Optical Turbulence in Nonlinear Schrödinder Equation

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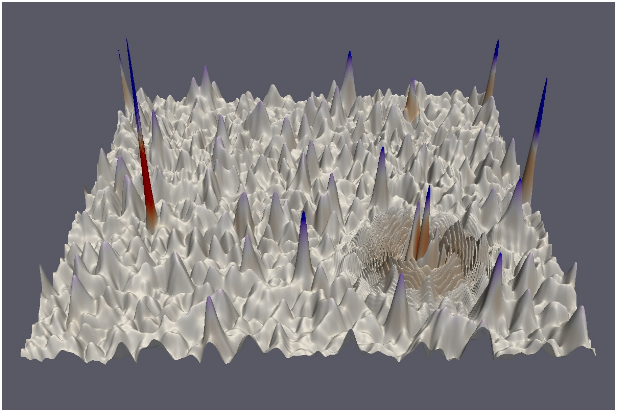
Nonlinear Schrödinger equation (NLS)

$$i\psi_t + \nabla^2\psi + |\psi|^2\psi = 0$$

- application: propagation of light through nonlinear media
- $\psi(x, y, t)$ — envelope of E
- focusing nonlinearity
- stabilization and forcing:

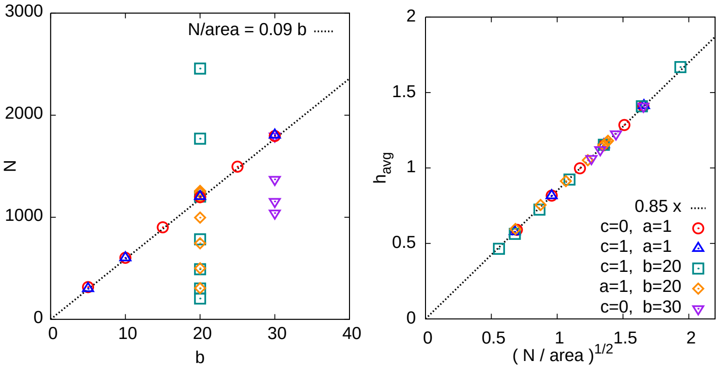
$$i\psi_t + (1 - i\epsilon a)\nabla^2\psi + (1 + i\epsilon c)|\psi|^2\psi = i\epsilon b\psi$$

Optical turbulence questions



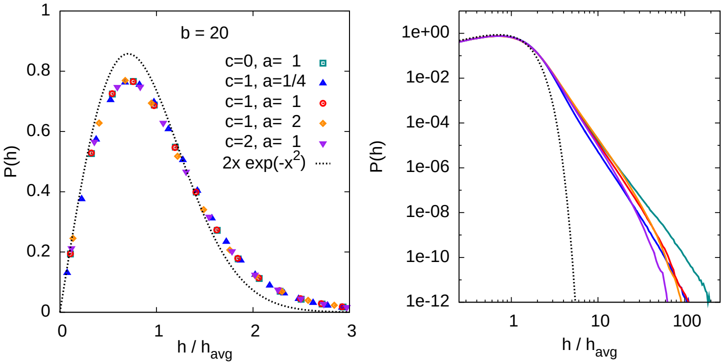
1. What is the statistics of collapse maximums?
2. How individual collapses are stabilized?
3. What is the statistics of $|\psi|$ a turbulent field?
   Can we explain (3) combining (1) and (2)?
   Can we predict the dependence on a, c, and b?

Parameter space: a, c, and b ($\epsilon = 0.01$).



- Notations: $h \equiv |\psi|$, $h_{avg} = \langle |\psi| \rangle$, $N = \int |\psi|^2 d^2 r$.
- No obvious scalings with a, c, or b...
  ...except $N \propto b$ and $h_{avg} \propto \sqrt{N}$.
- We will show that h_{avg} well describes background turbulence.
- What determines h_{avg}?

Distribution of $|\psi|$ in the field scales with h_{avg}



- PDF of $|\psi|$ scales with h_{avg}.
- Universal shape for $h \sim h_{avg}$, power-law (?) tails for $h \gg h_{avg}$.

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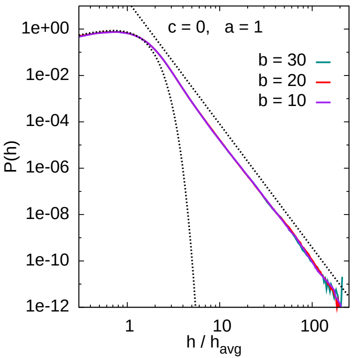

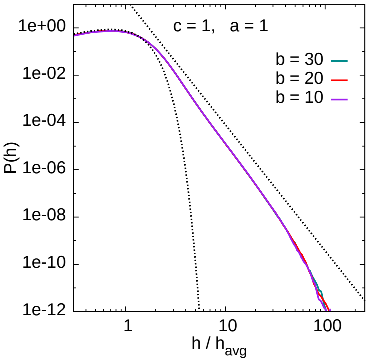

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- The tails do not depend on b.

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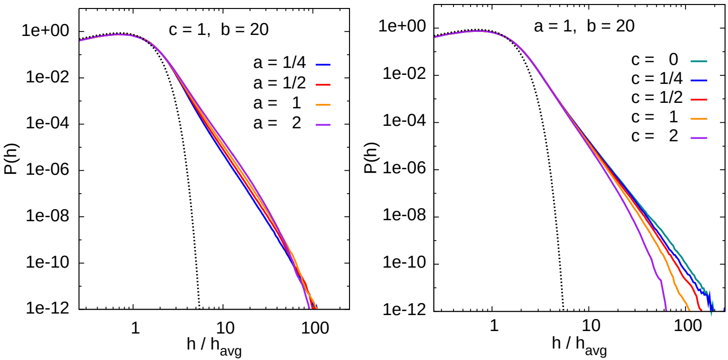
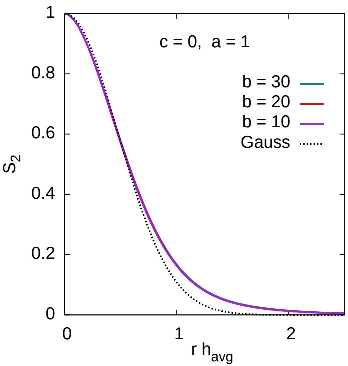
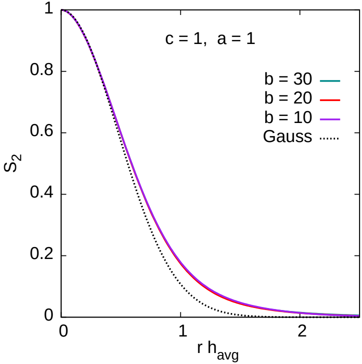


- PDF of $|\psi|$ scales with h_{avg}.
- Universal shape for $h \sim h_{avg}$, power-law (?) tails for $h \gg h_{avg}$.
- The tails do not depend on b.
- The tails do depend on a and c.

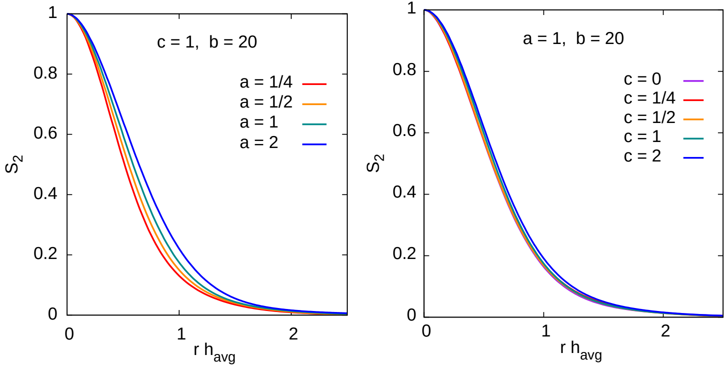
Spatial correlation function scales with h_{avg}





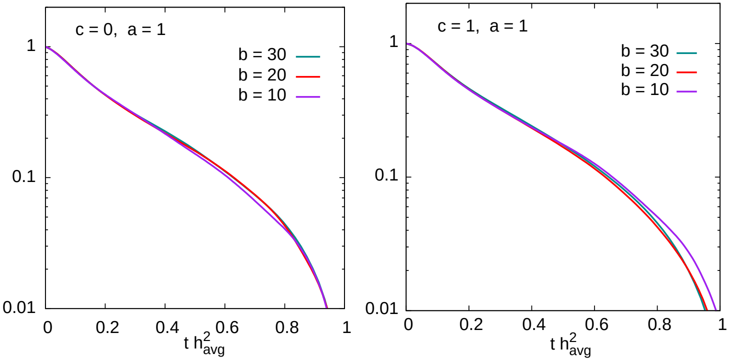
- Spatial correlation function scales with h_{avg}.
- Correlation length does not depend on b.

Spatial correlation function scales with h_{avg}



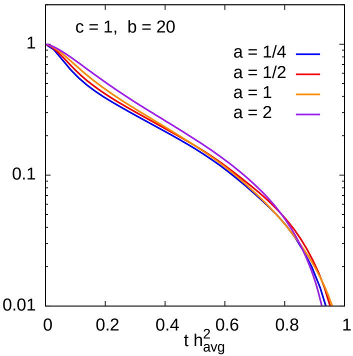
- Spatial correlation function scales with h_{avg}.
- Correlation length does not depend on b.
- Correlation length depend on a and c.
- The shape, when rescaled to correlation length, is universal.

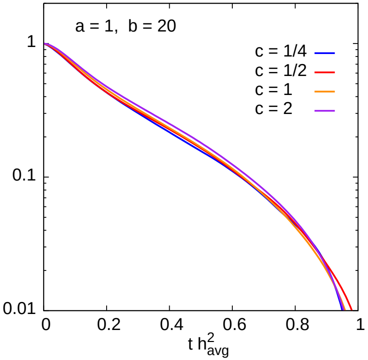
Temporal correlation function scales with h_{avg}^2



- Temporal correlation function scales with h_{avg}^2.
- Correlation time does not depend on b.

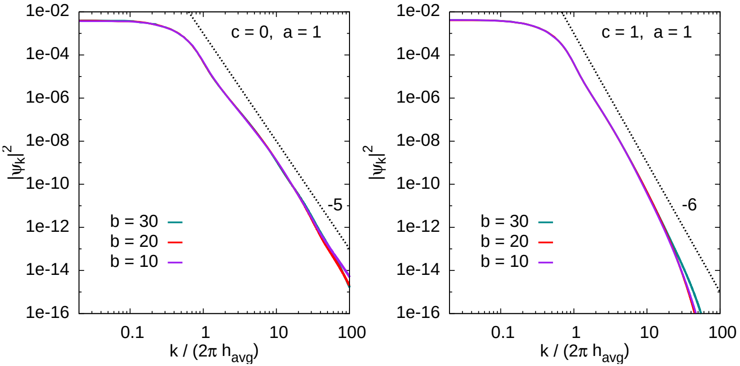
Temporal correlation function scales with h^2_{avg}





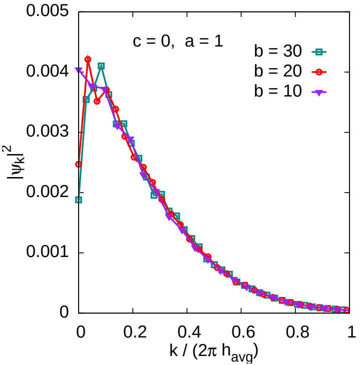
- Temporal correlation function scales with h^2_{avg}.
- Correlation time does not depend on b.
- Correlation time depend on a and c.

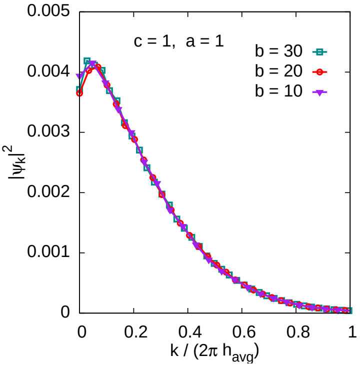
Energy spectra scale with h_{avg}



- Spectra of $|\psi|_k^2$ scale with h_{avg}.
- The tails do not depend on b.

Energy spectra scale with h_{avg}





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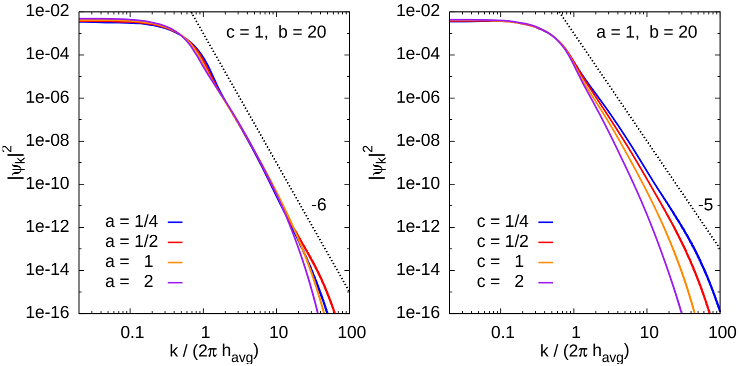

- Spectra of $|\psi|_k^2$ scale with h_{avg}.
- The tails do not depend on b.
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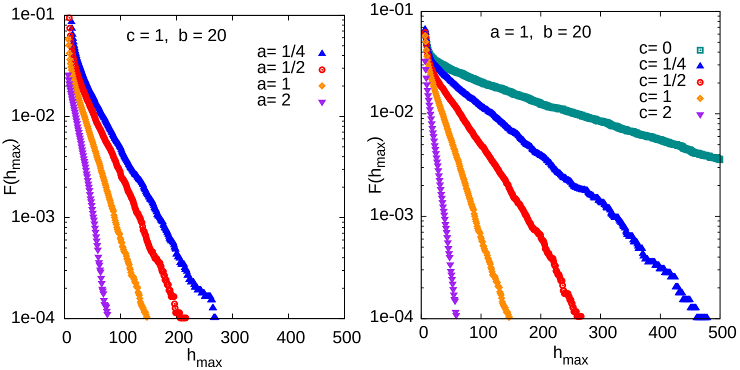
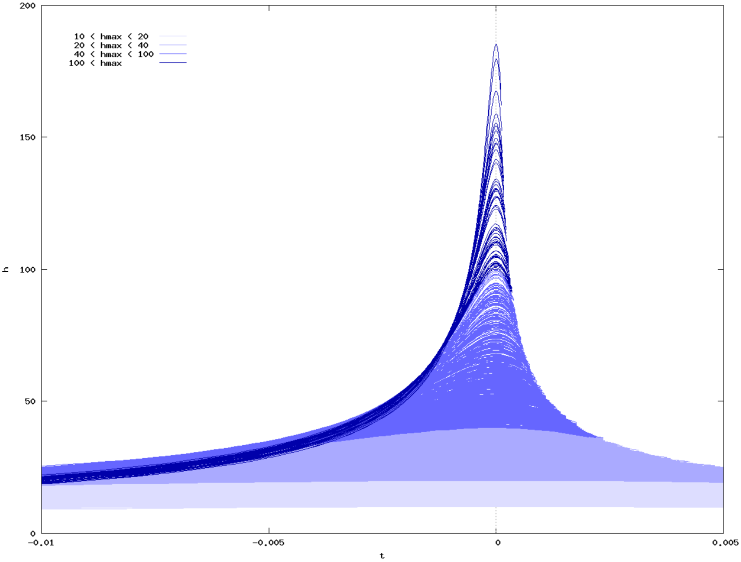
Frequency of collapses with $h > h_{max}$.



- Frequency of collapses with $h > h_{max}$ per unit area: $F(h_{max})$.
- The dependence is exponential.
- The exponent depends on a, c, and b.
- No obvious scaling with h_{avg}.

Universality of collapses in turbulence: a=1, c=1



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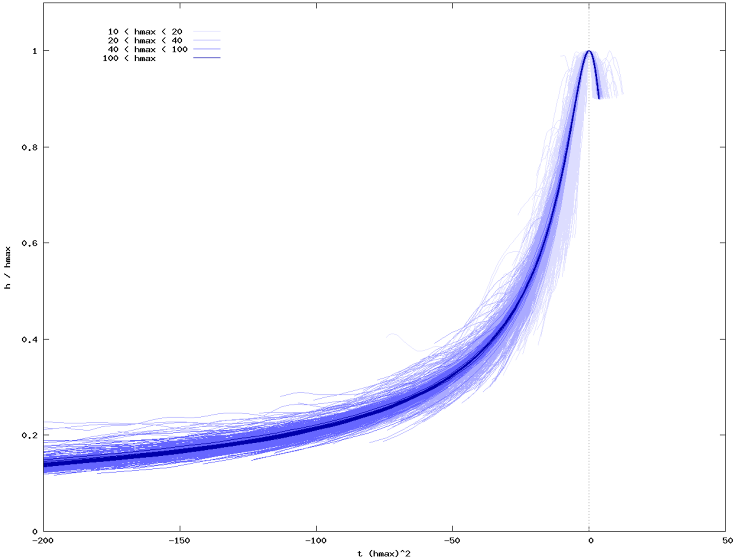

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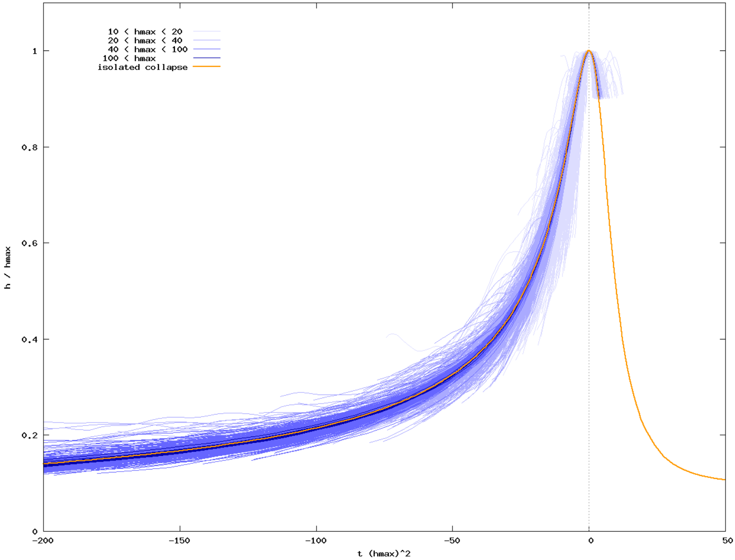

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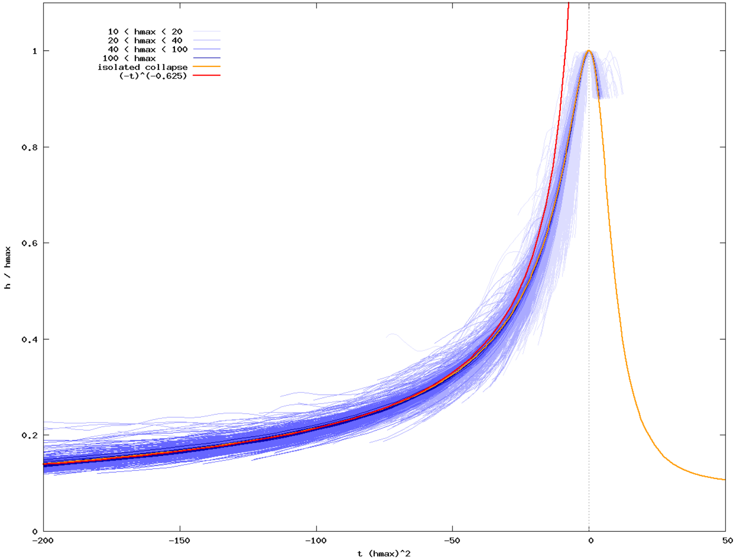
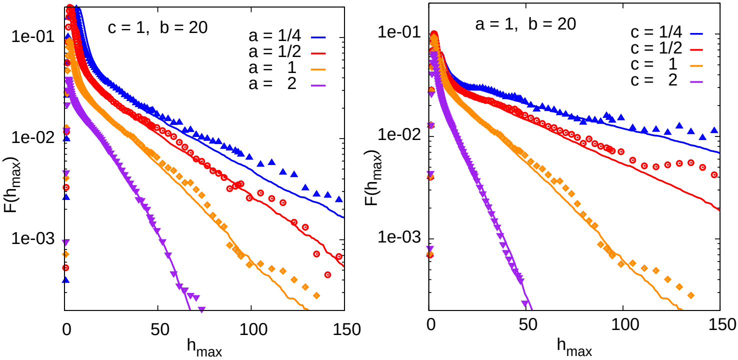


Connecting PDF of $|\psi|$ to frequency of collapses



- assume similarity among collapses, $h \sim (t_c - t)^{-\frac{1}{2}}$
- assume known frequency of collapses above $h_{\max}$: $F_{\max}(h_{\max})$
- conclude that PDF in the field $P(h) \sim h^{-5} F_{\max}(h)$
- Solid lines: $F(h_{\max})$. Points: $0.012\, h^5 P(h)$.

Collapse evolution in turbulence:

background turbulence: h_{avg}

- ➥ collapse seeds feed on background: h_{avg}
  - ➥ growth enhanced by forcing: b
    - ➥ collapse saturation: a, c
      - ➥ decay: h_{max}
        - ➥ background turbulence

Can theory explain and predict the following?

- similarities in background turbulence in different systems (might be specific to particular forcing)
- the level of background h_{avg}
- exponential tails for distribution of maximums
- evolution of a single collapse