Turbulent Mixing of Passive Tracers on Small Scales in a Channel Flow

Victor Steinberg

Department of Physics of Complex Systems,

The Weizmann Institute of Science, Israel

In collaboration with

Y. Jun, E. Segre, T. Burghelea and A. Groisman

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Transport of passive scalar

$$\frac{\partial \theta}{\partial t} + \vec{u} \cdot \vec{\nabla} \theta = D \nabla^2 \theta$$

Definitions: ε_s -rate of scalar fluctuations dissipation;
ε -rate of turbulent energy dissipation.

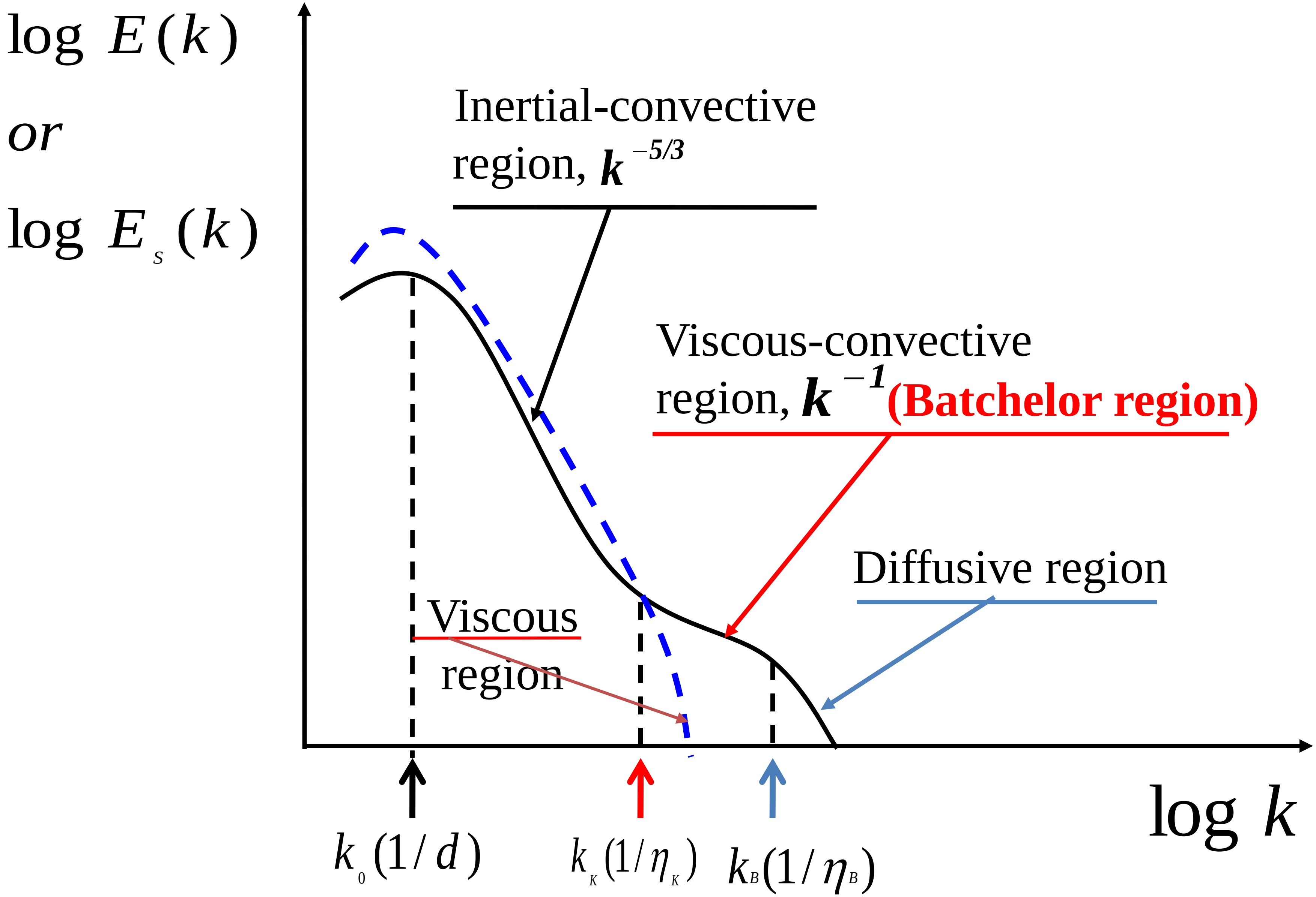
Kolmogorov (dissipation) wave number:

$$k_K = (\varepsilon / \nu^3)^{1/4}$$

Batchelor (dissipation) wave number:

$$k_B = (\varepsilon / \nu D^2)^{1/4}$$

Control parameters of the problem: $Sc=\nu/D \gg 1$ and $Pe=VL/D \gg 1$



Batchelor (1959), Kraichnan (1968)

Batchelor regime of mixing

- velocity field is statistically homogeneous, isotropic, and stationary
- random in time and spatially smooth velocity field
- energy spectrum is steeper than k^{-3}

Batchelor regime of mixing at S>>1

- Distribution of small blob of scalar by repeated stretching and folding, so that the scale reduces as

$$l(t) \approx L\exp(-\lambda_1 t),$$

where λ_1 is the Lyapunov exponent, and $\lambda_1 \approx \dot{\gamma}$

Then velocity gradient is smeared out, when the diffusion rate becomes comparable with the rate of stretching: $D / l(t_{mix})^2 \approx \dot{\gamma}$

This condition defines the Batchelor dissipation scale:

$$\eta_B \approx L Pe^{-1/2}, \quad Pe = \dot{\gamma} L^2 / D, \quad \text{and} \quad \dot{\gamma} \approx \frac{\Delta V}{L} \text{ - shear rate}$$

Then mixing time due to advection in the Batchelor regime is

$$\boxed{\mathbf{t}_{mix} = \dot{\gamma}^{-1} \cdot \ln(\mathbf{Pe})}$$

or mixing length $\mathbf{l}_{mix} = \mathbf{L} \cdot \ln(\mathbf{Pe})$

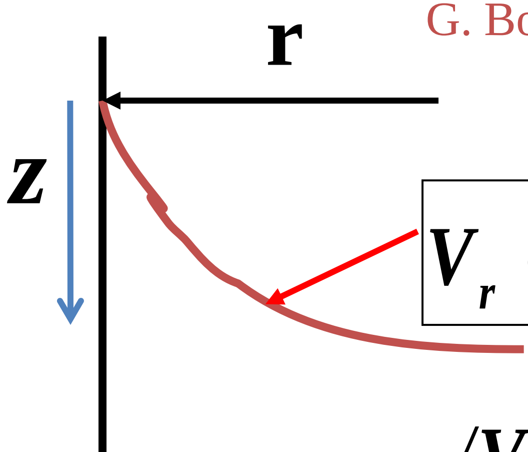
Theoretical predictions on turbulent mixing in Batchelor decay regime for unbounded systems

M. Chertkov, G. Falkovich, I. Kolokolov, V. Lebedev, Phys. Rev. E **51**, 5609 (1995)
D. Son, Phys. Rev. E **59**, R3811 (1999)
E. Balkovsky and A. Fouxon, Phys. Rev. E **60**, 4164 (1999)

1. Power spectrum of passive scalar is $E_s \propto k^{-1}$ at $k << k_B$ or for passive scalar correlation function: $\langle \theta(0)\theta(r) \rangle \propto \ln r$
2. PDFs of passive scalar concentration exhibit exponential tails, which range increases with Pe
3. Moments of PDFs decay exponentially along a channel
4. Exponents of the moments decay γ_i saturate at *i*>6. It means that PDFs are intermittent but not self-similar
5. PDFs of scalar concentration increments at different scales at the same bend have exponential tails and are self-similar
6. Structure functions of passive scalar concentration increments have logarithmic dependence on increments δr

Turbulent mixing in the Batchelor decay regime with no-slip boundaries

M. Chertkov and V. Lebedev, *PRL* ***90****, 034501 (2003)*
V. Lebedev and K. Turitsyn, Phys. Rev. E **69**, 036301 (2004)
A. Chernykh and V. Lebedev, JETP Lett. **87**, 682 (2008)
H. Salman and P. Haynes, Phys. Fluids **19**, 067101 (2007) num
G. Boffetta, F. De Lillo, A. Mazzino, J. Fluid Mech **624,** 151 (2009) sim



Influence of the boundaries on mixing:

$$\boxed{V_r \propto (d-r)^2} \qquad V_z \propto d$$

$$\langle V_r \rangle = 0$$

Boundary layer is a sink for a passive tracer

As the result the mixing is less effective than in an unbounded system, and the mixing time (or length) varies as:

$$\mathbf{t}_{mix} = \dot{\gamma}^{-1} \cdot \mathbf{Pe}^{1/4}, \qquad \text{or} \qquad \mathbf{l}_{mix} = \mathbf{d} \cdot \mathbf{Pe}^{1/4}$$

Predictions of the decay theory in peripheral region (finite width channel flow with no-slip boundaries)

1. Decay of the i-th moment $M_i \equiv \langle |\theta - \bar{\theta}|^i \rangle / \bar{\theta}^i$

$$M_i \propto \exp(-\gamma_i z / V_z), \ \gamma_i \propto \lambda_1 \cdot Pe^{-1/4}, \ \lambda_1\text{-Lyapunov exponent}$$

2. Decay becomes algebraic in transient regime, when boundary layer width δ is still wider than diffusive r_{bl} (two-stage mixing process)
3. Non-uniform spatial distribution of passive scalar: peripheral region and diffusive boundary layer r_{bl}

$$\Delta r \propto d \cdot Pe^{-1/4}$$

4. Correlation function

$$F_2(r) \propto M_2(r)^{-\beta}, \beta \propto Pe^{-1/4} << 1 \ \text{at Pr>>1}$$

and close to logarithmic dependence

5. Quantitative predictions about mixing length in a channel flow

Quantitative predictions on mixing length in the passive scalar decay

V. Lebedev and K. Turitsyn, Phys. Rev. E **69**, 036301 (2004)

In the Batchelor decay regime of mixing in a channel flow, scalar dynamics is given as

$$\partial_t \theta + V_\theta \partial_z \theta + \delta v_r \nabla_i \theta = D \nabla^2 \theta, \qquad (1)$$

V_θ is average velocity along channel;

$\delta v_r \equiv V_r - \langle V_r \rangle$ is fluctuating part of transversal velocity

In 2D flow in channel $\langle V_\theta \rangle$ is independent of z, $\langle V_\theta \rangle \propto r$ and $\delta v_r \propto r^2$ near the wall

Then one gets

$$sr \partial_z \langle \theta \rangle = \left[\mu \partial_r r^4 \partial_r + D \partial_r^2 \right] \langle \theta \rangle, \qquad (2)$$

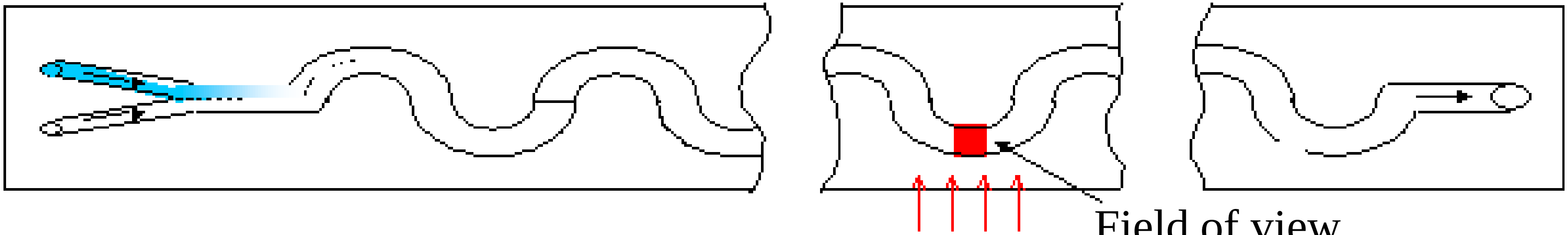
$$\langle V_\theta \rangle = sr; D_{rr}(r) \equiv \mu r^4 = \int_0^\infty \langle \delta v_r(t,r) \delta v(0,r) \rangle dt$$

In the first stage peripheral region has width $\delta = s/(\mu z) >> r_{bl}$ and at the second stage $\delta = r_{bl} = (D/\mu)^{1/4}$

Then for the first moment of passive scalar PDF at large z one gets for δ-correlated velocity

$$\langle \theta \rangle \propto \exp\left(- l_{mix}^{-1} z\right), l_{mix}^{-1} \approx 3.72 D^{1/4} \mu^{3/4} s^{-1} \qquad (3)$$

Mixing in a channel flow: experiment



Coordinate along the channel - N, the number of bend
Curved flow trajectories are crucial for the elastic turbulence

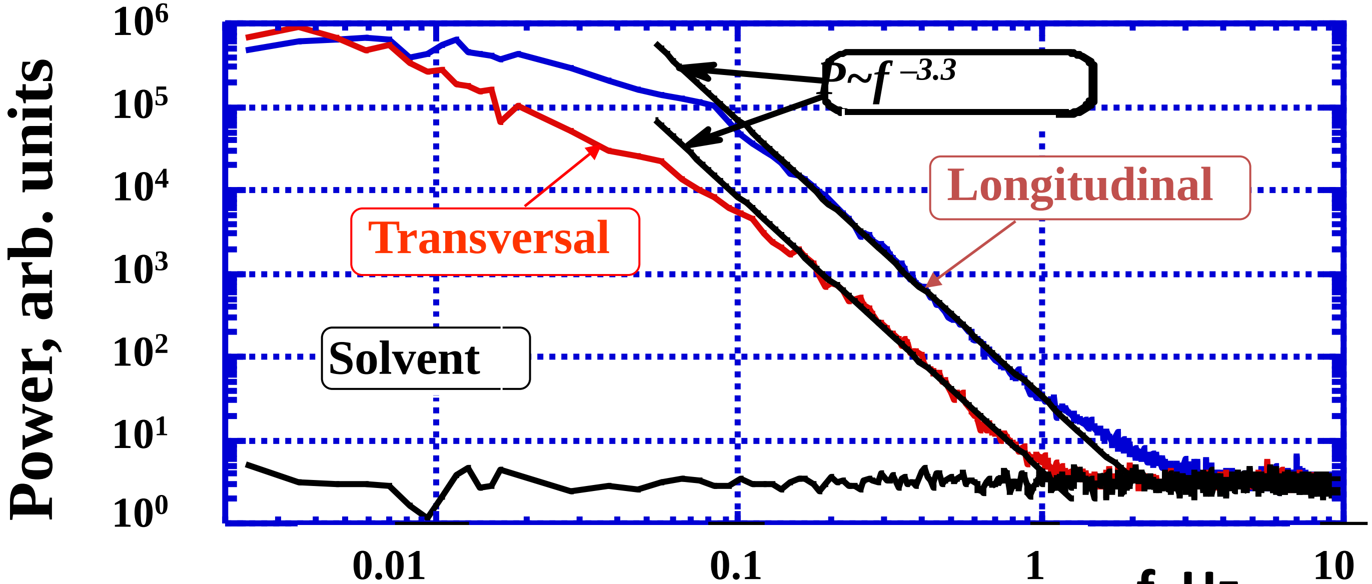
Solution: 80 ppm PAAm, (M=18Mda), 65% sucrose –water solvent with viscosities
$\eta_s = 113.8 mPas, \eta = 137.6 mPas$ of solvent and solution, respectively
The longest polymer relaxation time λ=11.5 s

Dye (2 ppm)	D (mm^2/s)	Wi	Pe=$(\partial V_r/\partial r)^{rms} d^2/D$
Fluorescein	3.1×10^{-6}	354	2.5×10^{6}
Fluorescein	3.1×10^{-6}	1006	3.6×10^{6}
FITCD 40Kda	3.1×10^{-7}	354	2.4×10^{7}
FITCD 40Kda	3.1×10^{-7}	1224	4×10^{7}
FITCD 2Mda	5.4×10^{-8}	1224	2.3×10^{8}

ν=150 mm^2/s
Sc up to 3×10^9

A. Groisman & V.Steinberg, ***Nature* 410**, 905 (2001)-3 mm-CCD camera/photo-objective
T. Burghelea, E. Segre, and V. Steinberg, *PRL* **92**, 164501 (2004)-100 μm-confocal microscope
Y. Jun and V.Steinberg, *Phys. Fluids* **22,** 123101 (2010)-1mm-CCD camera/microscope objective

Spectra of velocity fluctuations, measured in the center of the channel with LDV (N=12) in 3mm channel



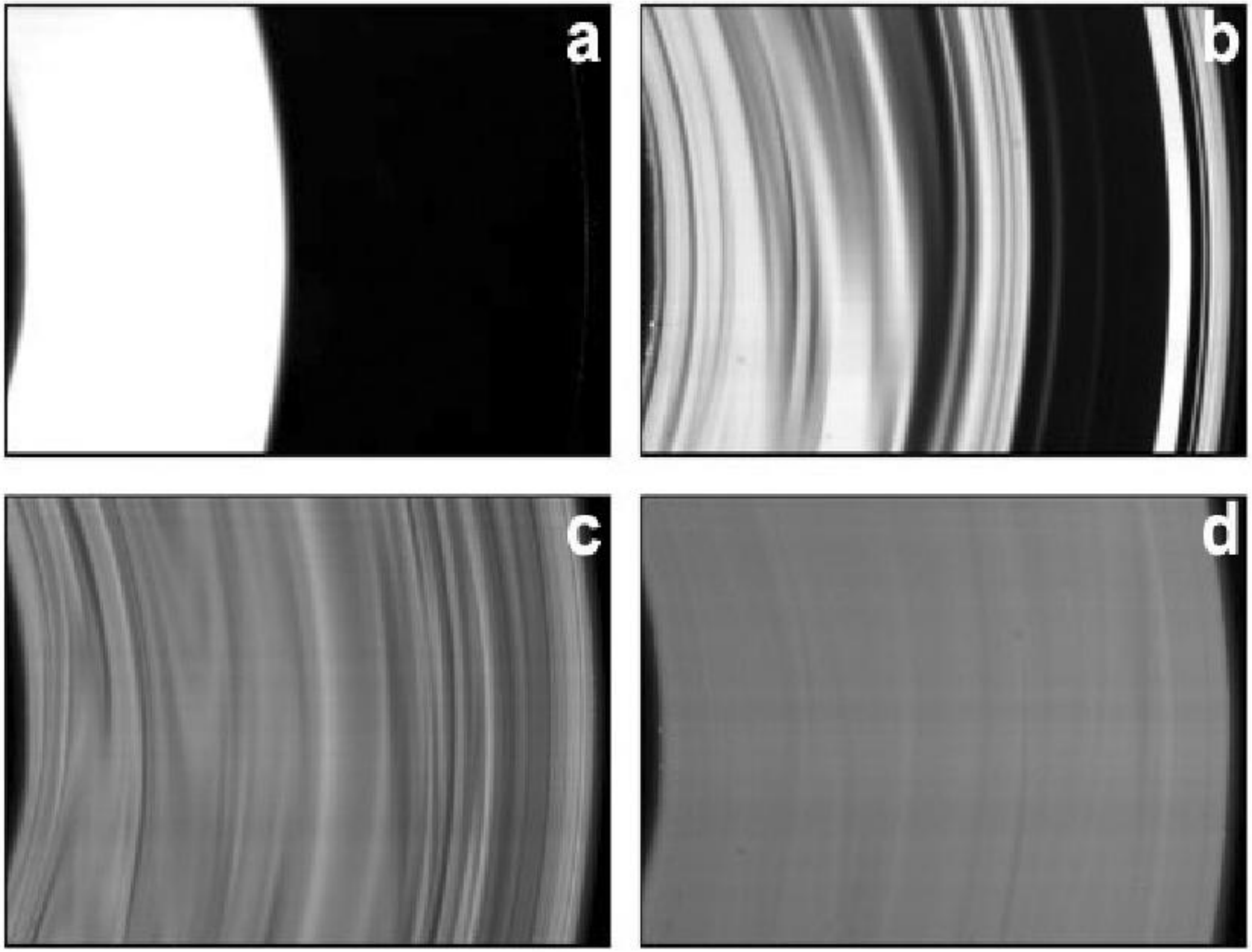
Average velocity - V = 6.6 mm/s;
fluctuations: longitudinal - V_{rms} = 0.09V, transversal - V_{rms} = 0.04V.

<u>Taylor hypothesis:</u> velocity fluctuations in time are due to fluctuations in space $\Rightarrow$ $\mathbf{P \sim k^{-3.3}}$

<u>The Batchelor regime of mixing!</u>

Mixing in 3 mm channel

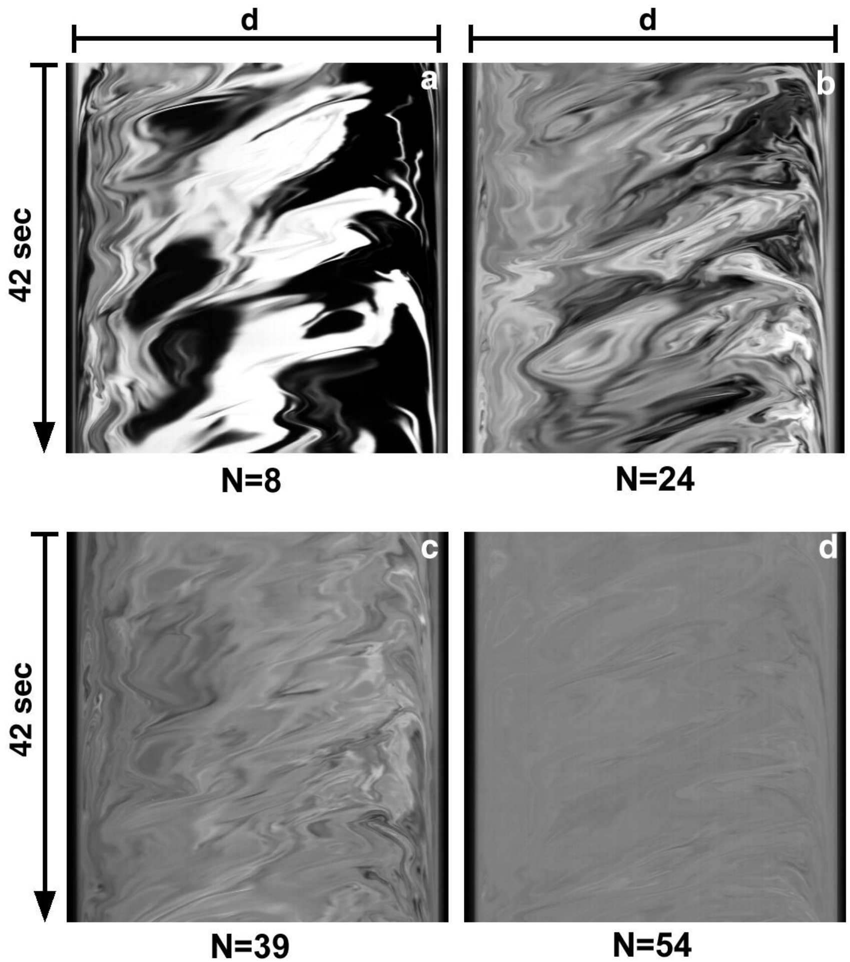
A.Groisman and V. Steinberg, *Nature* **410**, 905 (2001)



Photographs of the flow taken with laser sheet at different N. Bright regions correspond to high fluorescent dye concentration

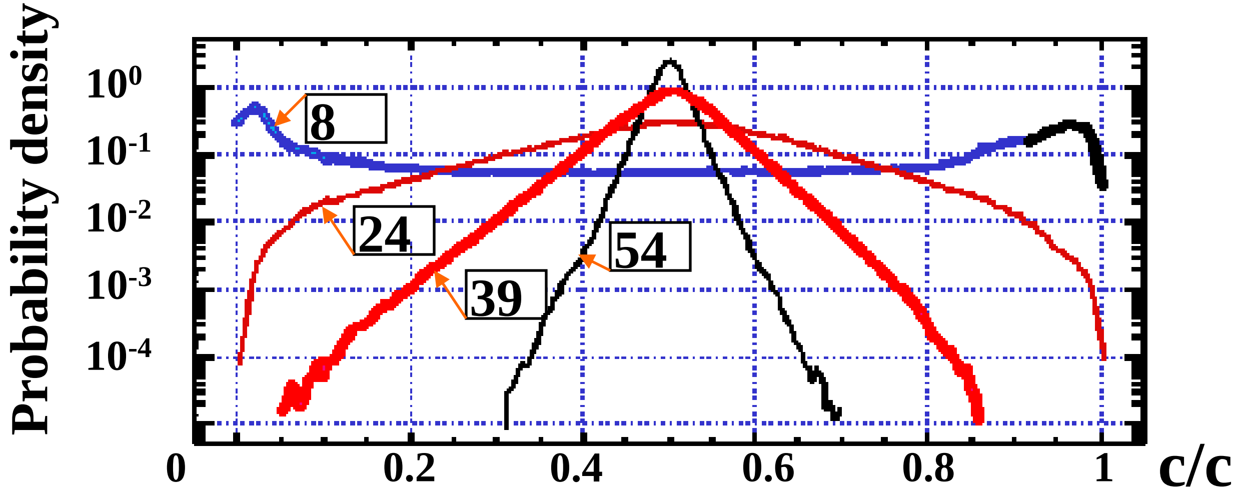
(a) N=29-pure solvent
(b) N=8
(c) N=29
(d) N=54
(polymer solution)

Space-time plots at different positions, N

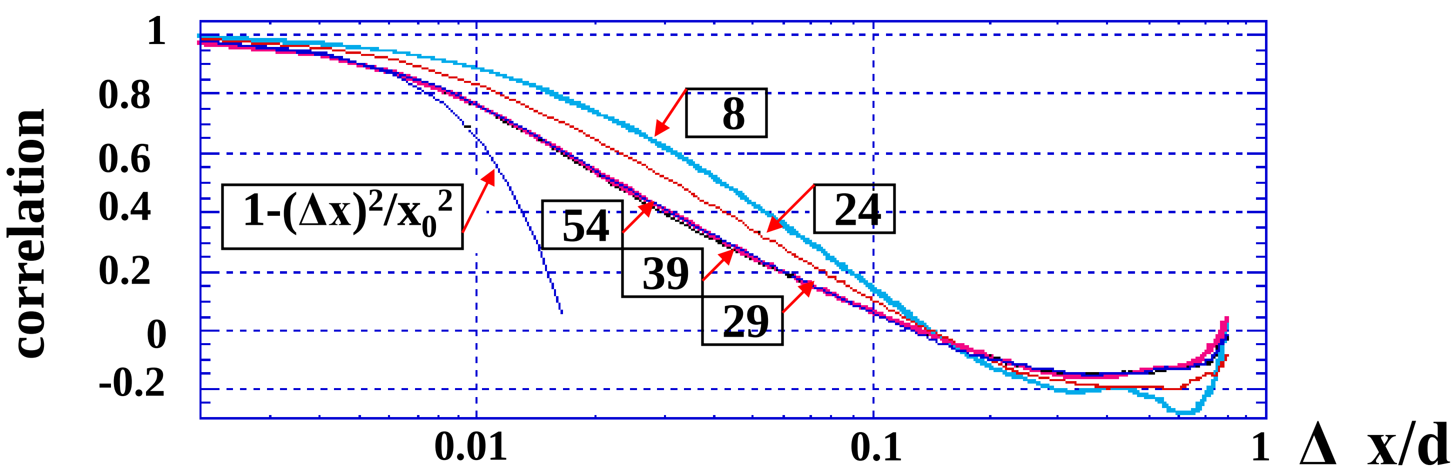


As fluid advances downstream and gets mixed, the contrast decreases but the characteristic size of the structures does not seem to change very much

Probability density functions for the passive scalar in a point at different positions, N, along the channel (3 mm channel)



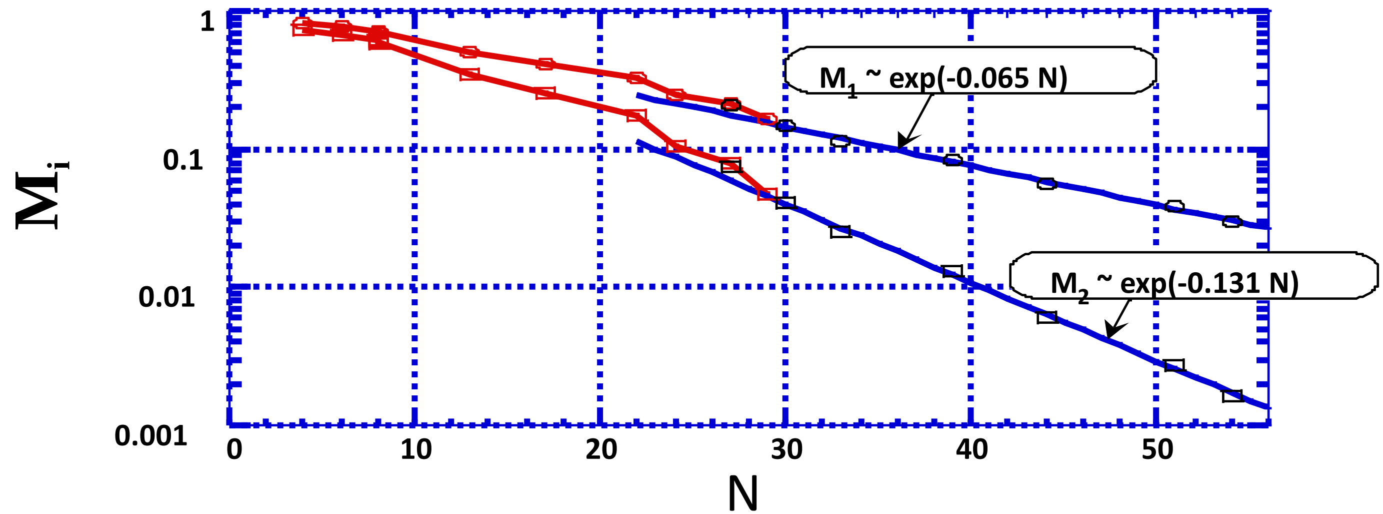
Spatial correlation of the concentration across the channel at different N. Each line - statistics over about 10^7 points, and over the total discharge of $2 \cdot 10^3\ d^3$



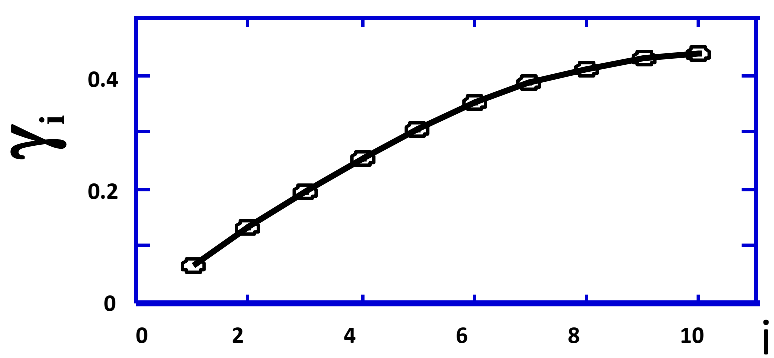
$x_0 = 50\ \mu m$; molecular diffusion scale $(D/(V_{rms}/d))^{1/2} = 25\ \mu m$, illuminating light sheet - 40 μm thick

Dependence of the moments, M_i, of the passive scalar PDF on the position, N, along the channel (3 mm channel)

$$M_i \equiv \langle |\theta - \bar{\theta}|^i \rangle / \bar{\theta}^i$$



Exponential decay of the moments with the position, N, along the channel and with the mixing time, $t = 7.6\ 2\pi\ 4.5/V = 21.5$ ***sec.***
Mixing time in the flow – 21.5 sec; diffusion time $d^2/D \approx 10^5$ sec. $\mathbf{M_i \sim exp(-\gamma_I N)}$ was also found for higher order moments.

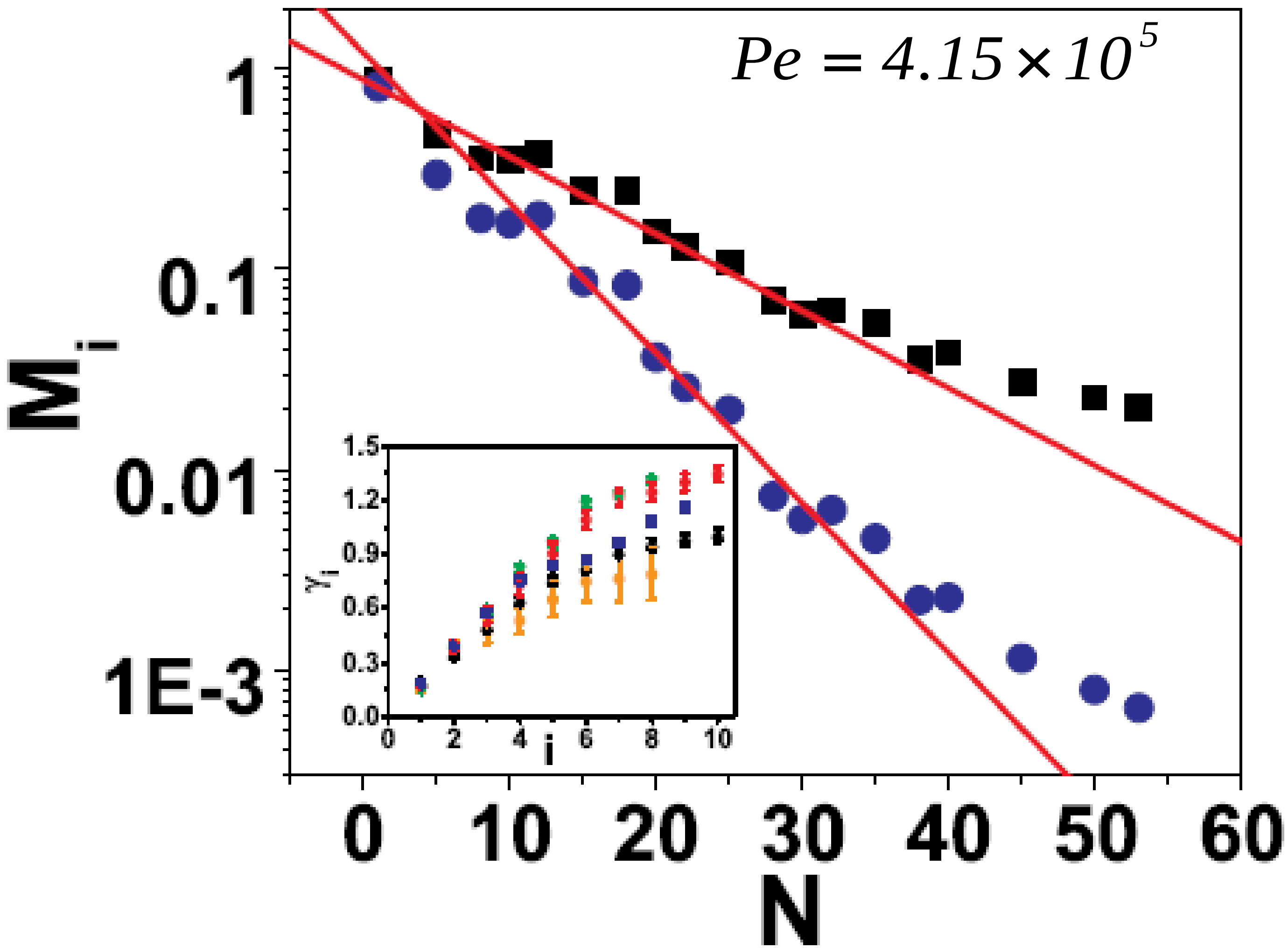


Dependence of the exponents, γ_i on the order, *i*, of the momentum

Linear growth of γ_i at small *i* and saturation at larger *i*, in complete agreement with the theoretical predictions

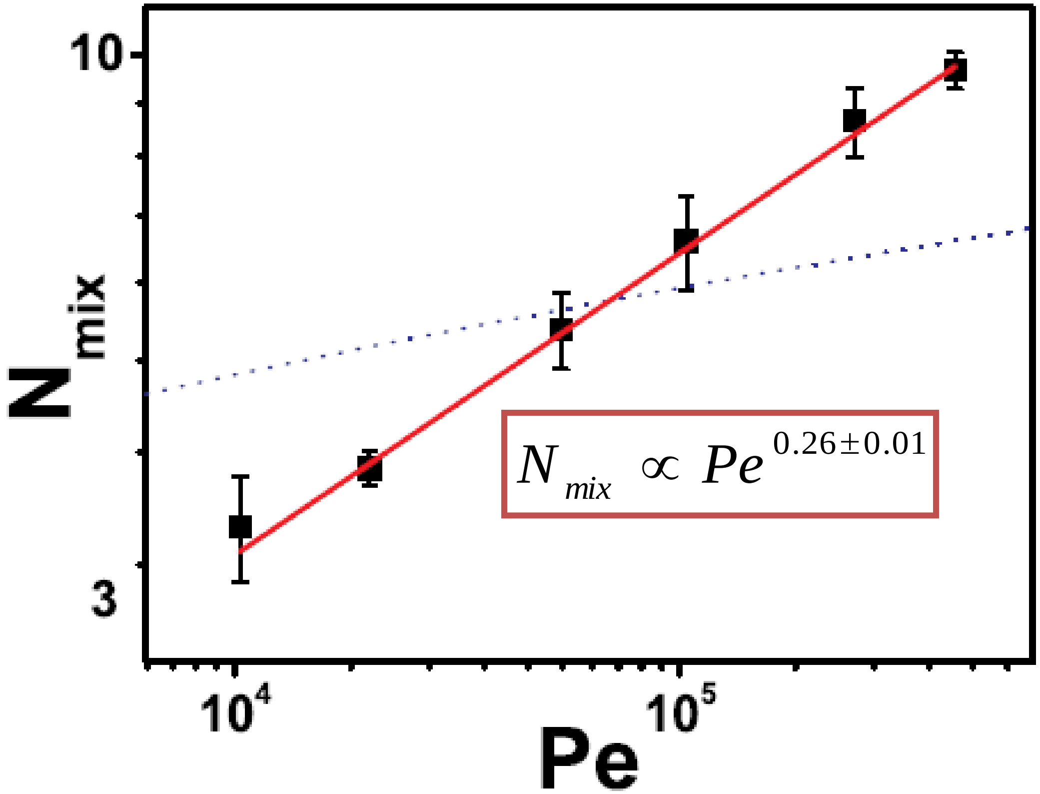
Moments of the scalar PDF decay exponentially downstream (100 μm channel)

T. Burghelea, E. Segre, and V. Steinberg, *PRL* **92**, 164501 (2004)



Inset: decay exponents of i-th moments at different Pe

Mixing length as function of Peclet number
(100 μm channel)

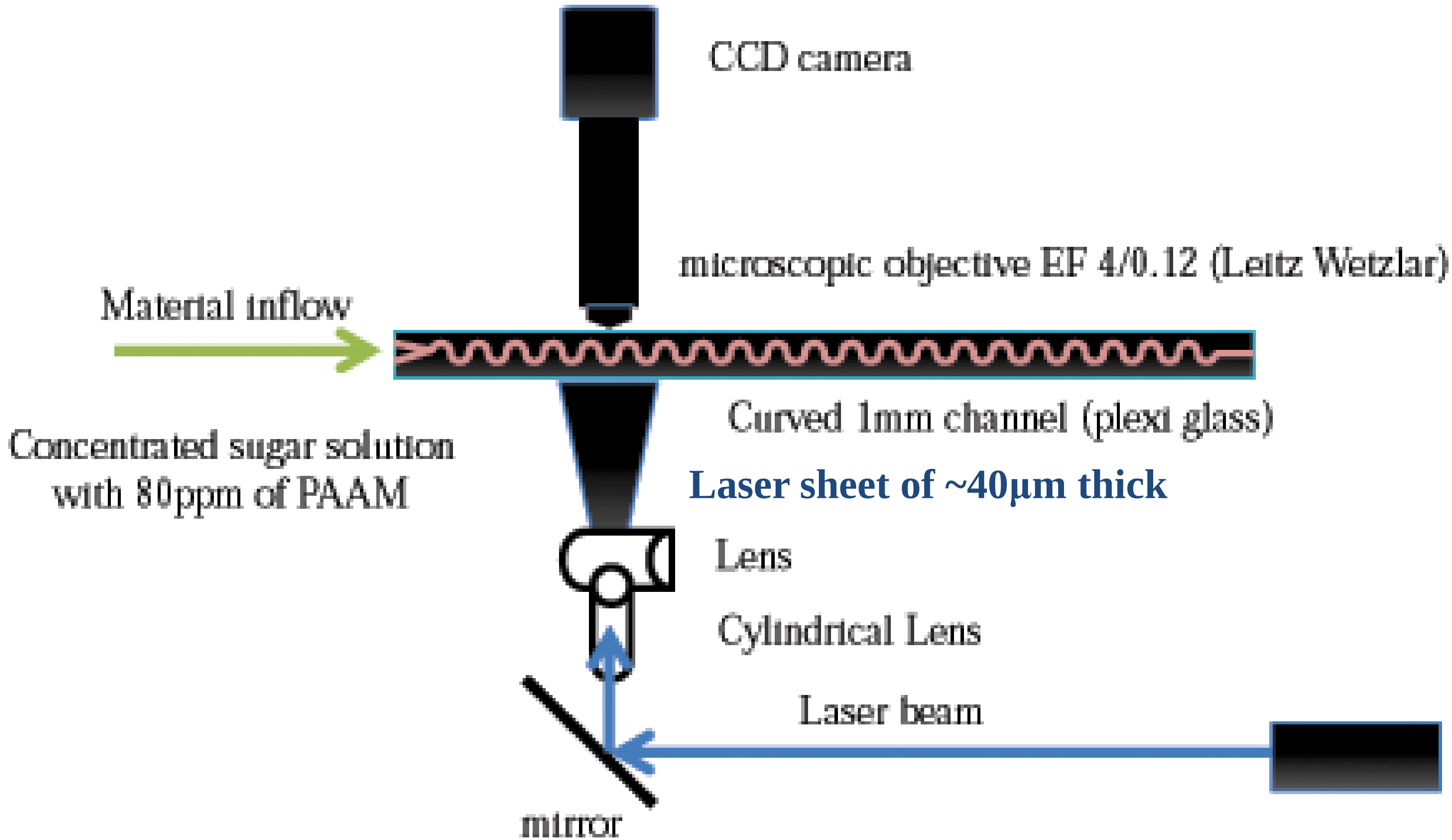


Batchelor regime in the bounded system: boundary layers are a sink for the tracer. This affects scaling: theory--Chertkov and Lebedev (2003), $L_{mix} \propto Pe^{1/4}$ instead of lnPe in unbounded system. We get in the experiment with 100 μm channel

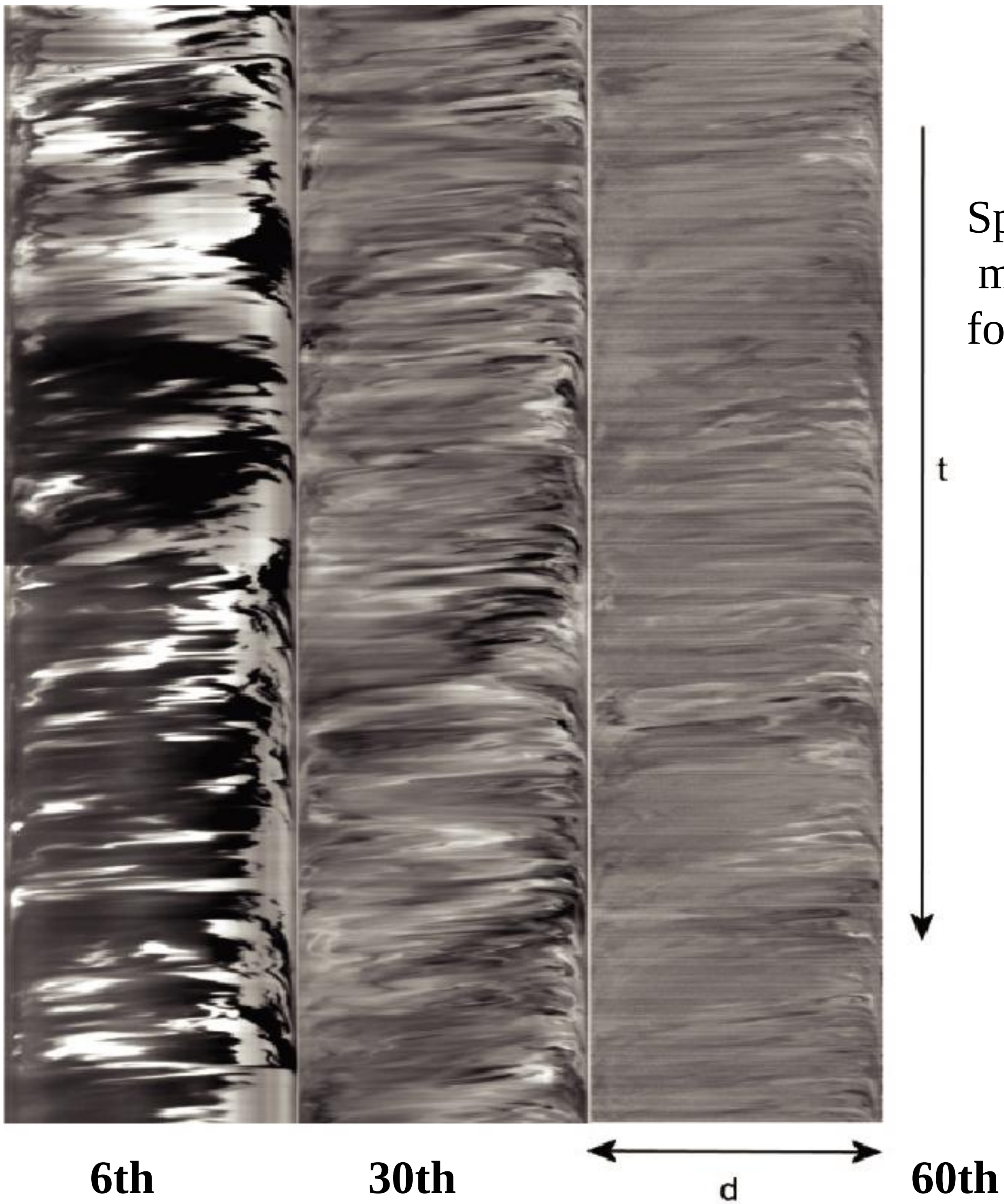
$$N_{mix} \propto Pe^{0.26\pm0.01}$$

Schematic drawing of experimental set-up with 1 mm channel

Y. Jun and V.Steinberg, *Phys. Fluids* **22,** 123101 (2010)

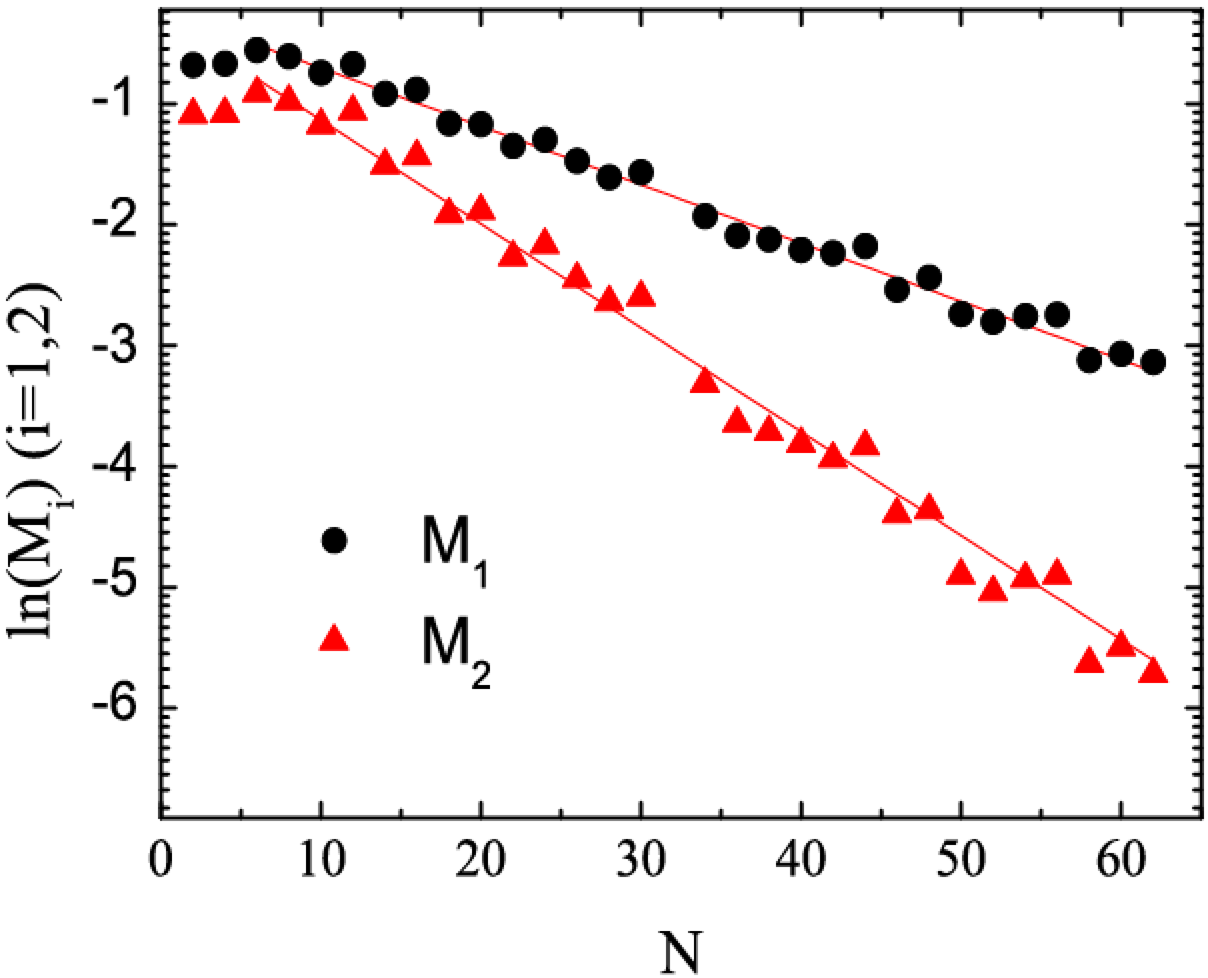


PIV with 2 µm red fluorescent particles and passive scalar concentration measurements



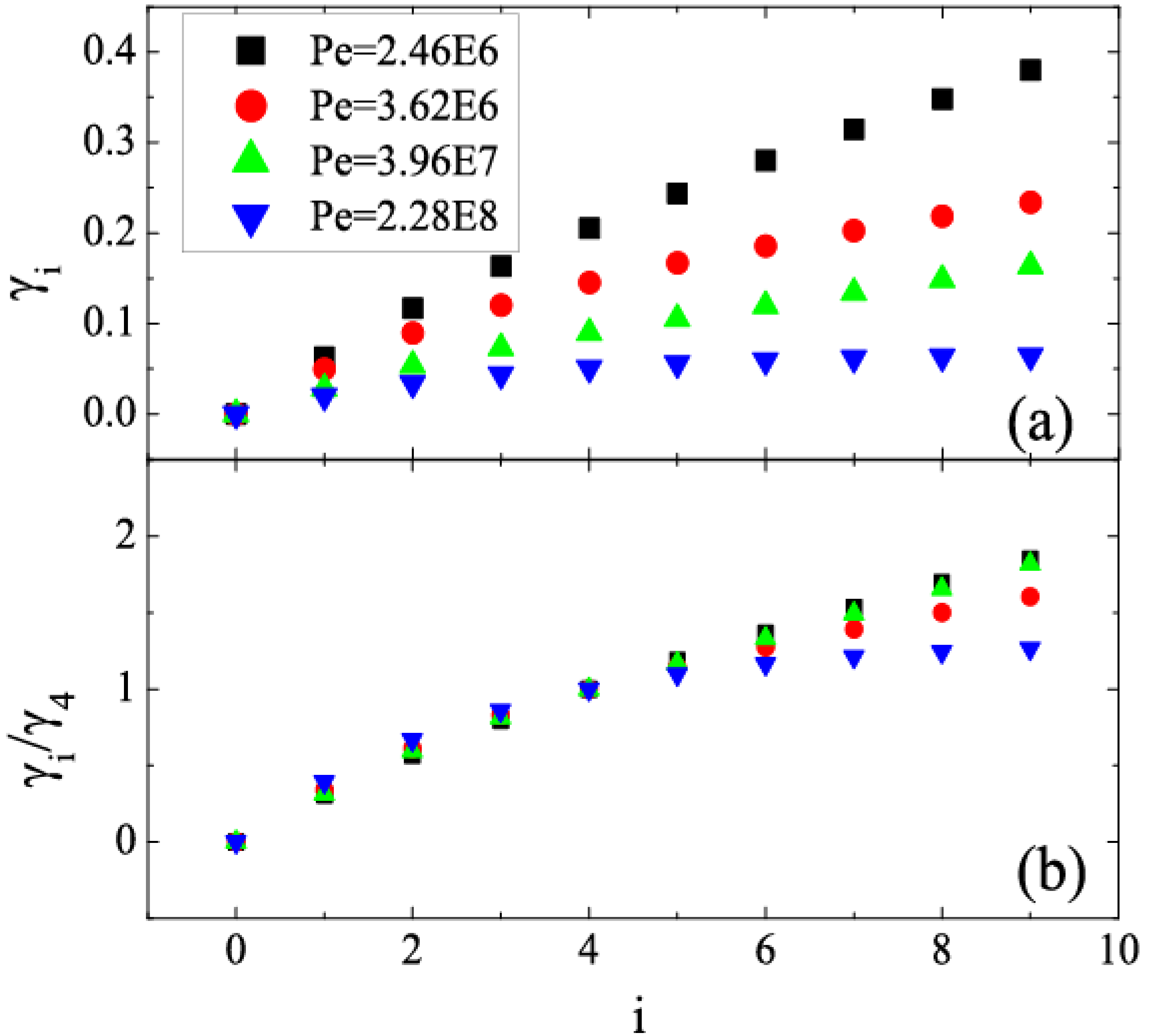
Space-time plots of time series of mixing across the channel width for 6, 30, and 60 bends

First and second moments of PDfs of fluorescent intensities as a function of N

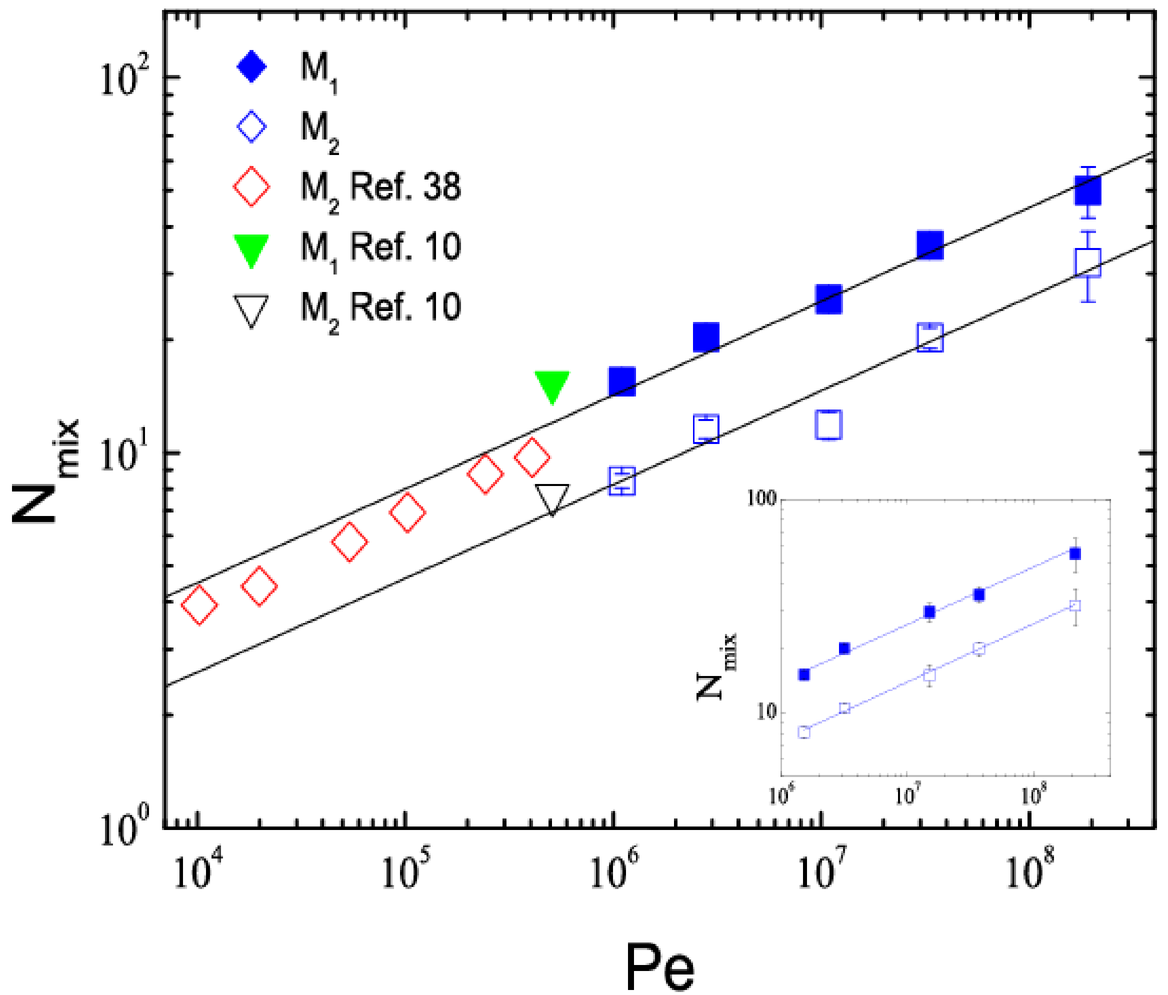


For Wi=354 and Pe=3.62x10^{8}

Decay exponents versus the number of the moment ***i*** *(a)* and normalized values (b) for 4 values of ***Pe***



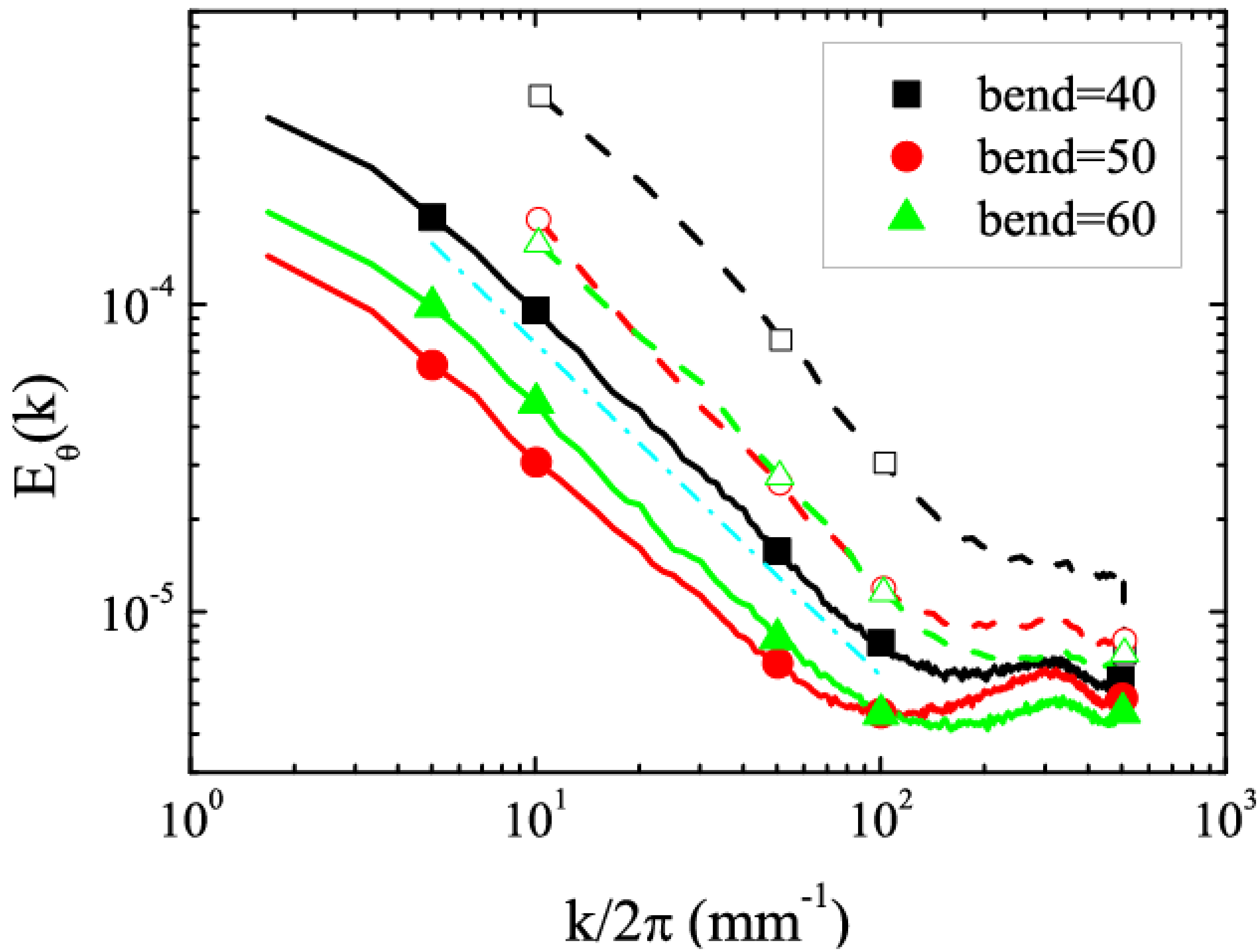
Mixing length as a function of ***Pe*** in the bulk (r/d=0.2-08) for three experiments



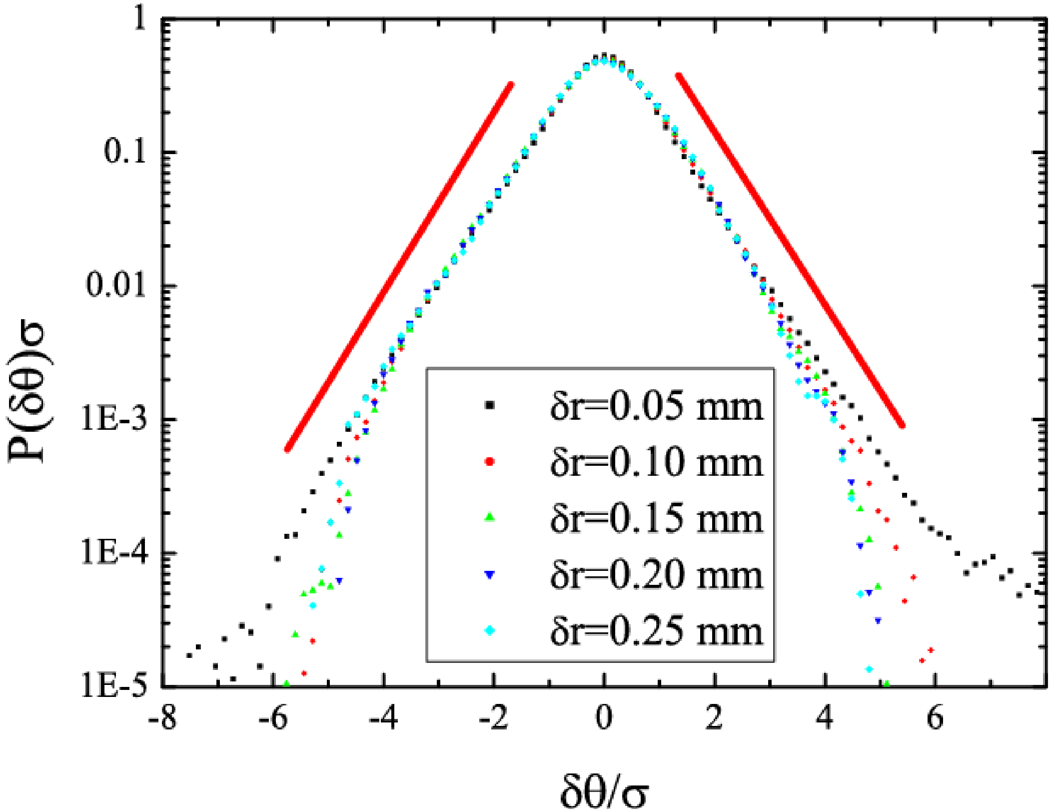
Inset: the data only for 1 mm channel $\boldsymbol{N_{mix}}$ vs $\boldsymbol{Pe}=(\partial V_r/\partial r)_{mix}d^2/D$

Linear fits have slopes of **0.27±0.02** for $\mathbf{M_1}$ and **0.27±0.01** for $\mathbf{M_2}$

Spatial scalar power spectra

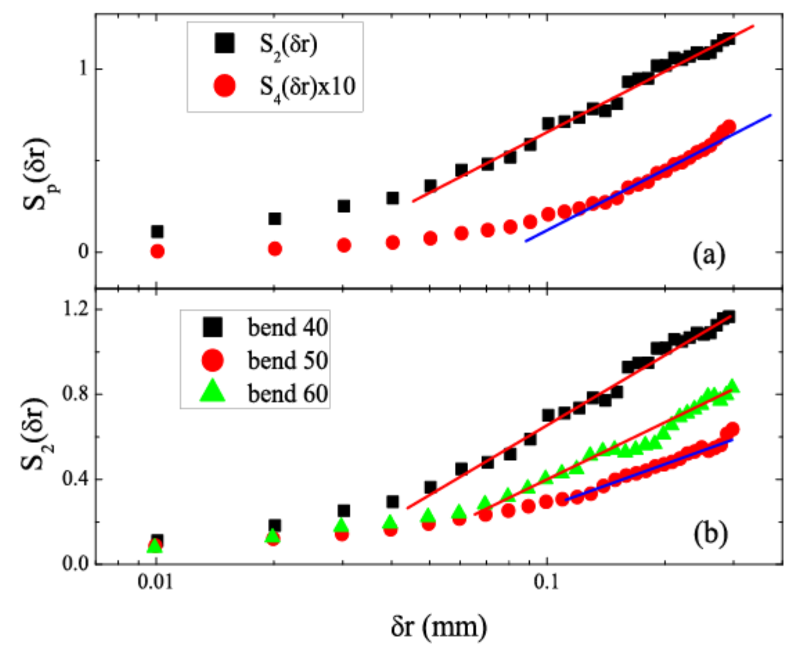


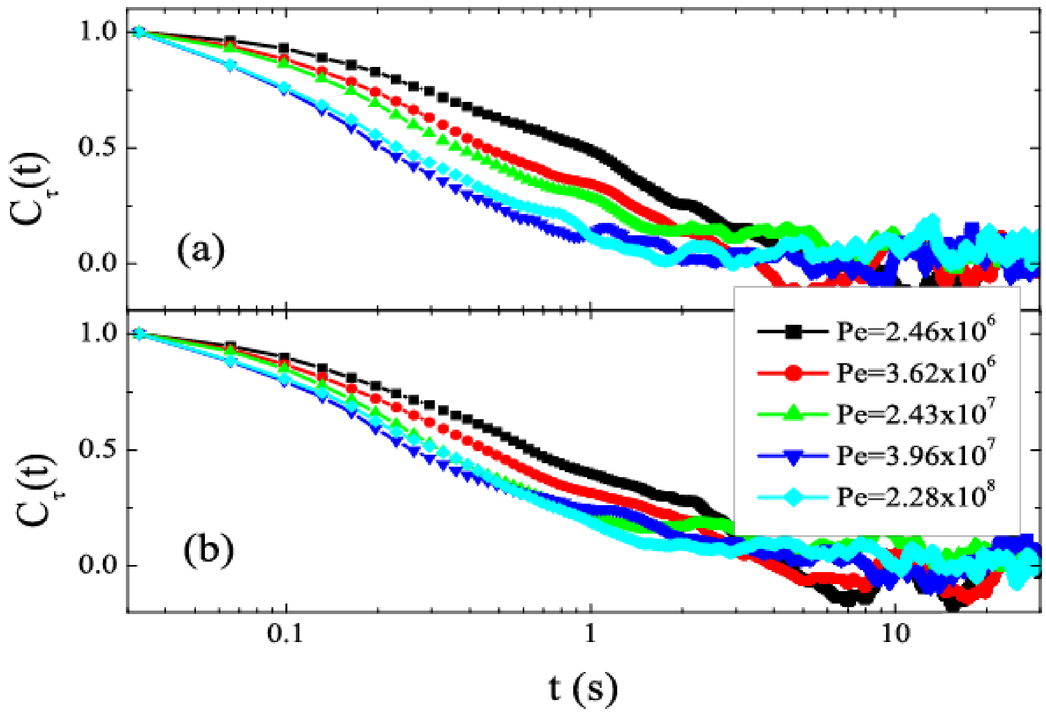
at N=40, 50 and 60 bends and Pe=2.28x10^8 in the bulk (full lines) and peripheral region (dashed lines). Dash-dotted line is power-law fit $k^{-\alpha}$ with α=1.08



PDFs of passive scalar increments at different δr for N=40 and Pe=2.28x10^8

(a) Second and fourth moments of PDFs
(b) Second moments at N=40,50 and 60



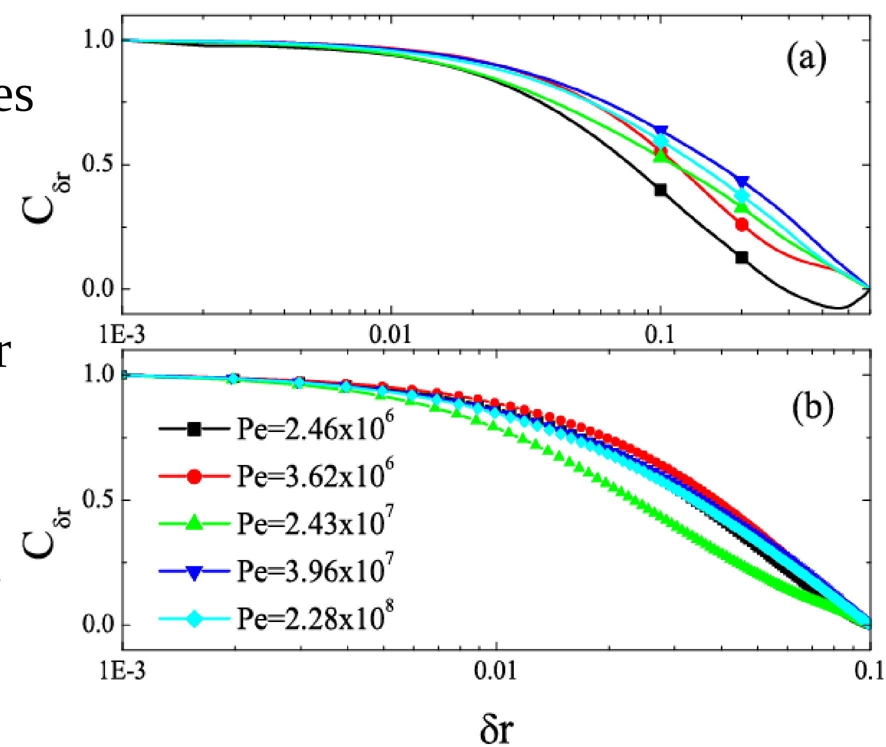


Time correlation functions at N=40
And 5 values of Pe in the bulk (a)
and the same in peripheral region (b)

Space correlation functions at N=40 for 5 values of Pe in **linear-log** coordinates in the bulk (a) and peripheral region (b)

Logarithmic decay is obvious and corresponds to **$\sim k^{-1}$** decay in spatial power spectra for scalar

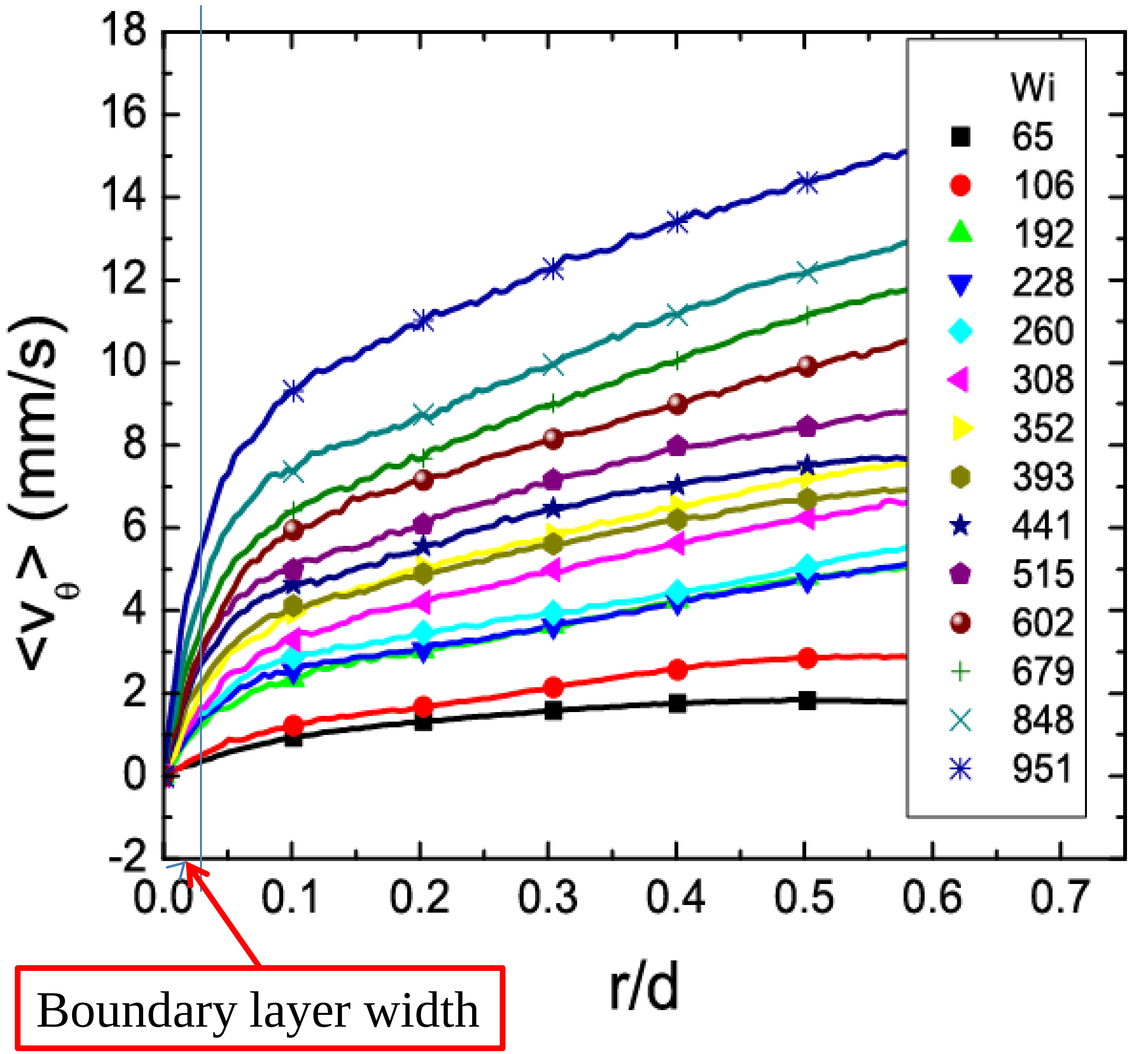
One cannot resolve difference between log – dependence from $C_{\delta r} \sim (\delta r)^{-\beta}$ with $\beta \sim Pe^{-1/4}$ at *Pe>>1 (Pe~* 10^8 *)*, suggested by recent theory for bounded system



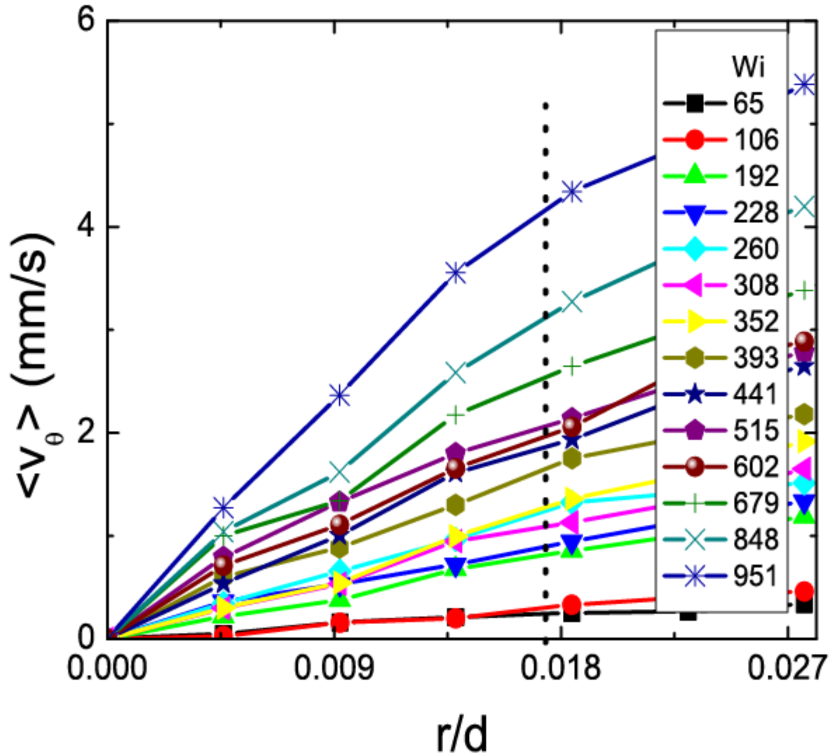
Quantitative experimental determination of mixing length in the decay Batchelor regime

Y. Jun and V.Steinberg, *Phys. Fluids* **22,** 123101 (2010)

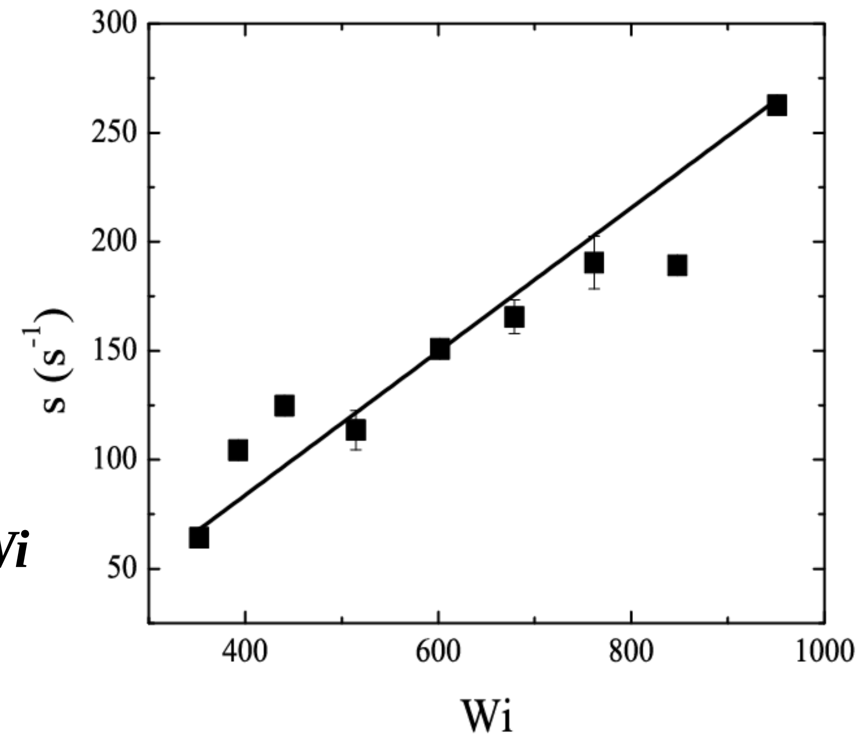
Mean longitudinal velocity profiles across the channel



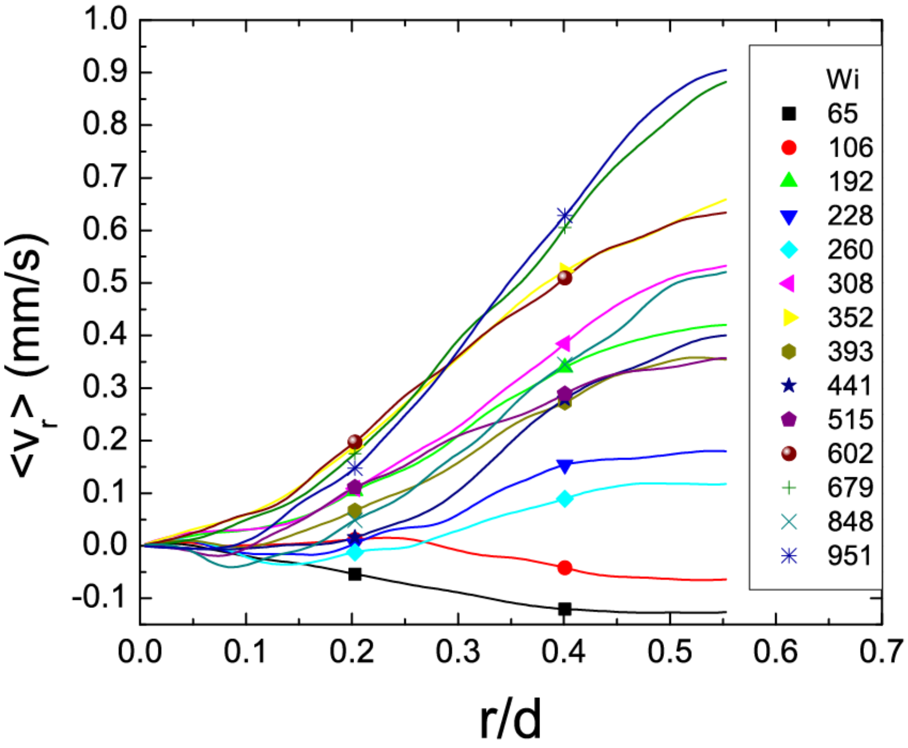
At high ***Wi***> $\boldsymbol{Wi_c}$ the boundary layer is obviously visible



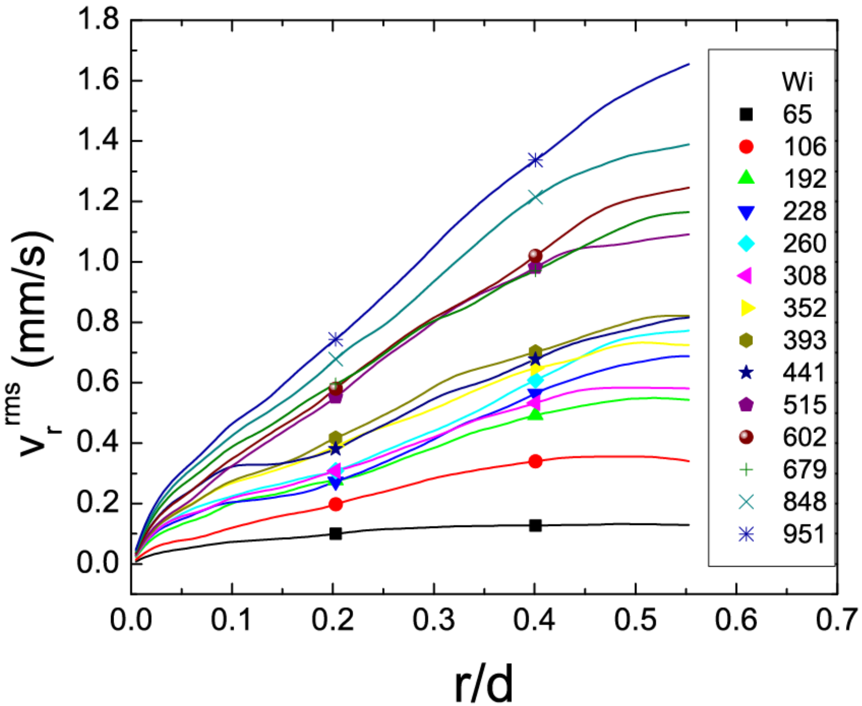
Mean longitudinal velocity profiles in the narrow boundary layer region near the wall. Vertical dotted line indicates range of linear fitting to get the slope ***s***



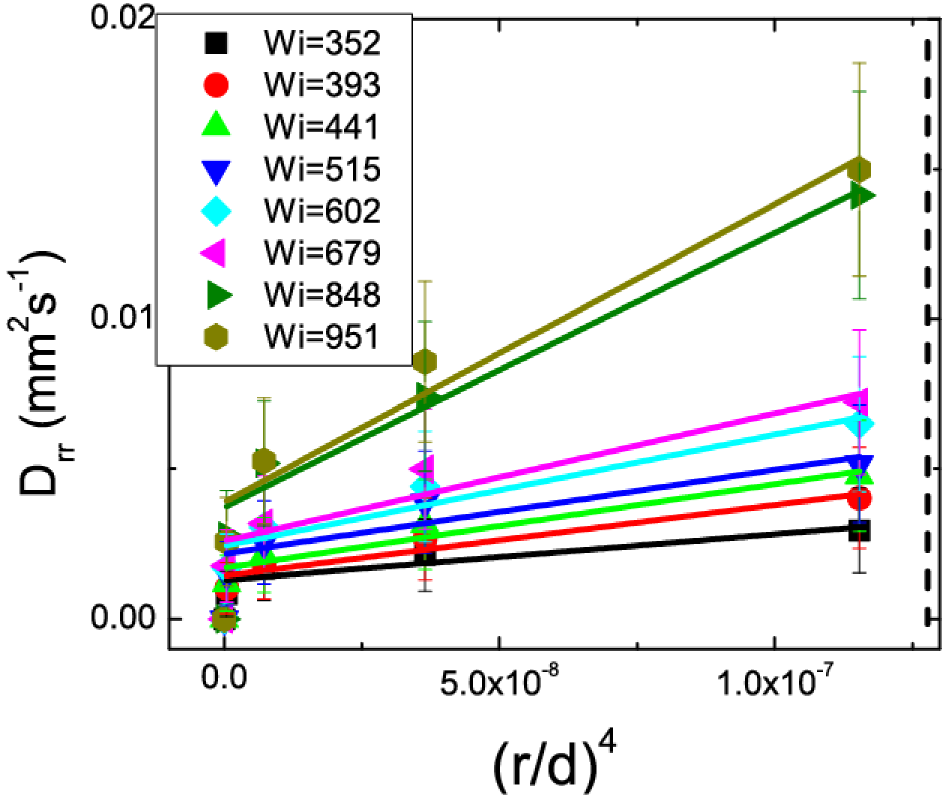
Slope ***s*** vs ***Wi***. Linear fit is ***s =-48+0.329 Wi***



Mean transversal velocity profiles across the channel for various Wi



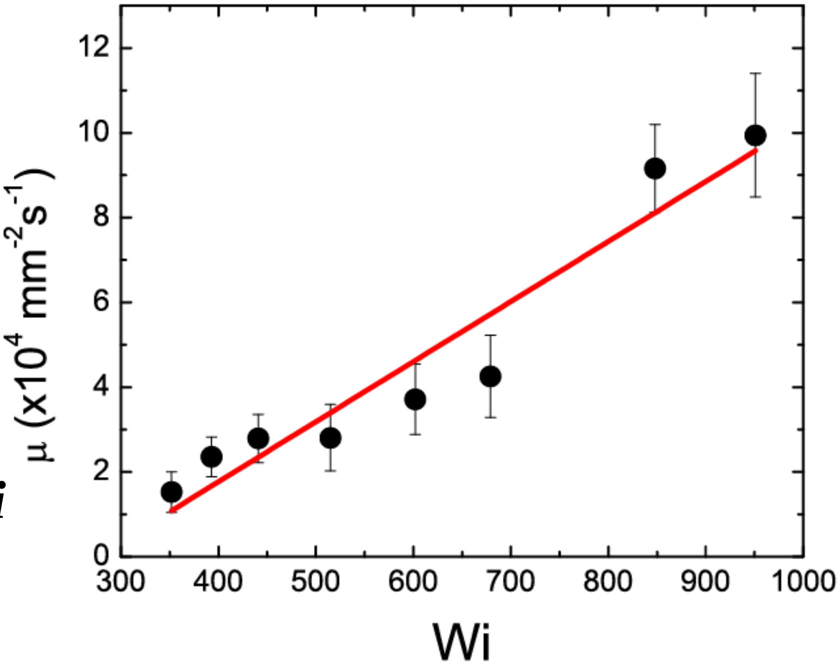
RMS fluctuations of transversal velocity profiles across the channel for various Wi



Transverse component of turbulent diffusion tensor $\boldsymbol{D_{rr}}$ vs $\boldsymbol{(r/d)^4}$
Lines are linear fit with slope **μ**. Vertical dotted line defines the range of fitting in the boundary layer.

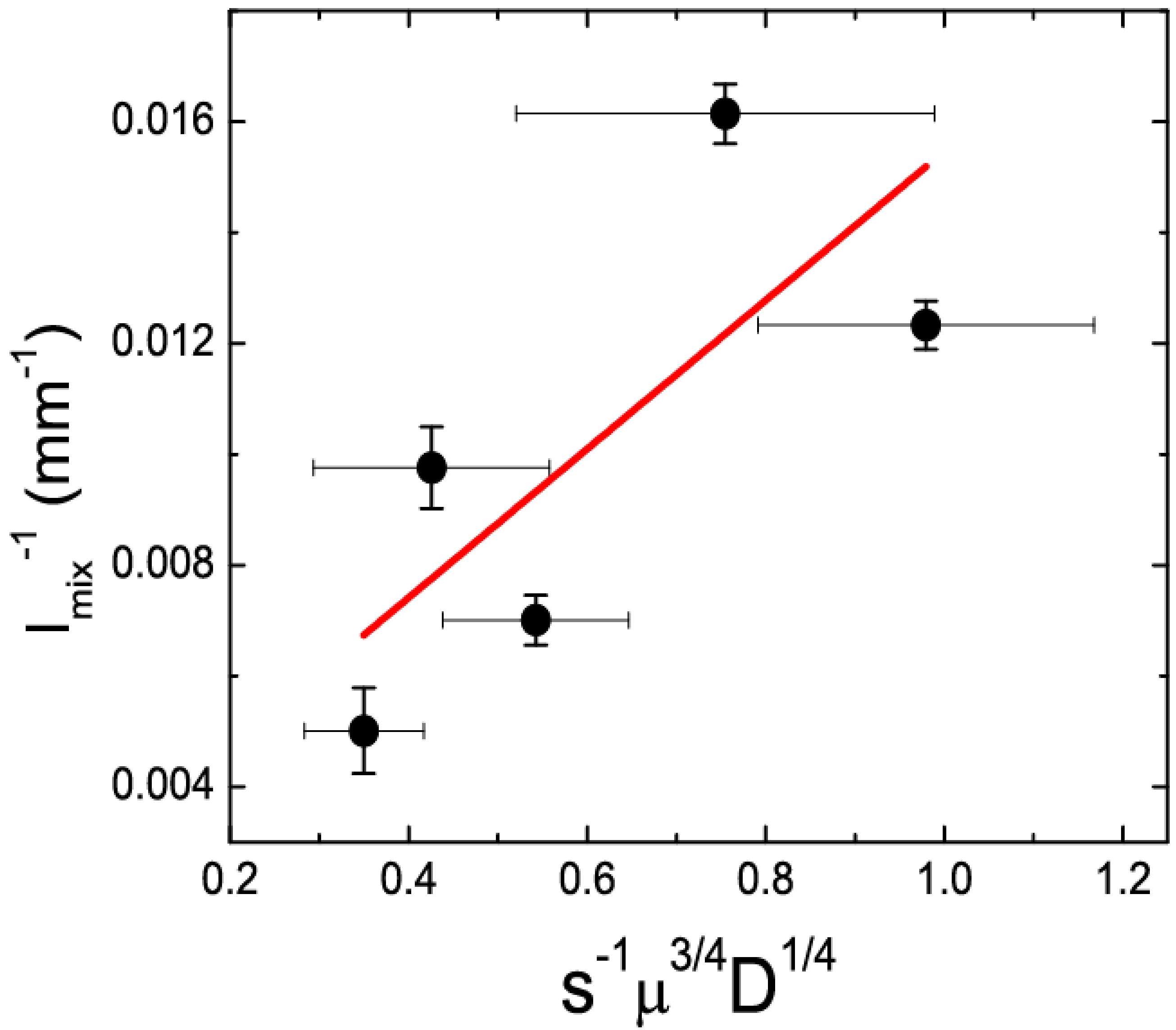
$$D_{rr}(r) \equiv \mu r^4 = \int_0^\infty \langle \delta v_r(t,r)\delta v(0,r)\rangle dt$$

$$\delta v_r \equiv V_r - \langle V_r \rangle$$



Slope **μ** vs ***Wi***. Linear fit is ***μ=-38.9+141.7 Wi***

Inverse mixing length vs parameters of the problem



Linear fit to the data gives coefficient to compare with theory: C=0.013

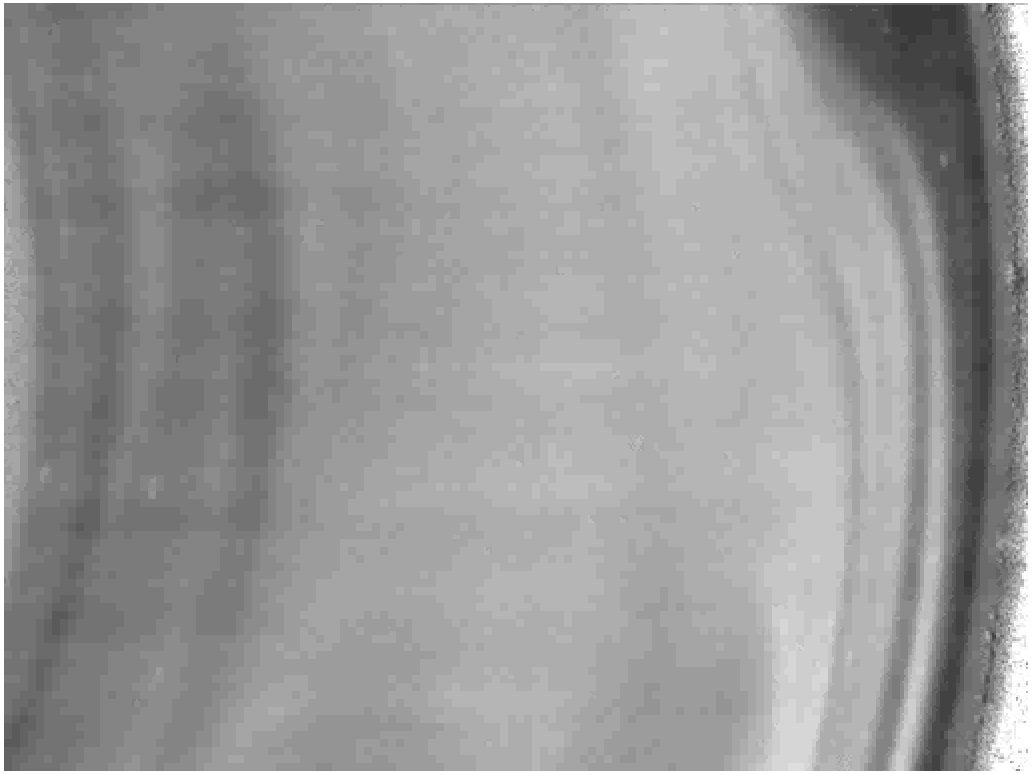
Comparison with theoretical predictions:

Theory predicts C=3.72

Experiment gives C=0.013±0.007 ?

Important difference between theory and the experiment is jets (plumes) observed in numerical simulations and experiment

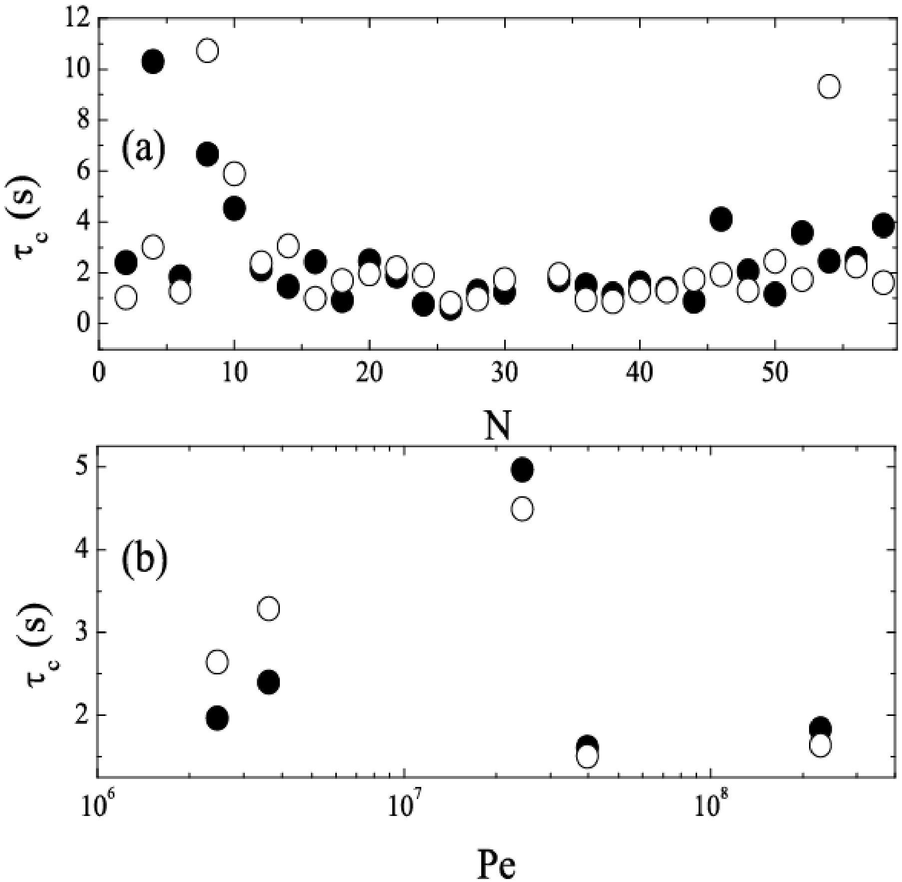
A. Chernykh and V. Lebedev, *JETP Lett*. **87**, 682 (2008) numerical
H. Salman and P. Haynes, *Phys. Fluids* **19**, 067101 (2007) simulations



Dimension of the image 1x0.768 mm^2

Y. Jun and V.Steinberg, *Phys. Fluids* **22,** 123101 (2010)

Correlation time of passive scalar vs N for Pe= 3.62x10^6 (a), and vsPe at N=40 (b) in both bulk (solid) and peripheral (open)



The scalar correlation time is of the order of velocity correlation time! So δ-correlated in time velocity field approximation is not valid

Conclusions

- All qualitative and functional dependencies of statistical and scaling properties of passive scalar behavior in the Batchelor decay regime were verified
- The main result of the quantitative test of the value of the mixing length shows huge discrepancy with theoretical predictions on the coefficient. It is of about 200 times smaller in the experiment that means that the experimental mixing length is about 200 times larger
- It was suggested two possible sources of this discrepancy:

(i) assumption made in the theory about δ-correlated velocity field is in odds with the experimental measurements

(ii) probably the most relevant observation in this respect is jets, rare localized and vigorous ejection of scalar trapped near the wall into the peripheral and bulk, which drastically reduces mixing efficiency and increases mixing length