Capillary wave turbulence

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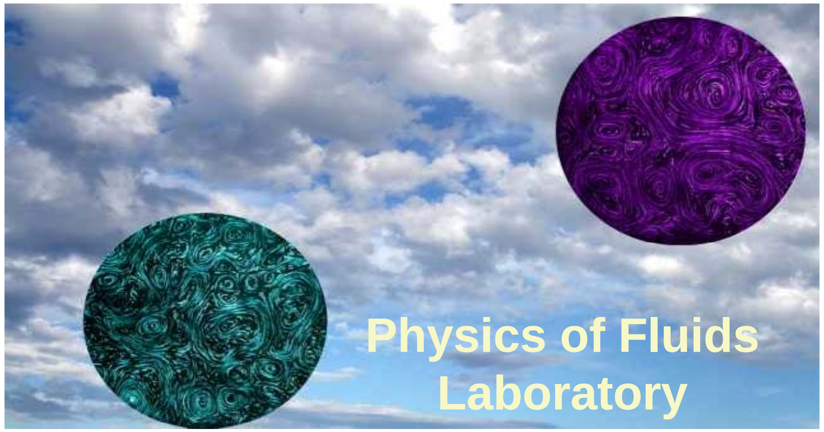
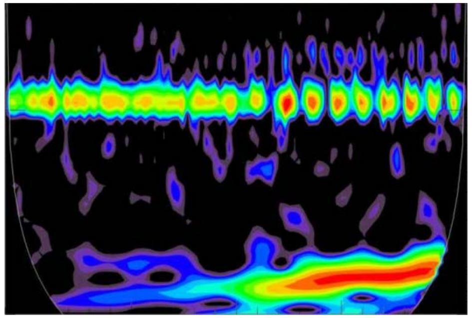


Physics of Fluids Laboratory

D. Byrne, Dr. H. Xia, Prof. M. Shats, Dr. H. Punzmann

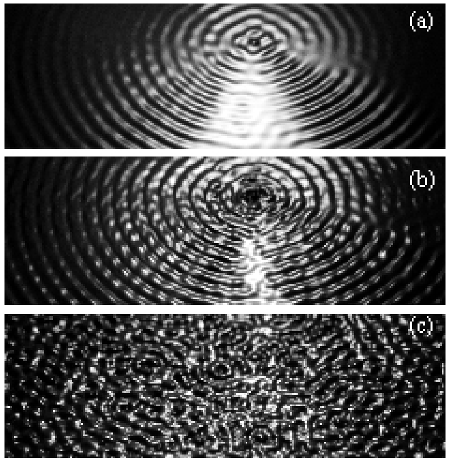
Turbulence in fluids

Plasma Turbulence



Rotating flows

Surface waves



Publications

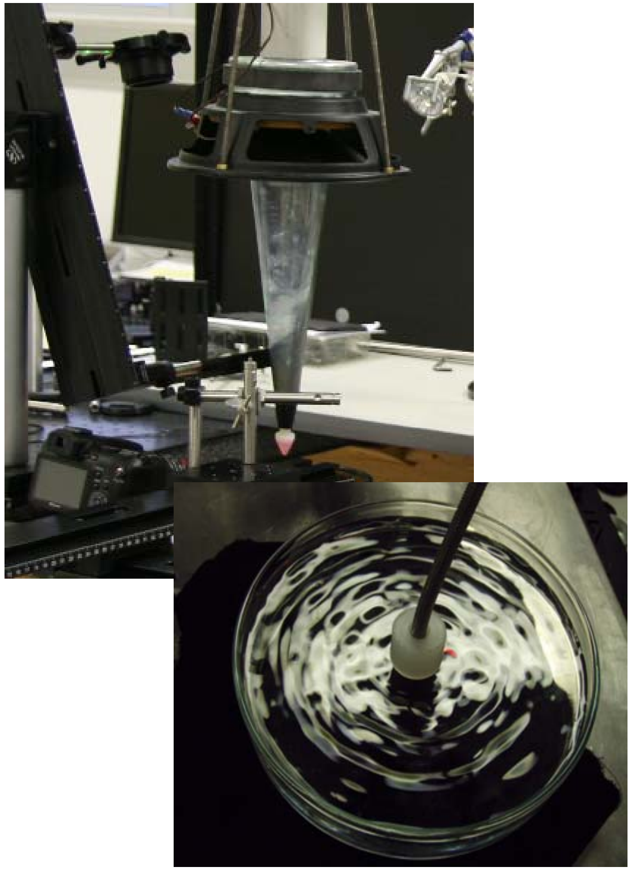
- Punzmann H., Shats M. and Xia H.
  Phase randomization of three-wave interactions in capillary waves,
  Physical Review Letters, **103**, 064502 (2009)

- Xia H., Shats M., Punzmann H.
  Modulation instability and capillary wave turbulence,
  EPL, **91**, 14002 (2010)

- Shats M., Punzmann H., Xia H.
  Capillary rogue waves,
  Physical Review Letters, **104**, 104503 (2010)

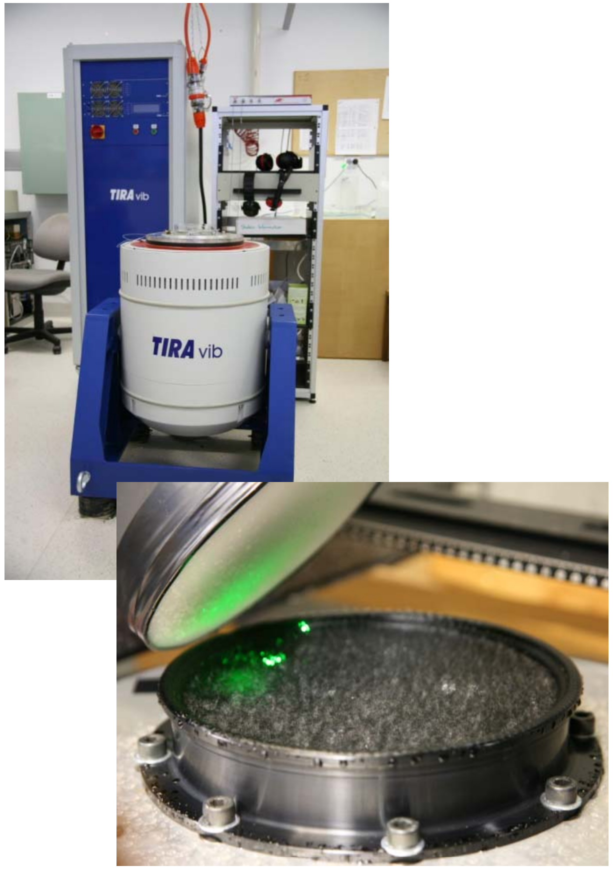
Laboratory setup

Plunger



direct wave excitation,
spatially localized

Shaker



parametric excitation,
spatially isotropic

Laboratory setup

Waves excited parametrically in vertically shaken container, or using conical plunger

Parametric excitation of surface waves

Faraday waves are excited in shaker experiments using fluid tanks of different shapes (round, square), depth (~30mm) , sizes (ϕ=100-300mm) in the frequency range 40Hz < f_s < 4kHz.

surface wave dispersion relation:

$$\omega^2 = \underbrace{gk}_{\text{gravity waves}} + \underbrace{\frac{\sigma}{\rho}k^3}_{\text{capillary waves}}$$

capillary waves:

$$\omega = \sqrt{\sigma/\rho}\, k^{3/2}$$

for short wave length λ<10 mm

Parametric excitation:

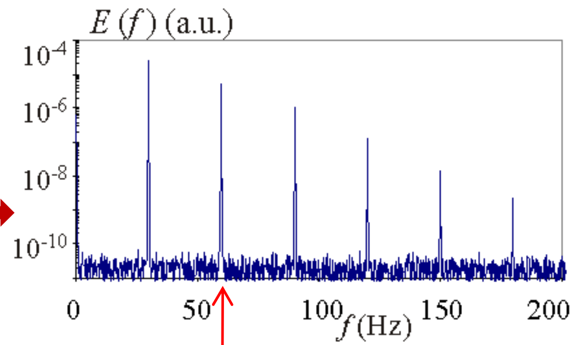
$$\omega_0 = \omega_1 + \omega_2 = 2\omega_1$$

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2 \approx 0$$

$$\vec{k}_1 = -\vec{k}_2$$

$$\omega_1 = \omega_0 / 2$$

Small amplitude capillary wave



Forcing frequency

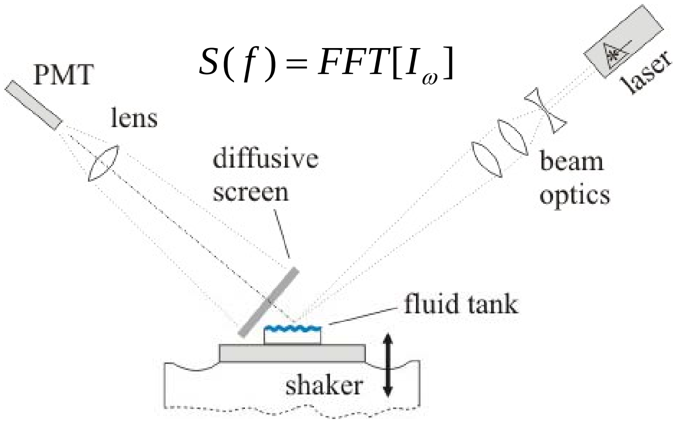
Measurement techniques

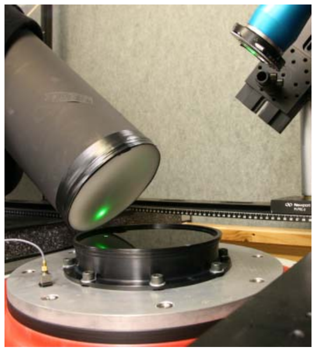
Laser reflection

Reflected laser light intensity proportional to gradient of the surface elevation

$$I_\omega \sim \nabla \eta(r,t)$$

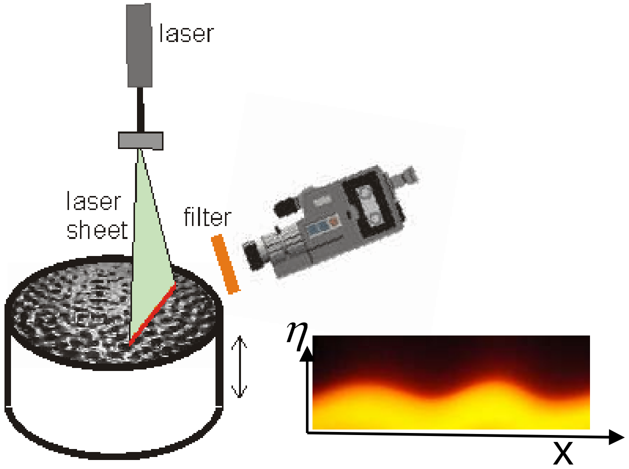
$$S(f) = FFT[I_\omega]$$

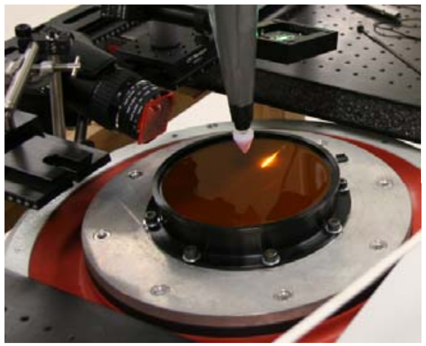




Surface imaging using thin laser sheet

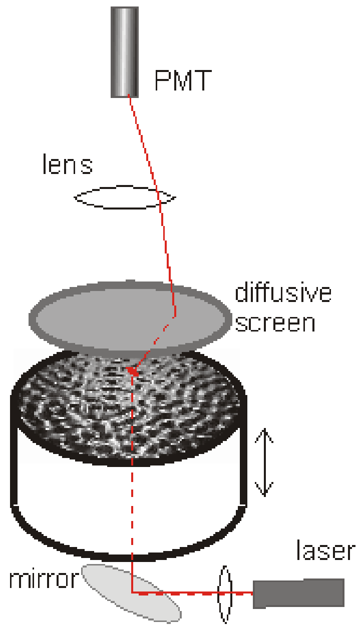
Fluorescent dye used to visualize perturbed surface





Laser transmission through diffusive liquid

Transmitted laser light intensity proportional to the surface elevation



Experimental tests of weak turbulence theory

Assumptions in the weak turbulence theory

1. Capillary wave dispersion relation allows three-wave interactions

$$\omega_1 = \omega_2 + \omega_3$$

$$\vec{k}_1 = \vec{k}_2 + \vec{k}_3$$

2. Waves have random phases

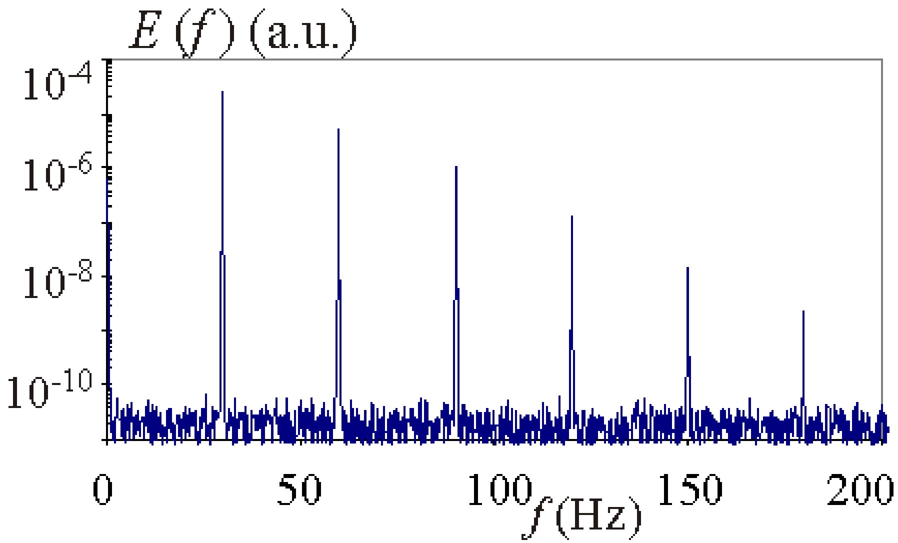
$$\omega_k = \sqrt{\frac{\sigma}{\rho} k^3}$$

3. Infinite domain

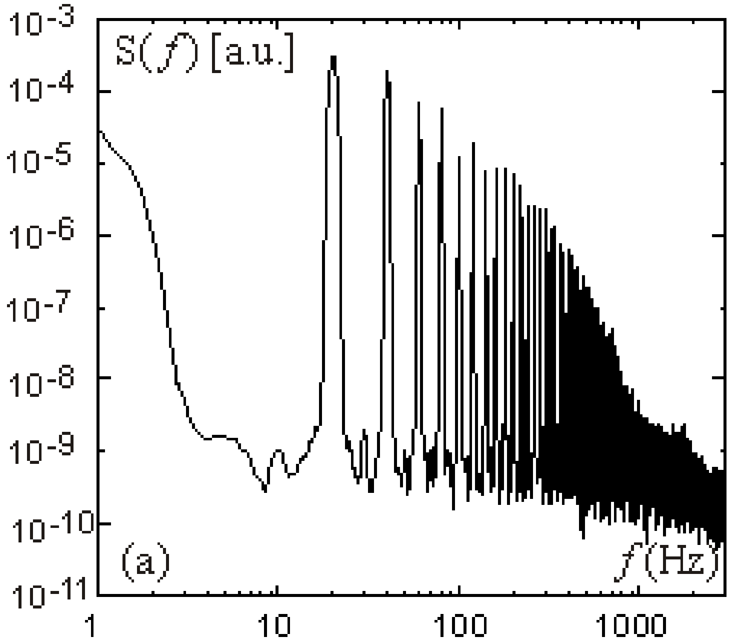
WTT predicts $$E_\omega \sim P^{1/2} \rho^{-2/3} \alpha^{1/6} \omega^{-17/6}$$

Assumptions in the weak turbulence theory

1. Capillary wave dispersion relation allows three-wave interactions



low forcing



higher forcing

No 3-wave interactions possible in such spectra

$$\omega_k = \sqrt{\frac{\sigma}{\rho} k^3}$$

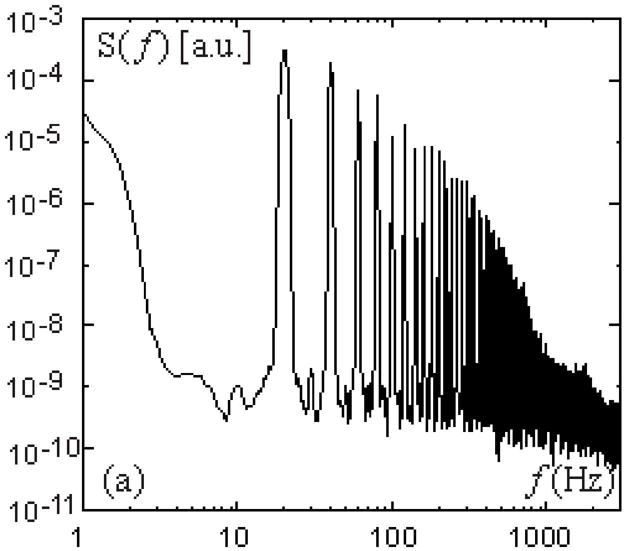
$$\omega_1 = \omega_2 + \omega_3$$

$$\vec{k}_1 = \vec{k}_2 + \vec{k}_3$$

Matching rules for *k* and ω cannot be satisfied simultaneously

Assumptions in the weak turbulence theory

2. Wave have random phases

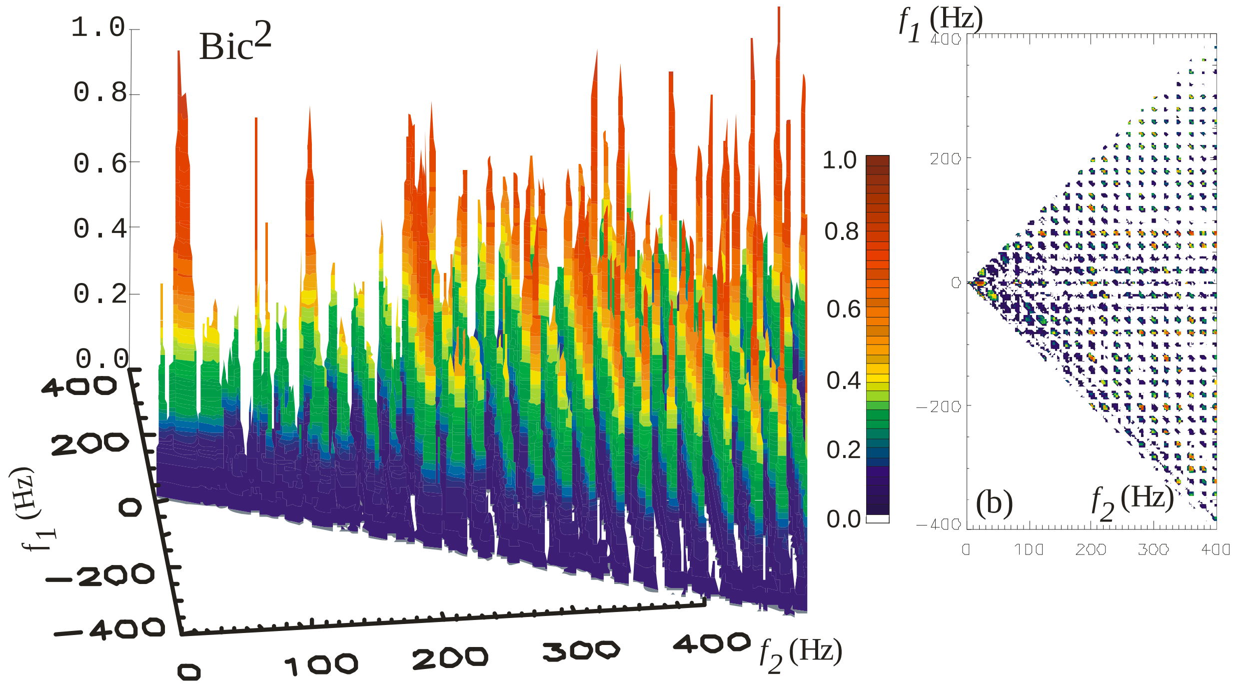


Bicoherence:

$$b_f^2(f_1, f_2) = \frac{\left|\left\langle F_f F_{f1}^* F_{f2}^* \right\rangle\right|^2}{\left\langle F_f F_f^* \right\rangle \left\langle F_{f1} F_{f1}^* \right\rangle \left\langle F_{f2} F_{f2}^* \right\rangle}$$

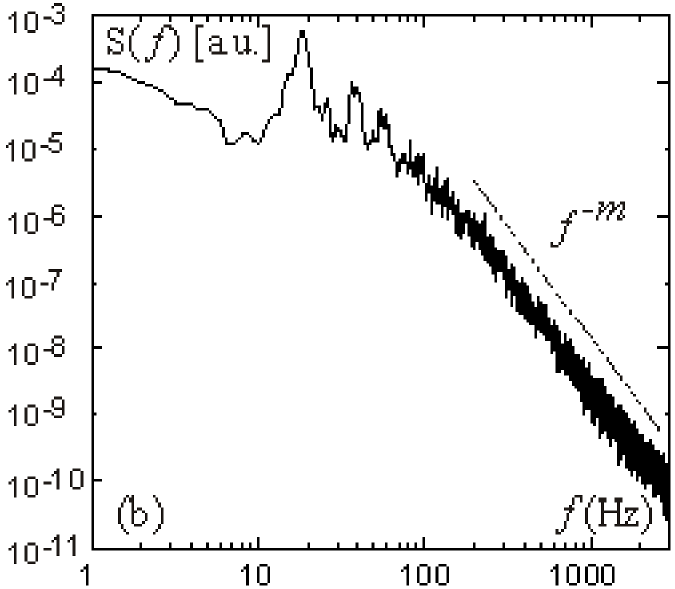
High bicoherence > 0.5

Phase coupled coherent harmonics:



Assumptions in the weak turbulence theory

2. Wave have random phases

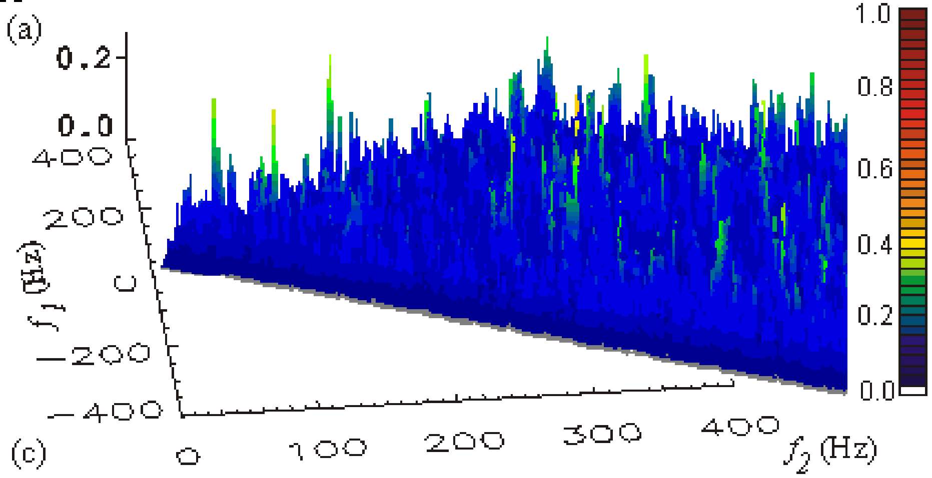


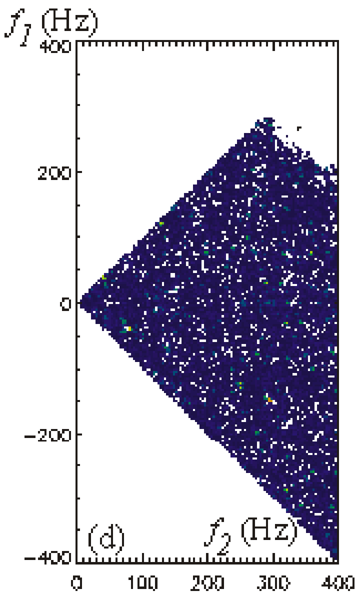
Bicoherence:

$$b_f^2(f_1, f_2) = \frac{\left|\left\langle F_f F_{f1}^* F_{f2}^* \right\rangle\right|^2}{\left\langle F_f F_f^* \right\rangle \left\langle F_{f1} F_{f1}^* \right\rangle \left\langle F_{f2} F_{f2}^* \right\rangle}$$

Low bicoherence < 0.3

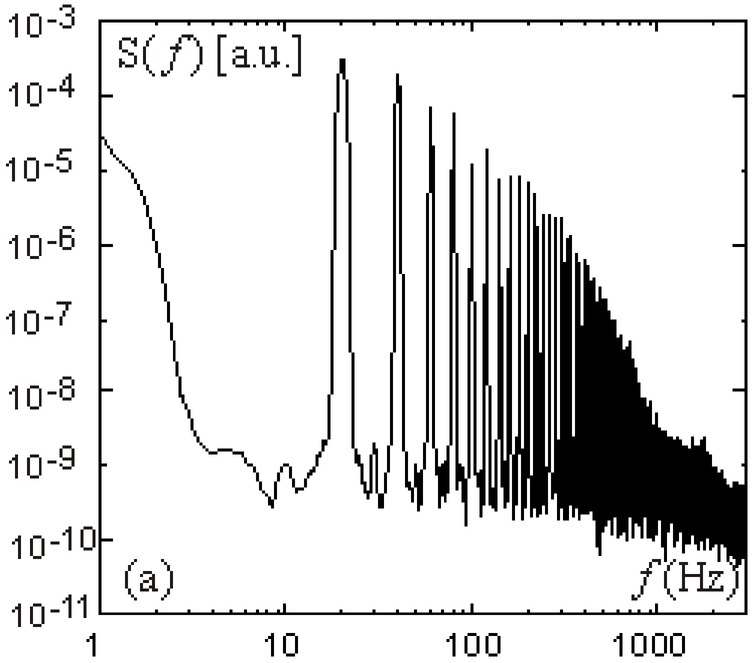
Random phases?



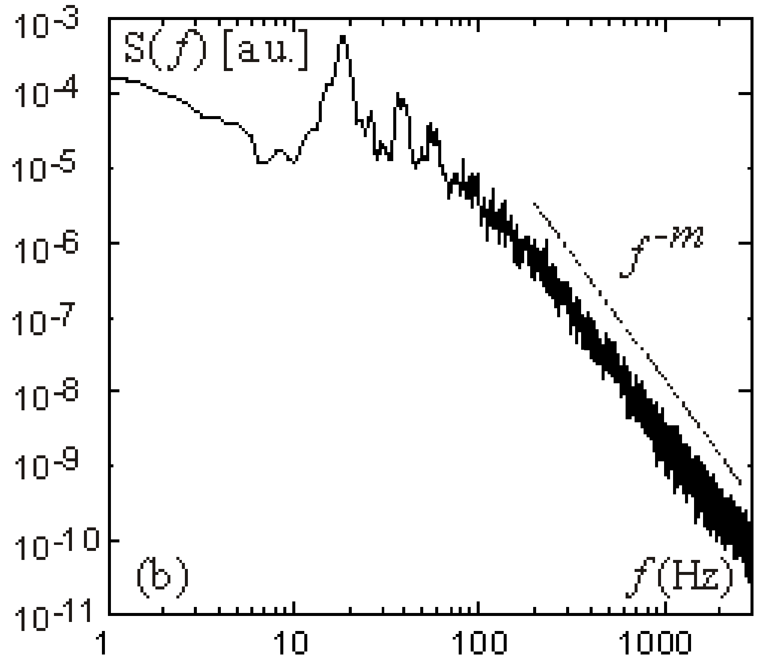


Transition to turbulence?

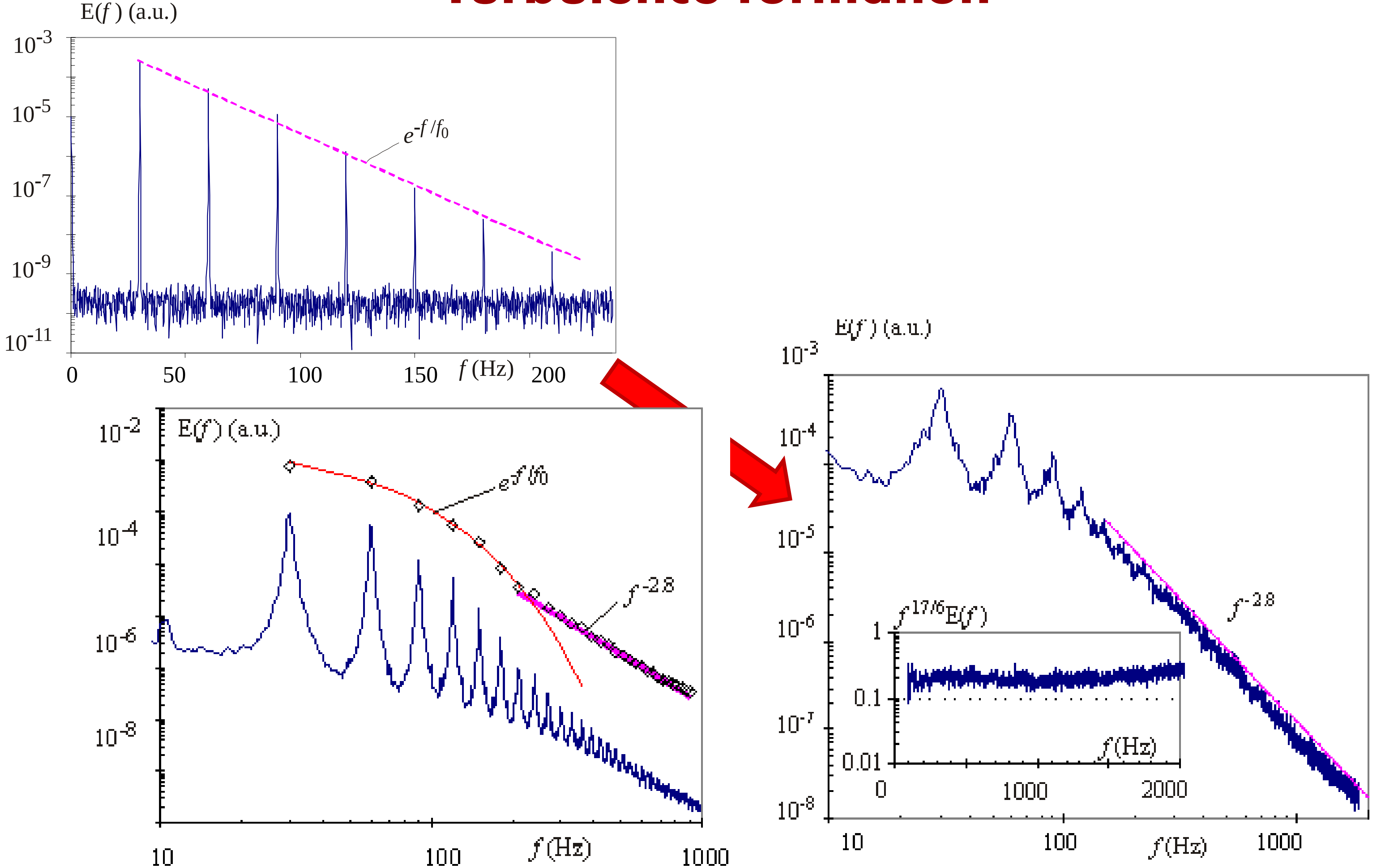
low-ish forcing



strong forcing



Turbulence formation



Turbulence formation

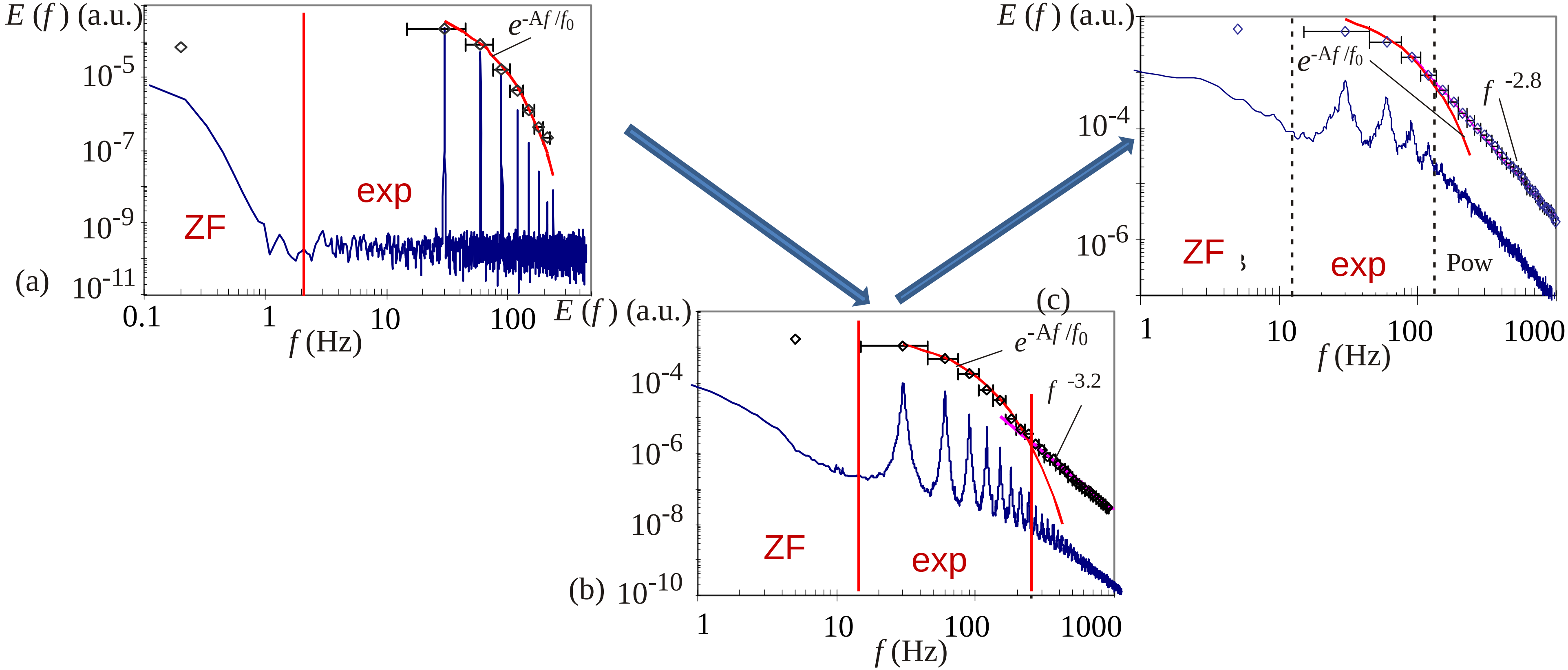
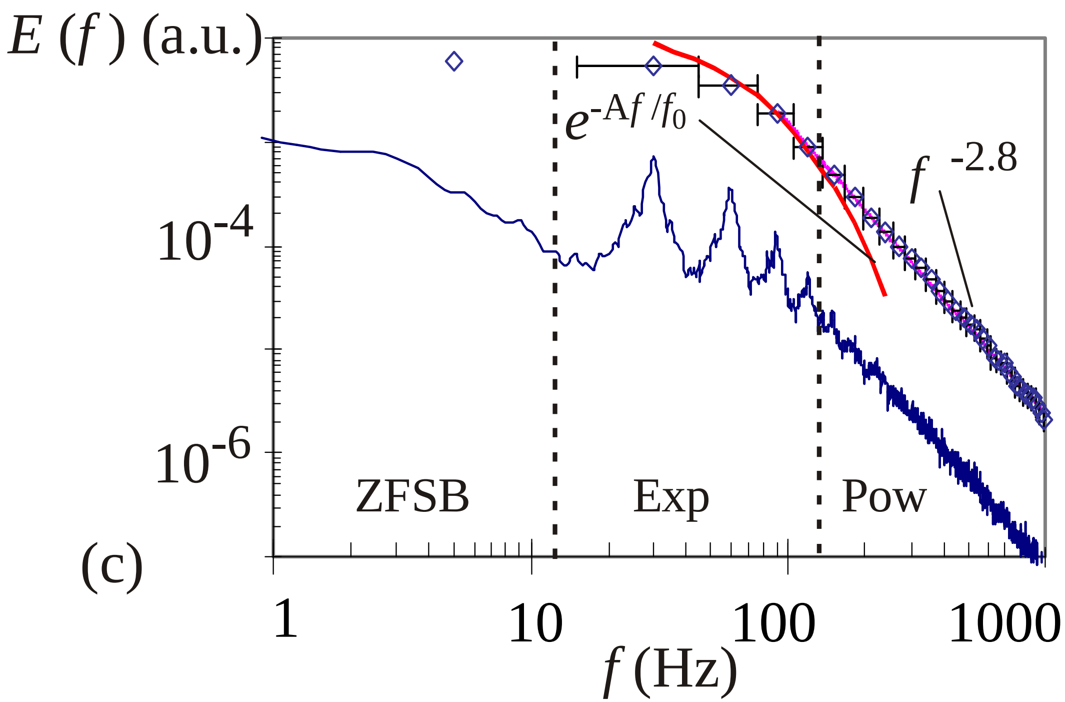
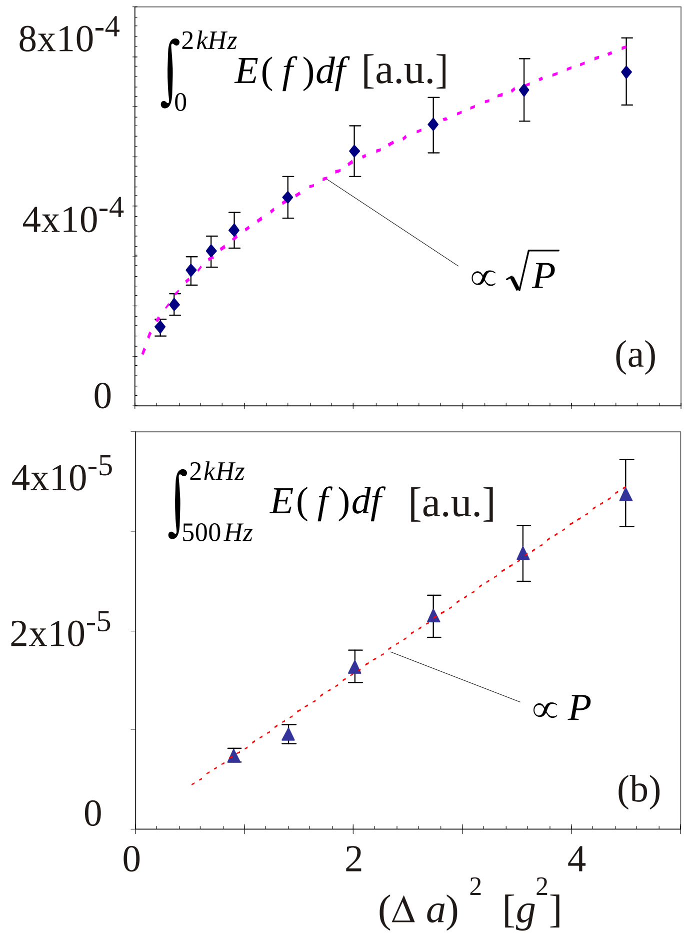


Figure	$\Delta a(g)$	E_{ZF}/E_T	E_{Exp}/E_T	E_{Pow}/E_T
2(a)	0.5	0.18	0.82	0.0
2(b)	1.4	0.48	0.51	0.01
2(c)	2.1	0.31	0.46	0.23

Comparison with WTT

$$E_\omega \sim P^{1/2}\rho^{-2/3}\alpha^{1/6}\omega^{-17/6}$$





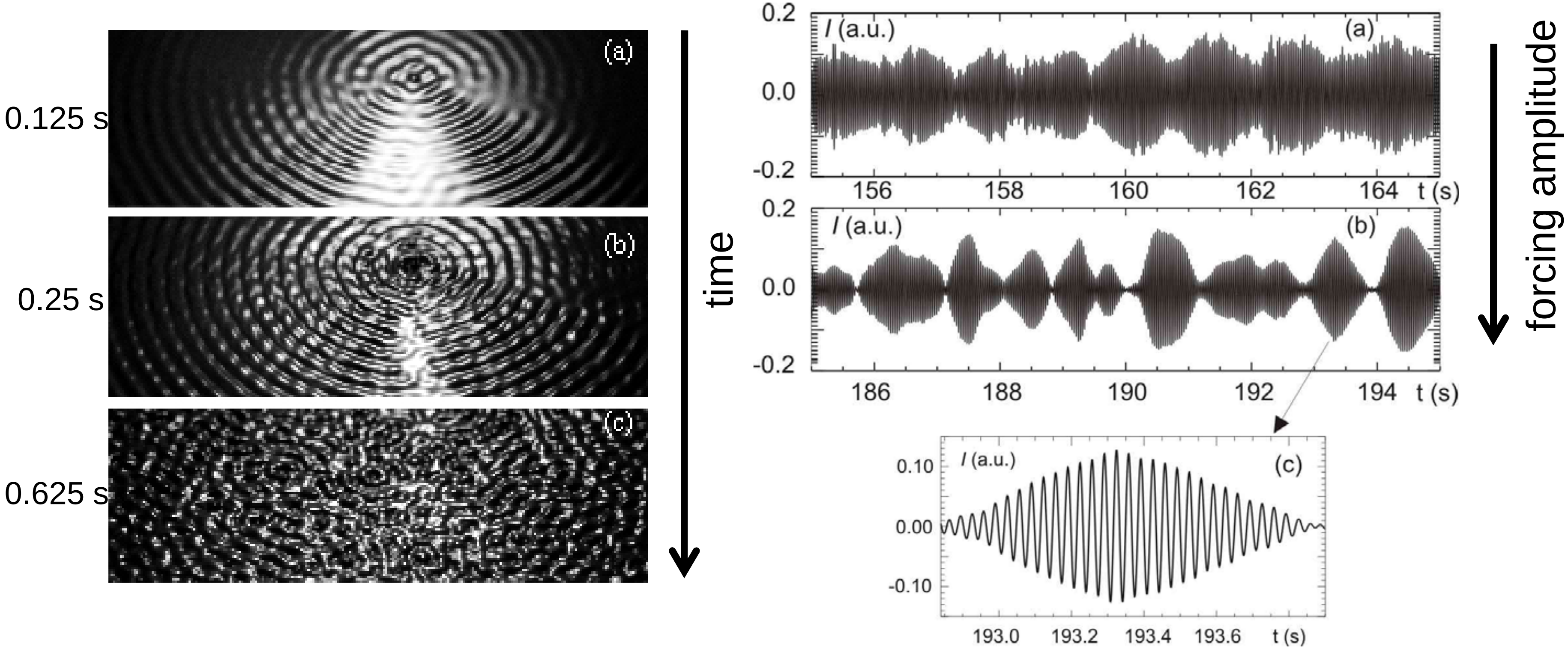
Transition to turbulence

Which mechanism(s)

- Broaden wave spectra
- Generate spectral continuum to allow 3-wave interactions?
- Randomize wave phases?
- Detach wave field from the boundary (infinite domain)?

Development of modulation instability

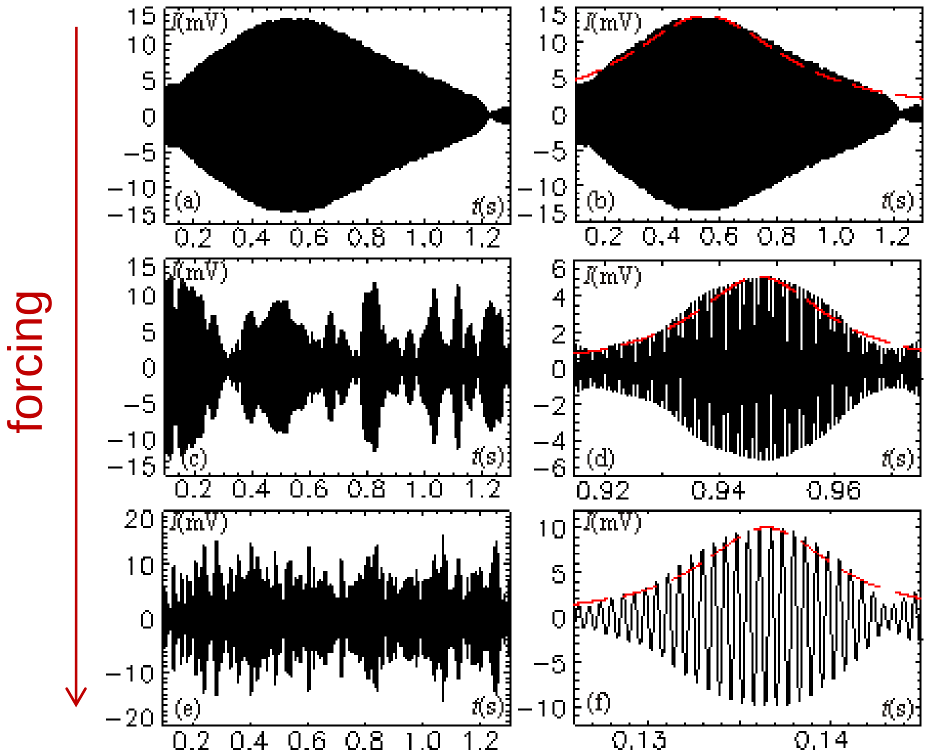
At modest damping (distilled water) parametrically excited waves are unstable to small perturbations of the wave amplitude.



Growing amplitude modulation is signature of **modulation instability**

Development of modulation instability

movie



Punzmann, Shats, Xia, Phys. Rev. Lett. (2009)

Xia, Shats, Punzmann, EPL (2010)

$$s(t) \sim \mathrm{sech}(bt)$$

Modulation instability

Lighthill criterion of the instability: $$\frac{\partial \omega}{\partial |a|^2}\frac{\partial^2 \omega_k}{\partial k^2} < 0$$

Capillary waves	$\omega_k = \sqrt{\frac{\sigma}{\rho}k^3}$	$\omega = \omega_k\left[1 - \frac{(ka)^2}{16}\right]$	$\frac{\partial \omega}{\partial \|a\|^2} < 0$	$\frac{\partial^2 \omega_k}{\partial k^2} > 0$
Gravity waves Benjamin-Feir inst	$\omega_k = \sqrt{gk}$	$\omega = \omega_k\left[1 + \frac{1}{2}(ka)^2\right]$	$\frac{\partial \omega}{\partial \|a\|^2} > 0$	$\frac{\partial^2 \omega_k}{\partial k^2} < 0$

Unstable

Modulation instability and envelope solitons

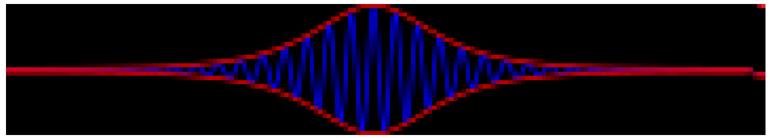
Modulationally unstable waves described by nonlinear Schrodinger equation

$$i\frac{\partial a}{\partial t}+\frac{1}{2}\frac{\partial^2\omega_0}{\partial k^2}\frac{\partial^2 a}{\partial x^2}-\gamma\left|a^2\right|a<0 \qquad \gamma=\left(\frac{\partial\omega}{\partial|a|^2}\right)_{a^2=0}$$

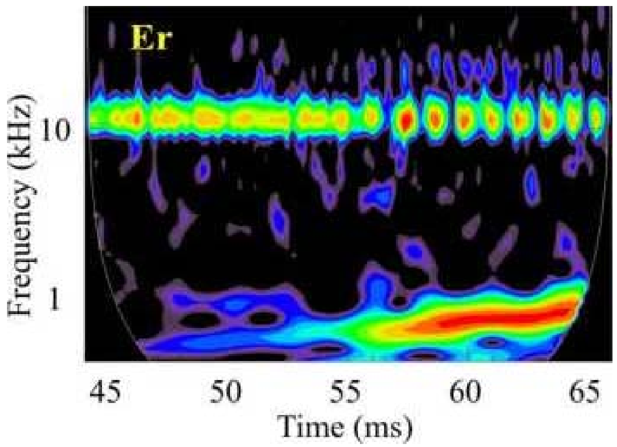
One exact solution of the NLS is the hyperbolic secant envelope soliton

Zakharov–Shabat, 1970

$$s(t)=\mathrm{sech}\left(\frac{\pi^2}{b}t\right)e^{if_n t}$$



Modulation instability found in plasma (Langmuir waves, drift waves), nonlinear optics (optical fibers), gravity surface waves (Benjamin-Feir instability)



Modulation instability in magnetically confined plasma:

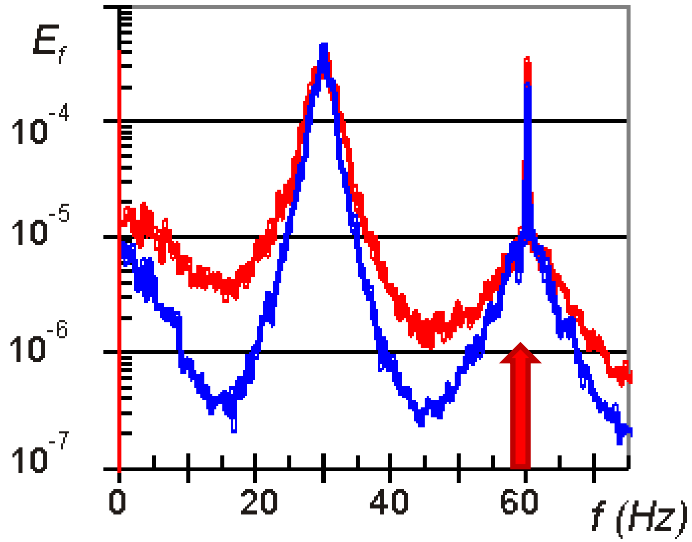
The onset of the low-frequency zonal flow coincides with the strong amplitude modulation of the parent wave.
Morlet wavelet analysis

M. Shats and W. Solomon, New J. Phys. (2002)

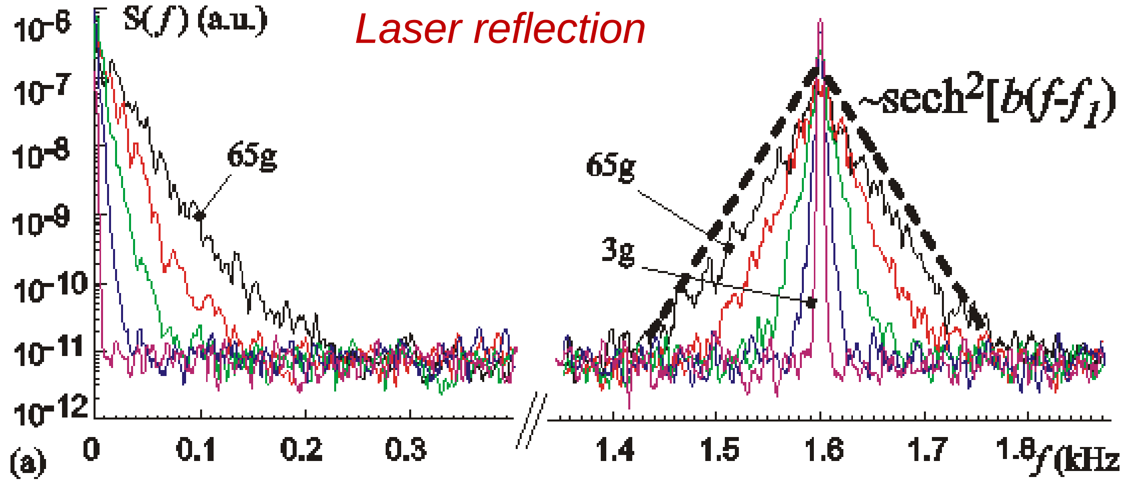
Nonlinear spectral broadening

Capillary waves

Video imaging



Punzmann, Shats, Xia, Phys. Rev. Lett. (2009)



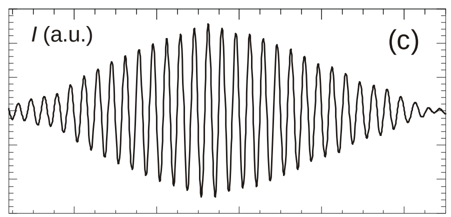
"Triangular shape" in lin-log spectra described by

$$F(f) = \mathrm{sech}\left[b(f - f_n)\right]$$

inverse

Fourier transform

$$s(t) = \left(\frac{\pi}{b}\right)\mathrm{sech}\left(\frac{\pi^2}{b}t\right)e^{if_n t}$$



Nonlinear spectral broadening

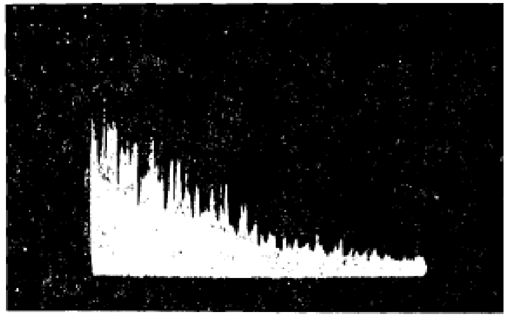


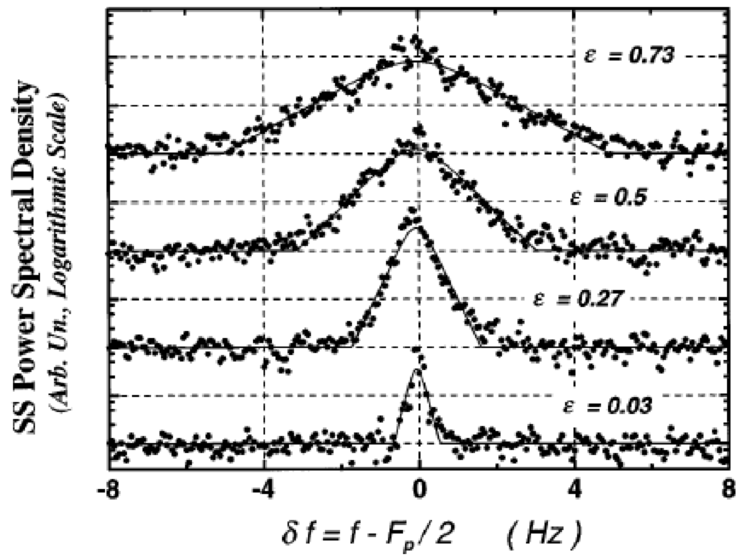
FIG. 3. Photograph of one random realization of the frequency dependence of the spectral density $I(\Omega)$. The orizontal sweep is linear with a maximum value 20 kHz.

Spin waves in magnetics

Krutsenko, L'vov, Melkov, Sov. Phys. JETP 48, 561 (1978)].

Second sound waves in superfluid He4

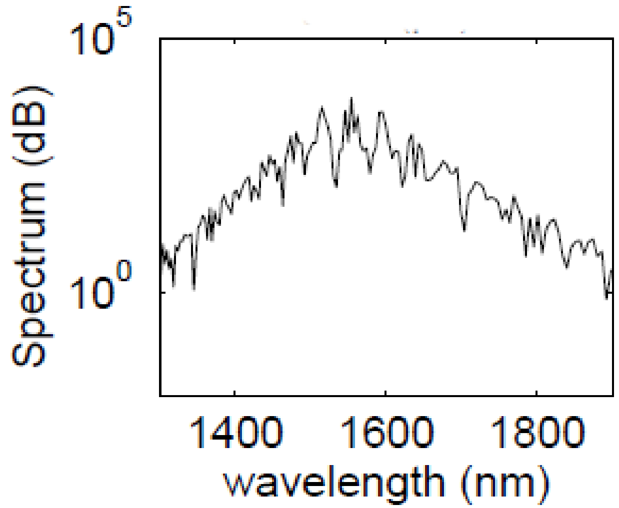
Rinberg, Cherepanov, Steinberg, Phys. Rev. Lett. (1997)



Nonlinear spectral broadening

Supercontinuum generation from CW pumping

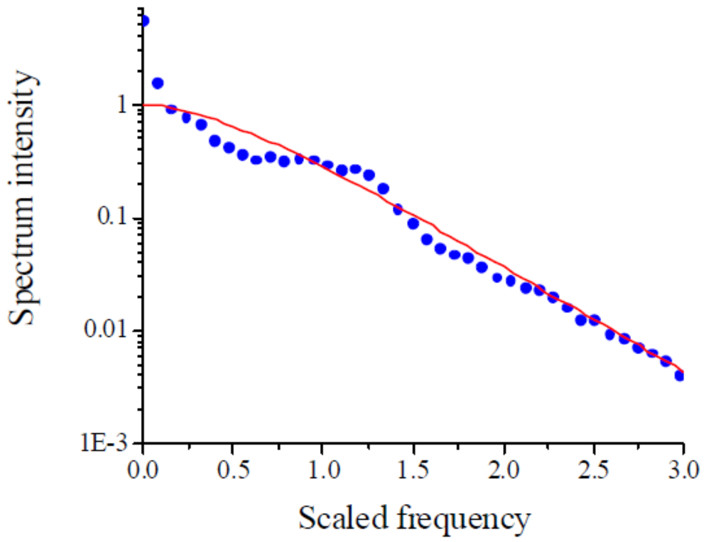
Kutz et al., Opt. Express 16, 3989 (2005)



Supercontinuum in fibers

N. Korneev et al., Opt. Express 16, 2636 (2008)

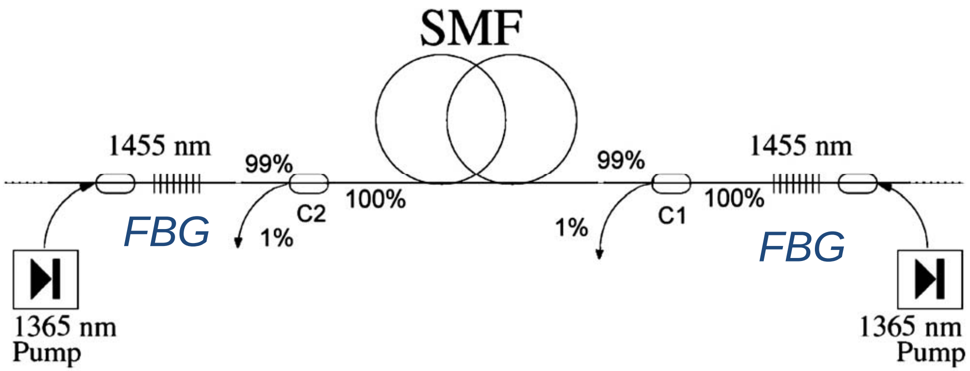
The spectrum of a sum of solitons according to Zakharov-Shabat statistics of soliton sea (red line), and the result of numerical simulation for breakup of a long pulse with initially small white noise (blue circles). Approximately 18 solitons are formed. It is seen, that the high-frequency tail is well described by a spectrum resulting from the superposition of independent solitons.



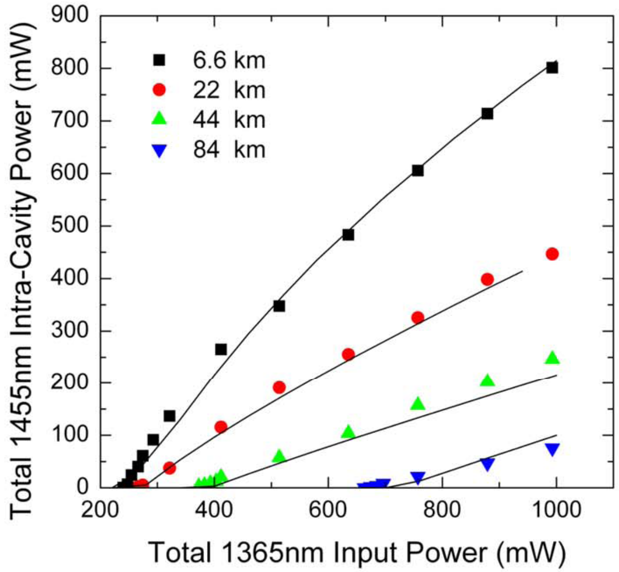
Nonlinear spectral broadening

Raman fiber lasers (RFLs)

Babin et al., PHYSICAL REVIEW A 77, 033803 (2008)

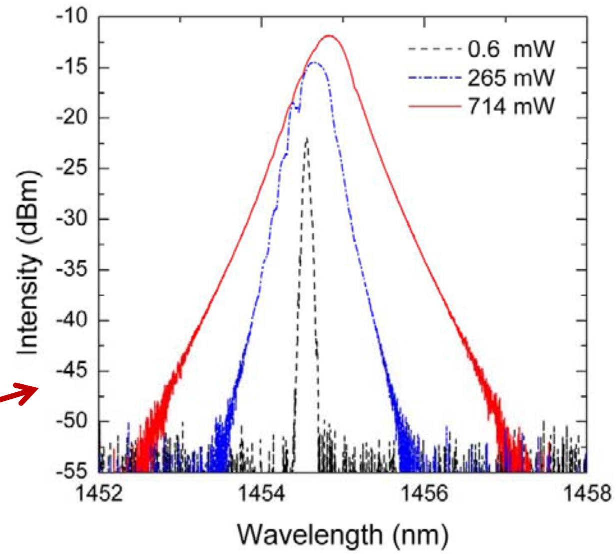


Stokes (red shifted) power



Pumping wavelength = 1365 nm;
Stokes wavelength = 1455 nm
Spectrum measured at point *C2;*
Stokes power 0.6, 265, and 714 mW for *L=6.6 km.*
SMH = single mode fiber
FBG = fiber Bragg gratings

Stokes line spectrum



Rogue waves in the ocean and elsewhere

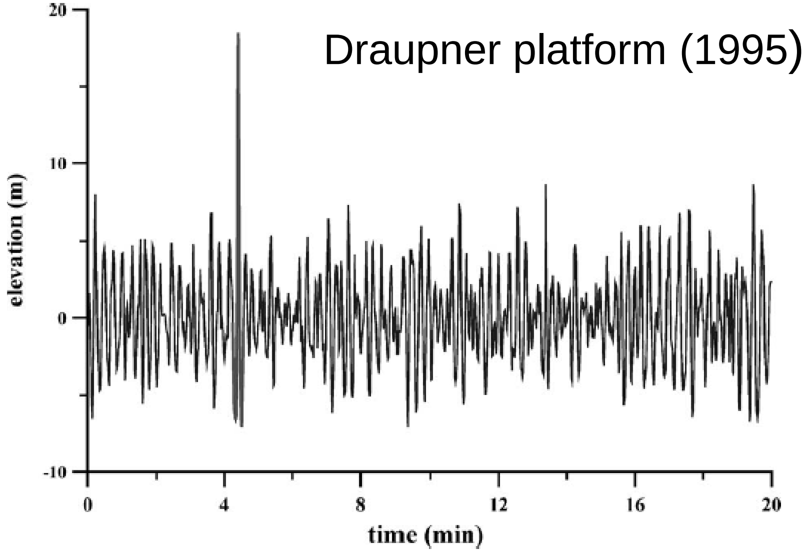


Fig. 3. Time record of the "New Year wave" in the North Sea.

nature 2007

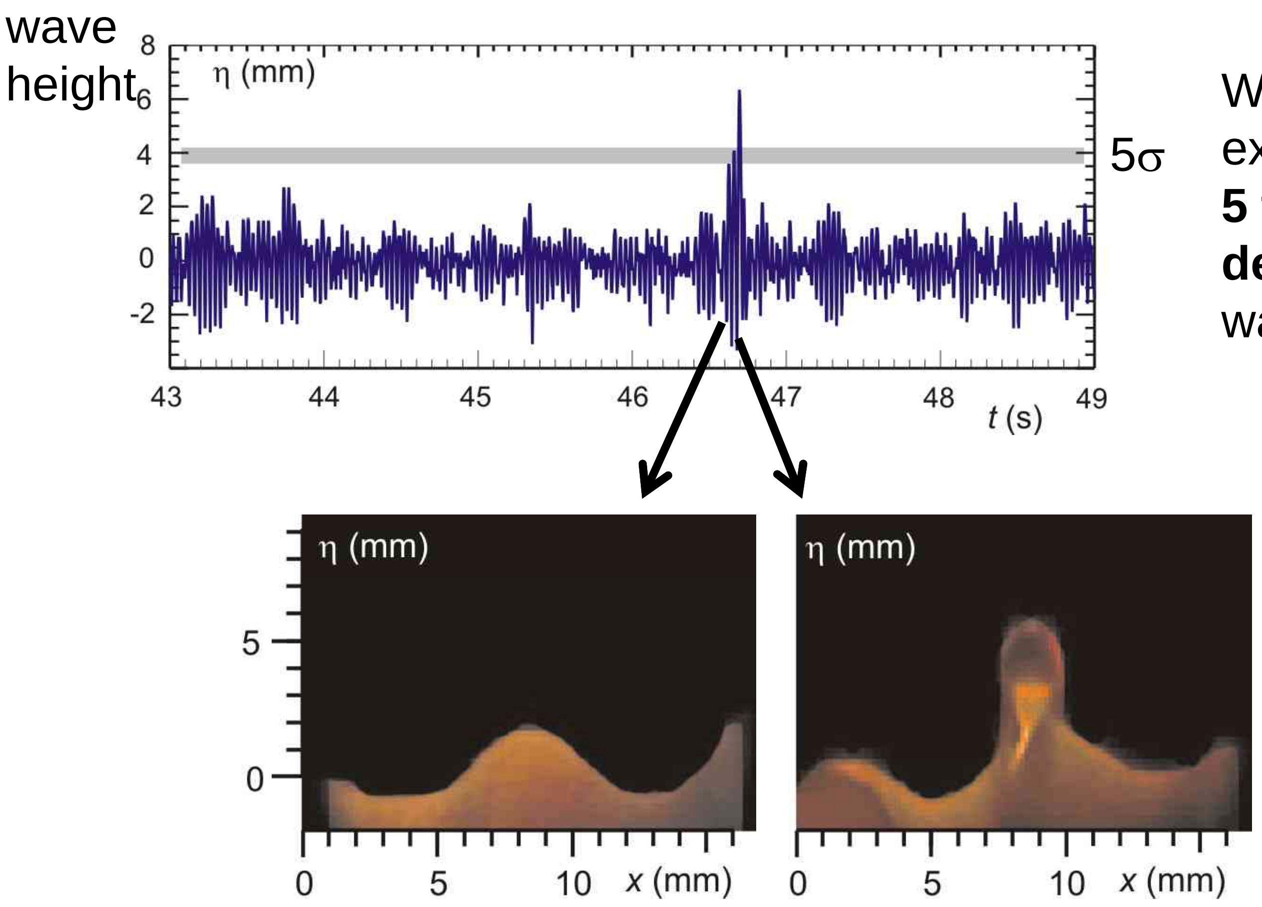
LETTERS

Optical rogue waves

D. R. Solli[1], C. Ropers[1,2], P. Koonath[1] & B. Jalali[1]

A Rogue wave is characterized by steep **wave front** ("a wall of water")

Capillary rogue waves



Wave crest height of this extreme event exceeds **5 times standard deviation** of background wave field.

Rogue wave is characterized by almost **vertical wave front**.

Shats, Punzmann, Xia, Phys. Rev. Lett. (2010)

Droplet generation by rogue waves

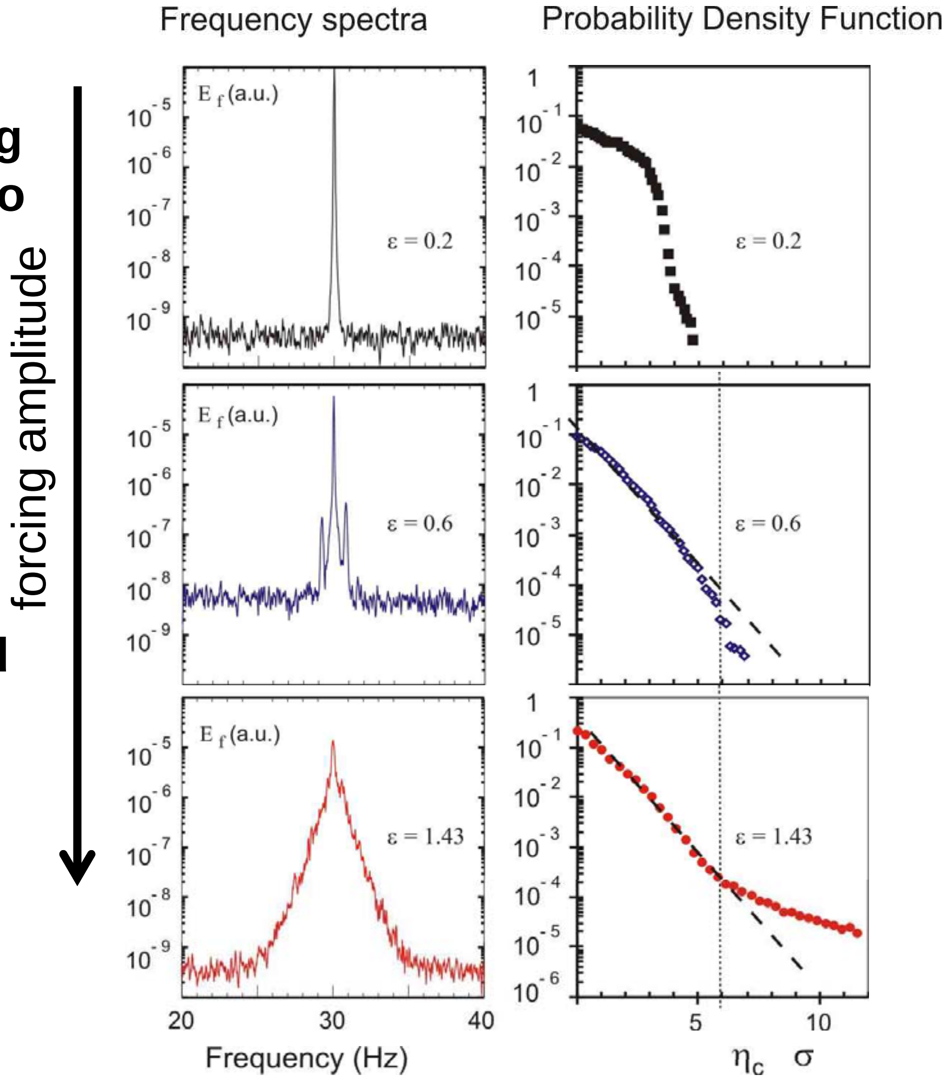
Wave visualization

Droplet formation

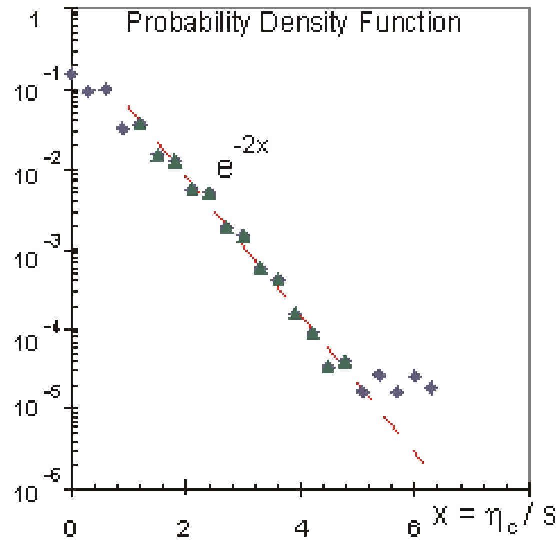
Probability of capillary rogue waves

increased forcing amplitude **leads to** increase in amplitude modulation and **spectral broadening** with exponential tails above a **critical threshold** ε **>~ 1.2**

Video imaging



Laser transmission



Rogue wave probability

$\eta_c/\sigma > 5$ is **1-2 orders of magnitude higher than expected** from the exponential trend of the wave background

Evidence of 4-wave interactions

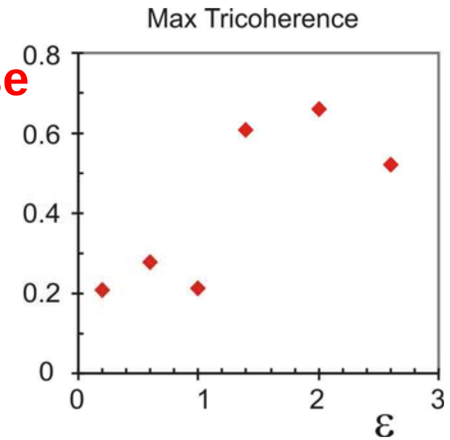
The degree of the four-wave coupling in the surface wave spectrum can be characterized by the tricoherence (= normalized trispectrum)

$$t^2(\omega_1, \omega_2, \omega_3) = \frac{|\langle F_1 F_2 F_3 F^*_{1+2-3}\rangle|^2}{\langle |F_1 F_2 F_3|^2\rangle\langle |F_{1+2-3}|^2\rangle},$$

coherent phase

$$0 < t^2 < 1$$

random phase

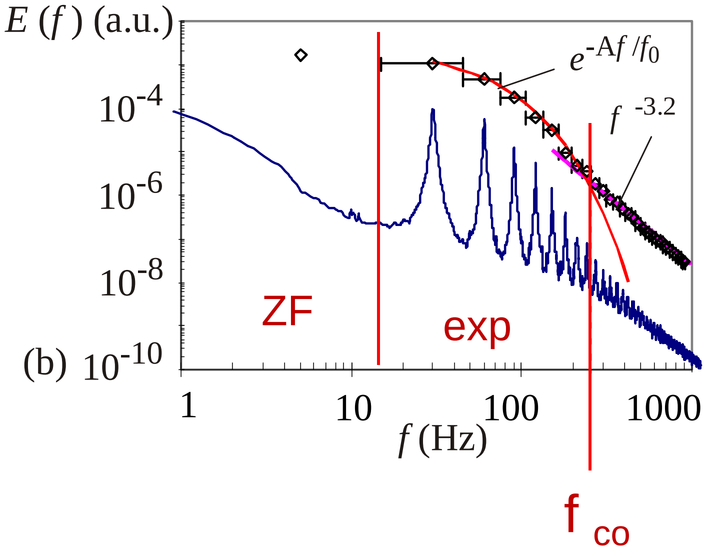


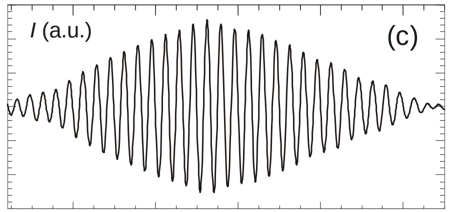
evidence of strong four-wave interactions

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$
$$k_1 + k_2 = k_3 + k_4$$

four-wave interaction process is
key ingredient of **modulation instability**

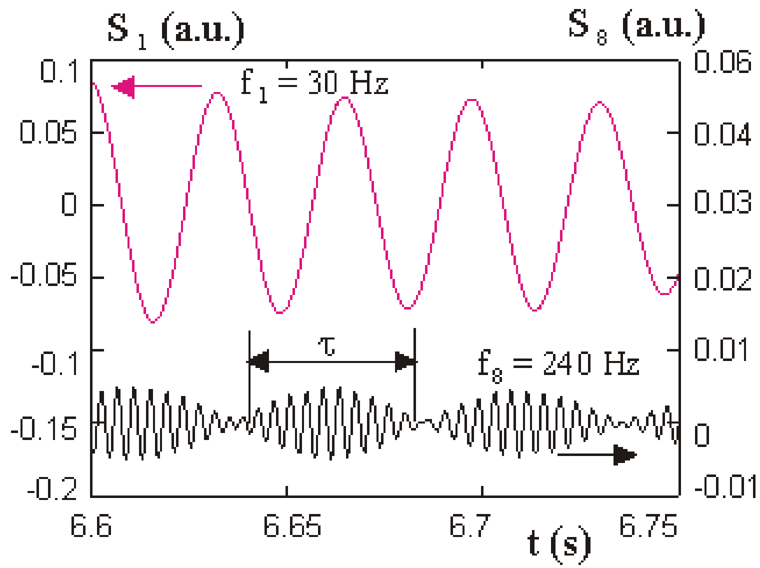
Random phases in 3-wave interactions





$$T_1 = 1/f_1 \quad \longleftrightarrow \quad \tau_n = b_n / \pi^2$$

compare

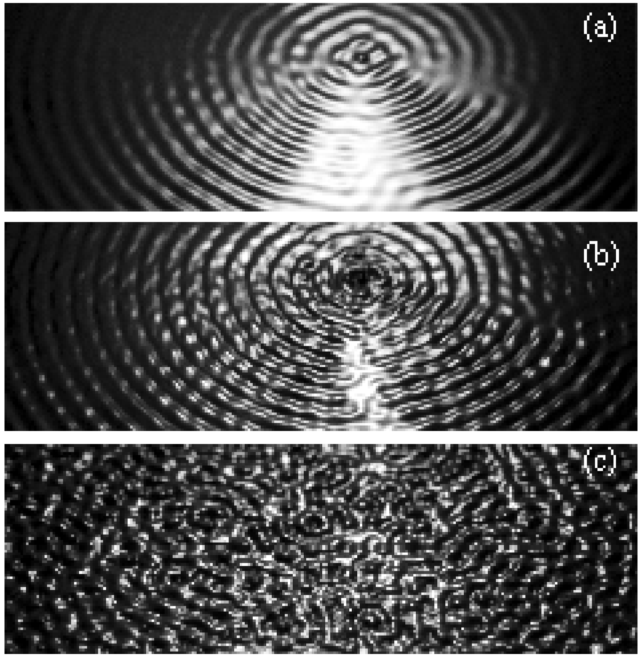


Transition to turbulence at f > f $_{co}$
coincides with phase randomization

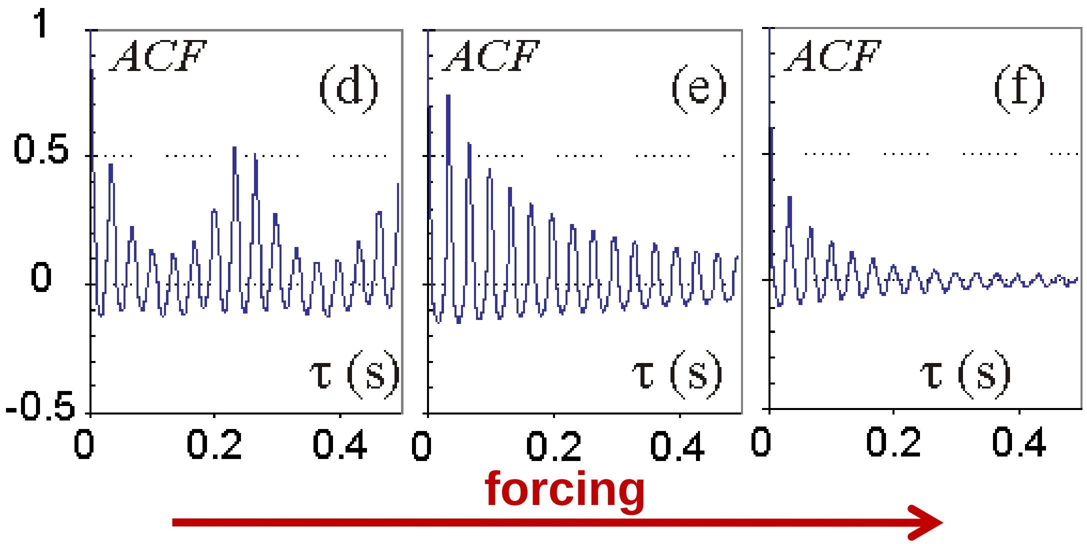
$$T_1 \tau < 1$$

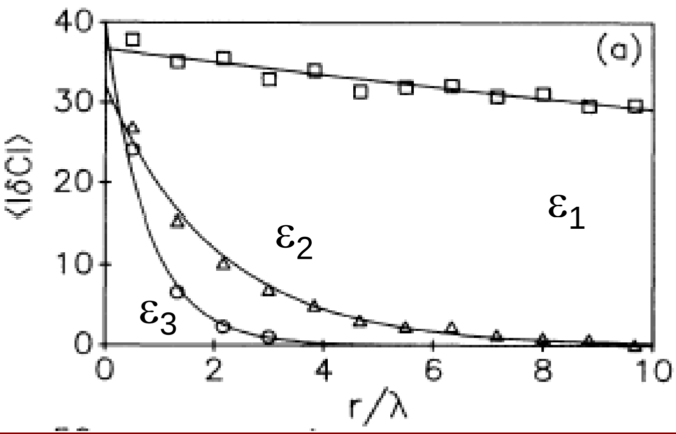
Finite container size effects

0.125s
0.250s
0.625s



Autocorrelation function





Order-disorder transition

Tufillaro, Ramshankar, Gollub
Phys. Rev. Lett. 1989

Conclusions

- Modulation instability found in capillary waves
- MI is responsible for breaking continuous waves into envelope solitons
- MI leads to spectral broadening of wave harmonics - *sech*-spectra formation
- MI development correlated with increased probability of capillary rogue waves
- MI is responsible for order-disorder transition, detachment of wave field from container
- MI responsible for phase randomization in 3-wave interactions
- MI provides conditions for transition to turbulence in parametrically-driven waves