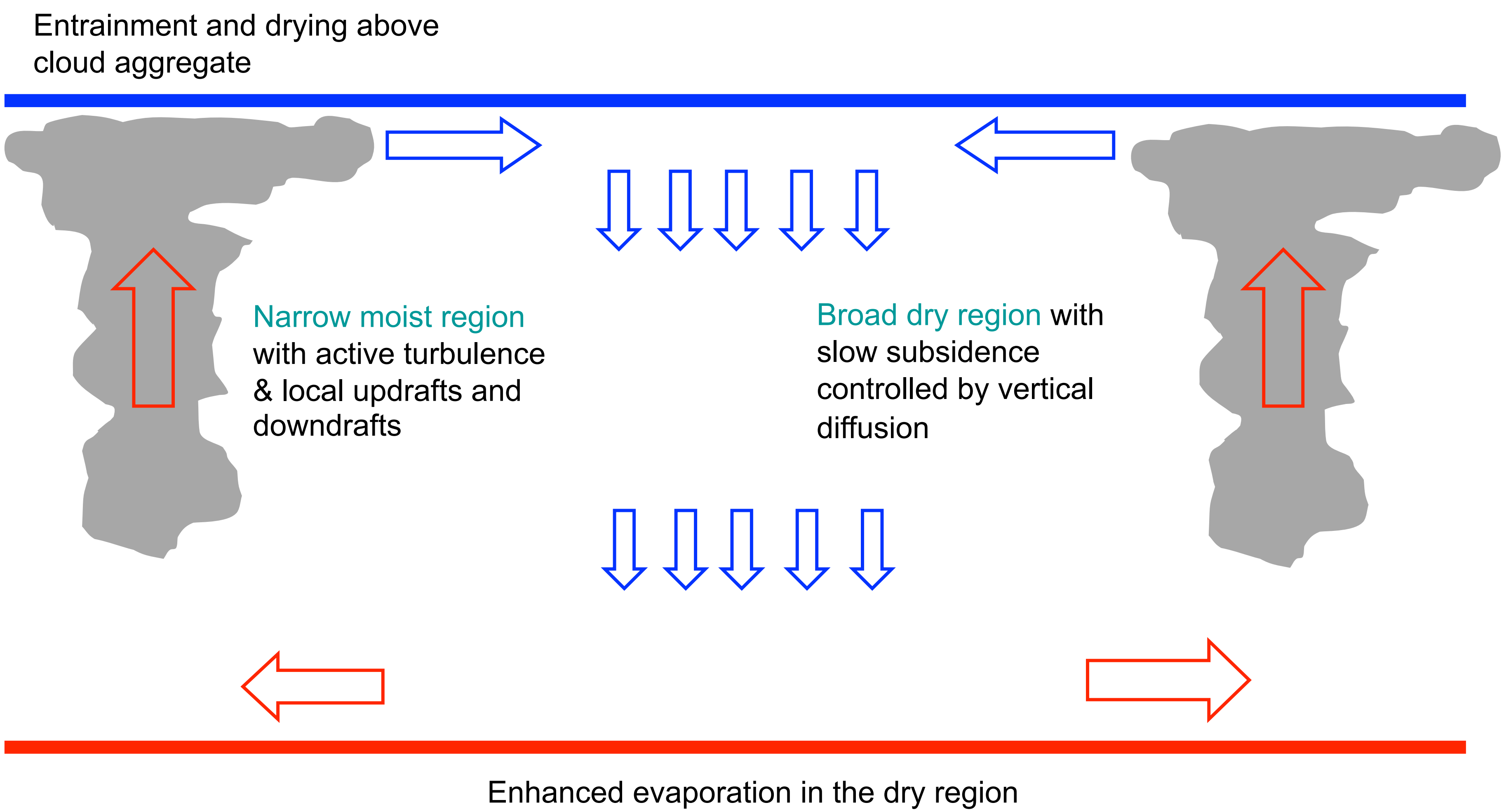
FIG. 2.

$$b > \frac{m'}{m} a$$

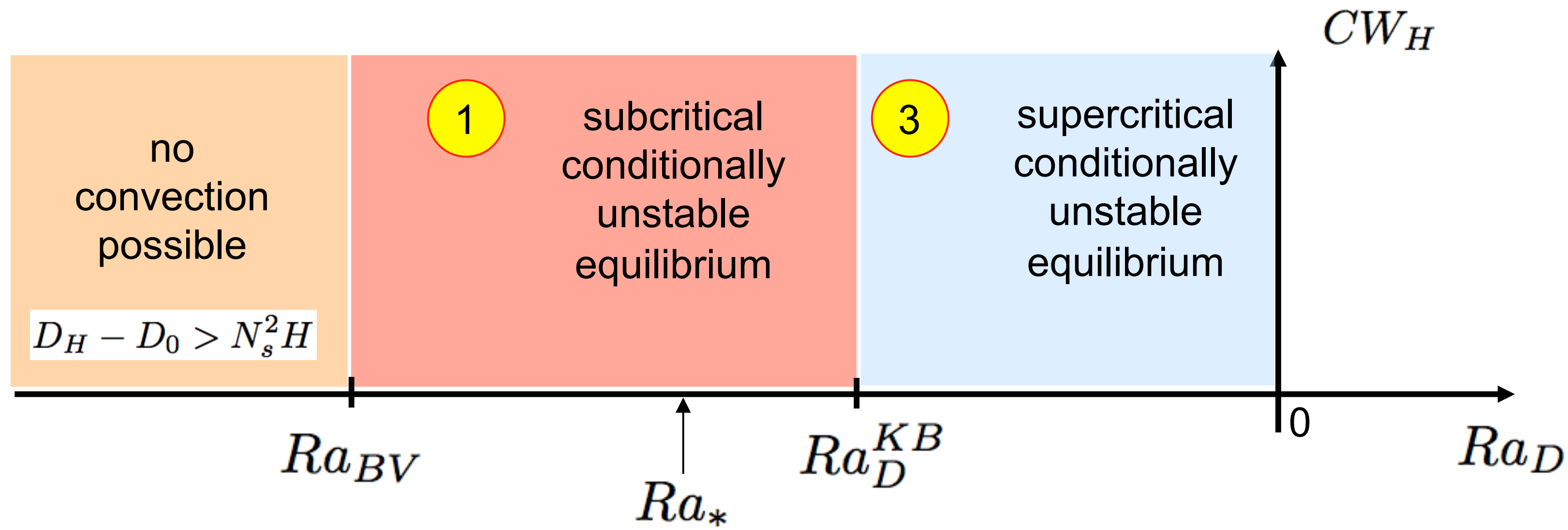
Can be achieved by $m' \ll m$

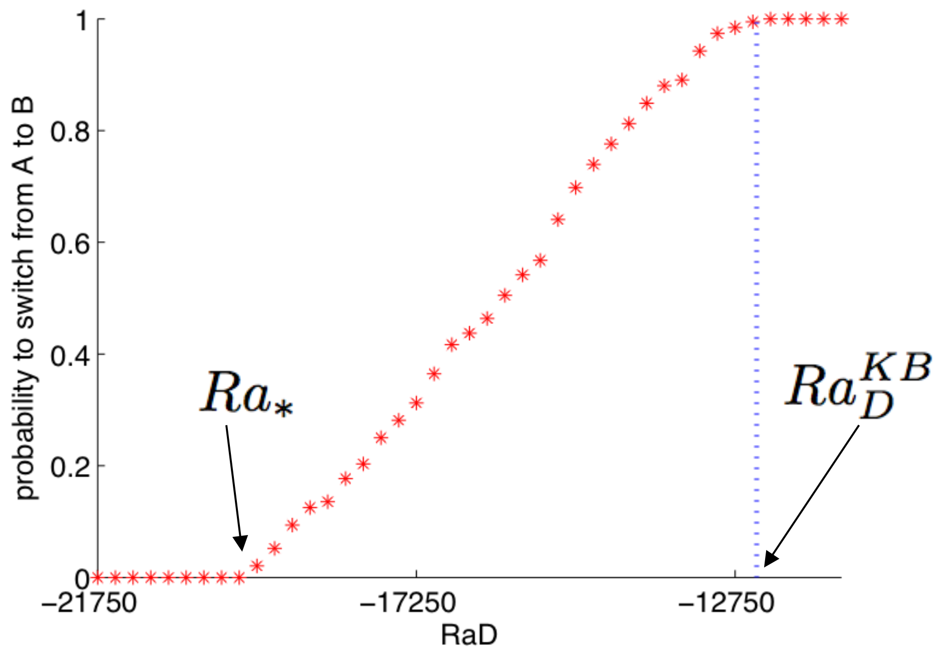
Idea of the Bjerknes model

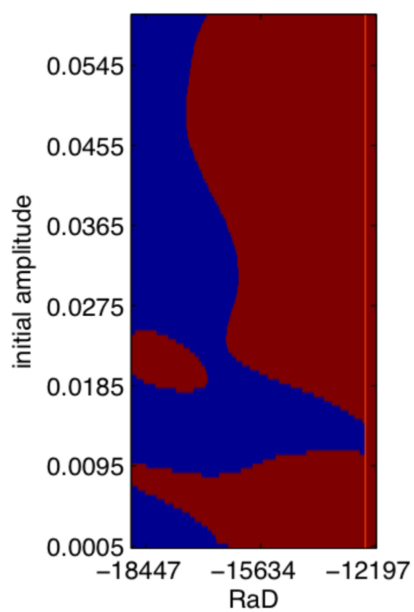
Bjerknes, Quat. J. Royal Meteor. Soc. 1938

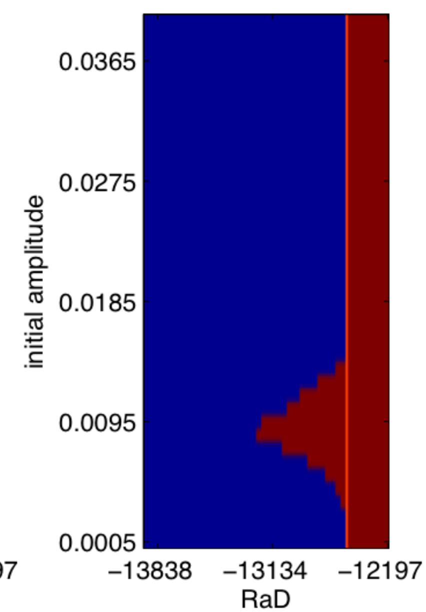


Fourier-Galerkin model



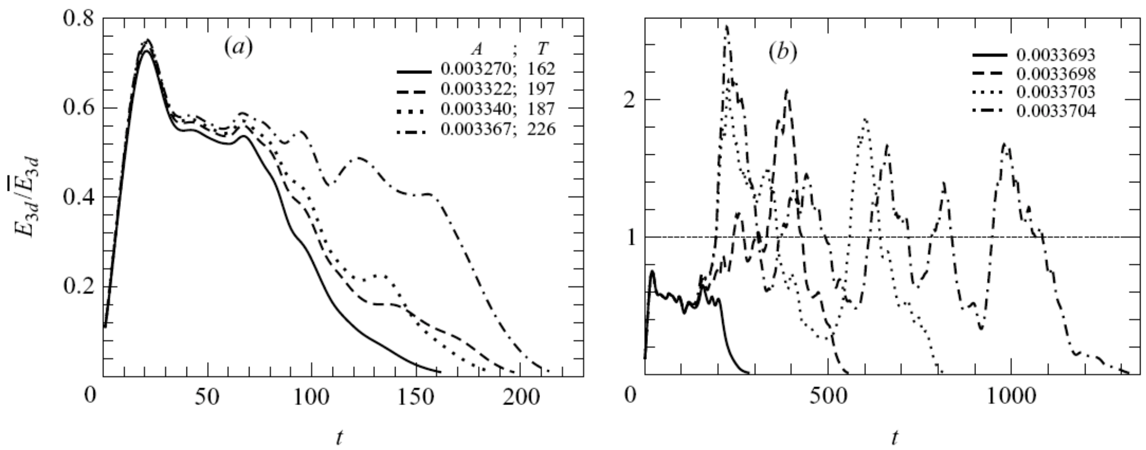




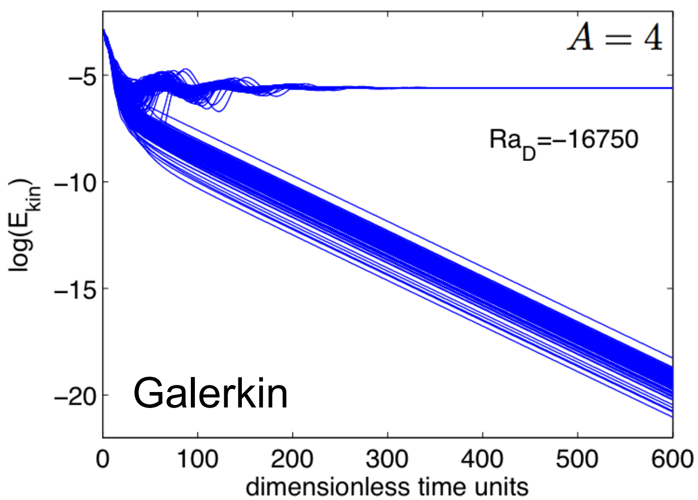


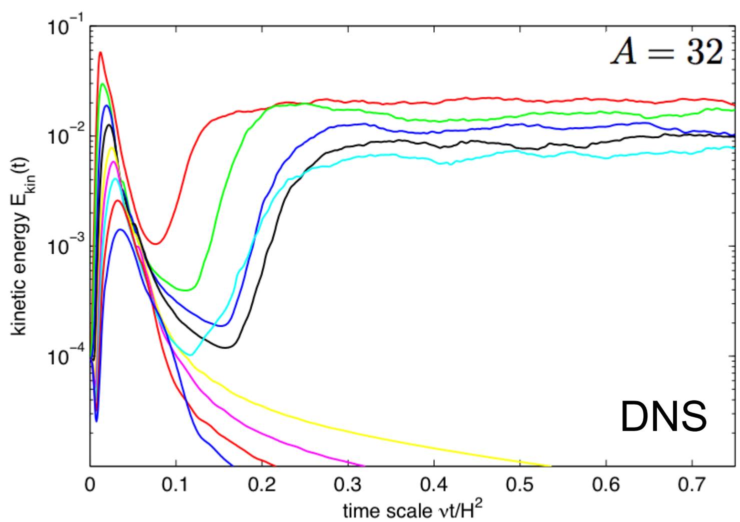
Lifetime of flow states

Pipe flow: transient states (*Faisst & Eckhardt, J. Fluid Mech. 2004)*

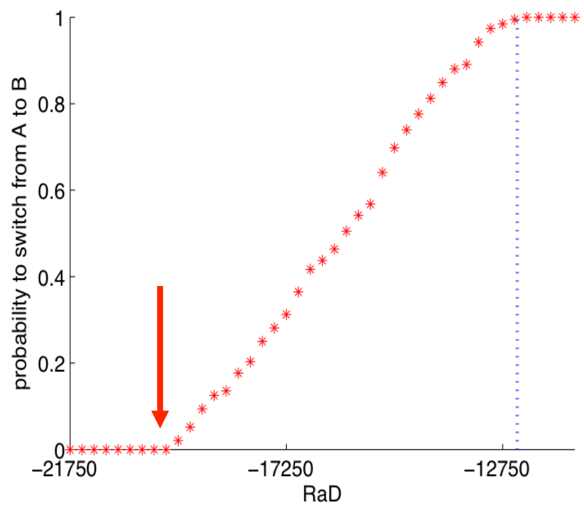


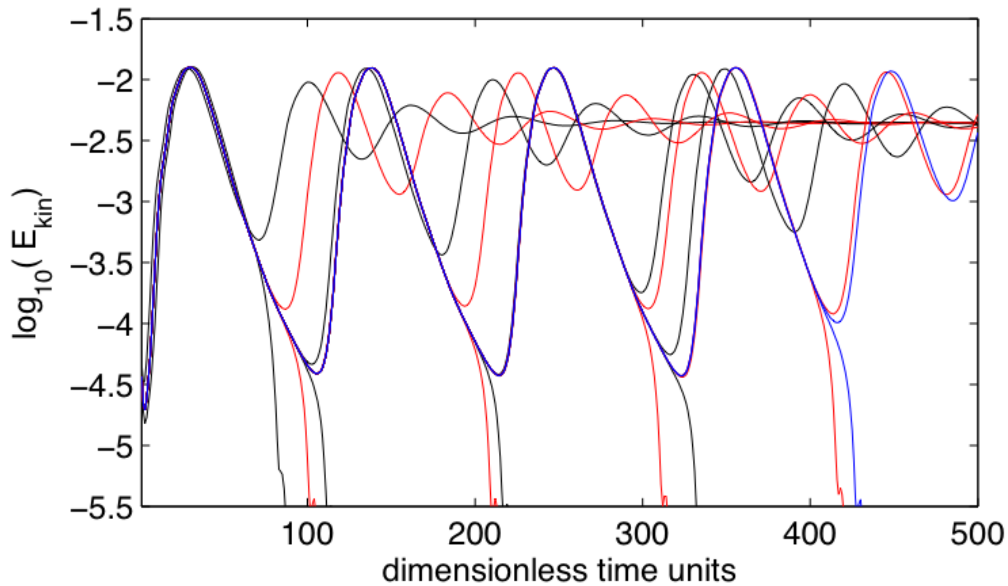
Conditionally unstable moist convection: no transient states

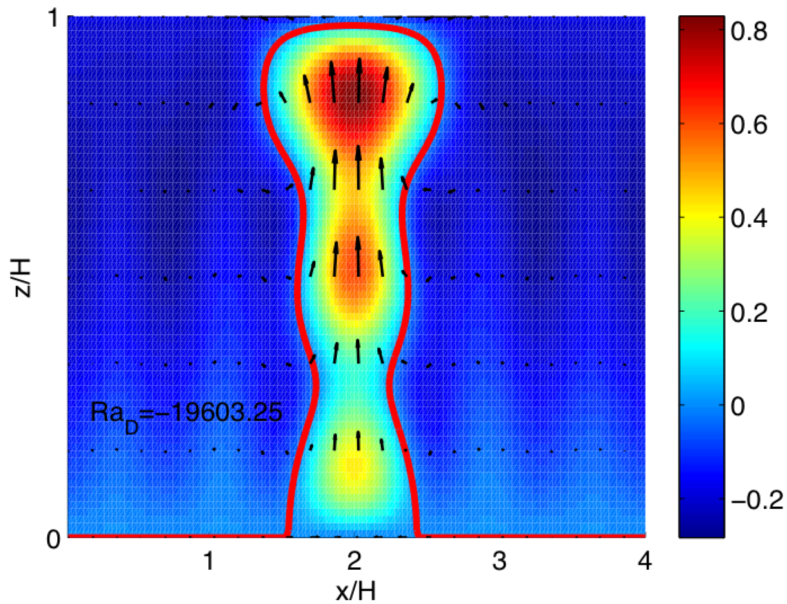




Tracking the edge state





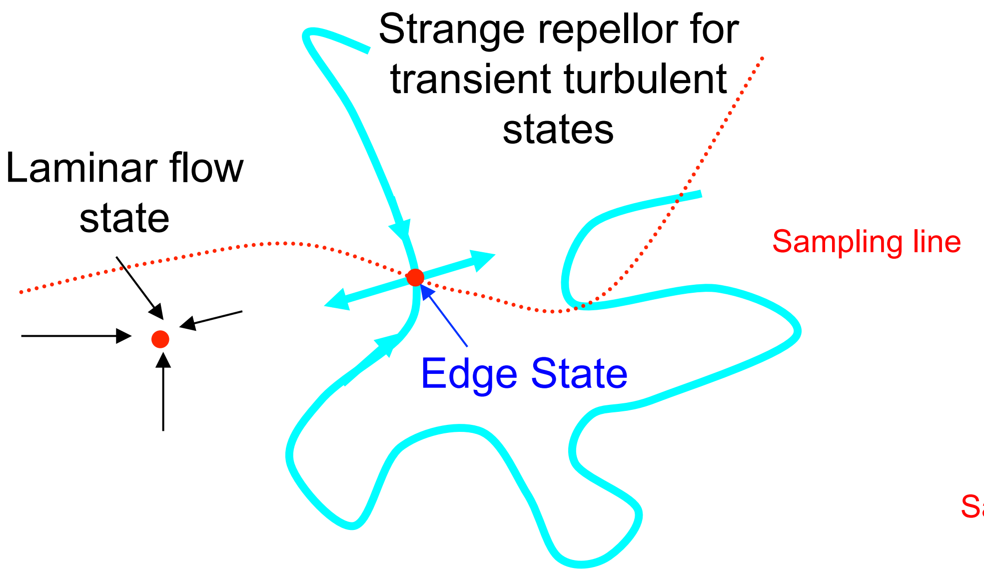


$$\mathbf{X}(t=0) = \lambda \mathbf{X}_A^* + (1-\lambda)\mathbf{X}_B^*$$

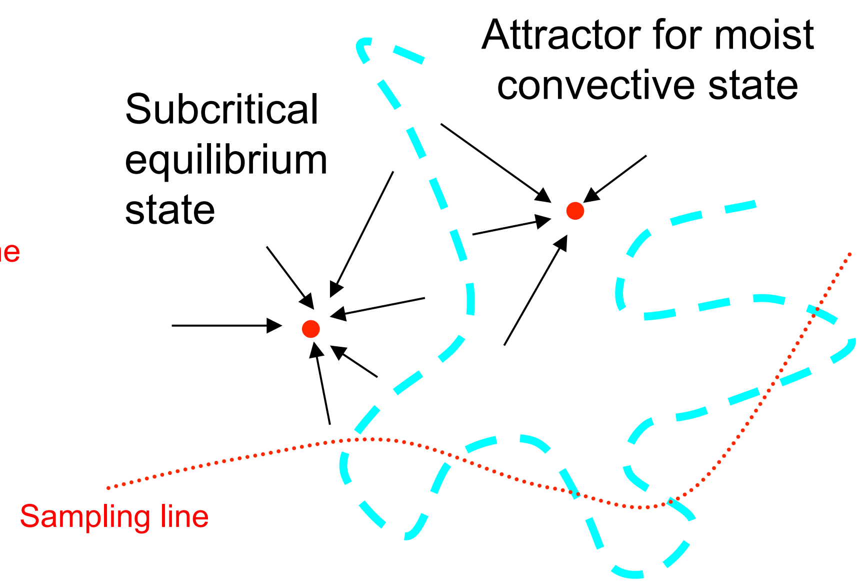
Edge state is a periodic orbit

Our present picture

Plane shear flow



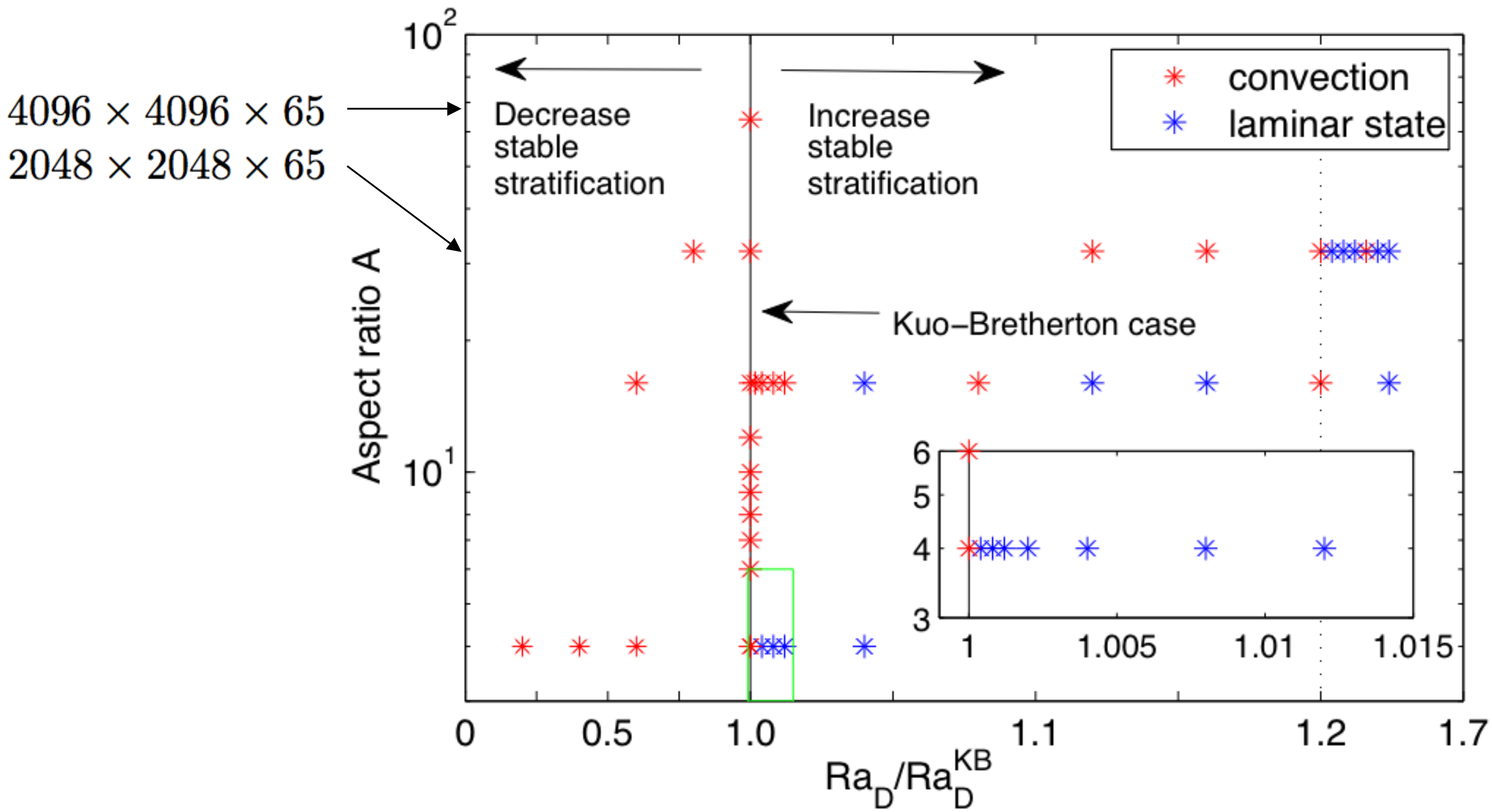
CU moist convection



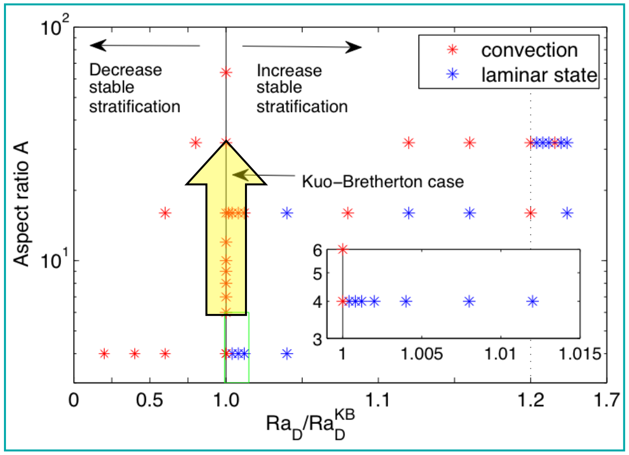
Skufca, Yorke & Eckhardt, Phys. Rev. Lett. 2006
Schneider, Marinc & Eckhardt, J. Fluid Mech. 2010

- Sensitive dependence of transition on initial perturbation amplitude
- Sensitive dependence of transition on aspect ratio of cell
- No transient flow patterns

Parameter plane for DNS

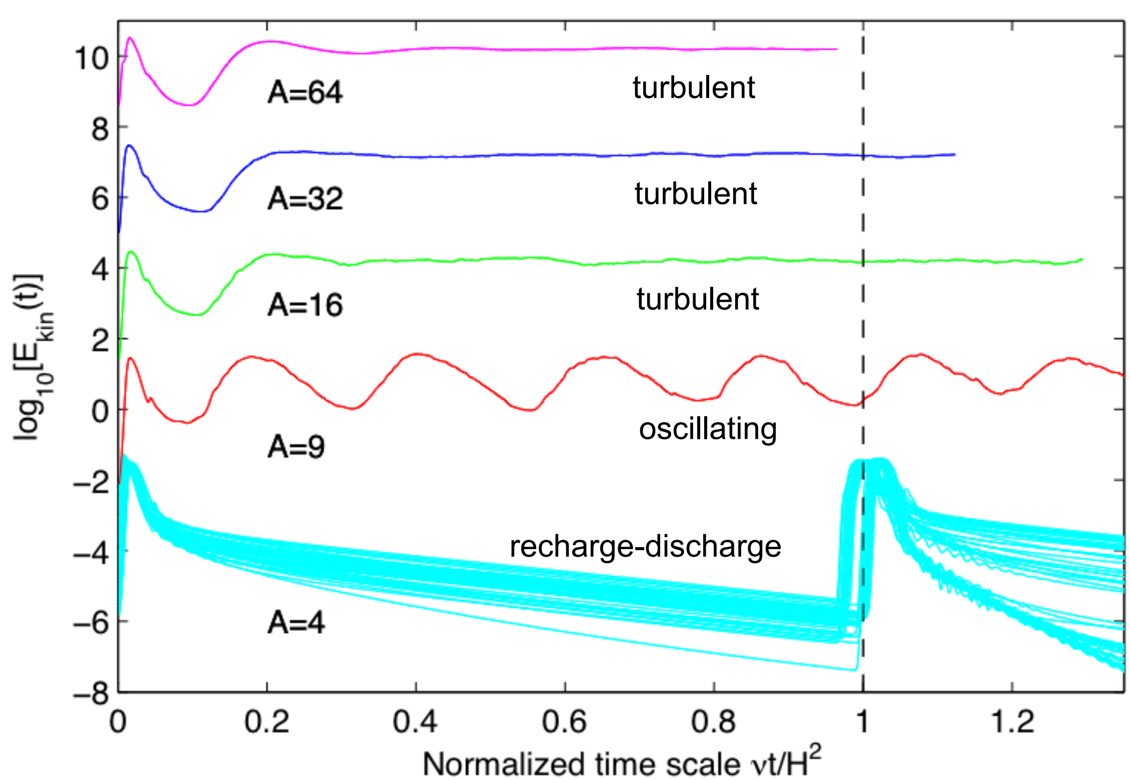


Aspect ratio dependence

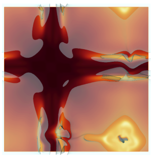


Moist convection is constrained by aspect ratio

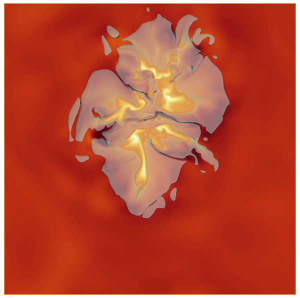
$$Ra_M = 373420$$



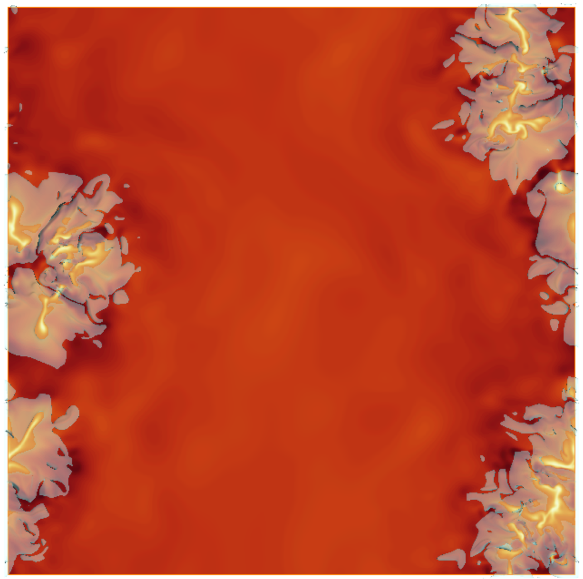
Cloud aggregation



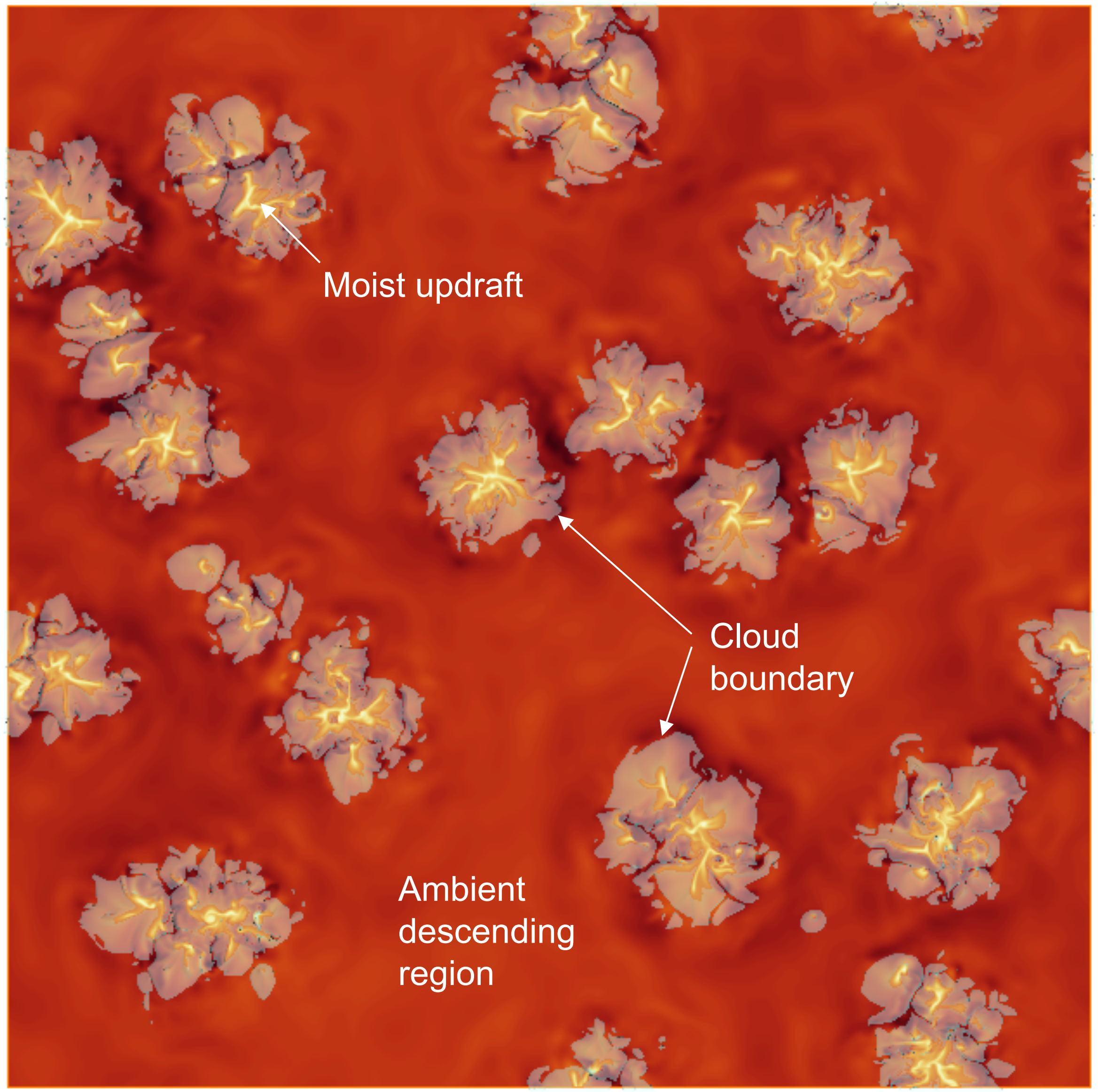
$\Gamma = 8$



$\Gamma = 16$

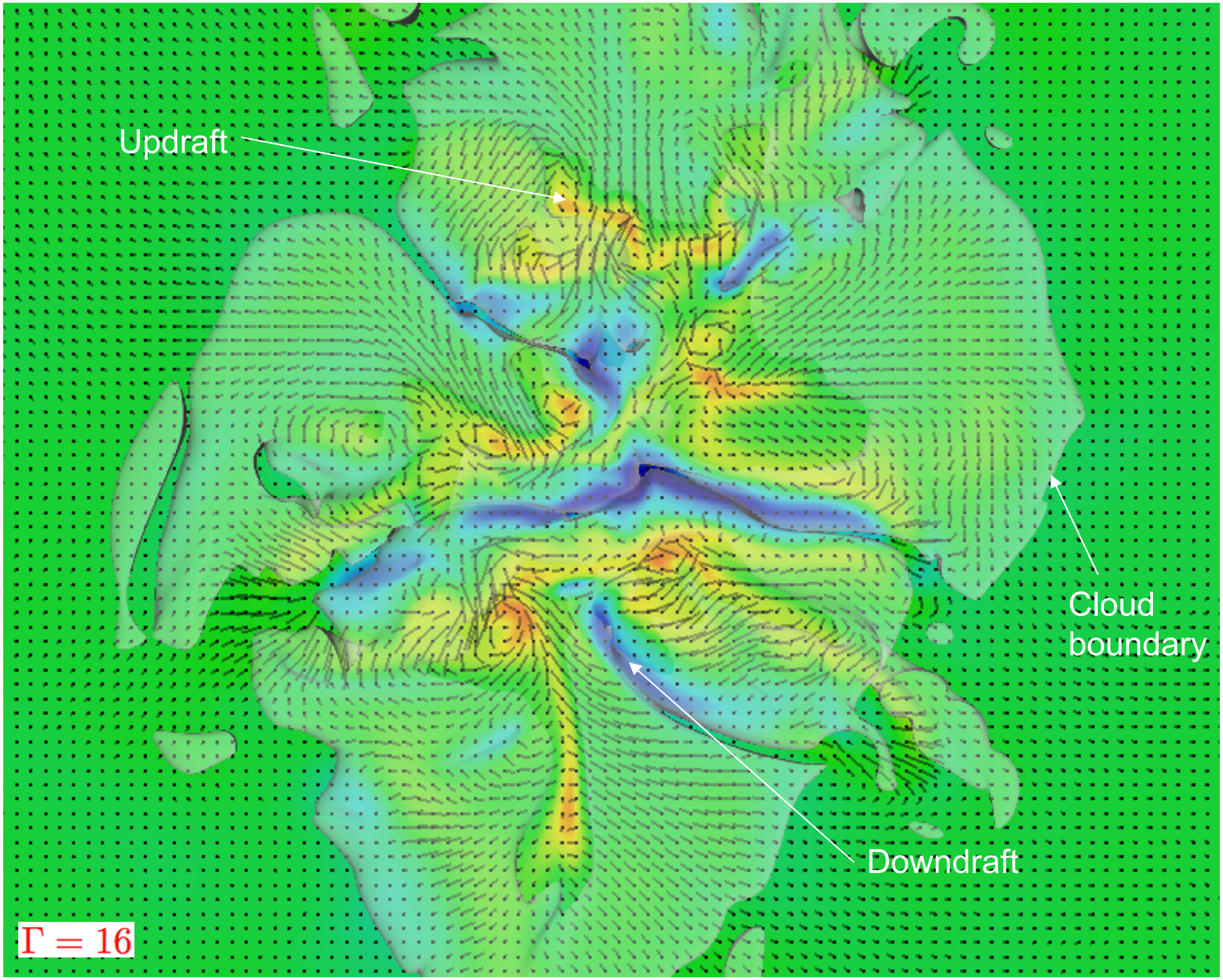


$\Gamma = 32$

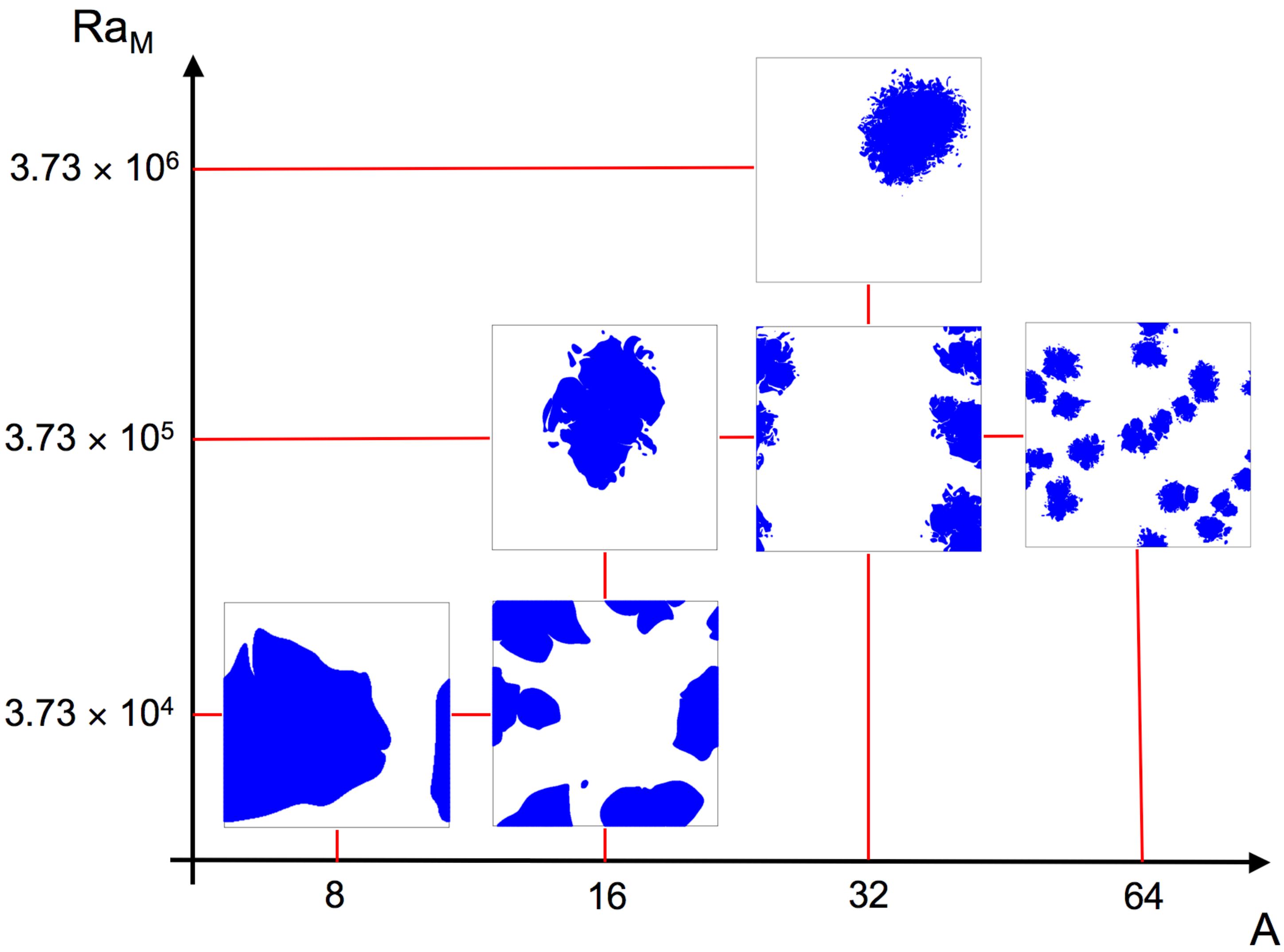


$\Gamma = 64$

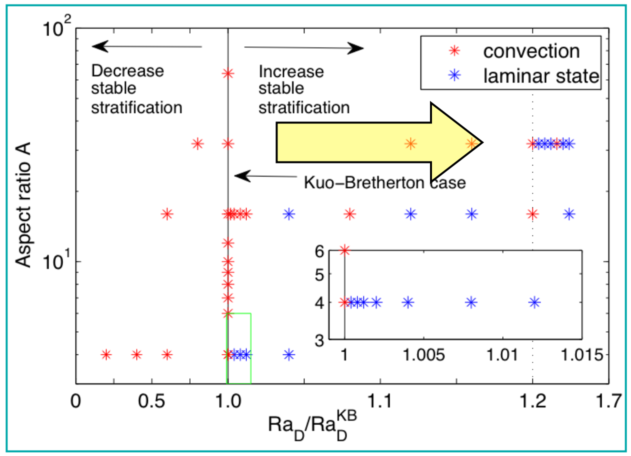
Turbulent velocity field inside cloud aggregate



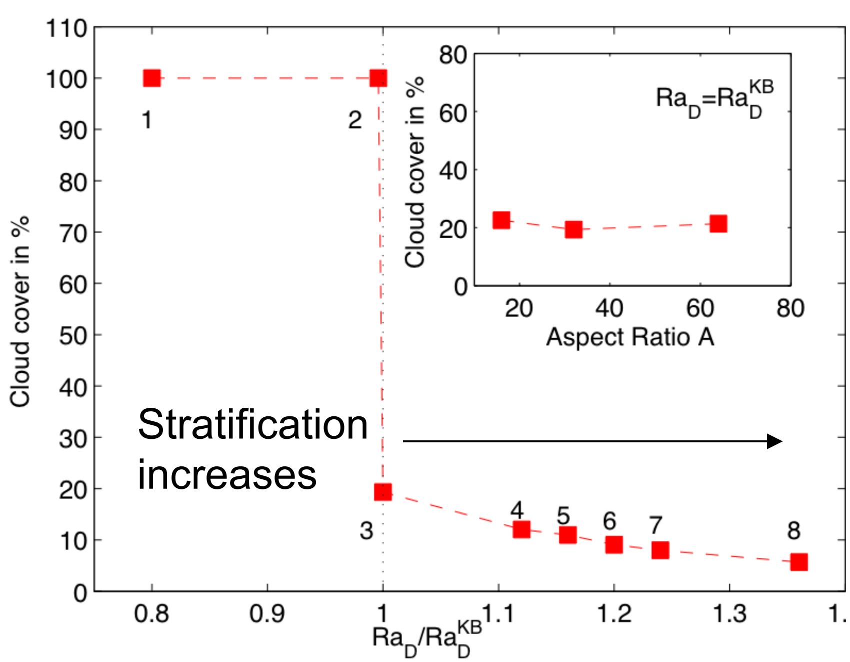
Rayleigh number vs. aspect ratio

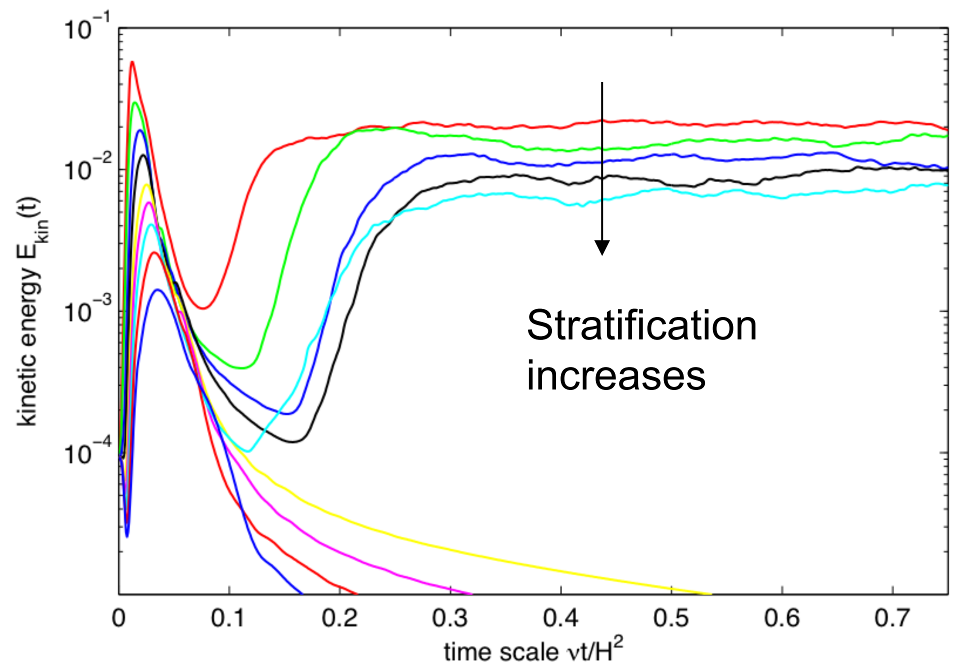


Increase stable stratification



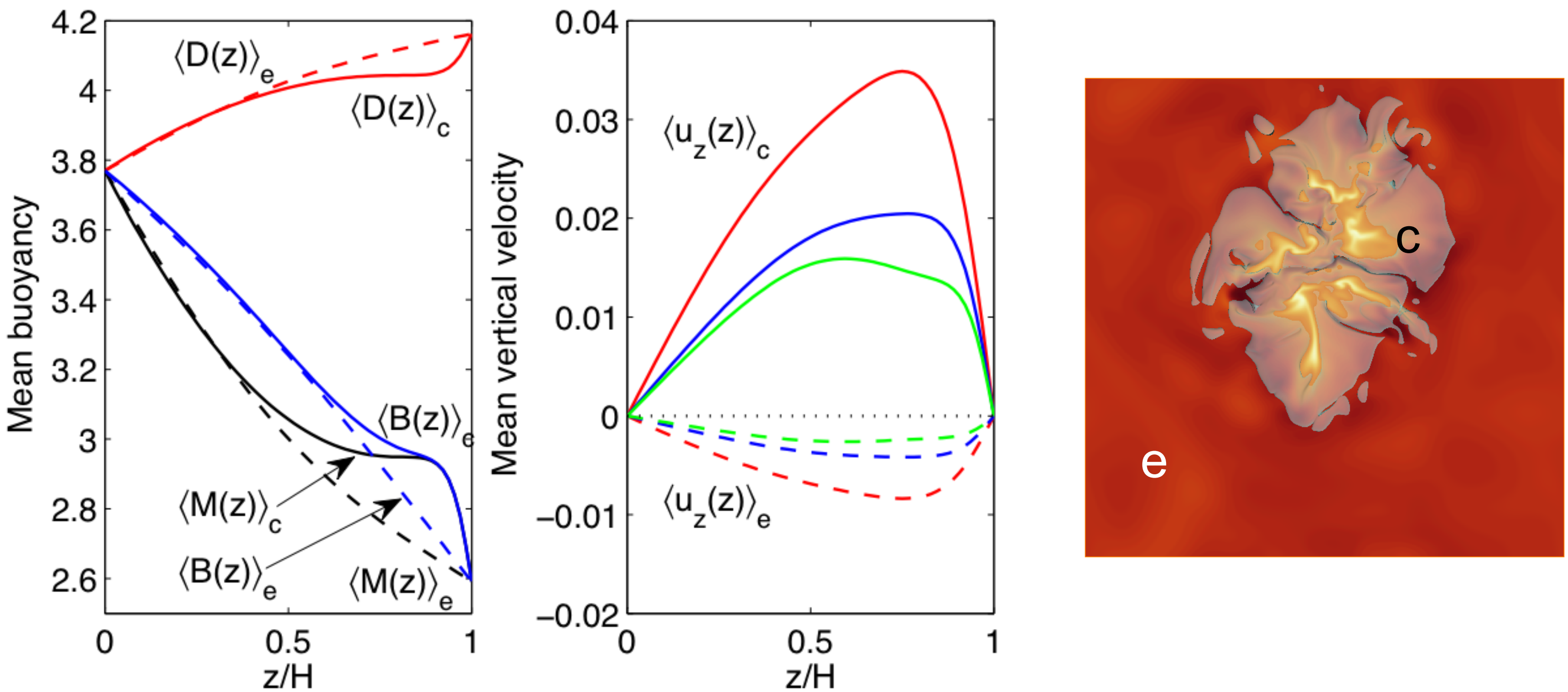
All runs at A=32





Statistics inside/outside clouds

What is the high-Ra limit?

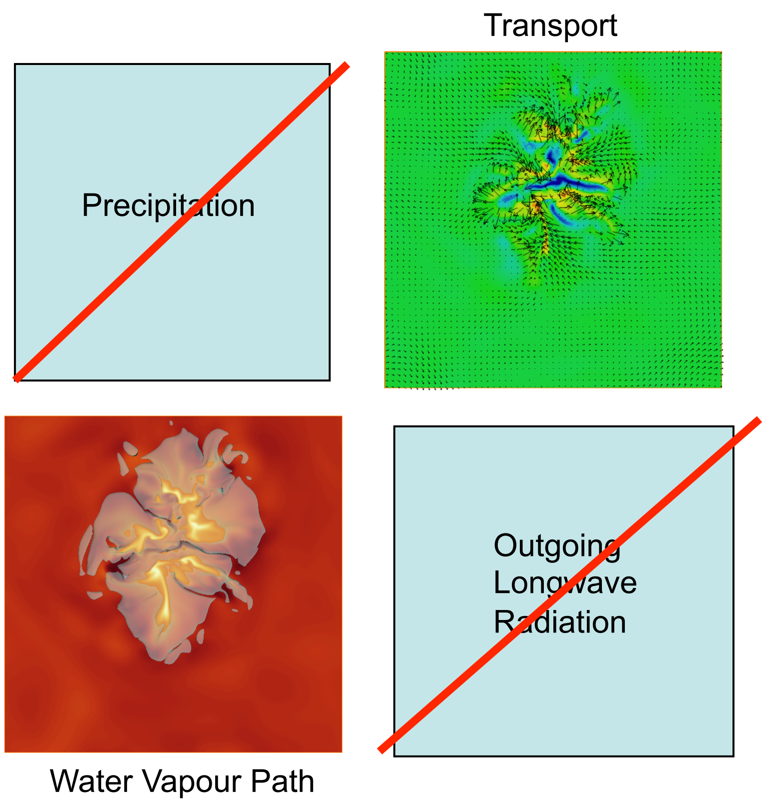


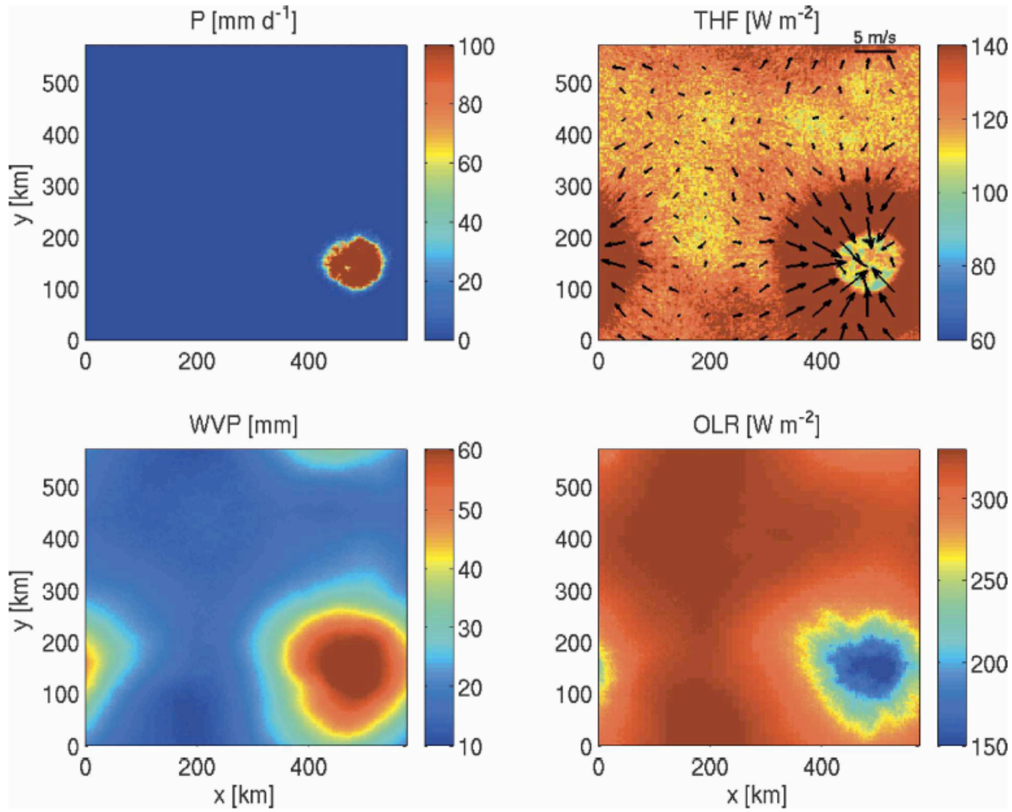
$Ra_M = 373416$

$Ra_M = 1867080$

$Ra_M = 3734160$

Comparison with LES





Aggregation after 350 convective time units

$$16H \times 16H \times H$$

$$1024 \times 1024 \times 65$$

Aggregation of 50 days

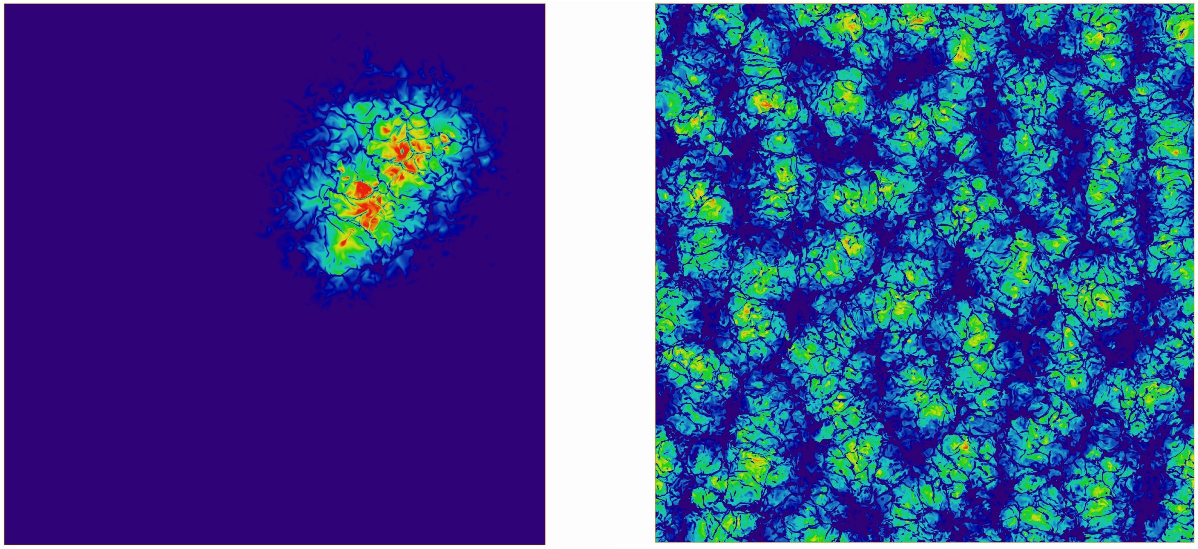
$$576\text{km} \times 576\text{km} \times 28\text{km}$$

$$\Delta_h = 3\text{km} \qquad \min(\Delta_z) = 500\text{m}$$

Bretherton et al., J. Atmos. Sci. 2005

Summary and outlook

- Our shallow moist convection model yields different degrees of cloud cover and makes a systematic statistical analysis as a function of the Rayleigh number possible
- Up-down dynamics is highly asymmetric and transition to convective regime depends on aspect ratio
- Turbulent cloud aggregates in conditionally unstable case for sufficiently large aspect ratio



- Effects of precipitation and radiative cooling

References & Thanks to

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JS & O. Pauluis, J. Fluid Mech. **648**, 509 (2010)

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http://www.tu-ilmenau.de/tsm

HIL02

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