Elastic turbulence: Overview

Victor Steinberg
Department of Physics of Complex Systems,
The Weizmann Institute of Science, Israel

Two effects of polymers on flow

Turbulent drag reduction-
modification of ***turbulent*** flow at Re>>1 by adding polymers

?

Elastic turbulence-
Elastic instabilities and transition from ***laminar to random*** flow by adding polymers

Both effects result from interaction of polymers with flow, causing polymer stretching, and back reaction of polymers on the flow

Both flows are characterized by two fields: velocity and elastic stresses, similar to magneto-hydrodynamics, where velocity and magnetic fields are necessary to know in order to understand the flow

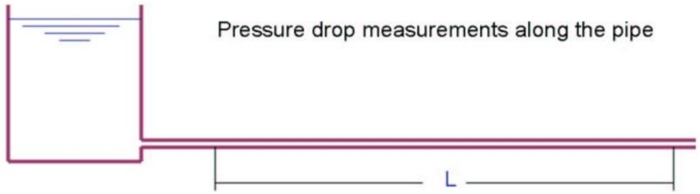
Two ways to investigate these phenomena:

- ***macroscopic approach to study hydrodynamics and stresses***
- ***microscopic approach to study dynamics and statistics of polymer molecules***

Macroscopic approach in turbulent drag reduction problem

Turbulent Drag Reduction by
Dilute Addition of Polymers

(Toms 1948)

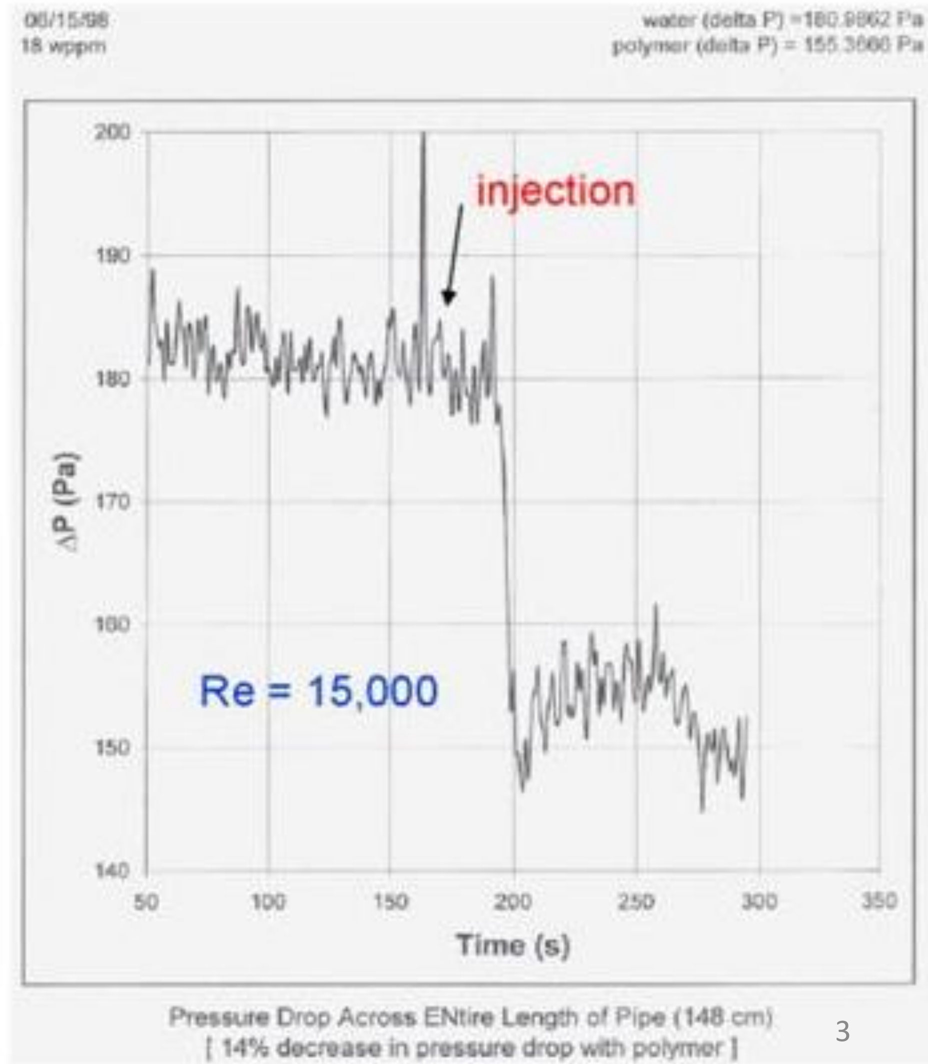


$\Delta P_s / \Delta P_w = O(1/5)$

for polymer concentrations of the order of
a few tens of parts per million by weight

Turbulent drag reduction **is characterized by the friction factor that can be reduced up to 5-6 times by adding high molecular weight polymer molecules (usually M=10^7 and concentration of 10-100 ppm)**

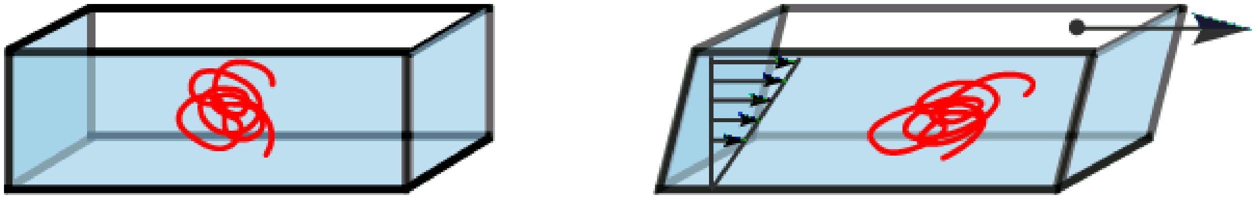
(i) No agreement that only small scales determines polymer dynamics @ stretching
(ii) Coil-stretch transition
(iii) No theory of turbulent drag reduction with elastic stresses and their distribution in the flow



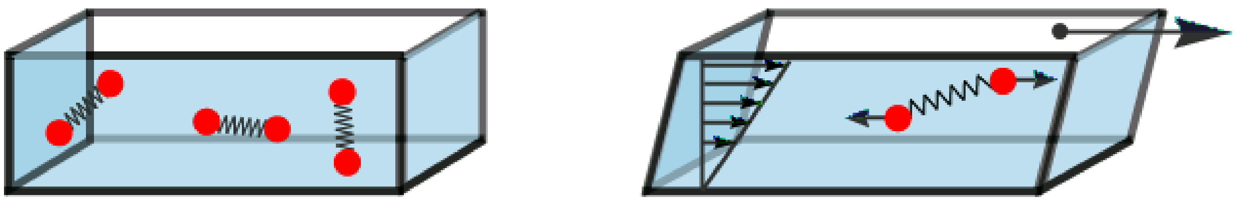
Elastic turbulence: what is it?

Dynamics and stretching of polymers in various flows and elastic stresses

Polymer molecules in a shear flow

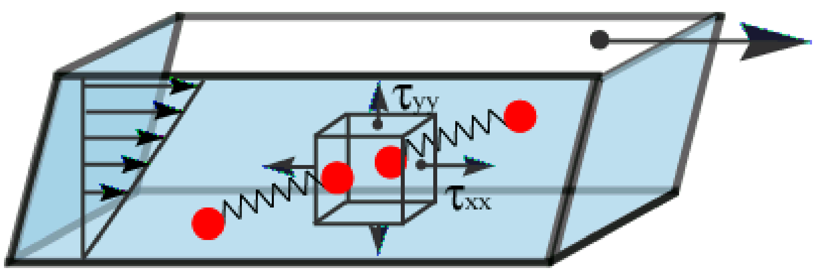


Elastic dumbbell model:



$$\lambda \propto \eta R/k$$ -polymer relaxation time

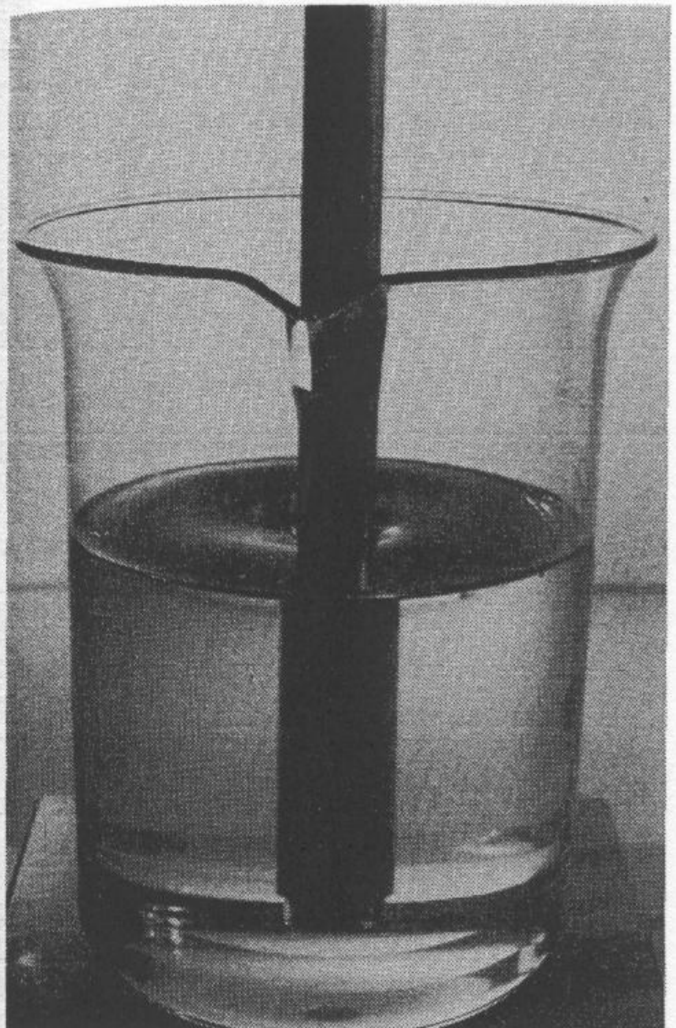
η-viscosity, R-molecule size; k~1/R^2-spring constant.

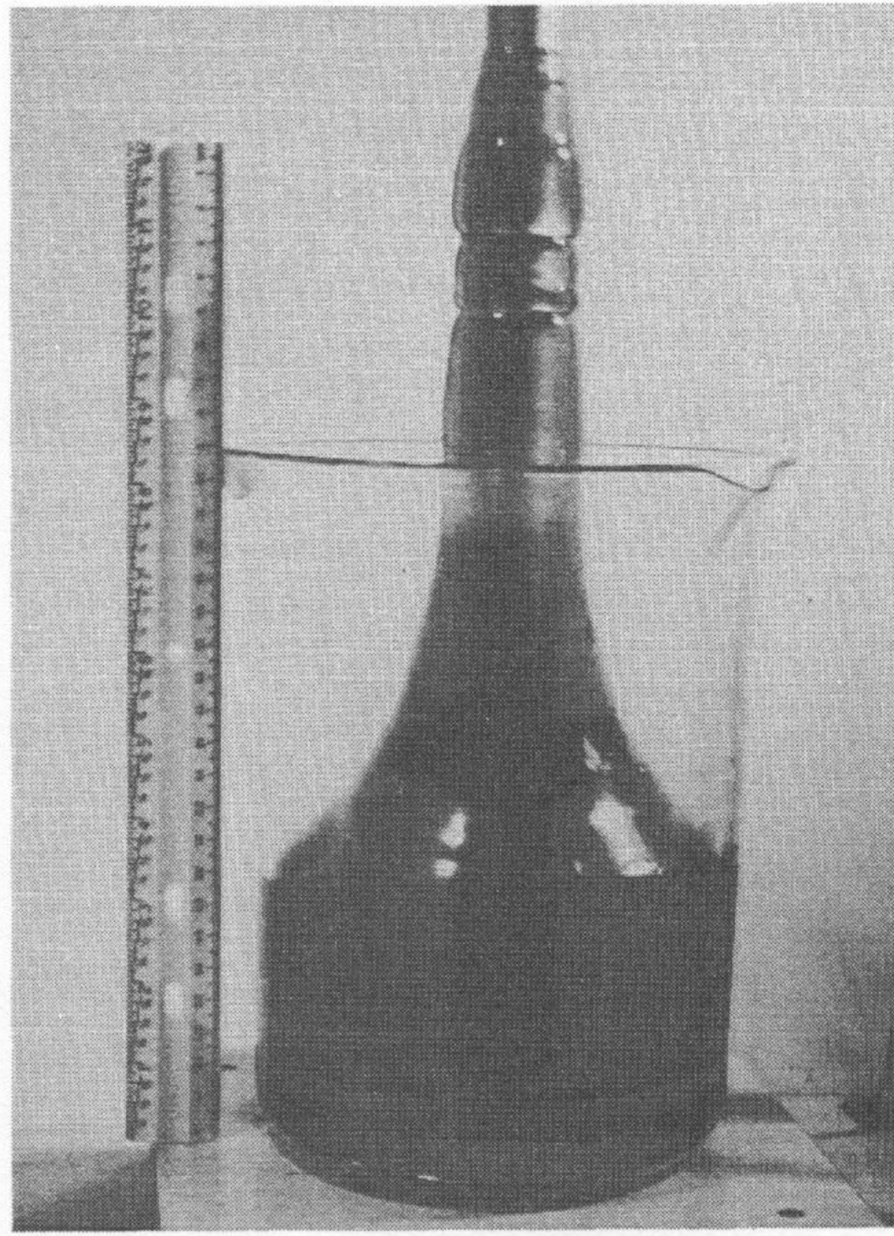


First normal stress difference:

$$N_1 = \sigma_{xx} - \sigma_{yy} \propto (\lambda\dot{\gamma})^2, \ \dot{\gamma}$$ - shear rate.

One of the striking effects and relevant to flow instability discussed further:





Ⓝ Rod climbing, or Weissenberg effect Ⓟ

Hydrodynamics of polymer solutions

(or more quantitative way to understand relation between two phenomena)

Equation of motion:

$$\frac{\partial \vec{\mathbf{V}}}{\partial \mathbf{t}} + (\vec{\mathbf{V}}\ \vec{\nabla})\vec{\mathbf{V}} = -\vec{\nabla}\mathbf{p}/\boldsymbol{\rho} + \nu_s \Delta \vec{V} - \underbrace{\vec{\nabla}\tilde{\sigma}_p/\boldsymbol{\rho}}_{\text{linear}}$$

Constitutive equation for Oldroyd-B model:

$$\tilde{\sigma}_{\mathbf{p}} + \underbrace{\boldsymbol{\lambda}\frac{\mathbf{D}\tilde{\sigma}_{\mathbf{p}}}{\mathbf{Dt}}}_{\text{linear relaxation}} = -\boldsymbol{\eta}_{\mathbf{p}}\left[\vec{\nabla}\vec{\mathbf{V}} + (\vec{\nabla}\vec{\mathbf{V}})^{\mathbf{T}}\right]$$

and convective derivative:

$$\frac{\boldsymbol{D}\tilde{\sigma}_p}{\boldsymbol{Dt}} \equiv \frac{\partial\tilde{\sigma}_p}{\partial \boldsymbol{t}} + \underbrace{(\vec{\boldsymbol{V}}\ \vec{\nabla})\tilde{\sigma}_p - \tilde{\sigma}_p(\vec{\nabla}\vec{\boldsymbol{V}}) - (\vec{\nabla}\vec{\boldsymbol{V}})^T\tilde{\sigma}_p}_{\text{elastic nonlinearity}}$$

$$\mathrm{Wi} = V\lambda/L = \frac{\text{nonlinearity}}{\text{relaxation}}$$ (or $Wi = \lambda\dot{\gamma}$ in case of simple shear)

Weissenberg number

Elastic stress is the main source of nonlinearity at large Wi and low Re.

$$\mathrm{Re} = VL/\nu = \frac{\text{nonlinearity}}{\text{dissipation}}$$

Reynolds number

Three limiting cases:

$$1.\ \mathrm{Re} \gg 1;\ Wi \Rightarrow 0$$

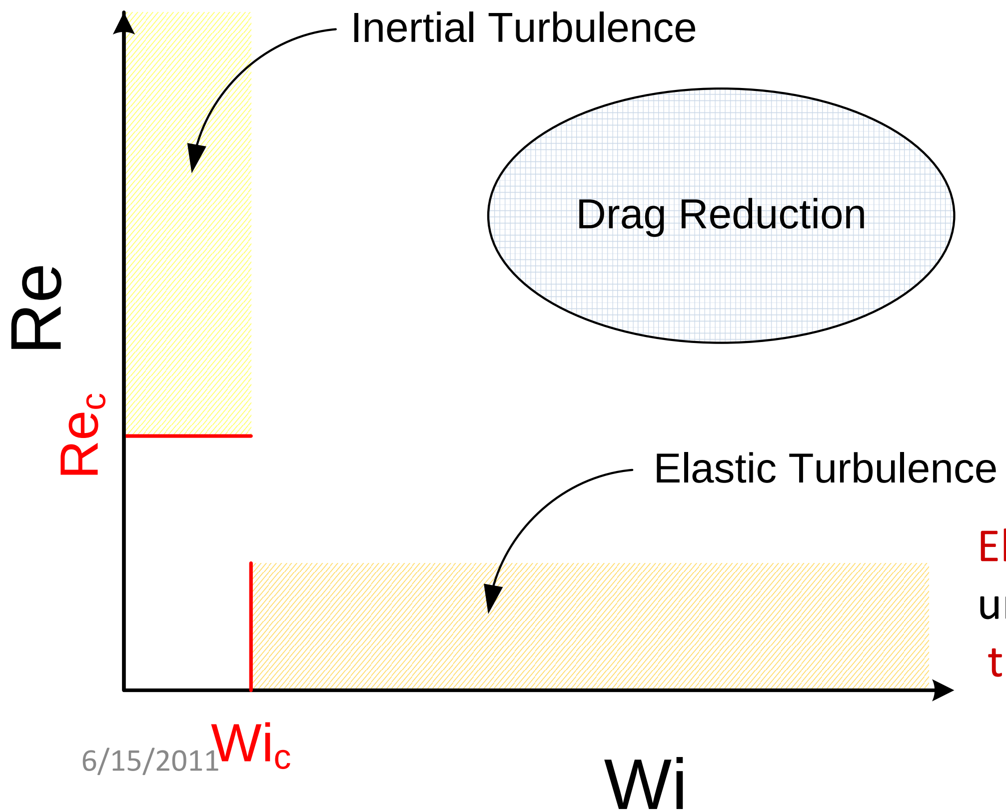
Hydrodynamic turbulence

$$2.\ Wi \gg 1;\ \mathrm{Re} \Rightarrow 0$$

Elastic turbulence

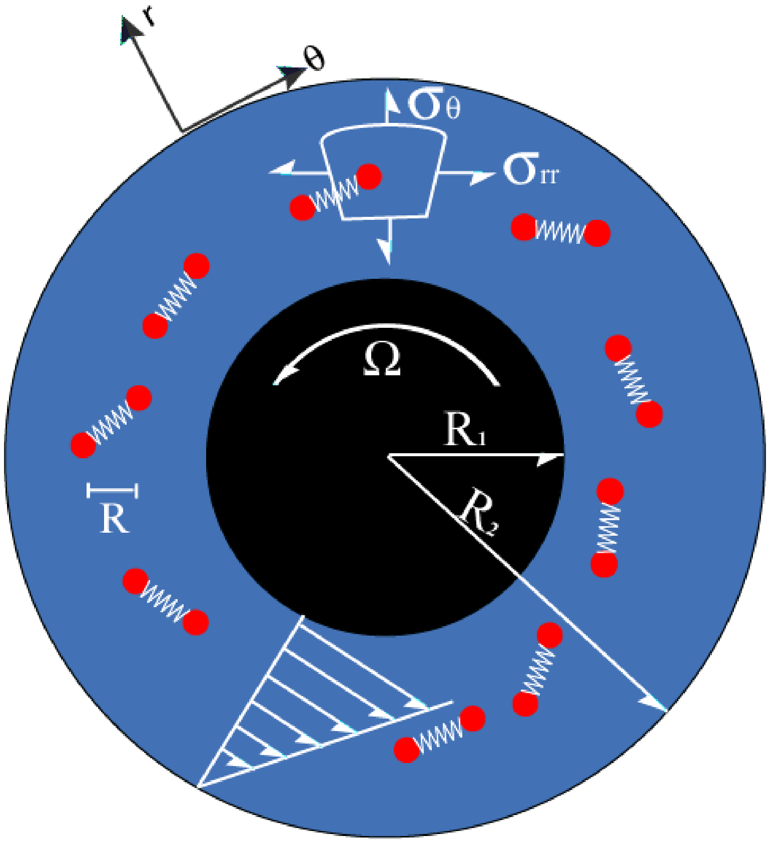
$$3.\ \mathrm{Re} \gg 1;\ Wi \gg 1$$

Turbulent drag reduction



Elastic turbulence is a model system to understand more complicated phenomenon- turbulent drag reduction

Back to: Rod Climbing (Weissenberg) Effect.



Volume force: $\frac{N_1}{r}$

(hoop stress).

$$N_1 = \sigma_{\theta\theta} - \sigma_{rr} \propto \langle R^2 \rangle Wi^2$$

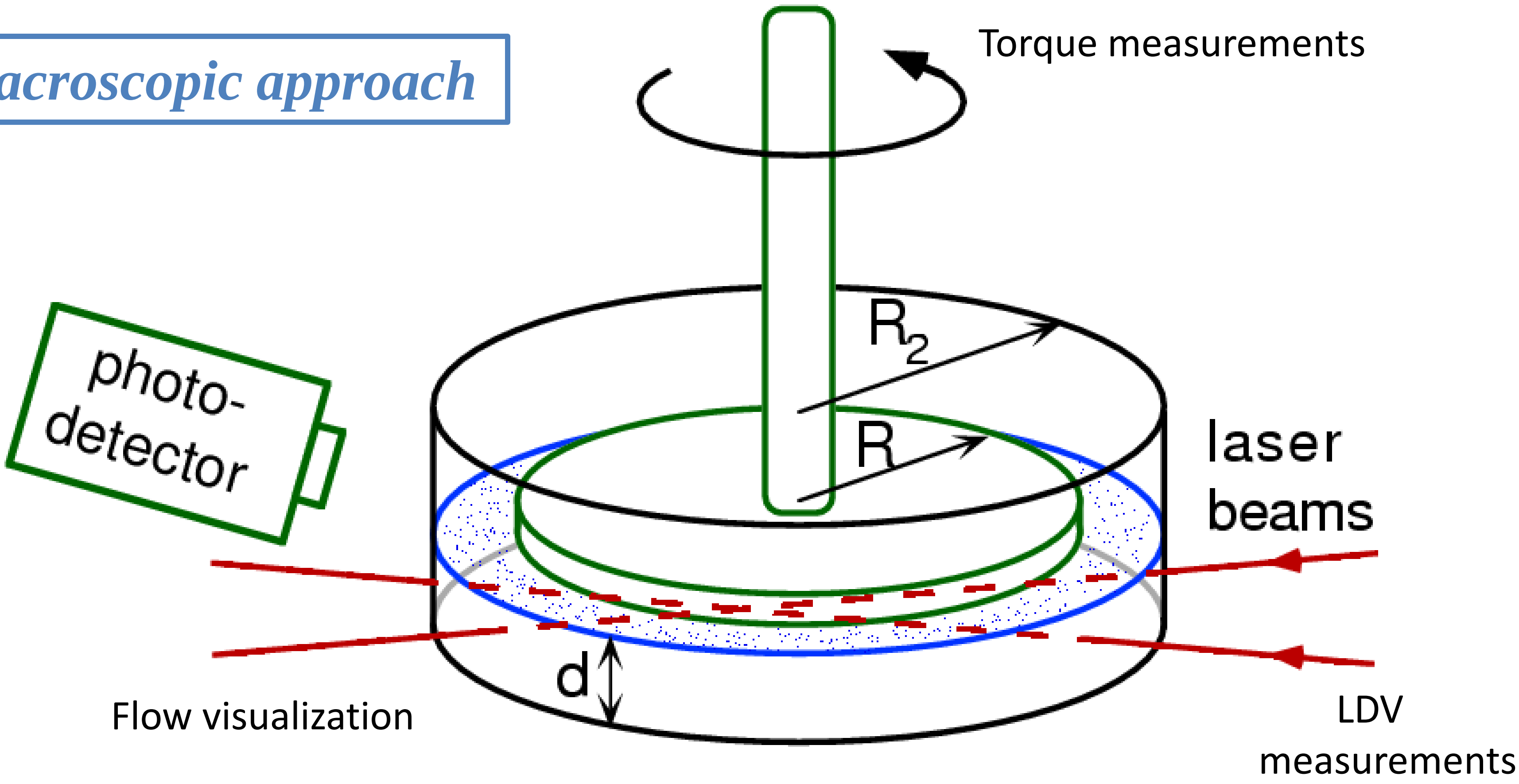
$$Wi = \lambda \dot{\gamma}, d = R_2 - R_1$$

$$\dot{\gamma} = \Omega \frac{R_1}{d}$$ **-shear rate.**

Elastic turbulence

A. Groisman & V.Steinberg, ***Nature* 405**, 53 (2000)

Macroscopic approach



- **Solution:** 80 ppm PAAm, 65% saccharose and 1% NaCl,

$$\eta_s = 0.324\ Pas\ , M = 18 \cdot 10^6\ da ; \eta = 0.424\ Pas \quad \text{at} \quad \dot{\gamma} = 1 s^{-1}$$

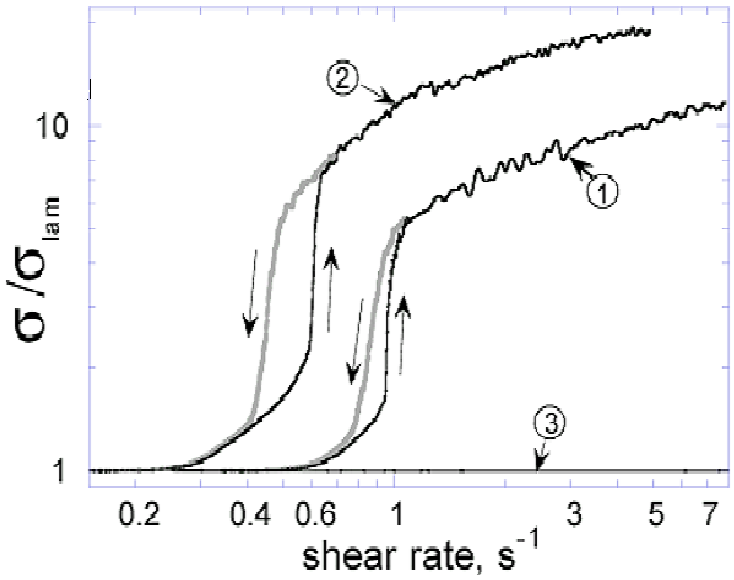
relaxation time λ=3.4 sec at 12C (temperature at which the experiment run)

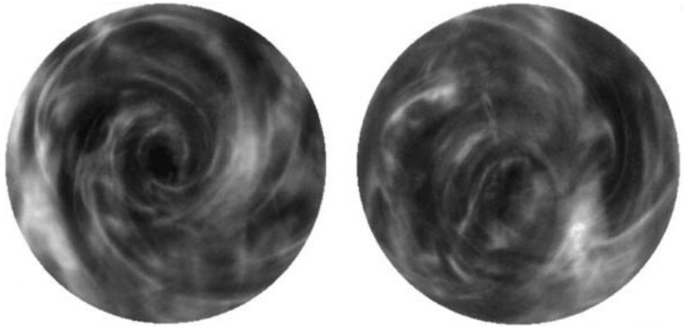
Elastic Turbulence: Experiment

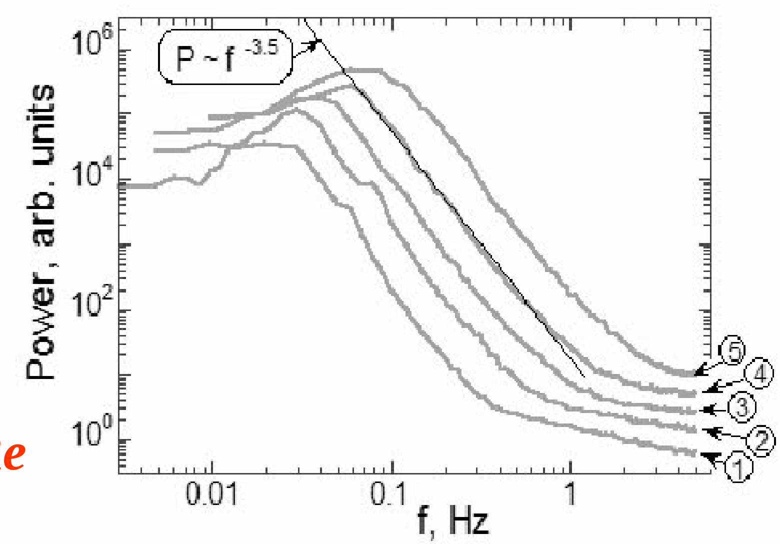
A. Groisman & V. Steinberg, ***Nature* 405**, 53 (2000)

Swirling flow between two disks

- Sharp growth of the flow resistance
- Power-law decay of power spectra with $f^{-\alpha}$ and $\alpha \approx 3.5$
- Efficient mixing
- As the result: Elastic turbulence **is** ***random in time, spatially smooth flow (its velocity power spectrum decays faster than*** k^{-3} ***)***



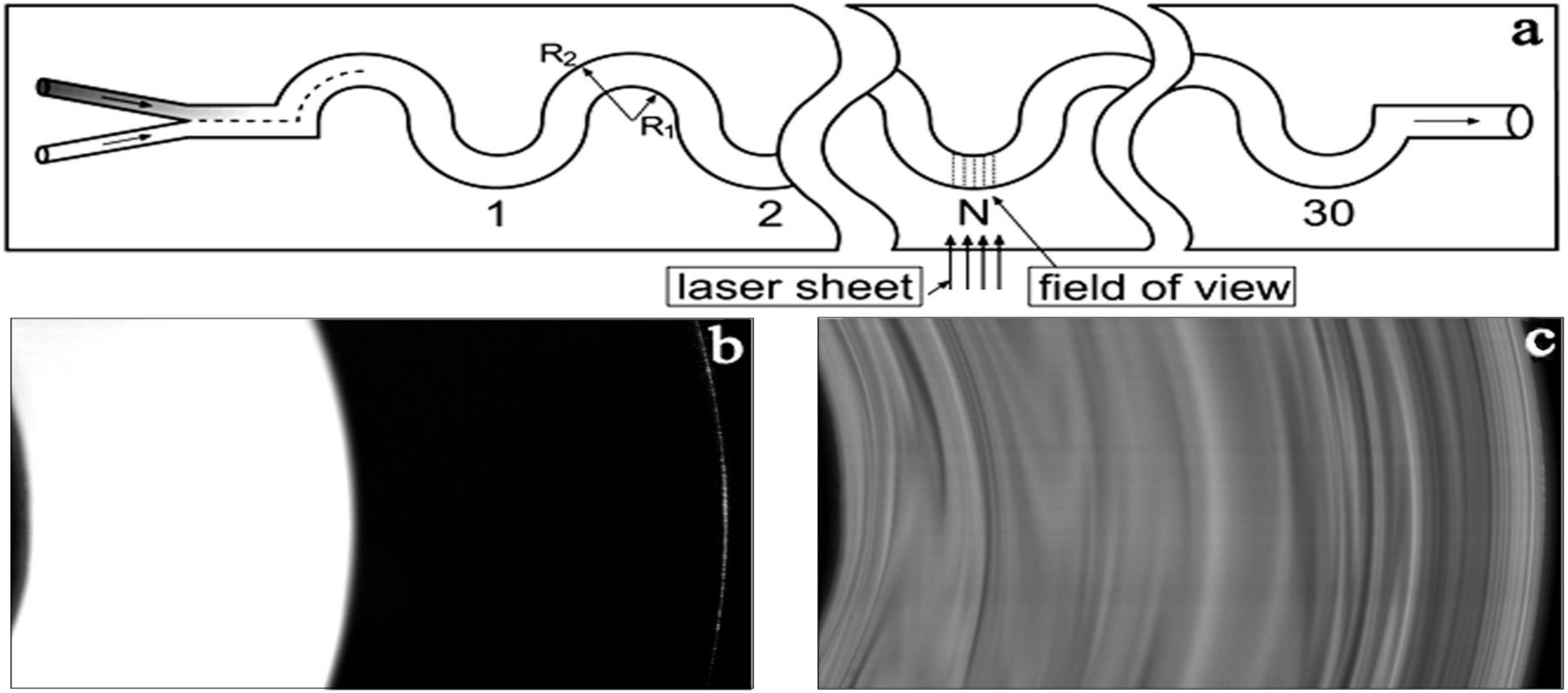




<u>Thus</u>, <u>elastic turbulence</u> is a dynamic state solely driven by the <u>nonlinear elastic stresses</u> at small Re (being independent of it) and at $Wi > Wi_c$

Mixing in a curvilinear channel

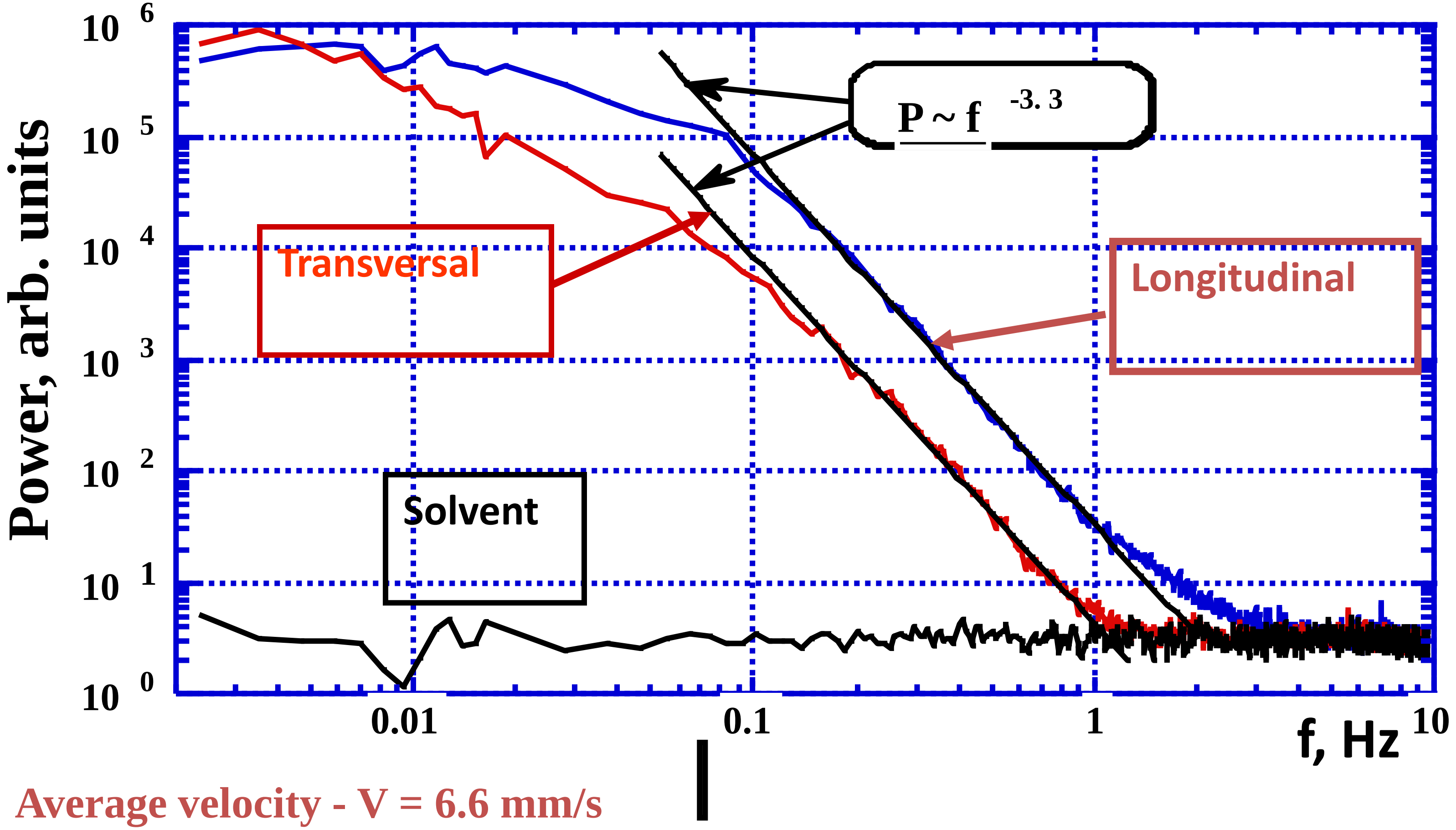
A. Groisman & V.Steinberg, ***Nature* 410**, 905 (2001)



without polymer

with polymer

Spectra of velocity fluctuations, measured in the center of the channel with LDV.



Average velocity - V = 6.6 mm/s

Taylor hypothesis: velocity fluctuations in time are due to fluctuations in space $\Rightarrow$ **$P \sim k^{-3.3}$**

Microscopic approach: effect of flow on polymer conformation

Single Polymer Dynamics:
coil-stretch transition in a random flow

Theory and experiment

Theory of coil-stretch transition in a random flow

E. Balkovsky, A. Fouxon, V. Lebedev, *PRL* **84,** 4765 (2000)
M. Chertkov, *PRL* **84**, 4761 (2000)

R< dissipation length, random smooth flow

Dynamics of stretching is governed by the velocity gradient only in smooth and random in time flow

$$\vec{V}(r,t) = \frac{\partial V_i}{\partial r_j}(t) \cdot \vec{r}$$

Dynamics of stretching of a fluid element in a flow is described as

$$\langle R(t) \rangle = R(0)\exp(\gamma t)$$ where γ is the Lyapunov exponent. ($\gamma \approx \left(\frac{\partial V_i}{\partial r_j}\right)^{rms}$)

Since polymer molecule follows deformation of a fluid element, it can be stretched significantly even in a random flow, if **the velocity correlation time is longer than the relaxation time**, λ (Lumley(1970))

Statistics of polymer molecule extensions:

Coupling between statistics of polymer stretching and dynamics of stretching of a fluid element in a random flow

$R_0 << R << R_{max}$

Dynamic equation for the end-to-end vector **R**=R**n** with ***linear elasticity:***

$$\partial_t R_i = R_j \nabla_j V_i - \frac{R_i}{\lambda}$$

Stationary probability distribution function of the end-to-end length R:

$$P(R_i) \propto R_0^{\alpha} R_i^{-\alpha-1}$$ where $$\alpha \propto (\lambda^{-1} - \gamma)$$

At α>0 of right tail the majority of molecules is in coil state.

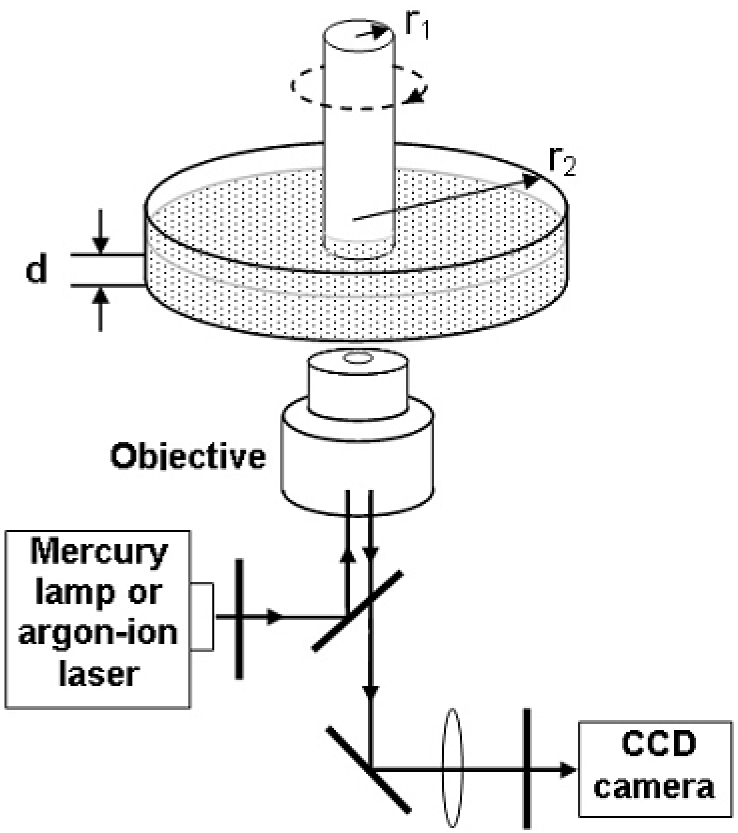
At α<0 of left tail the majority of molecules is strongly stretched.

α changes sign at γλ=1. At γλ<1 ⇒ α>0; at γλ>1 ⇒ α<0.

*α=0 can be interpreted as the **criterion for the coil-stretch** transition in a random flow that corresponds to* $Wi_{loc} \equiv \lambda \left(\frac{\partial V_i}{\partial x_j} \right)^{rms} \approx \lambda\gamma = 1$

Experiment: coil-stretch transition in a random flow.

S. Geraschenko, C. Chevallard, V. Steinberg, Europhys. Lett. **71**, 221 (2005)
Y. Liu and V. Steinberg, EPL **90**, 44002 and 44005 (2010)

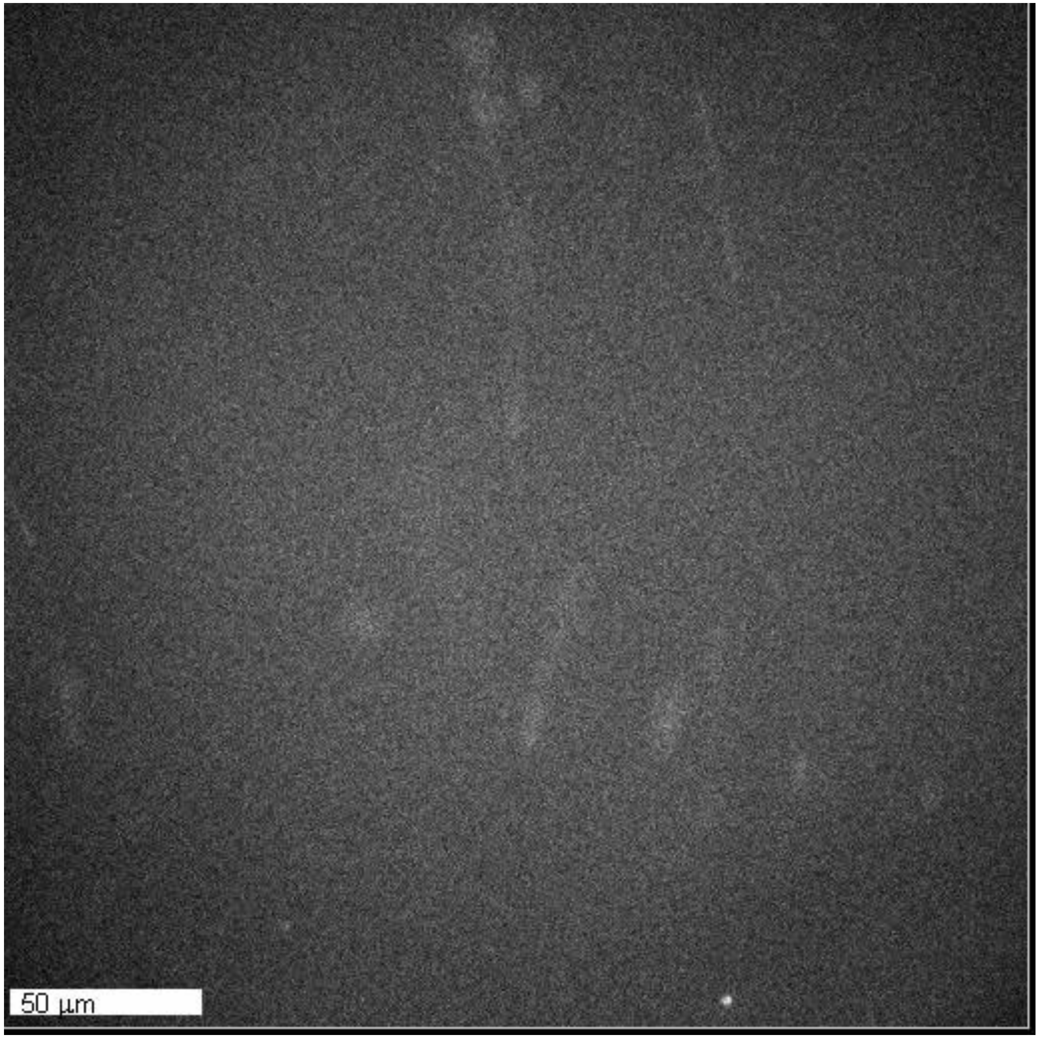


Experimental Setup

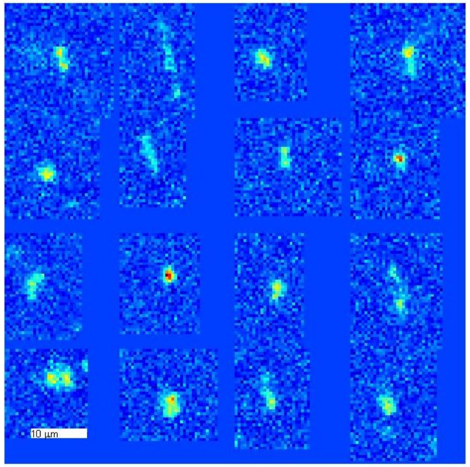
r_1 = 2.25 mm
r_2 = 6 mm
d = 675 μm
d/r_1 = 0.3
300 μm from center
100 μm above cover glass

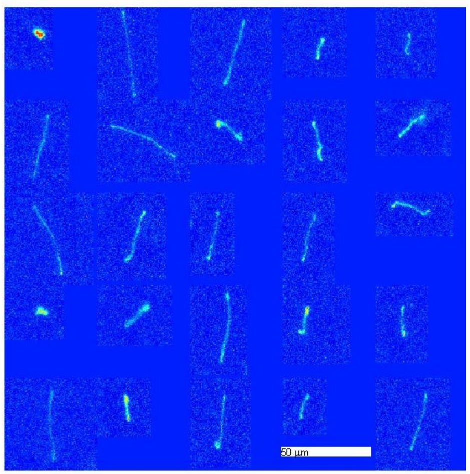
Polymers: λ-DNA and T4DNA

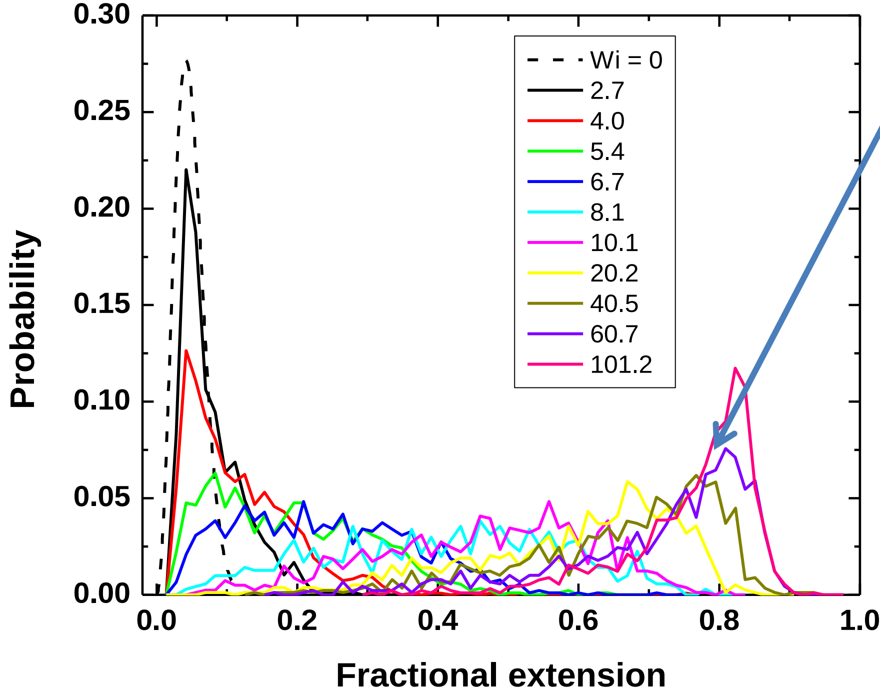
Polymer solution:
100.64 μg/mL of T4DNA with 35% sucrose;
Rotation speed 120 rpm;
Wi=45.8, local Wi=13.4



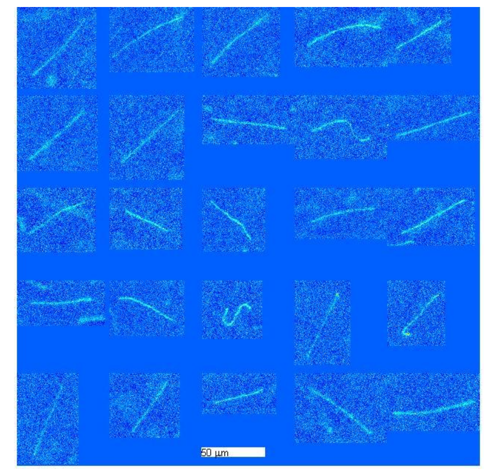
PDF of Polymer Extension at large Wi>>1 (at $V_{rms}/V_{av} > 1$)







Saturation of polymer stretching due to nonlinearity



PDF of polymer extension at different values of Wi in solution of 25 µg/mL T4 DNA with 50% sucrose. The dash line is the calculated distribution of end-to-end distance for a Gaussian chain in coiled state. The PDF is based on the statistics of 700~1100 molecules for each Wi. (typical polymer conformation at Wi = 2.7, 8.1, and 101.2)

Criterion of the coil-stretch transition:

(obtained from separate measurements of λ, average Lyapunov exponent at the coil-stretch transition defined from the scaling of PDF's tails of polymer stretching)

$$\bar{\gamma}_{cr} = 0.07 \pm 0.015 \quad 1/\mathrm{s}$$

$$\lambda = 11 \pm 0.1 \;\; \mathrm{s}$$

The experimental value for the critical Weissenberg number for coil-stretch transition

$$Wi_{loc}^{cr} \equiv \bar{\gamma}_{cr} \lambda = 0.77 \pm 0.2$$

and should be compared with theoretical prediction:

$$Wi_{loc}^{cr} = 1$$

Macroscopic approach to study flow structure, statistics and stresses

Elastic Turbulence: Theory and Experiment

Theory of elastic turbulence

E. Balkovsky, A. Fouxon, V. Lebedev, ***PRE* 64**, 056301 (2001)
A. Fouxon, V. Lebedev, ***Phys. Fluids* 15**, 2060 (2003)

In Hookean (linear)approximation of polymer elasticity and by neglecting thermal noise one gets uniaxial presentation of elastic stress tensor

$R/R_0 \gg 1, R/R_{\max} \ll 1$

$$\sigma_{p,ij} \propto c\lambda^{-1}\langle R_i R_j \rangle \longrightarrow \boxed{\sigma_{p,ij} = B_i B_j}$$

Then equations of motion can be rewritten in the form similar to MHD equations with zero magnetic resistance and linear damping at Re<<1 and Wi>>1

$$\frac{\nabla P}{\rho} = (\vec{B}\nabla)\vec{B} + \nu\Delta\vec{V}; \nabla\vec{V} = 0 \qquad \textbf{(1)}$$

$$\partial_t \vec{B} + (\vec{V}\nabla)B = (\vec{B}\nabla)\vec{V} - \frac{\vec{B}}{\lambda}; \nabla\vec{B} = 0 \qquad \textbf{(2)}$$

Eqs. (1,2) show instability at Wi>1 that leads to random statistically steady state.

Assumptions and results of the model:

i) Flow is stabilized due to back reaction of linear stretched polymers on flow velocity (Eq.(2)). From the flow stationarity one gets **1**

$$\left(\frac{\partial V_i}{\partial r_j}\right)^{rms} \propto \lambda^{-1} \longrightarrow \boxed{Wi_{loc} \approx 1}$$

The first key prediction:
Wi_{loc} **saturates at the value of order unity in elastic turbulence**

ii) Viscous and relaxation dissipations are of the same order **2**

$$\frac{\sigma_p}{\lambda} = \frac{B^2}{\lambda} \propto \eta(\nabla_i V_j)^2 \longrightarrow \boxed{\frac{\sigma_p \lambda}{\eta} \propto Wi_{loc}^2}$$

Then elastic stress can be estimated as $\boxed{\frac{\sigma_p \lambda}{\eta} \approx 1}$

The second key prediction:
σ_p **saturates in elastic turbulence**

Thus the elastic stress saturates in bulk as well as local Weissenberg number.

The main result of the theory is the velocity power spectrum $\boxed{E(k) \propto v^2 l (kl)^{-\beta},}$ $\beta>3$

Macroscopic hydrodynamics approach:

Experimental test of both assumptions and the results of the theory of elastic turbulence

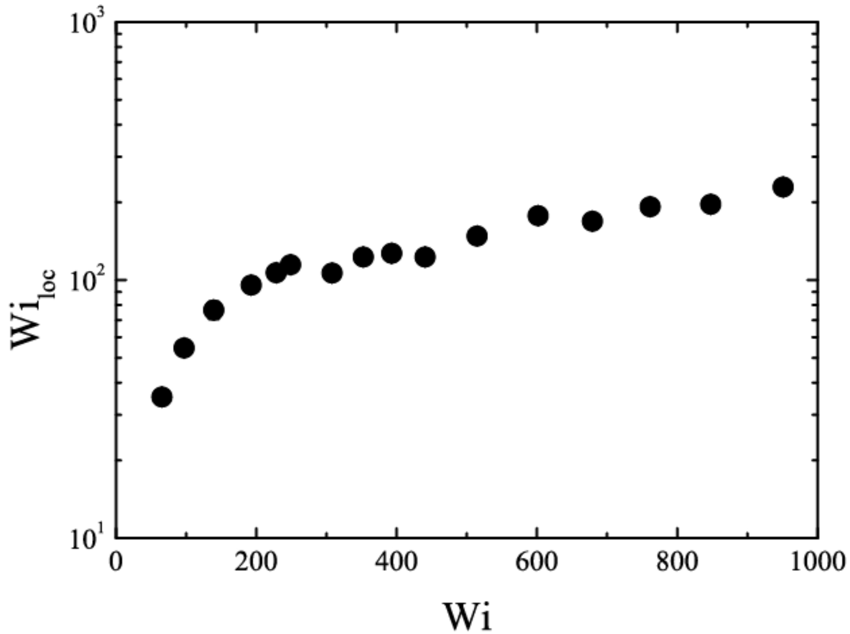
T. Burghelea, E. Segre, V. Steinberg, PRL **96**, 214502 (2006)
Phys. Fluids **19**, 053104 (2007)

Y. Jun and V. Steinberg, PRL **102,** 124503 (2009)
Phys. Fluids **22,** 123101 (2010)
Phys. Rev. E submitted (2011)

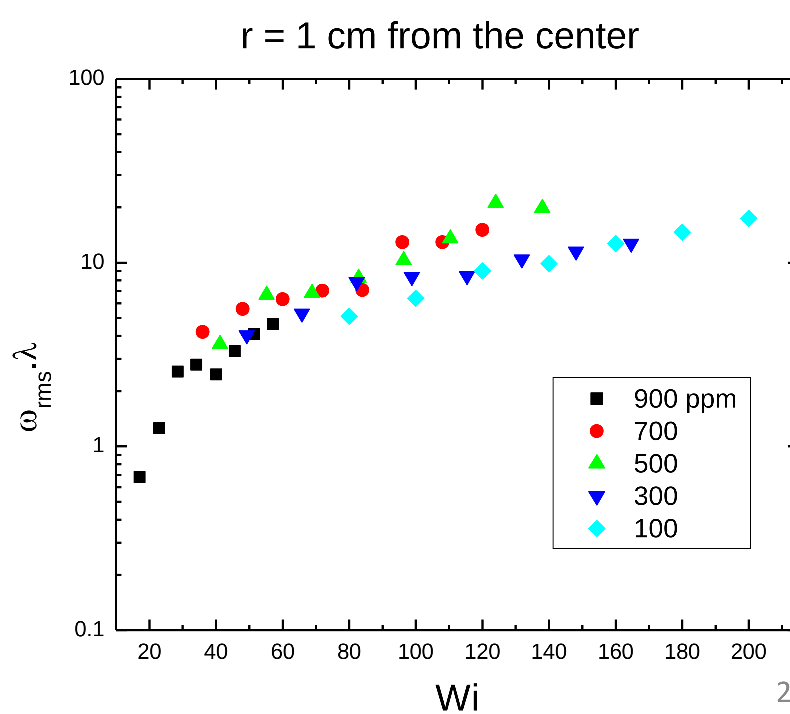
The first key prediction: Wi_{loc} **saturates at the value of order unity in elastic turbulence**

$$Wi_{loc} \approx 1$$

The data obtained in curvilinear channel
At 80 ppm PAAm and η=0.138 Pas, λ=11.5 s



Wi_{loc} varies with Wi in contrast to theory



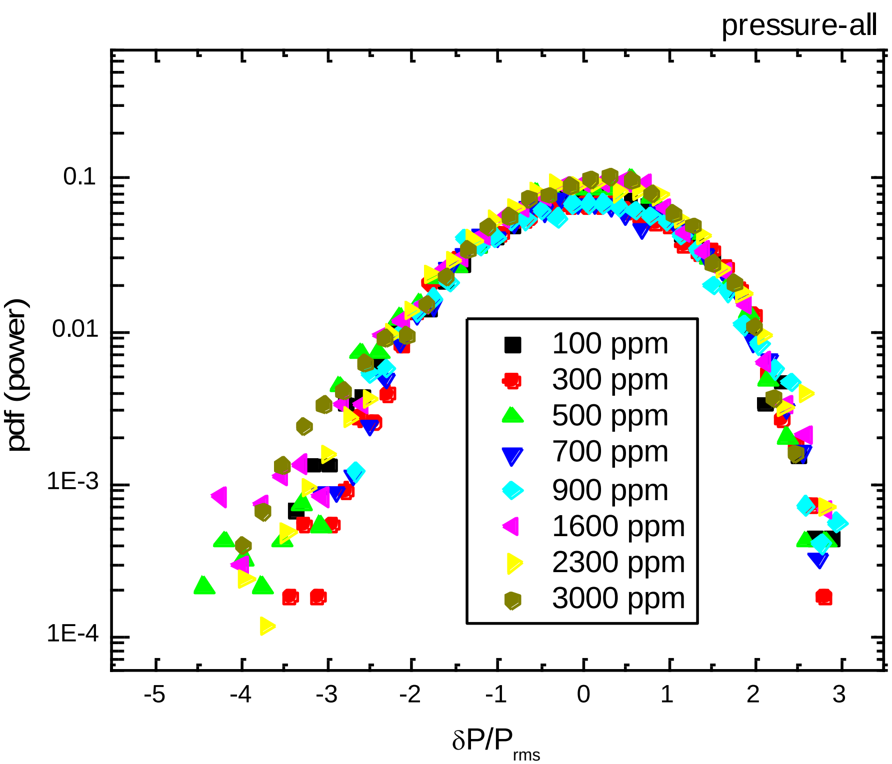
The data obtained in the bulk of swirling flow at various PAAm concentrations, solvent viscosities and polymer relaxation times.

$$Wi_{loc} \equiv \omega_{rms} \cdot \lambda$$

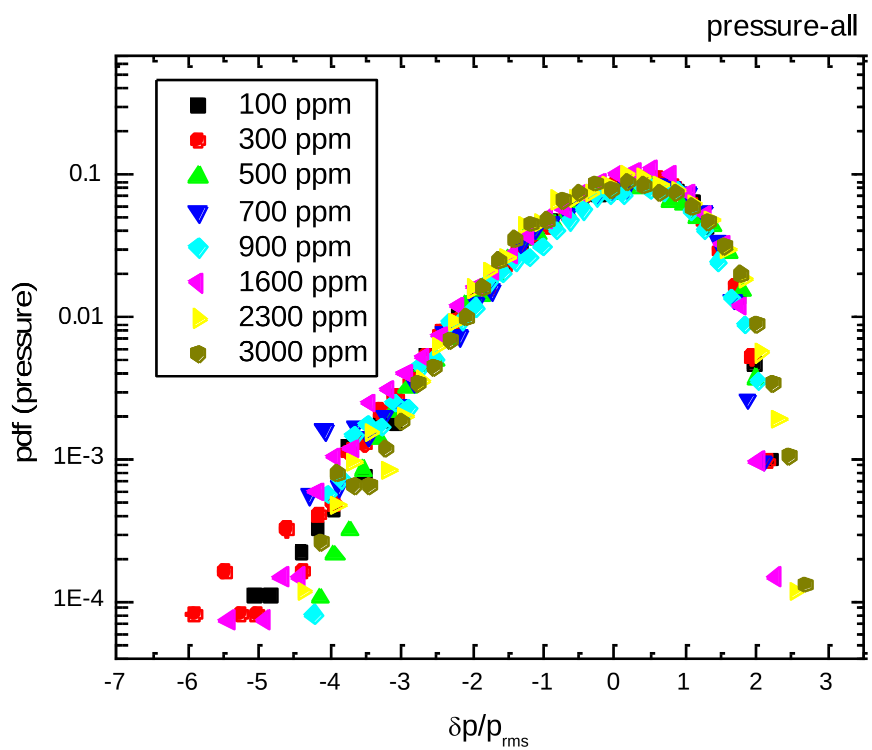
Measurements in elastic turbulence regime in a bounded vessel:

(in formal analogy with hydrodynamic turbulence)

Universal shape of PDFs for power and pressure fluctuations
in a wide range of Wi and polymer concentrations



PDF of injected power fluctuations

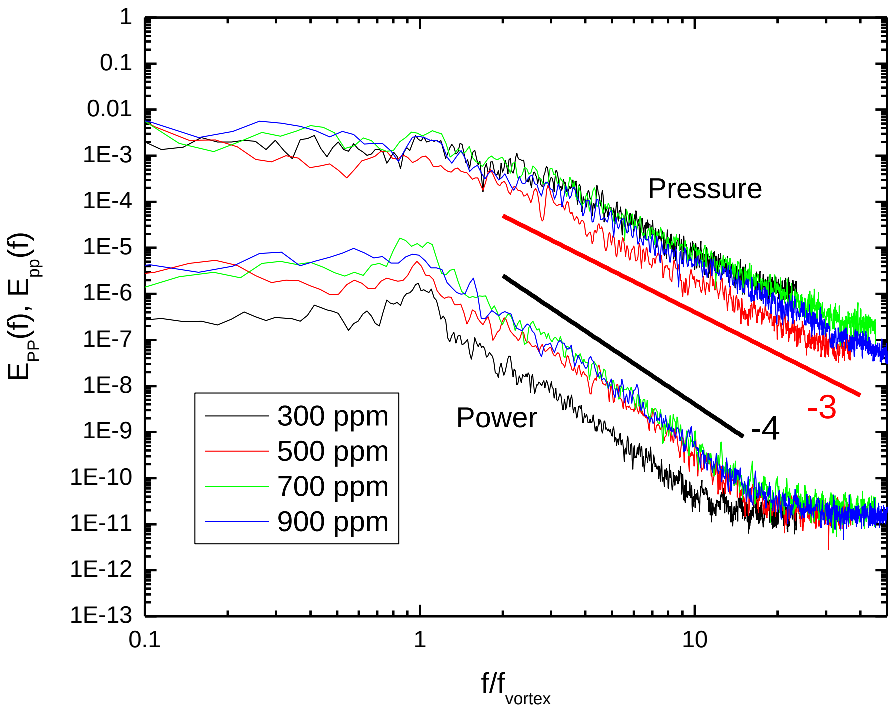


PDF of pressure fluctuations

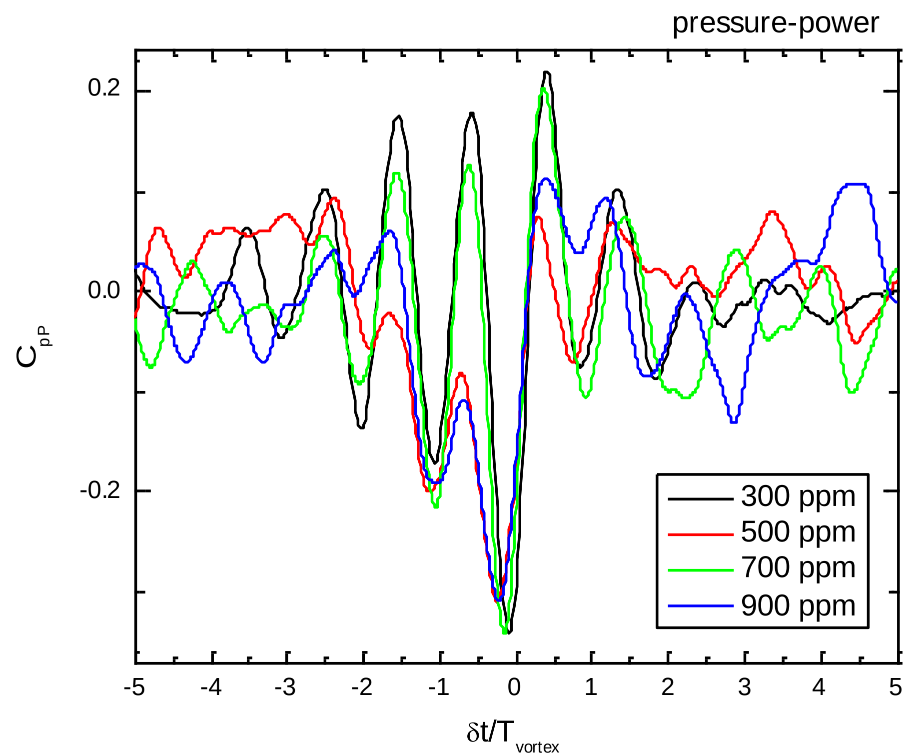
at maximum value of Wi

The source of intermittent statistics is an excess of elastic stresses near the upper disk and their intermittent injection into the bulk

Further statistical analysis of power and pressure fluctuations



Power spectra of injected power and pressure fluctuations

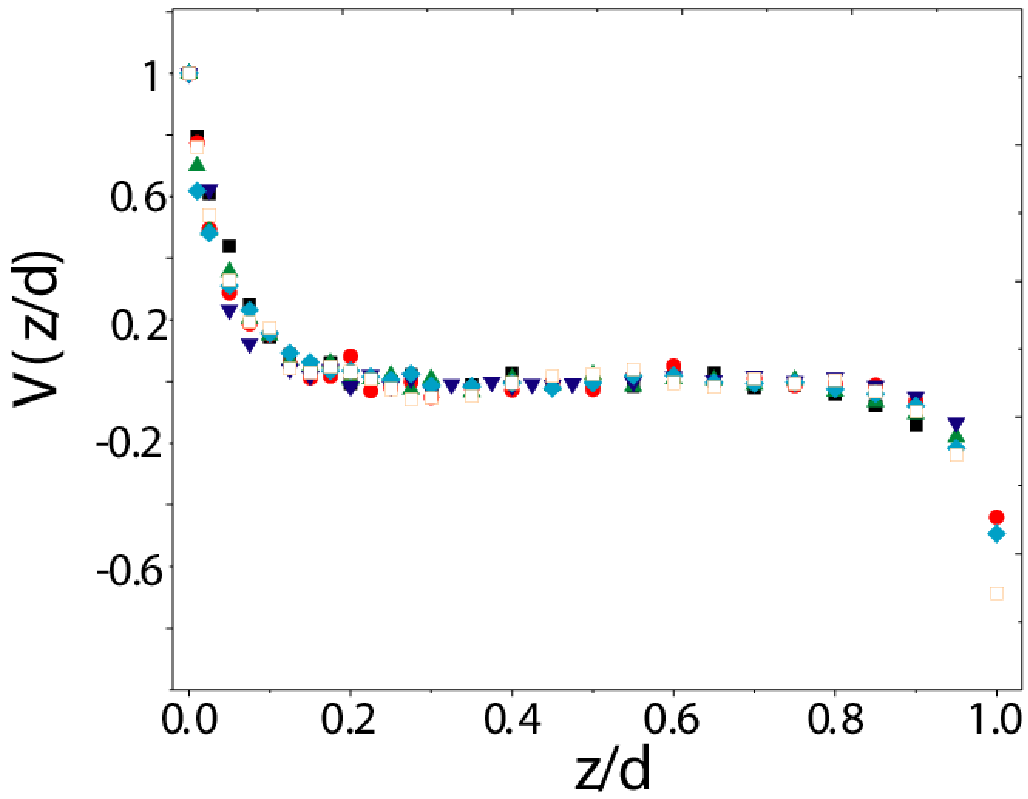


Cross-correlation function of power and pressure fluctuations

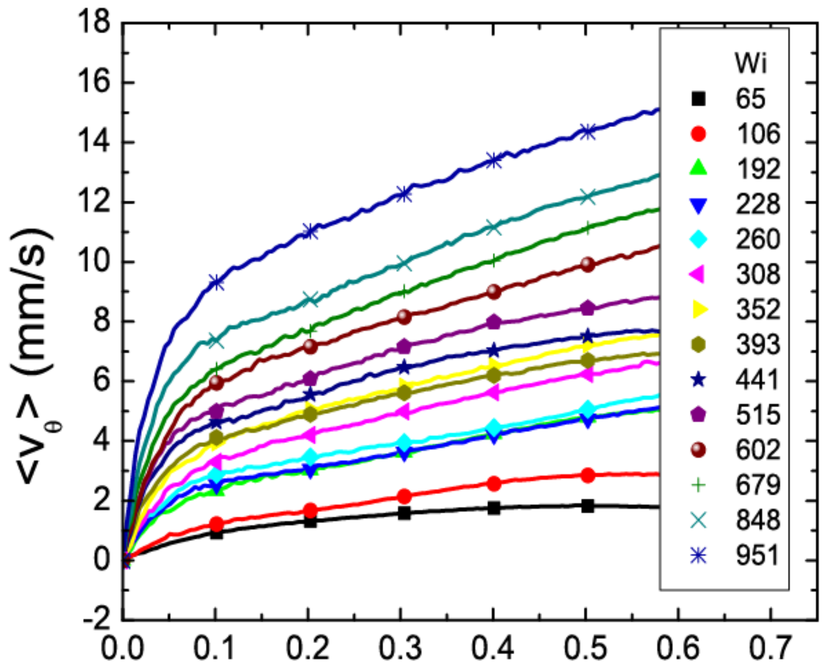
In elastic turbulence at Re<<1 and Wi>>1 $\Delta p / \rho = \partial^2 \sigma^p{}_{ij} / \partial x_i \partial x_j$,i.e. PDF and power spectra of p provides the way to study statistics of elastic stresses

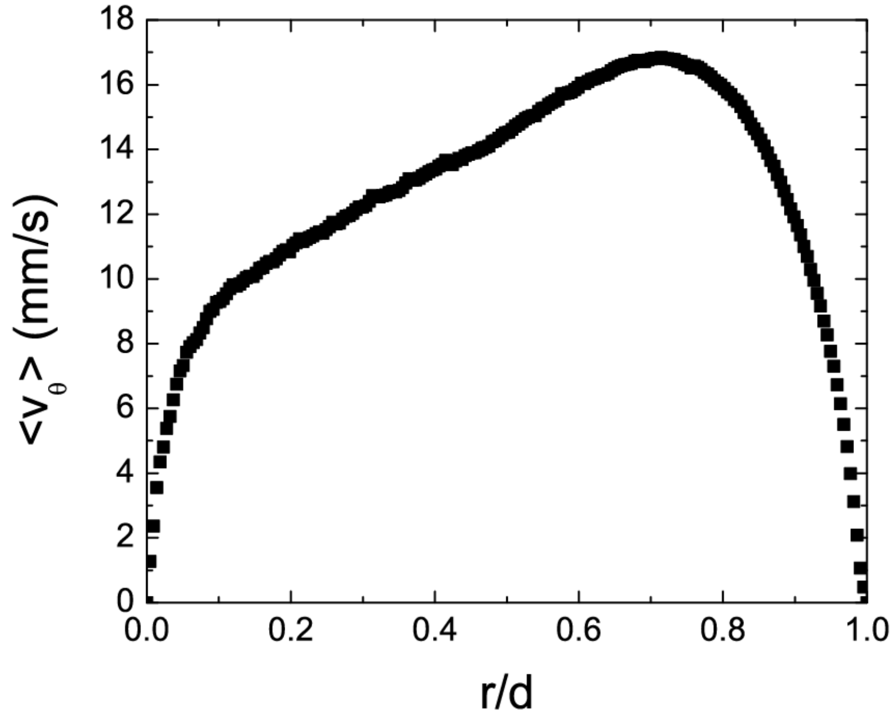
Universality of in statistical properties of P and p in a wide class of non-equilibrium hydrodynamic systems

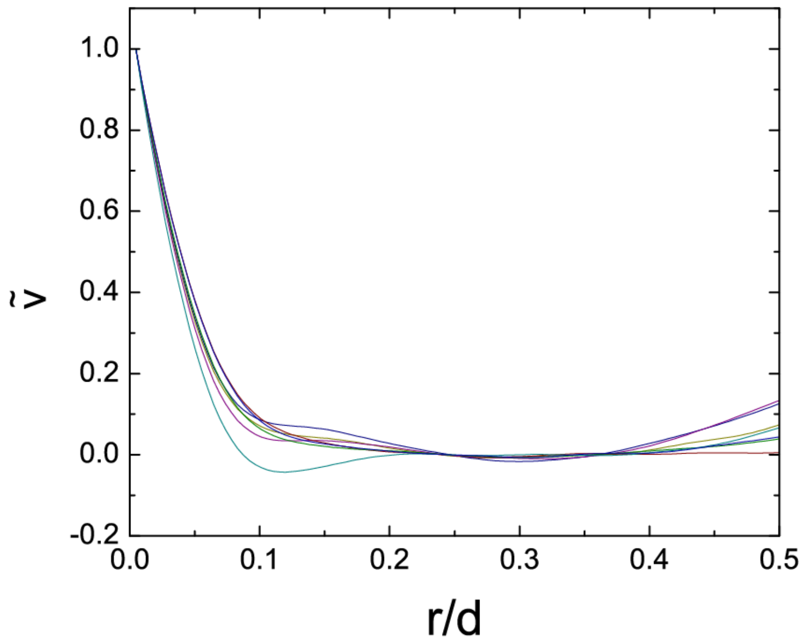
New characteristic length scale–boundary layer width



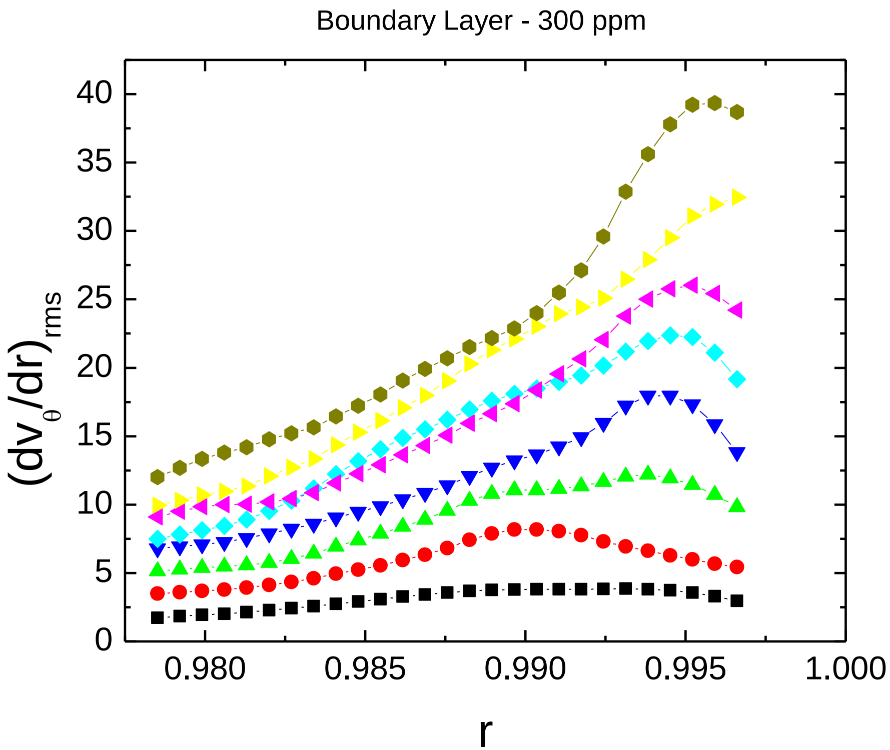
Normalized average azimuthal velocity profiles at different Wi at 80 ppm of PAAm ***in swirling flow***







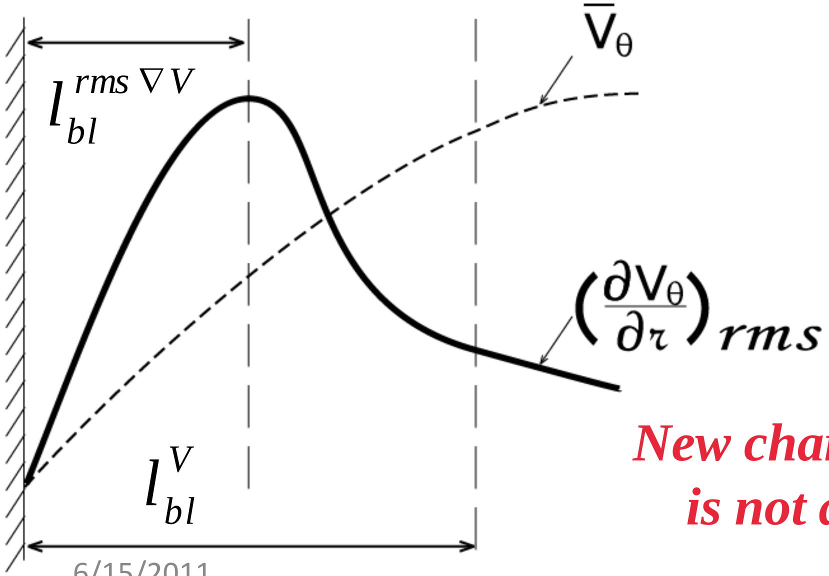
Normalized average azimuthal velocity profiles at different Wi at 80 ppm of PAAm ***in channel flow***



Profiles of $(\partial V_\theta / \partial r)_{rms}$ in horizontal plane in swirling flow at various Wi

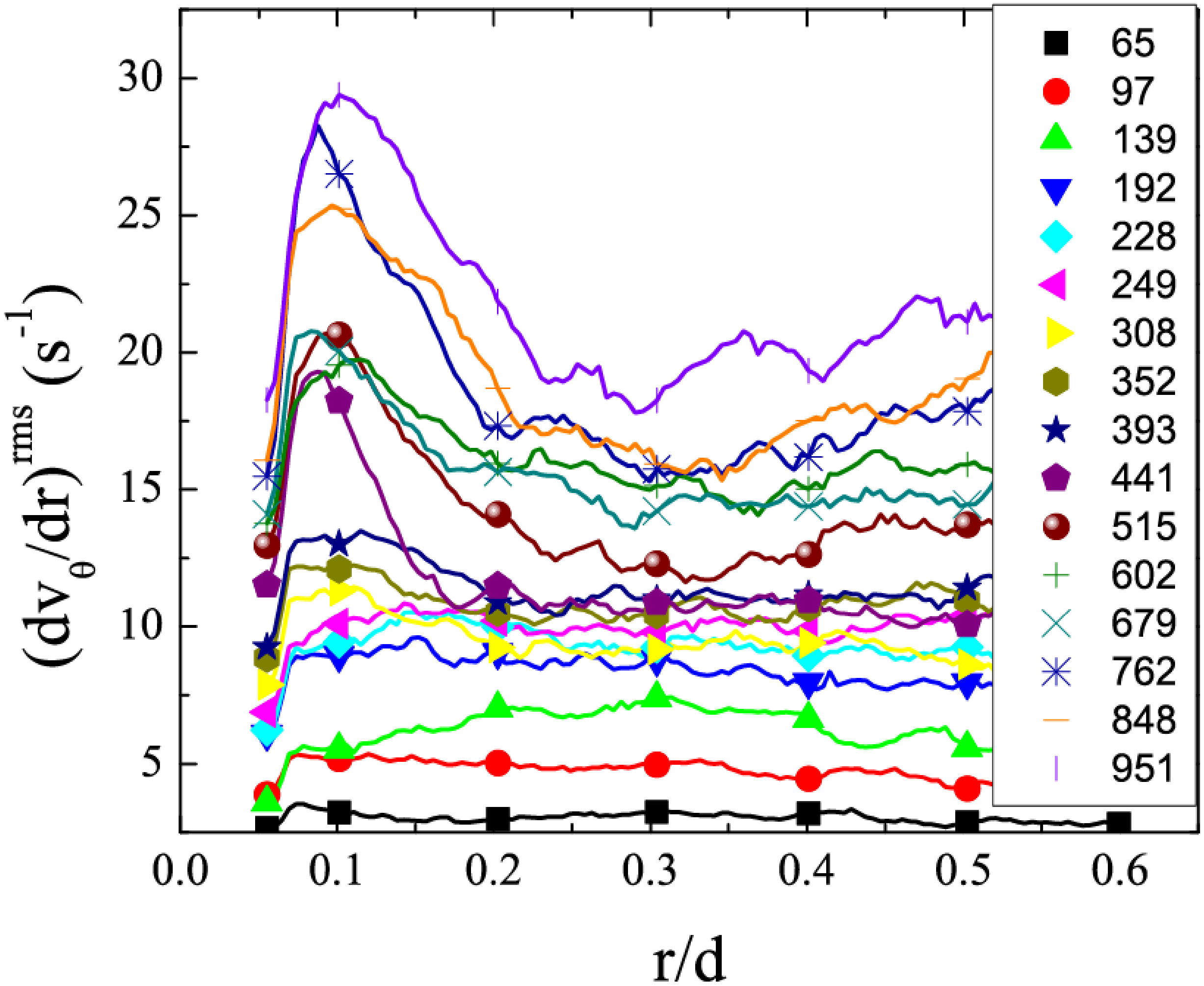
$$\left(\frac{\partial V_\theta}{\partial r}\right)^{rms}_{\max} \approx 100 \left(\frac{\partial V_\theta}{\partial r}\right)^{rms}_{bulk}$$

$$Wi^{\max}_{bl,loc} \approx 100\, Wi_{bulk,loc}$$



New characteristic spatial scale-boundary layer width- is not anticipated by theory

Profiles of $(\partial V_\theta / \partial r)_{rms}$ across the channel in curvilinear channel flow



The peaks in $(\partial V_\theta / \partial r)_{rms}$ profiles indicate peaks in elastic stress and boundary layer in the elastic stress distribution

Interim conclusions about elastic stress distribution and intermittent behavior macroscopic approach)

Role of elastic stress in statistical and scaling properties of elastic turbulence

First, elastic stress from BL is intermittently injected into bulk that leads to skewness and exponential tail in PDF of injected power (torque) and pressure fluctuations and exponential tails in PDF of velocity gradients in a bulk.

Second, new length scale-boundary layer width –appears as the result of this non-uniform stress distribution:

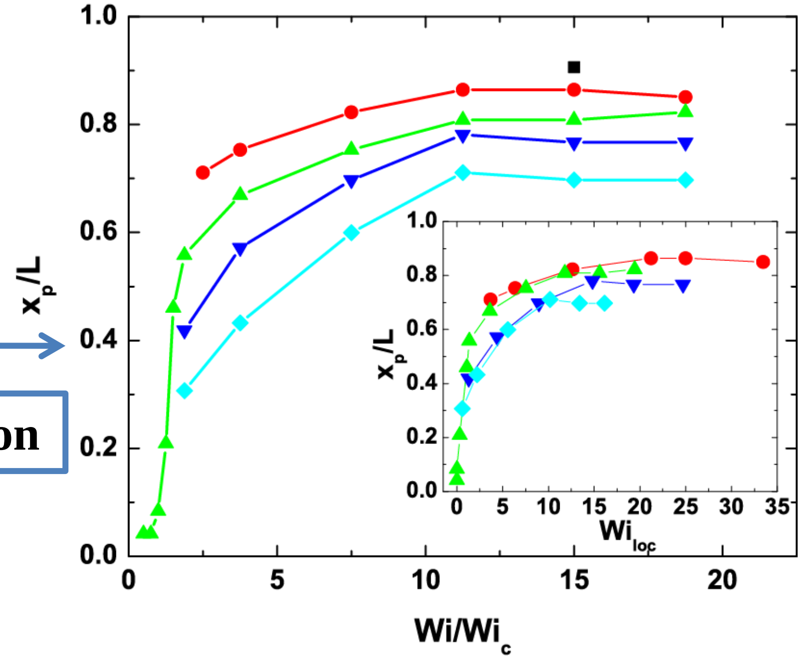
$$l_{bl}^{str} \propto \left(\frac{Wi_{loc}^{bulk}}{Wi_{loc}^{bl}} \right)^{\alpha} << 1$$

Third, the non-uniform distribution of elastic stress should result in non-uniform distribution of polymer stretching-the polymer molecules should be stretched considerably more in the boundary layer- verification of this prediction is presented further .

Microscopic approach to measure statistics and scaling of elastic stresses in elastic turbulence

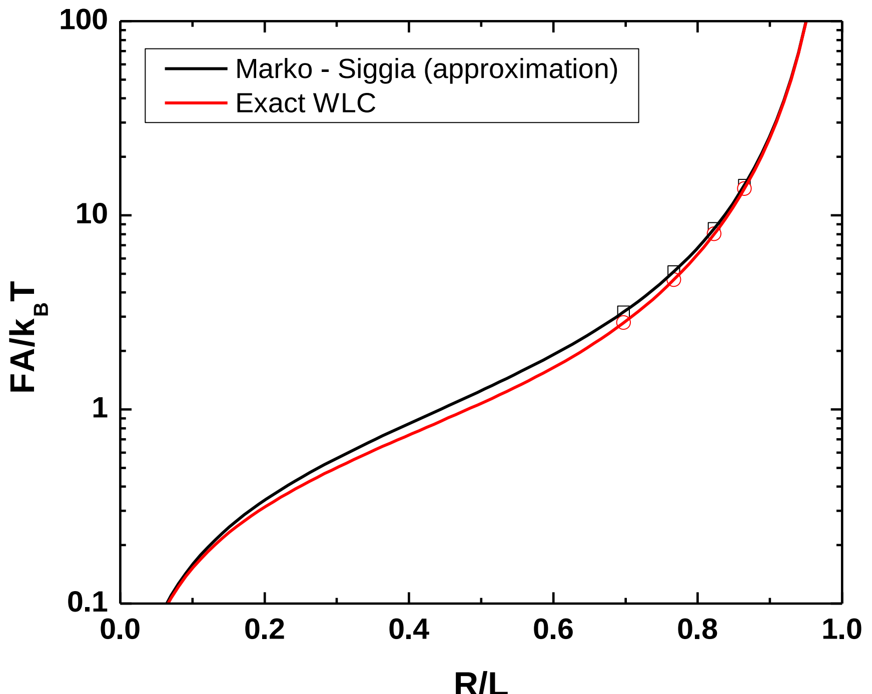
Y. Liu and V. Steinberg, EPL **90**, 44002 (2010)
EPL **90**, 44005 (2010)

Polymer stretching in elastic turbulence at large Wi>>1, measurements of elastic stresses and further test of theory



$$x_p / L \propto 1$$

in contrast to the theory assumption



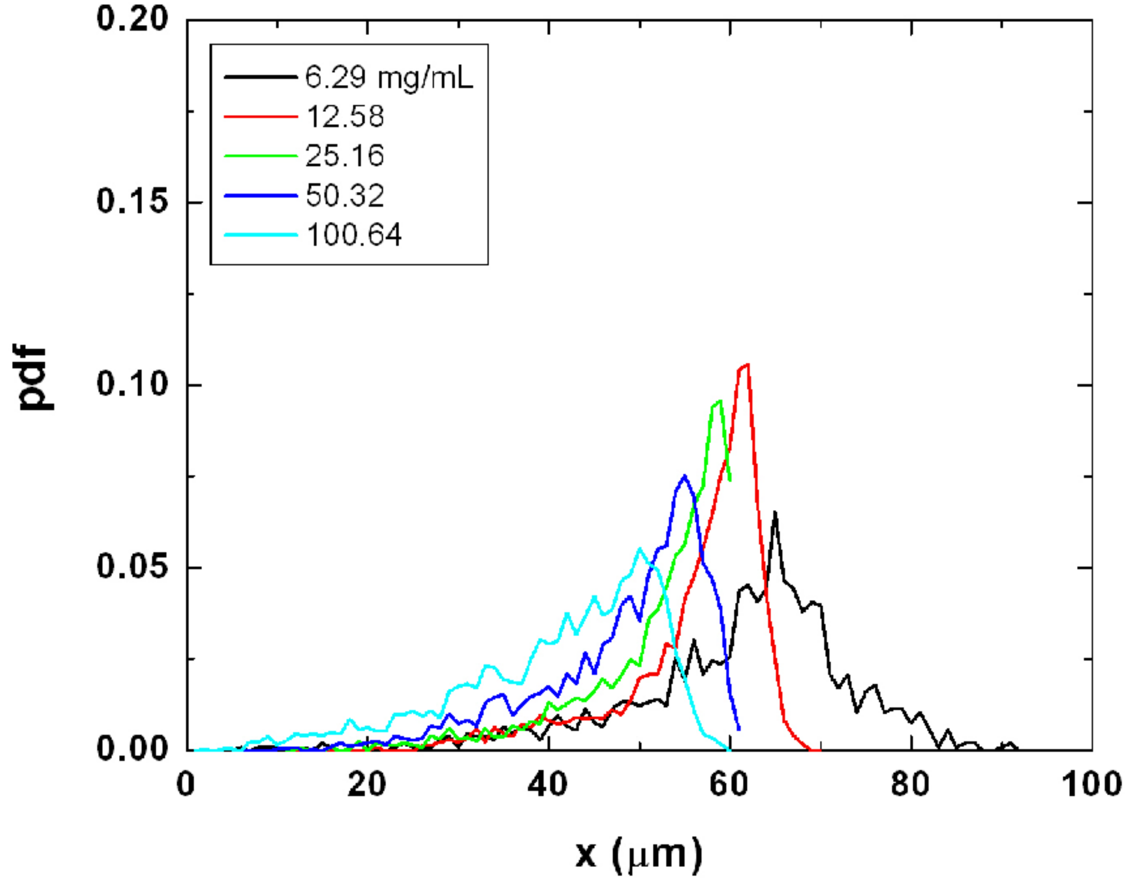
Force-Extension Relationship of WLC

C. Bustamante, J.F. Marko & E.D. Siggia, Science 258, 1126 (1992)

$$\frac{FA}{kT} \approx \frac{1}{4(1-R/L)^2} - \frac{1}{4} + \frac{R}{L}$$

Molecular sensor to measure elastic stress in a flow

PDF of Polymer Extension at large Wi>>1 (at $V_{rms}/V_{av} > 1$)



PDF of x/L in elastic turbulence at the highest Wi for five polymer concentrations

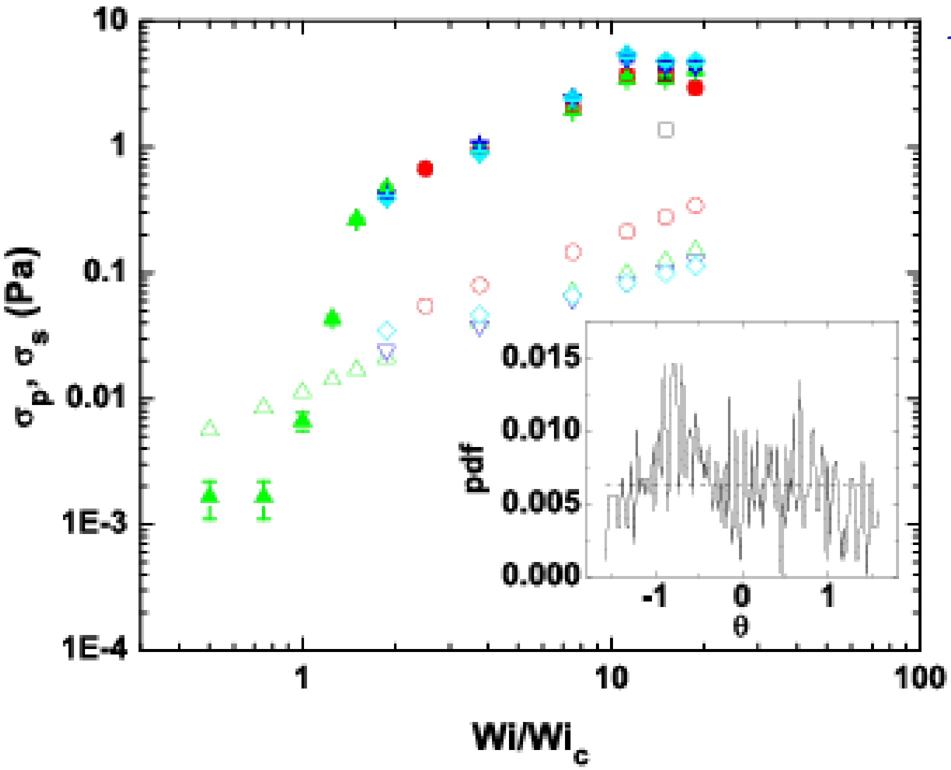
The second key prediction:
Saturation of elastic stress in elastic turbulence regime

$$\frac{\sigma_p \lambda}{\eta} \approx 1$$

that follows from the assumption that viscous and relaxation dissipations are of the same order

$$\frac{\sigma_p}{\lambda} \propto \eta (\nabla_i V_j)^2$$

Elastic and viscous stresses vs Wi

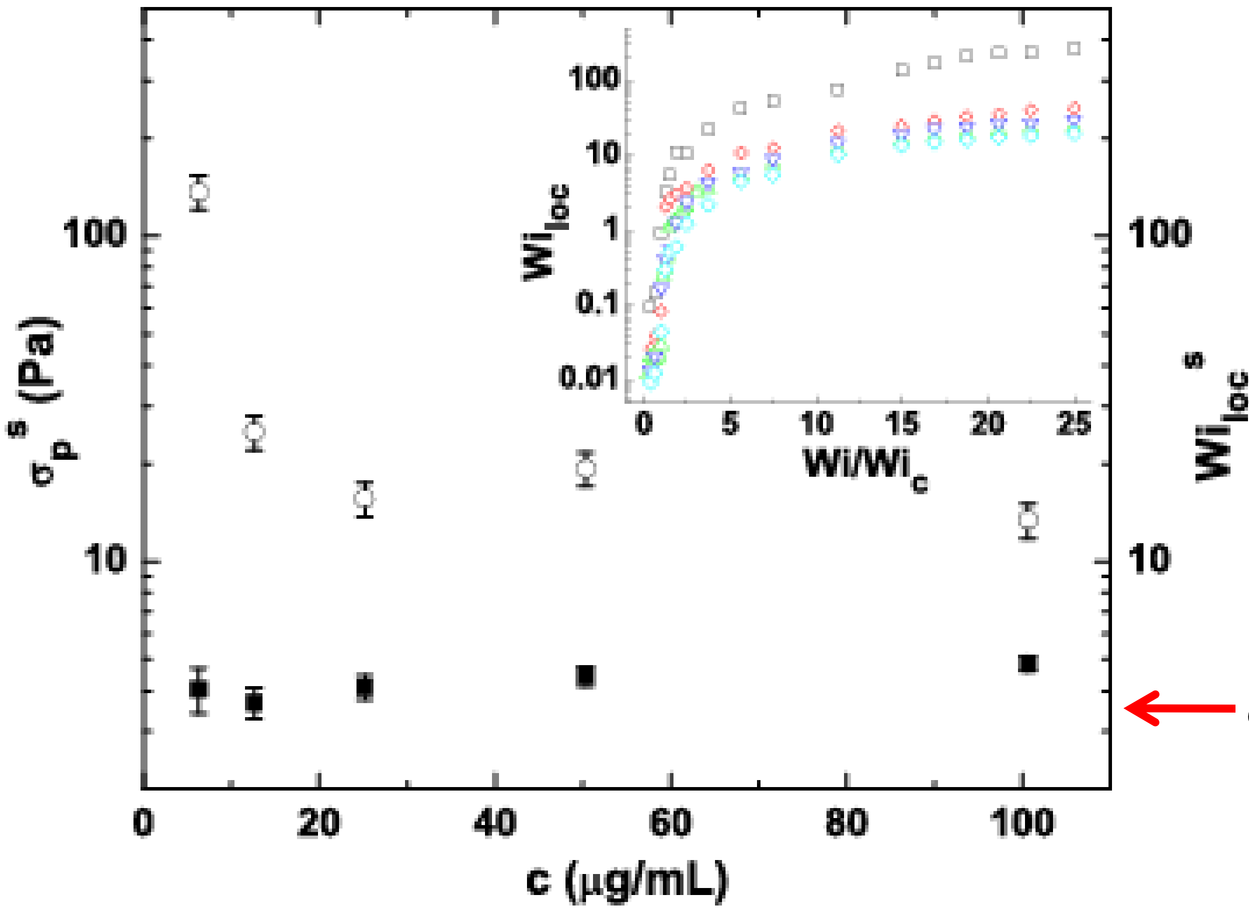


Elastic (full symbols) and viscous (open symbols) stresses vs Wi for five concentrations

Inset: PDF of inclination angle at Wi/Wi*=7.5 and c≈20ppm

Elastic stresses σ_p as well as Wi_{loc} do not saturated at a single value but change with Wi and then saturated at very high values in contrast to theoretical prediction.

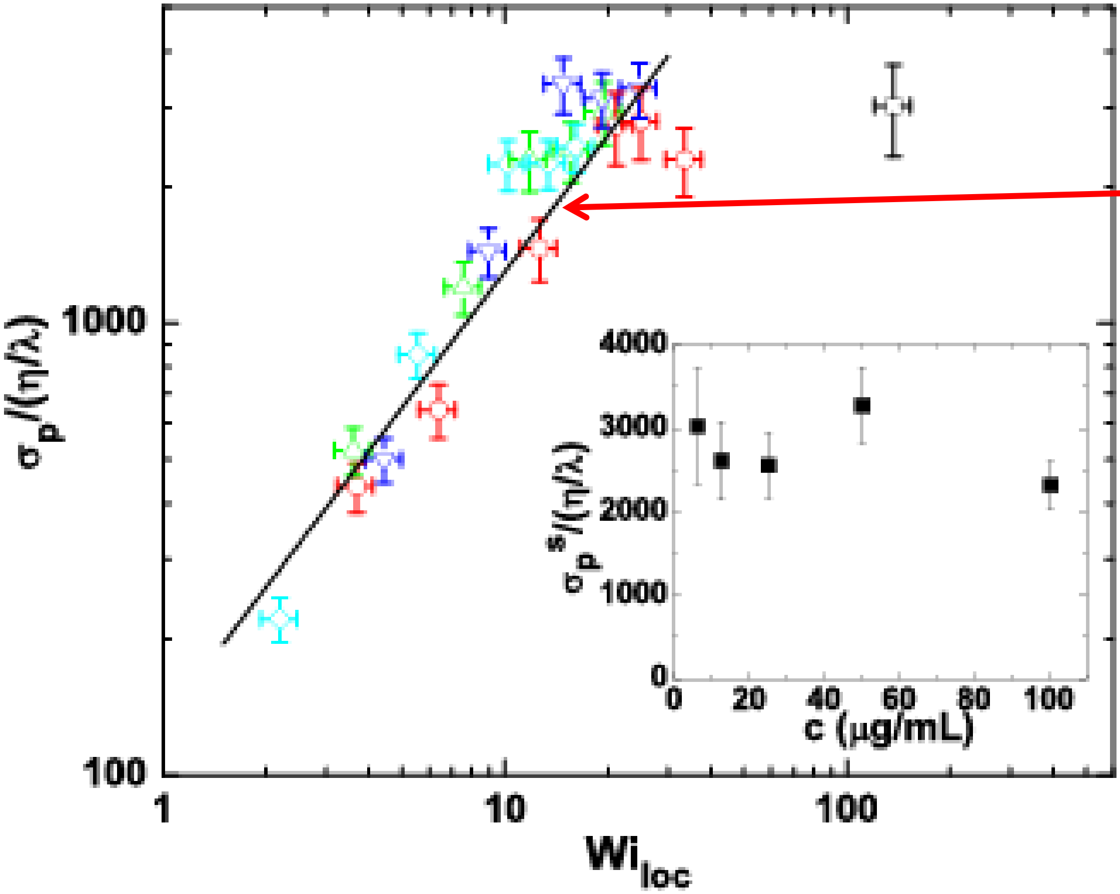
Saturated elastic stresses σ_p^s and local Weissenberg number Wi_{loc}^s as a function of polymer concentrations



$\sigma_p^s \approx 5Pa$

$\sigma_p^s \approx 5Pa$ ***is more than two orders of magnitude higher that predicted theoretically from*** $\sigma_p/(\eta/\lambda) \approx 1$ ***that gives*** $\sigma_p \approx 0.01Pa$

Normalized elastic stress versus local Weissenberg number for five polymer concentrations



$$\sigma_p /(\eta / \lambda) \approx 130\, Wi_{loc}$$

This linear relation could not be explained by the theory based on polymer with linear elasticity.

Possible relation to the problem of the drag reduction in a turbulent flow and experimental challenges in turbulent drag reduction experiments

What can be learnt from elastic turbulence relevant to turbulent drag reduction problem:

1. Polymer can be stretched (and even overstretched-DNA) in a random flow!

2. Elastic stresses exceed viscous stresses up to two orders of magnitude in elastic turbulence

3. Elastic stresses are about proportional to Wi_{loc} in elastic turbulence

4. Wi_{loc} in boundary layer (and so elastic stresses) are up to two orders of magnitude exceed those in a bulk!

5. **Theory based on linear elasticity of polymers cannot explain properties of elastic turbulence**

6. If non-uniform distribution of elastic stresses occurs also in hydrodynamic turbulence flow with polymers, then most of energy in the elastic stress boundary layer is stored in the stretched polymers (elastic stresses) rather than in the Reynolds stresses and not transfer to the wall. ***It can be an explanation to turbulent drag reduction.***

Short review on turbulent drag reduction

Main conjectures and predictions of theories of turbulent drag reduction

1. A polymer in turbulent flow is influenced only by small scales, below dissipation one, where flow is random in time and spatially smooth- ***similar to elastic turbulence***
2. Polymer coil-stretch transition can occur in random flow, if $Wi = \lambda / \lambda_{turb} > 1$ in full ***analogy with elastic turbulence***
3. Stretched polymers create elastic stresses, which can balance Reynolds stresses in a bulk turbulent flow at some scale and above some polymer concentration
4. First quantitative theory, which take into account the elastic stresses, made predictions about elastic waves and spectra and ***links turbulent drag reduction problem to elastic turbulence and mhd*** (Lebedev et al 2000, 2001, 2003) for linear model of polymers and in unbounded flow

Numerical simulations of the last decade give a great insight into the problem:

1. Observation of drag reduction in full scale numerical simulations in a turbulent channel flow of Oldroyd-B model of viscoelstic fluid (Beris et al 1997 and later on)
2. Role of elastic stresses and coil-stretch transition in drag reduction in turbulent Kolmogorov flow, which reflects in universal function for the friction coefficient (Boffetta, Celani, Mazzino 2004): $f(\mathrm{Re}^{\alpha}, Wi^{\beta})$
3. Stretching of polymer in turbulent shear f low (Davoudi, Schumacher 2006) and recently (T. Watanabe, T. Gotoh 2010; F. Bagheri et al 2010)

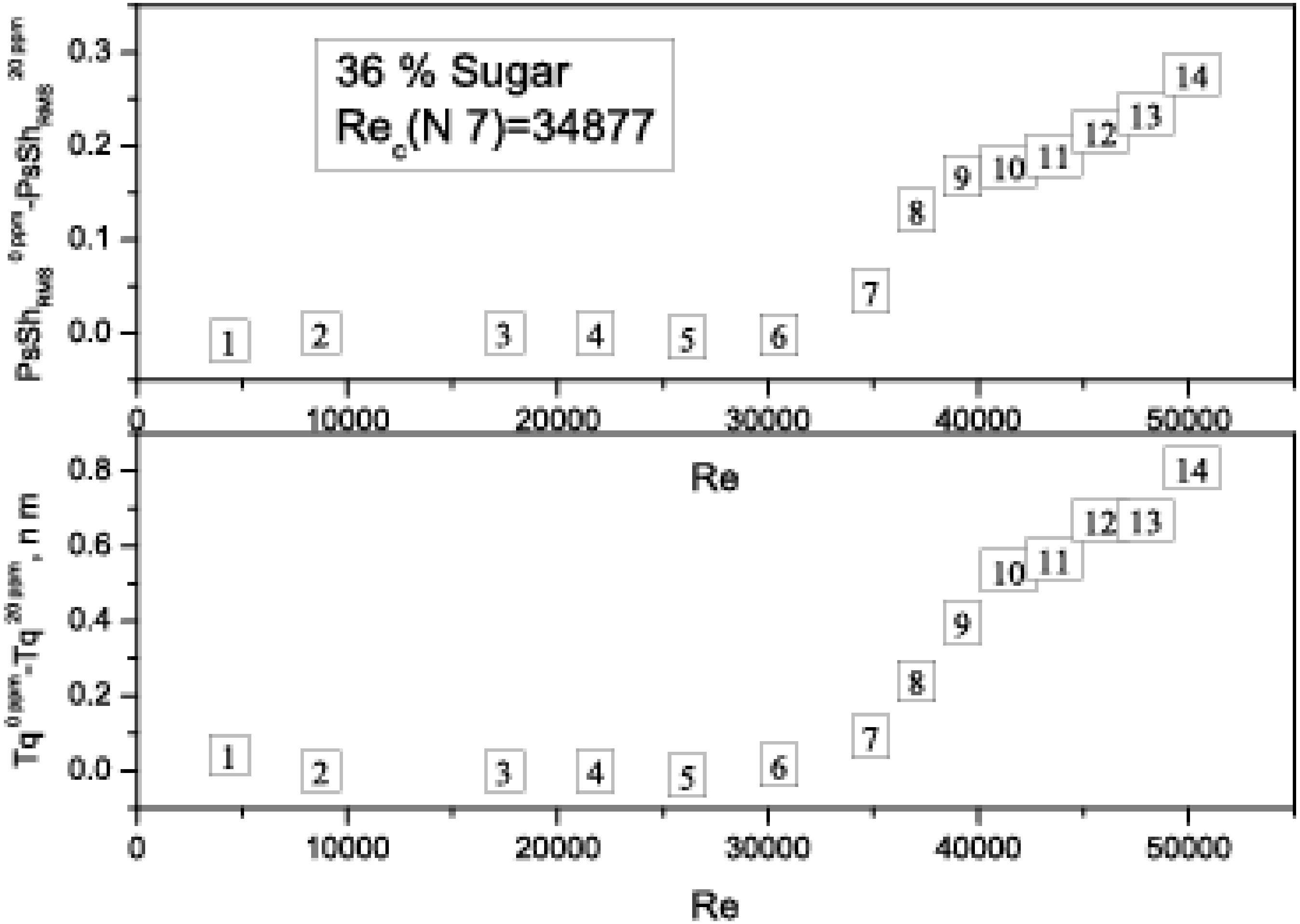
What experimental data and theoretical predictions are lacking to make further progress in the field?

- Measurements and characterization of polymer stretching and elastic stresses and a possibility of the coil-stretch transition in random and turbulent flows
- Scaling of elastic stresses with Re and Wi. Development of a new stress sensor to measure directly statistics of elastic stresses locally and their spatial distribution in elastic turbulence and further in turbulent flow
- Theory that makes quantitative predictions about scaling relations of the friction factor with Re and Wi: $f(\mathrm{Re}^{\alpha}, Wi^{\beta})$
- Dependence of the turbulent drag reduction onset and value on polymer molecular weight, elastic nonlinearity and concentration
- Distribution of elastic stresses in random and turbulent flows

- ***Conclusion***: measurements and characterization of elastic stresses in turbulent flow are the key issues to solve the turbulent drag reduction problem (two fields should be measured like V and H in MHD)

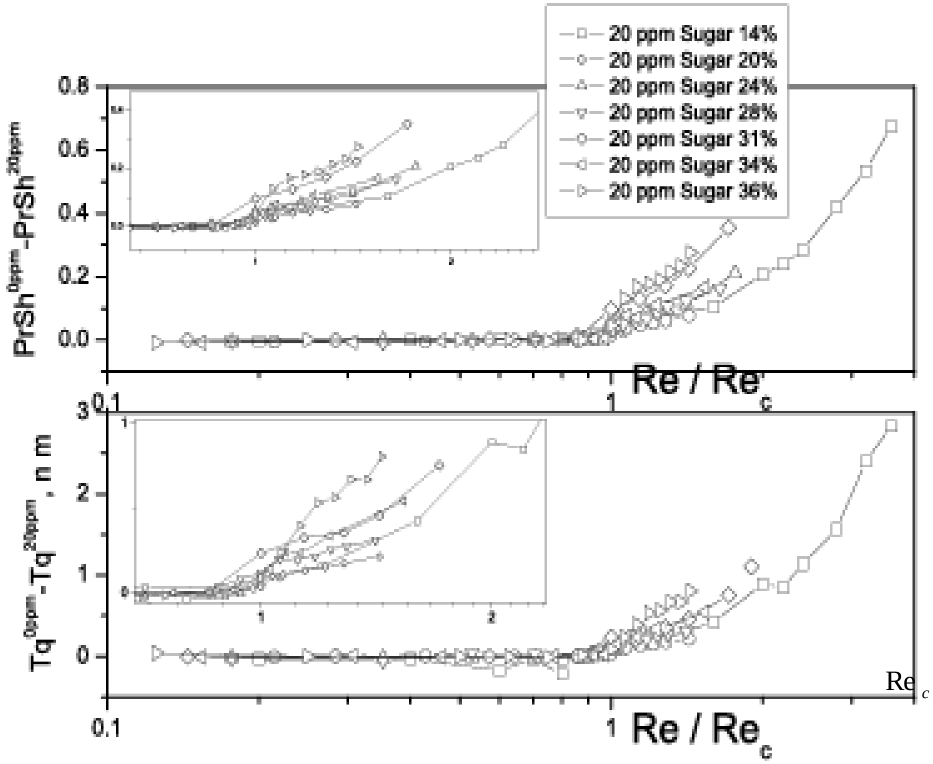
Recent experimental results on turbulent drag reduction in von Karman swirling flow with 20 ppm of PAAm 18MDa

Y. Burnishev and V. Steinberg, unpublished



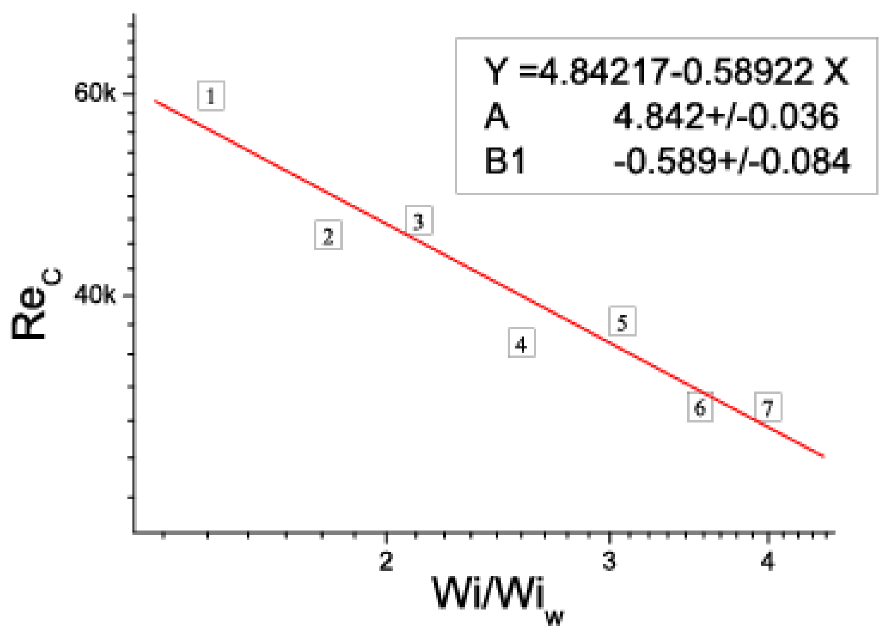
Reduced pressure fluctuations vs Re

Reduced average torque vs Re



Reduced pressure fluctuations and average torque for seven values of Wi (solvent viscosities) vs Re/Re_c

Dependence of Re_c at the onset of turbulent drag reduction on Wi/Wi_w



Conclusions

- Measurements and characterization of polymer stretching and elastic stresses in turbulent flow are the key issues to solve turbulent drag reduction (two fields should be measured like V and H in MHD)

What we expect from theory

- Quantitative predictions of scaling dependence of friction coefficient on Re and Wi: $f(\mathrm{Re}^{\alpha}, Wi^{\beta})$ and its universality
- Scaling of elastic stresses with Re and Wi
- Dependence of turbulent drag reduction onset and value on polymer molecular weight, elastic nonlinearity and concentration

What is the next in experiment

1. Distribution of polymer stretching on Lagrangian trajectories in 3D and elastic stresses in 3D in elastic turbulence
2. Development of a new stress sensor to measure directly statistics of elastic stresses locally and their spatial distribution in elastic turbulence
3. Direct measurements of statistics and spatial distribution of elastic stresses in turbulent flow (including elastic stress boundary layer and its relation to buffer layer(?), elastic waves and their spectra)

Collaborators during the last11 years of this research:

Macroscopic Experiments:

Y. Jun
T. Burghelea,
E. Segre
A. Groisman

Microscopic Experiments:

Y. Liu
S. Gerashchenko
C. Chevallard,

Theoretical support:

V. Lebedev, M. Chertkov, K. Turitsyn, A. Fouxon, G. Falkovich