# Turbulent Rotating Rayleigh-Bénard convection

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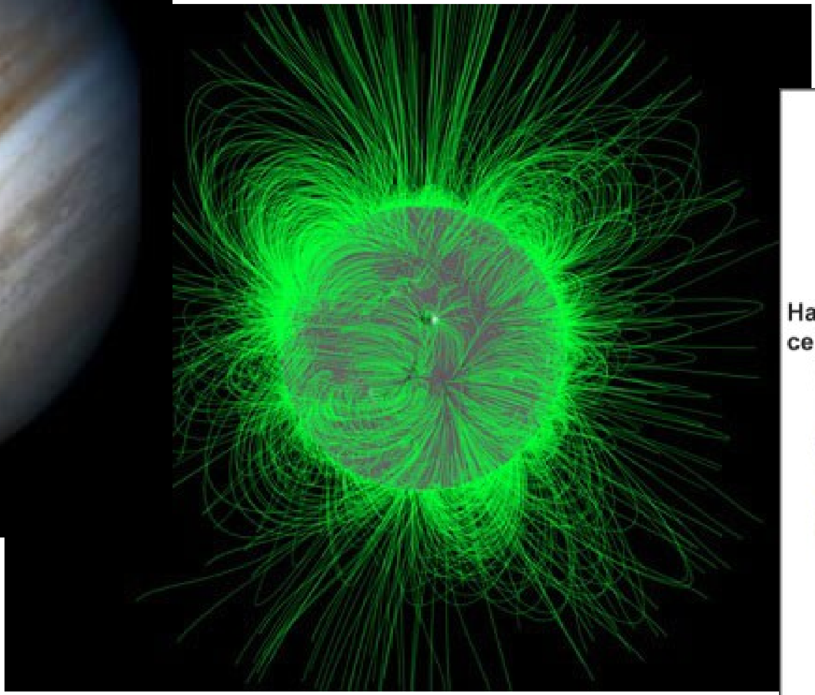
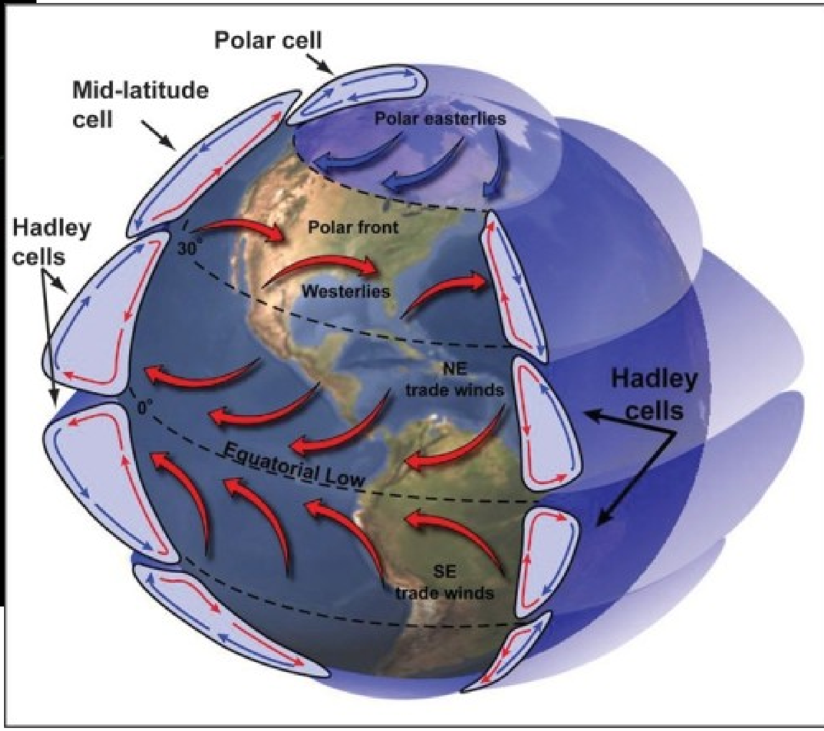


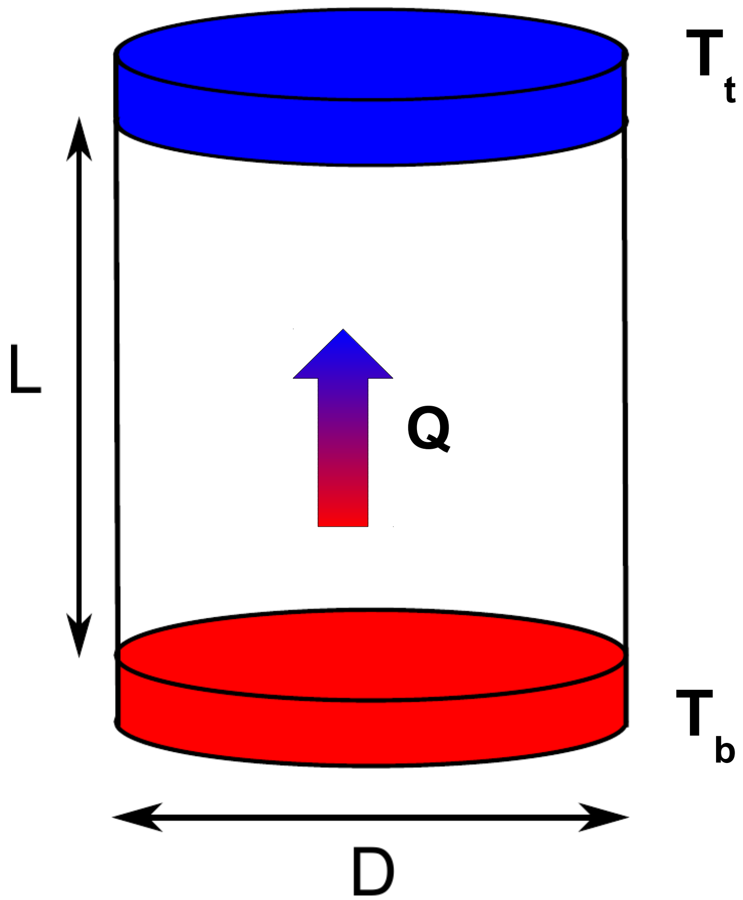



The work was supported by NSF Grant DMR07-02111

# Content

- Influence of rotation on the heat transport
- The large scale circulation in cylinders with aspect ratio $\Gamma=1/2$ when no rotation is present
- Influence of rotation on the large scale circulation

# Rayleigh Bénard convection in a cylinder




Aspect ratio: $\Gamma = \frac{D}{L}$

Rayleigh number: $Ra = \frac{g \alpha \Delta T L^3}{\kappa \nu}$

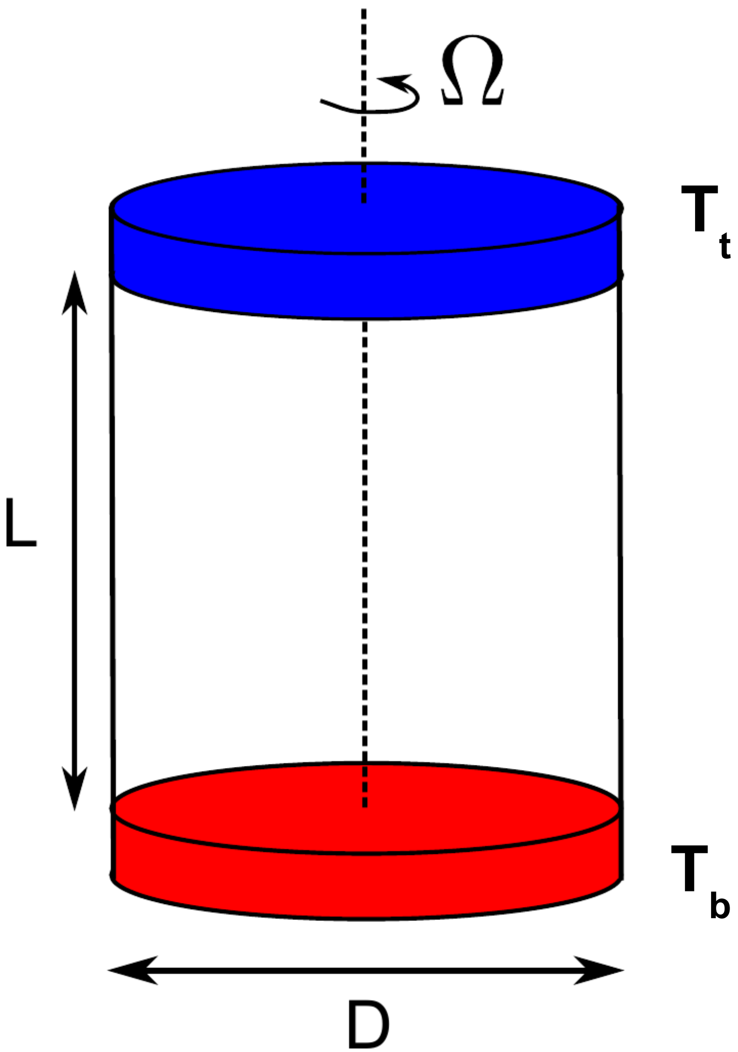
Prandtl number: $Pr = \frac{\nu}{\kappa}$

Nusselt number: $Nu = \frac{Q L}{A \lambda \Delta T}$

$$\Delta T = T_b - T_t$$

working fluid: water @40°C -> Pr=4.38

# RBC under rotation

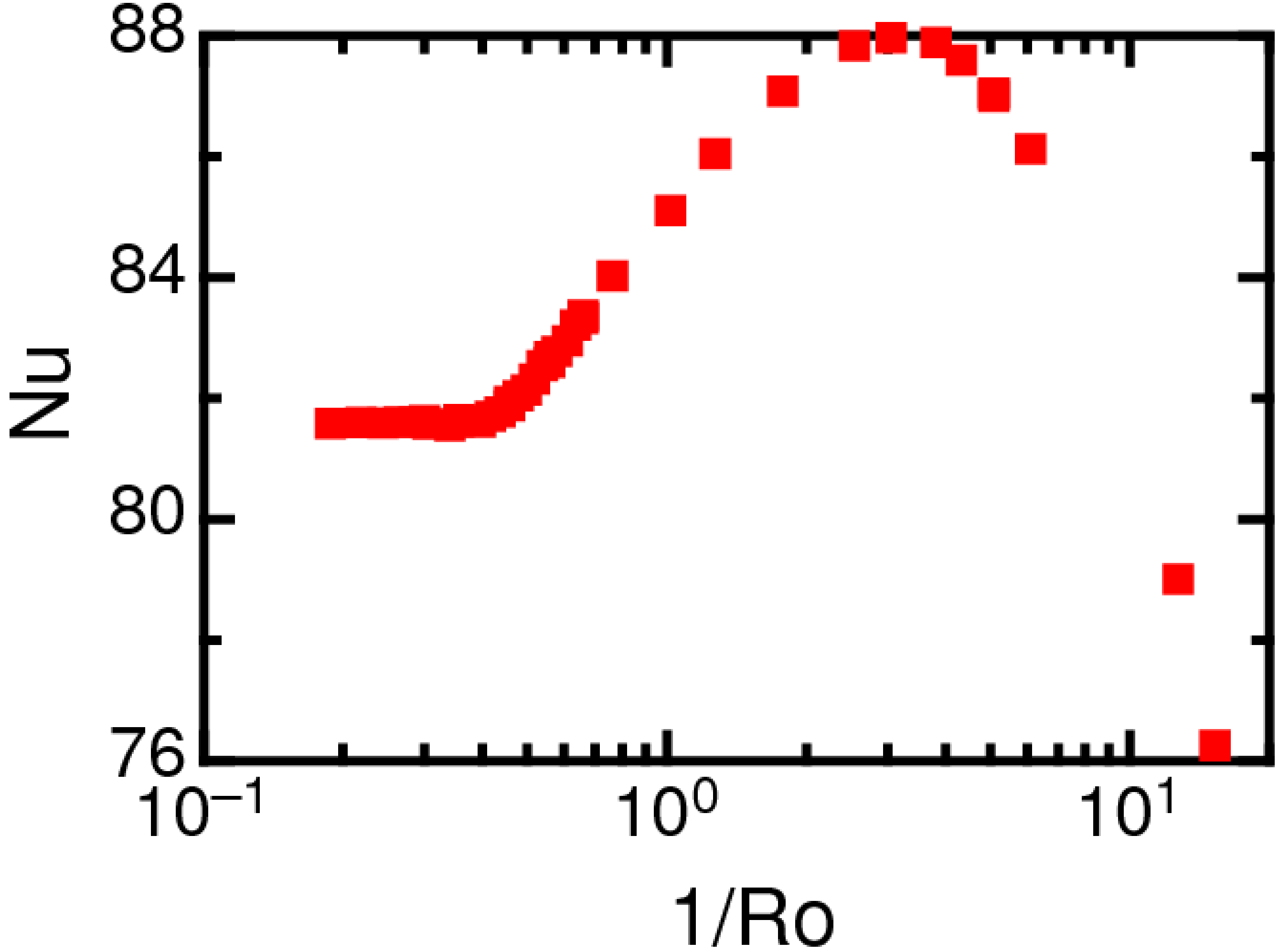



$$\Delta T = T_b - T_t$$

Inverse convective Rossby number:

$$\frac{1}{Ro} = \frac{Coriolis\ Force}{Buoyancy} = \frac{2\Omega}{\sqrt{g\alpha\Delta T/L}}$$

# Nu vs 1/Ro

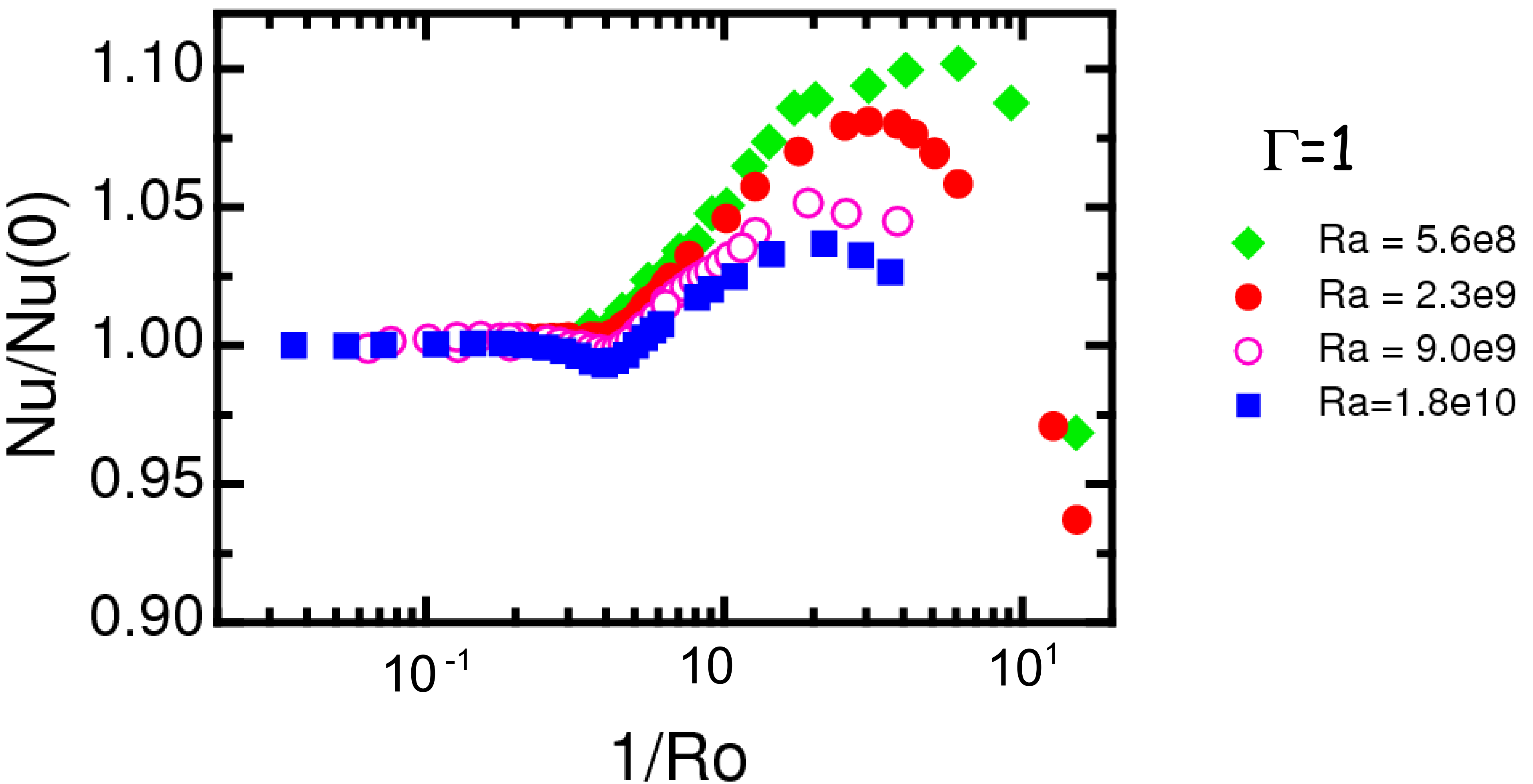



$\Gamma$=1

Ra = 2.3e9

Stevens, Zhong, Clercx, Ahlers, Lohse, *PRL* **103**, 024503 (2009)
Zhong & Ahlers, *JFM*, **665**, 300 (2010)

# Nu vs 1/Ro

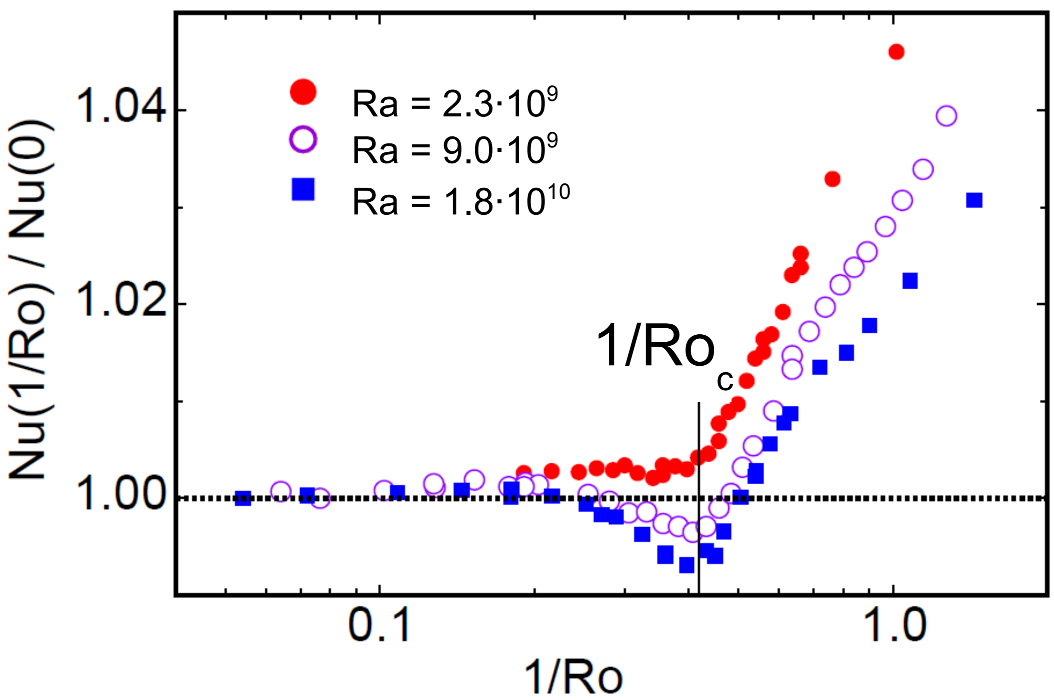



Stevens, Zhong, Clercx, Ahlers, Lohse, *PRL* **103**, 024503 (2009)
Zhong & Ahlers, *JFM*, **665**, 300 (2010)

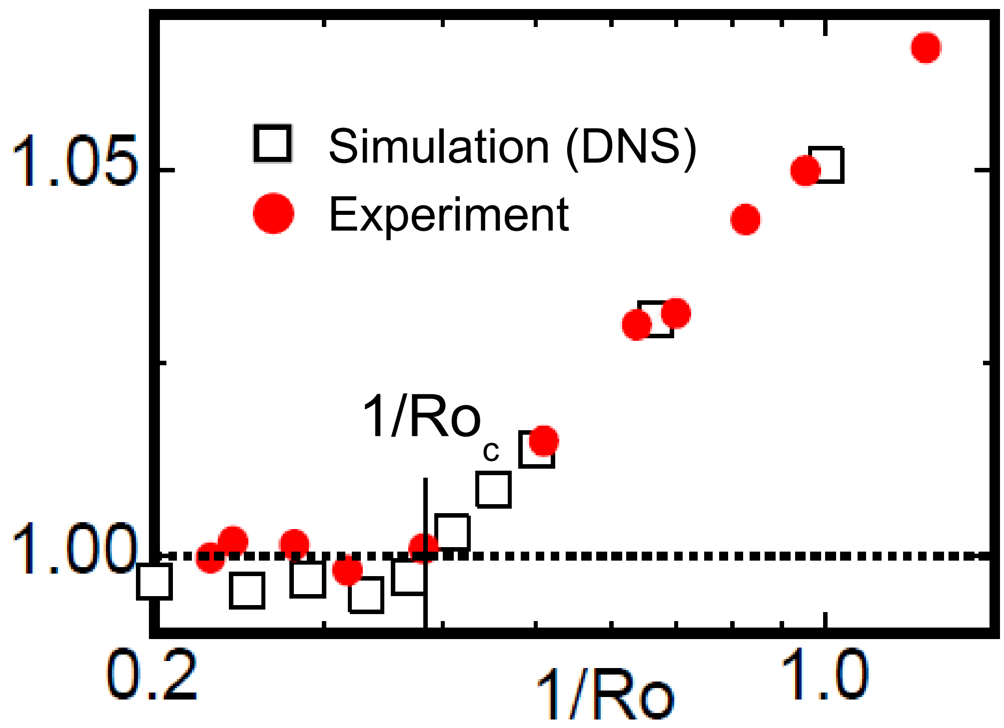
# Nusselt vs Rayleigh

Γ=1

Pr = 4.38

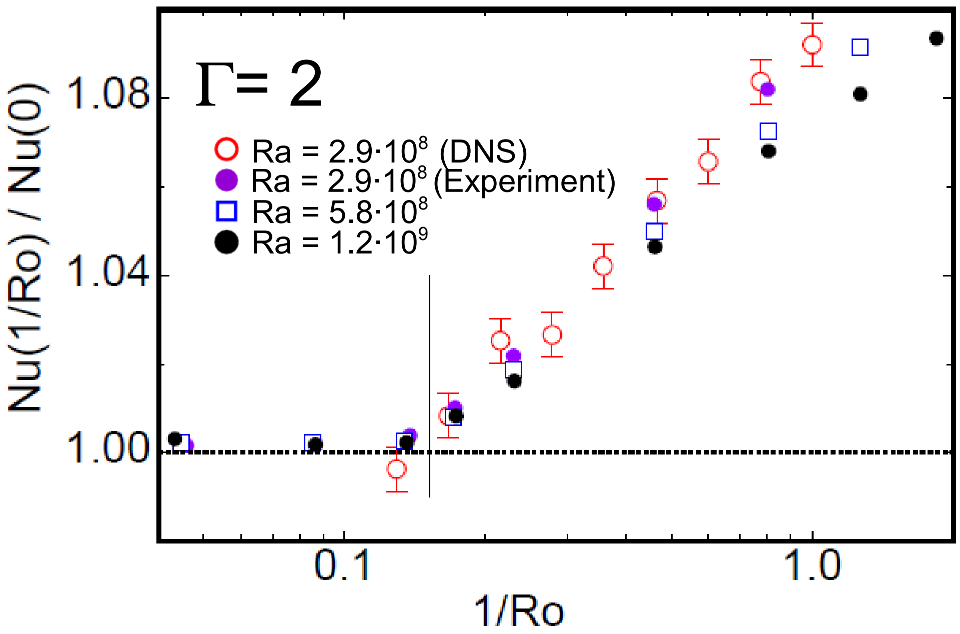


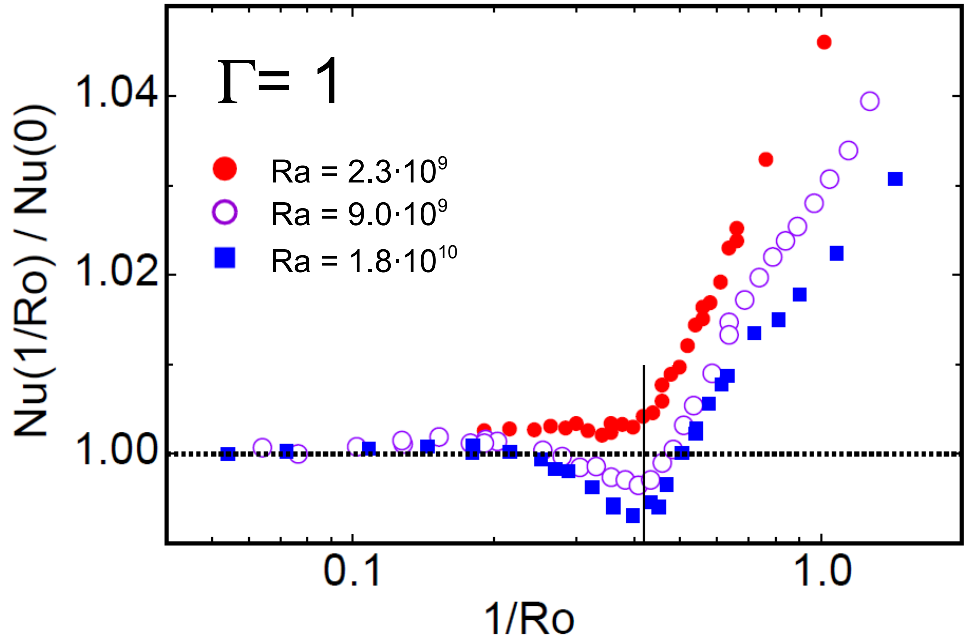

Pr = 6.26 Ra = $2.73 \cdot 10^8$

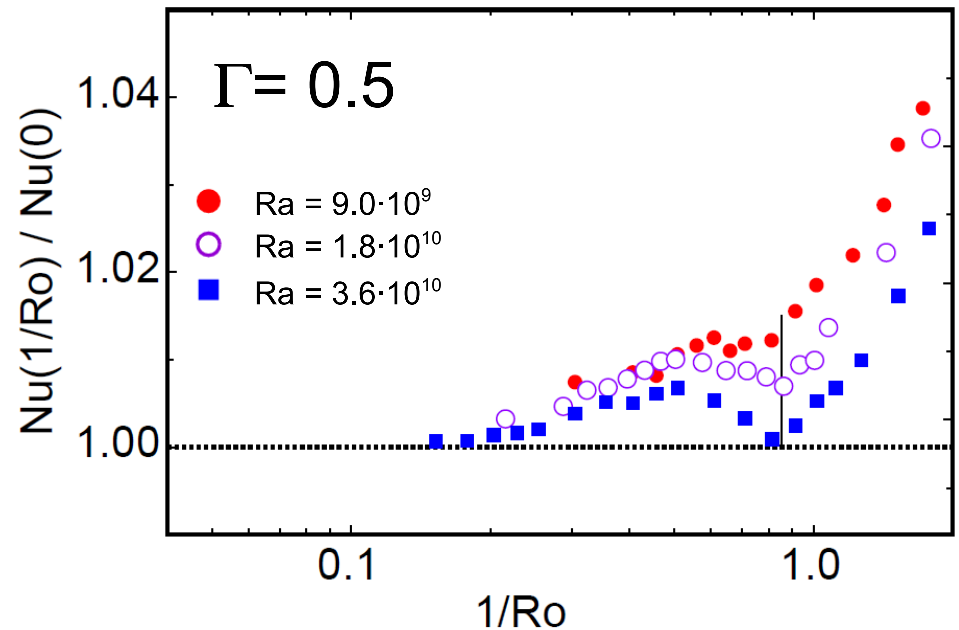



Stevens, Zhong, Clercx, Ahlers, Lohse, PRL **103**, 024503 (2009)
Zhong & Ahlers, JFM, **665**, 300 (2010)

# Nusselt vs Rayleigh

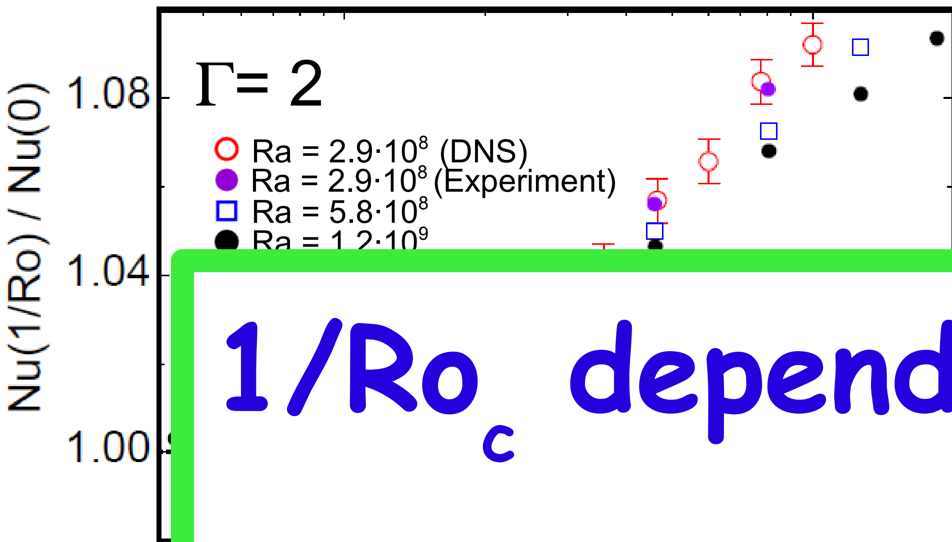


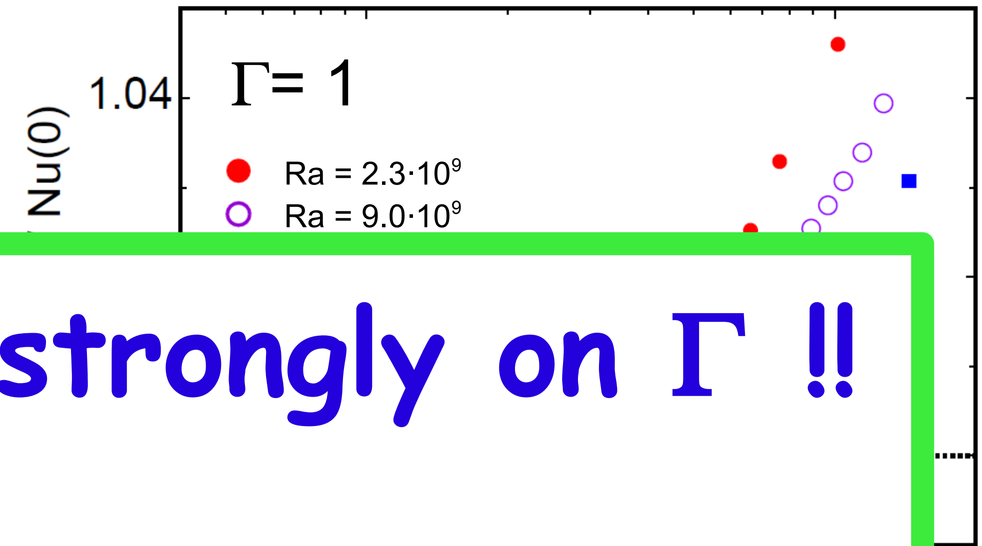




From: Weiss, Stevens, Zhong, Clercx, Lohse, Ahlers, *PRL*, **105**, 224501, (2010)

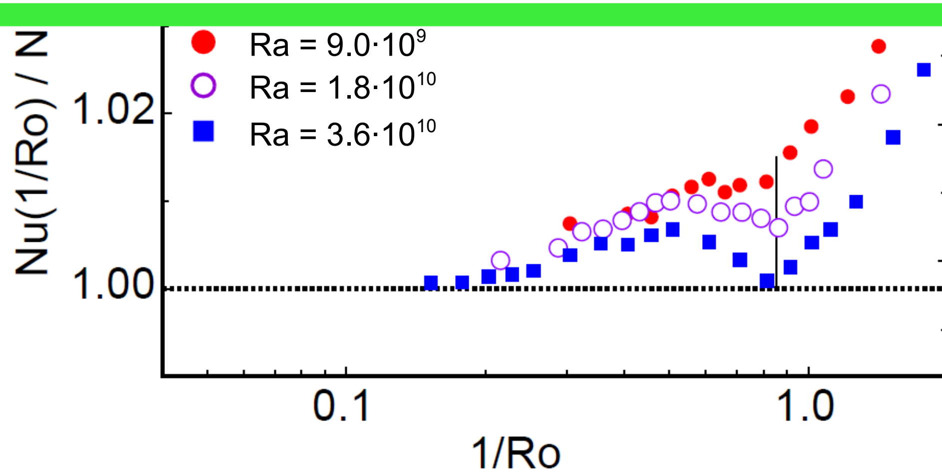
# Nusselt vs Rayleigh



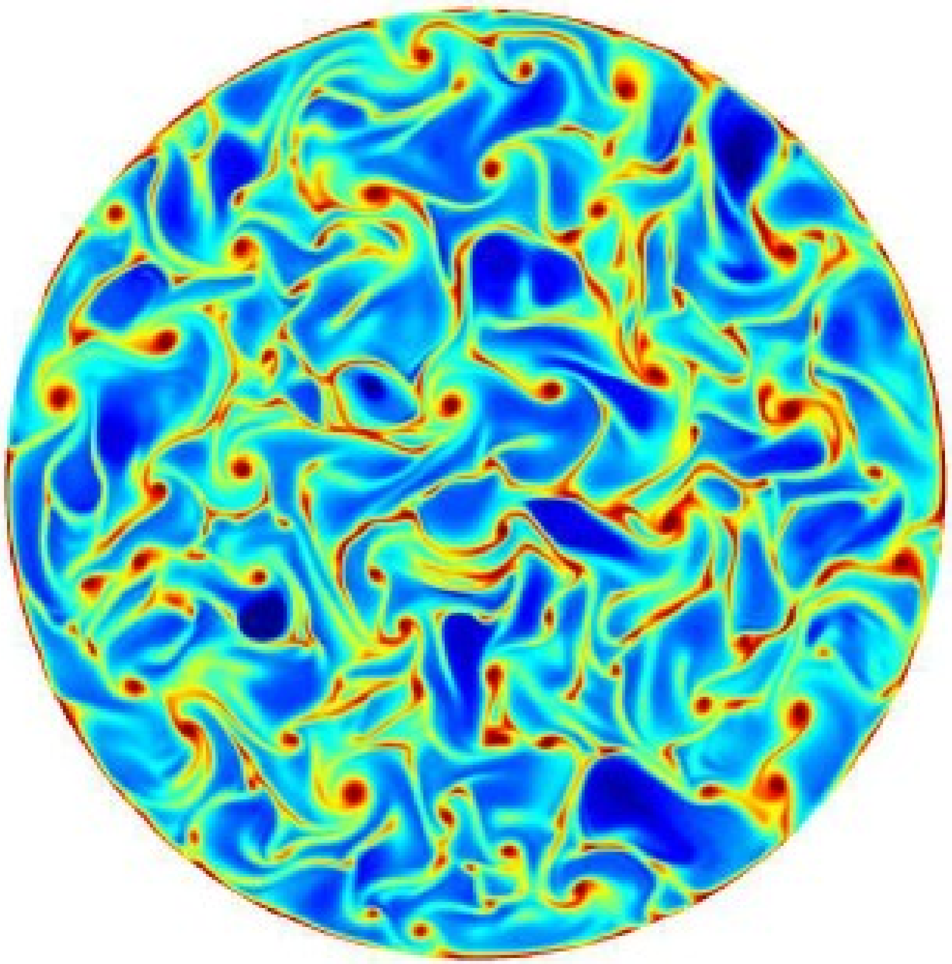

**$1/Ro_c$ depends strongly on $\Gamma$ !!**

**$1/Ro_c$ is independent of Ra !!**



# Vortex formation and Ekman pumping

Temperature field at the kinetic boundary layer
(from: Stevens, Overkamp, Lohse, & Clercx, *JFM*, submitted)



$\Gamma = 2$

$Ra = 2.92e8$

$1/Ro = 3.33$

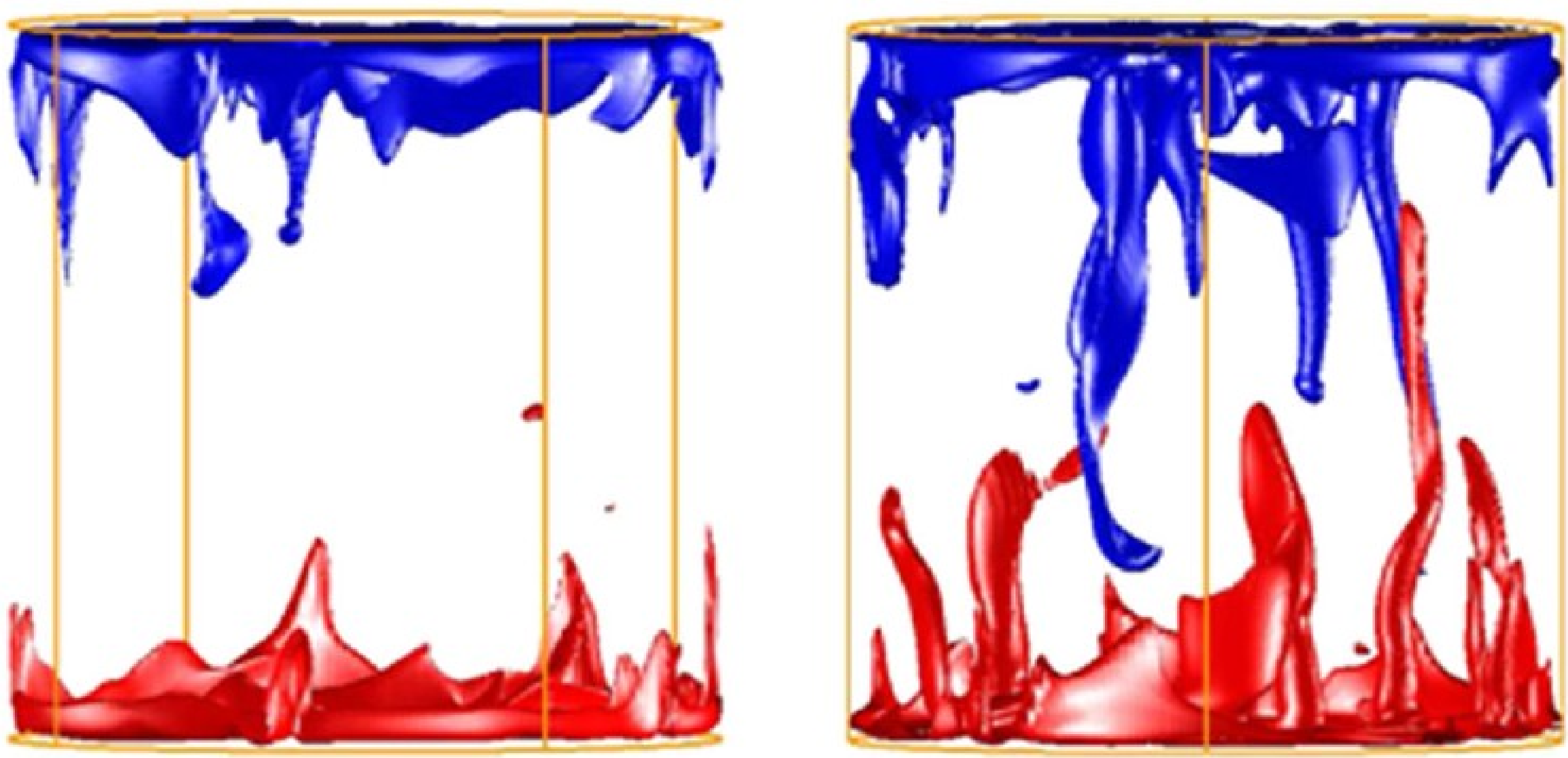
# Vortex formation and Ekman pumping

$\Gamma=1$

Pr = 6.4

Ra = $10^8$

1/Ro = 0 | 1/Ro=3.3

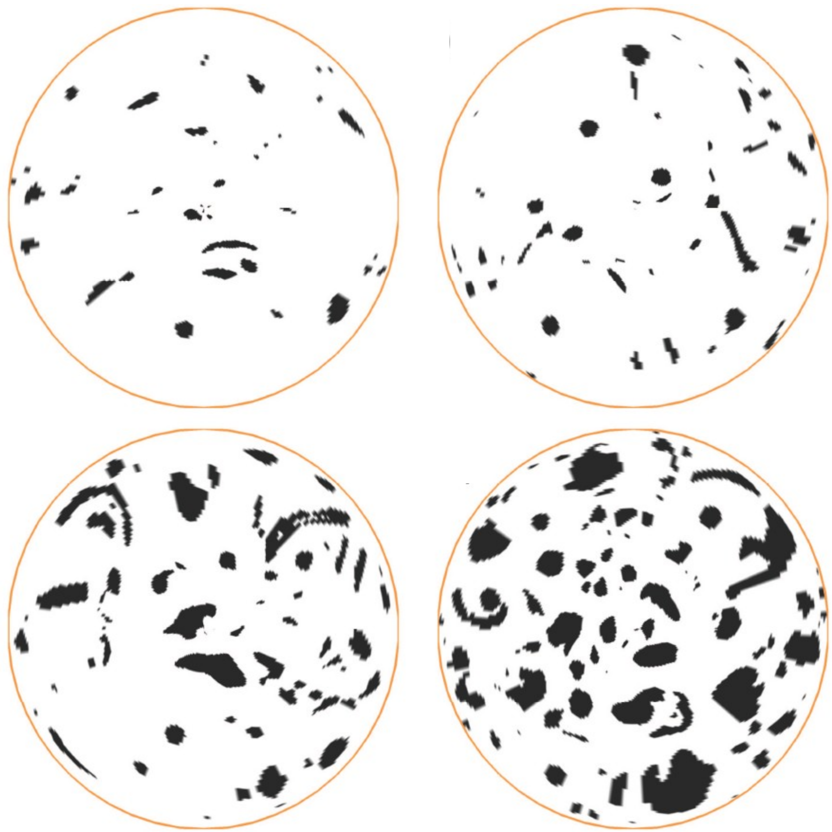


Zhong, Stevens, Clerx, Verzicco, Lohse, Ahlers. , *PRL* **102**, 044502 (2009).

# Vortices close to the thermal boundary

DNS for Ra = $2.73 \cdot 10^8$ and Pr = 6.26

1/Ro = 2/3 1/Ro = 1



1/Ro = 1.54 1/Ro = 3.33

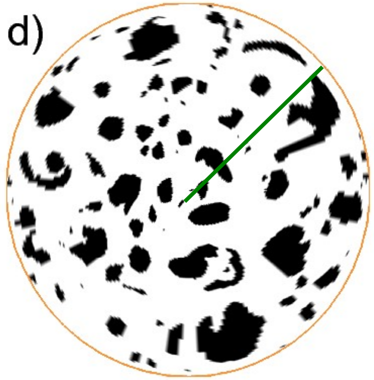
$\overline{A}$... Fraction of area that is covered by vortices

From: Weiss, Stevens, Zhong, Clercx, Lohse, Ahlers, *PRL*, **105**, 224501, (2010)

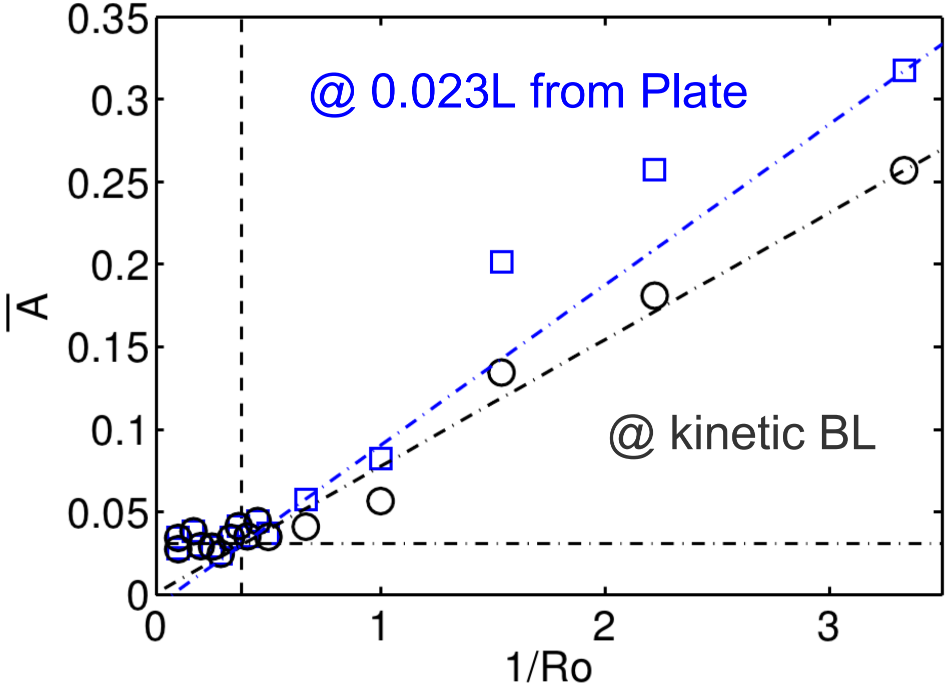
# Vortices close to the thermal boundary

DNS for Ra = $2.73 \cdot 10^8$ and Pr = 6.26

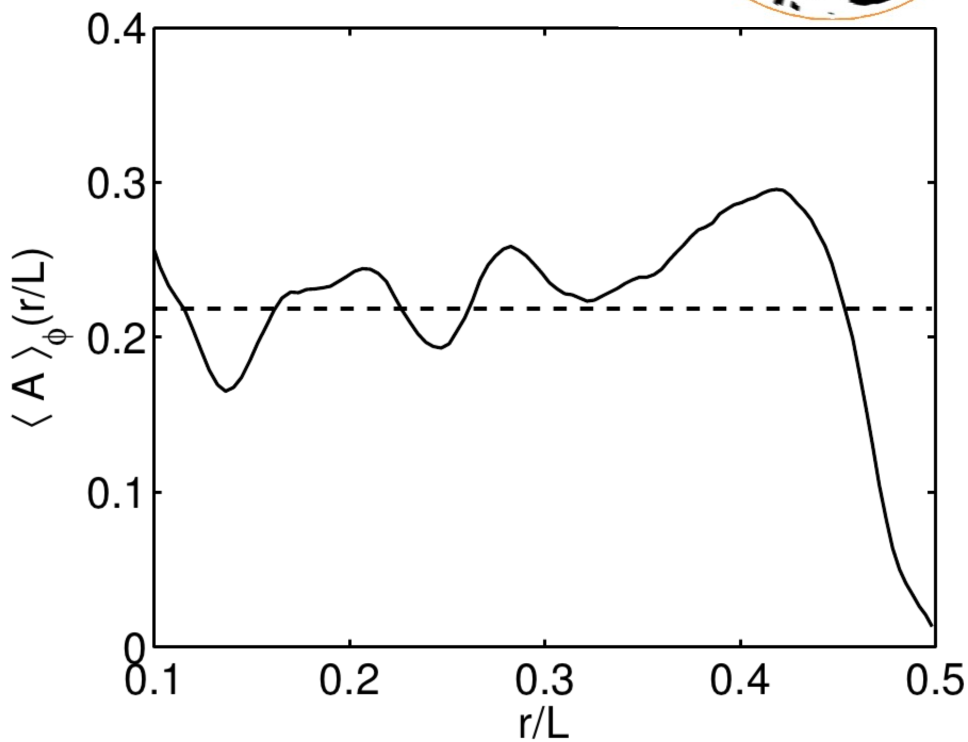
d)



Fraction of area that is covered with vortices




Azimuthally averaged vortex density




(2.73< 1/Ro <3.33; averaged over 8 snapshots)

# A Ginzburg Landau model

$$\frac{\partial A}{\partial t} = \left(\frac{1}{Ro}\right)^2 A - gA^3 + \xi_0^2 \nabla^2 A$$

$A$ … local vortex density

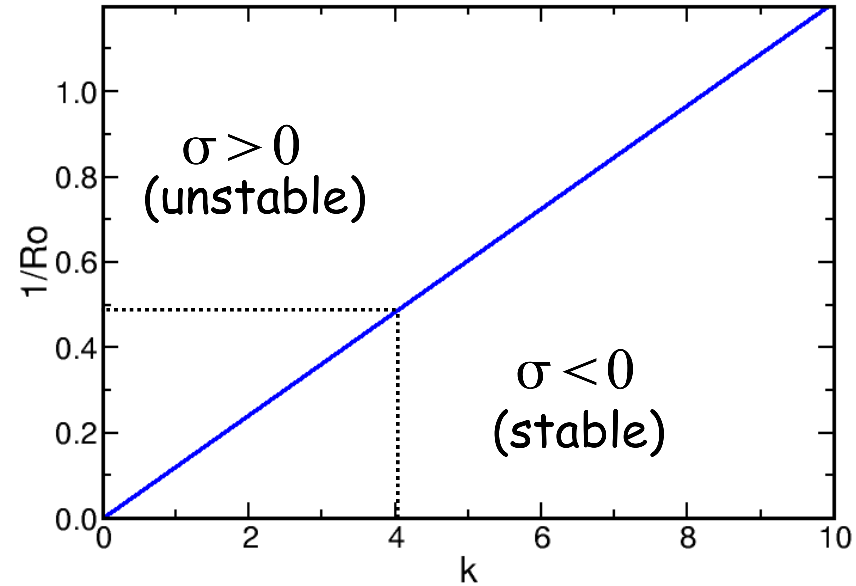
## For an infinite system:

A solution: $A_1(\vec{x}, t) = 0$

Small spatial perturbation: $A = \delta_A \exp(\sigma t) \exp(i \vec{k} \vec{x})$

Growth rate: $\sigma = \left(\frac{1}{Ro}\right)^2 - \xi_0^2 k^2$ with $k = |\vec{k}|$

Neutral curve: $$\frac{1}{Ro} = \xi_0 k$$




Homogeneous solution: $A_2 = \frac{1}{Ro} \frac{1}{\sqrt{g}}$

# A Ginzburg Landau model

$$\frac{\partial A}{\partial t} = \left(\frac{1}{Ro}\right)^2 A - gA^3 + \xi_0^2 \nabla^2 A$$

$A$ … local vortex density

**For a cylindrical finite system:**

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$$

Rescaling:

$$r = \xi_0 Ro \cdot \tilde{r}$$

Steady case:

$$0 = A - g A^3 + \frac{\partial^2 A}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial A}{\partial \tilde{r}}$$

# A Ginzburg Landau model

$$\frac{\partial A}{\partial t} = \left(\frac{1}{Ro}\right)^2 A - gA^3 + \xi_0^2 \nabla^2 A$$

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Rescaling:

$$r = \xi_0 Ro \cdot \tilde{r}$$

Steady linear case:

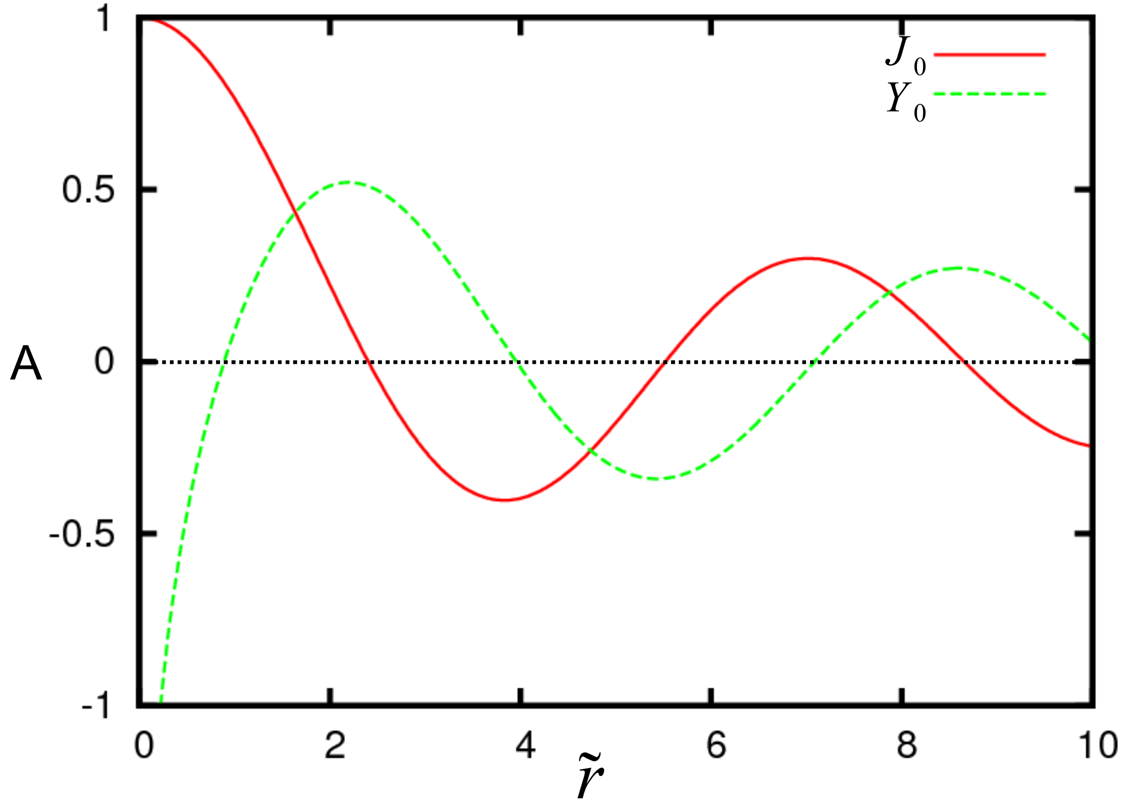
$$0 = A - gA^3 + \frac{\partial^2 A}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial A}{\partial \tilde{r}}$$

Solutions:

$$A(\tilde{r}) = J_0(\tilde{r})$$

$$A(\tilde{r}) = Y_0(\tilde{r})$$

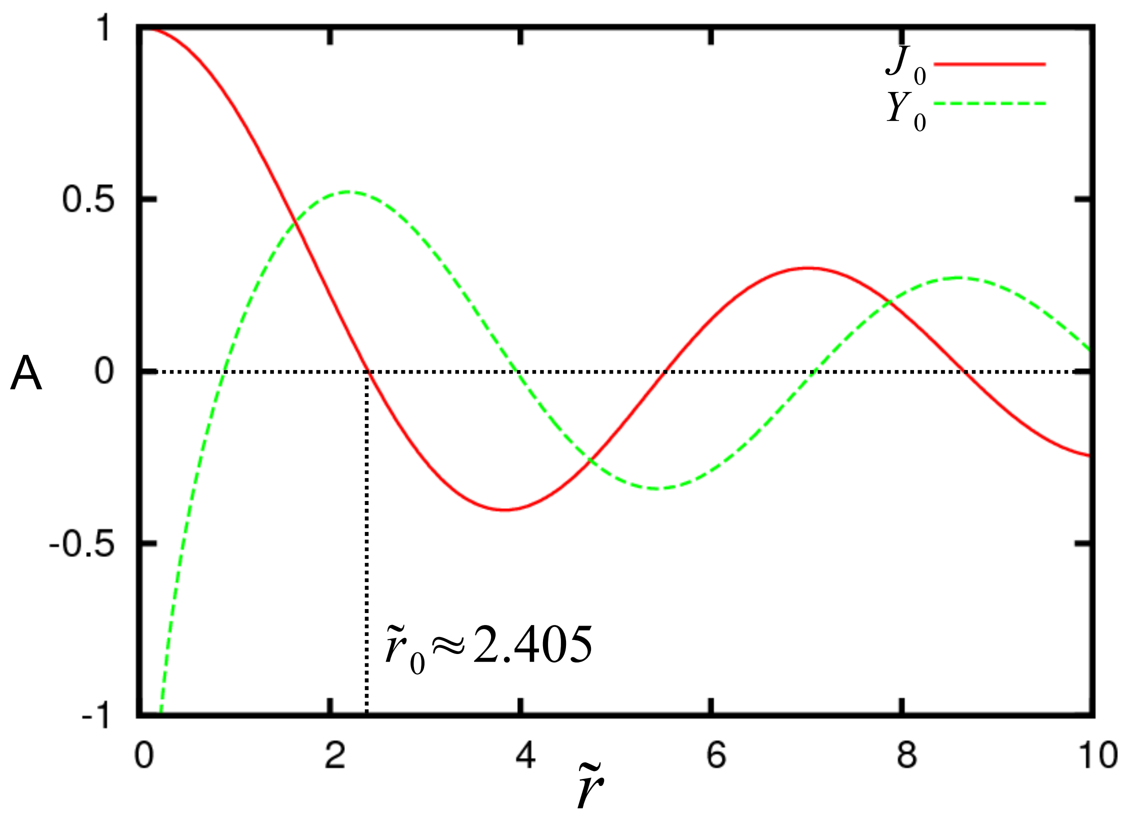
Bessel function




- $Y_0$ unphysical !!
- $J_0$ has to fullfill BC

No vortices at the side wall !!

BC: $A(r=\Gamma/2) \overset{!}{=} 0$




- $Y_0$ unphysical !!
- $J_0$ has to fullfill BC

No vortices at the side wall !!

BC: $A(r=\Gamma/2) \stackrel{!}{=} 0$

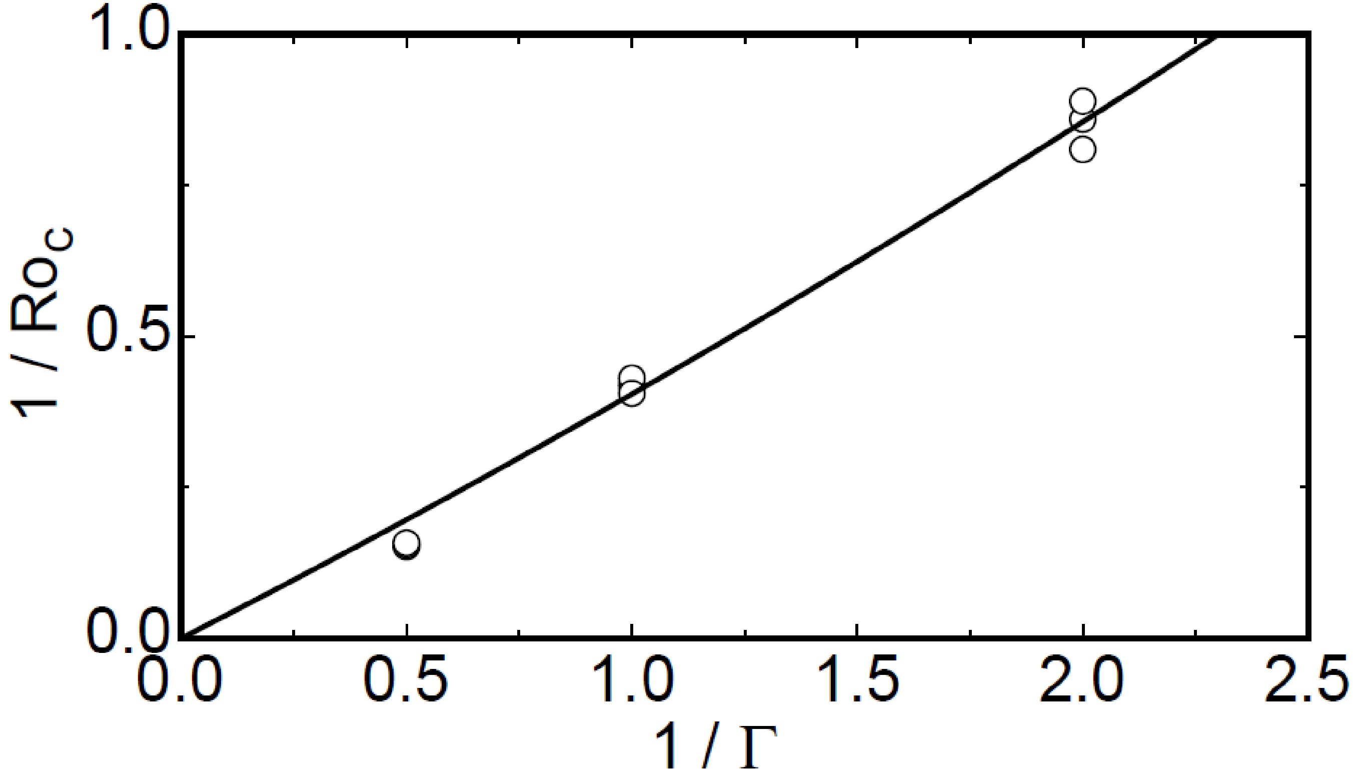
$\Gamma/2 = \xi_0 Ro \cdot \tilde{r}_0$

Critical inverse Rossby number: $$\frac{1}{Ro_c} = 2\xi_0 \cdot \tilde{r}_0 \frac{1}{\Gamma}$$

No solution A>0 for 1/Ro < 1/Ro$_c$ !!

# A Ginzburg Landau model

$$\frac{1}{Ro_c} = 2\xi_0 \cdot \tilde{r}_0 \frac{1}{\Gamma}$$




Fit:

$$\frac{1}{Ro_c} = \frac{a}{\Gamma}\left(1 + \frac{b}{\Gamma}\right)$$

$$a = 0.381$$

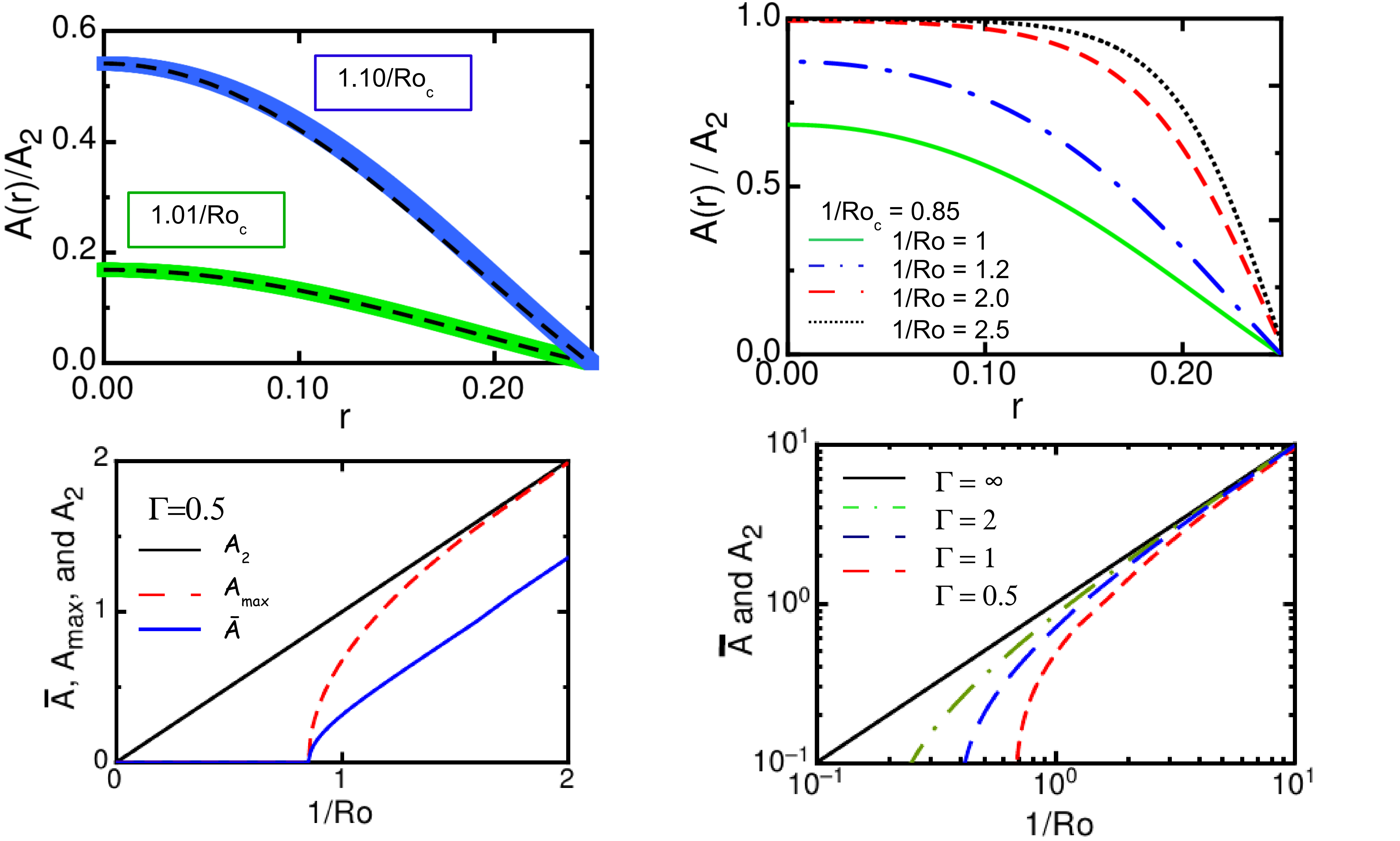
$$b = 0.061$$

$$\xi_0 = \frac{a}{2\tilde{r}_0} = 0.079$$

# Radial Vortex Density Profiles

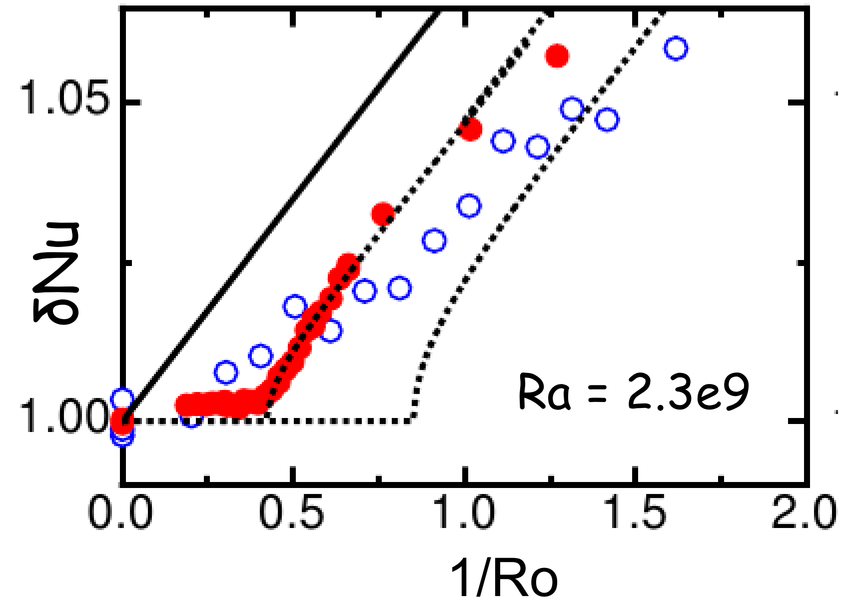
$$0=\left(\frac{1}{Ro}\right)^2 A - gA^3 + \xi_0^2 \nabla^2 A$$

$A_2$ … homogeneous solution of the infinite system

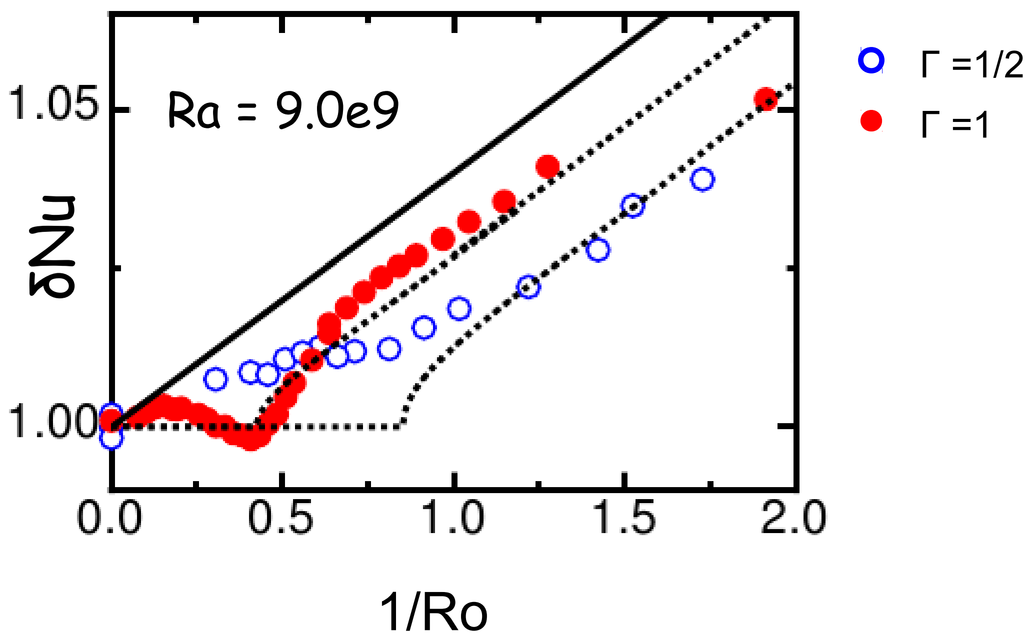


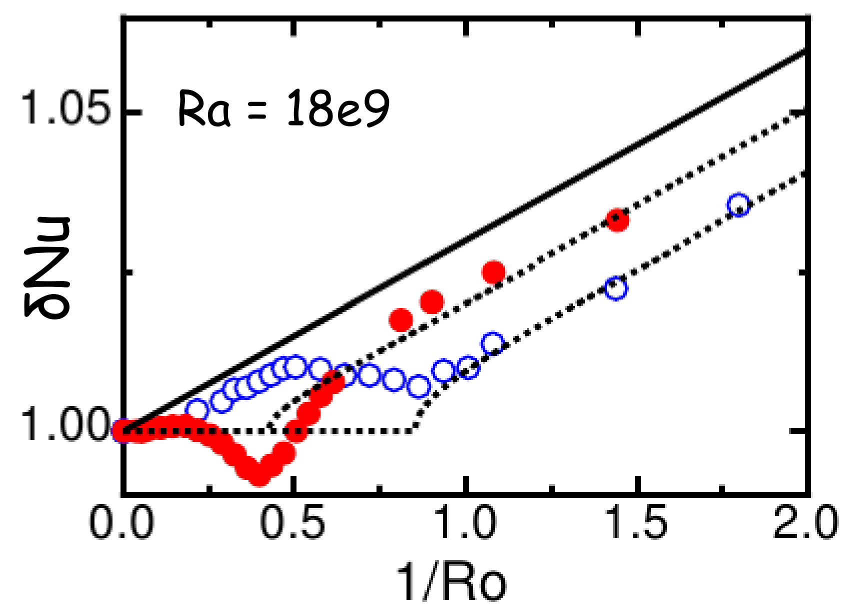
# Γ = 1 vs Γ = 1/2

$$\delta\, Nu = \frac{Nu(1/Ro)}{Nu(0)} = \beta\, \bar{A} + 1$$




Assuming βindependent of Γ



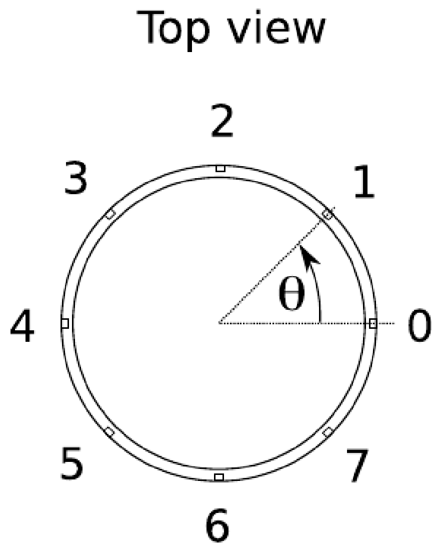



# Short Summary

- Rotation leads to enhanced heat transport due to Ekman pumping
- This enhancement depends on vortex formation and sets in at a critical rotation rate $1/Ro_c$
- $1/Ro_c$ depends on aspect ratio $\Gamma$
- The transition can be described by a Ginzburg-Landau model with local vortex density as order parameter
- Bifurcation exists because of the finite extension of the system

# The Large Scale Circulation in cylinders with $\Gamma = 1/2$

# Side wall temperature measurements




$$T = T_k + \delta_k \cos(i\pi/4 - \theta_k)$$

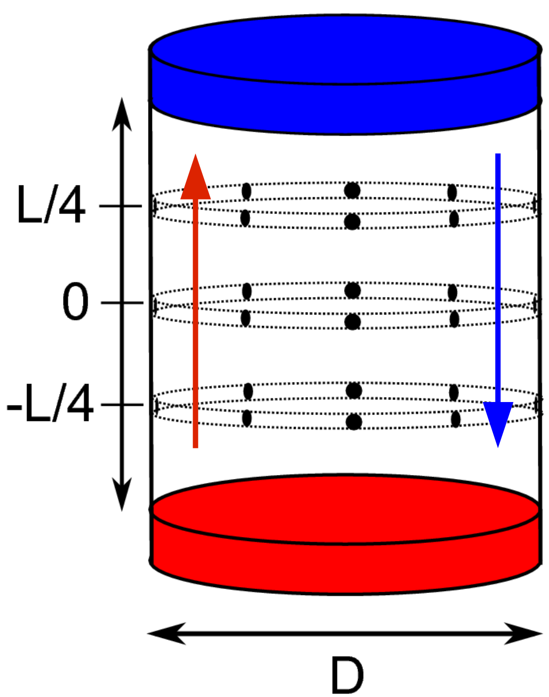
k = [t,m,b] (top, middle, (bottom)

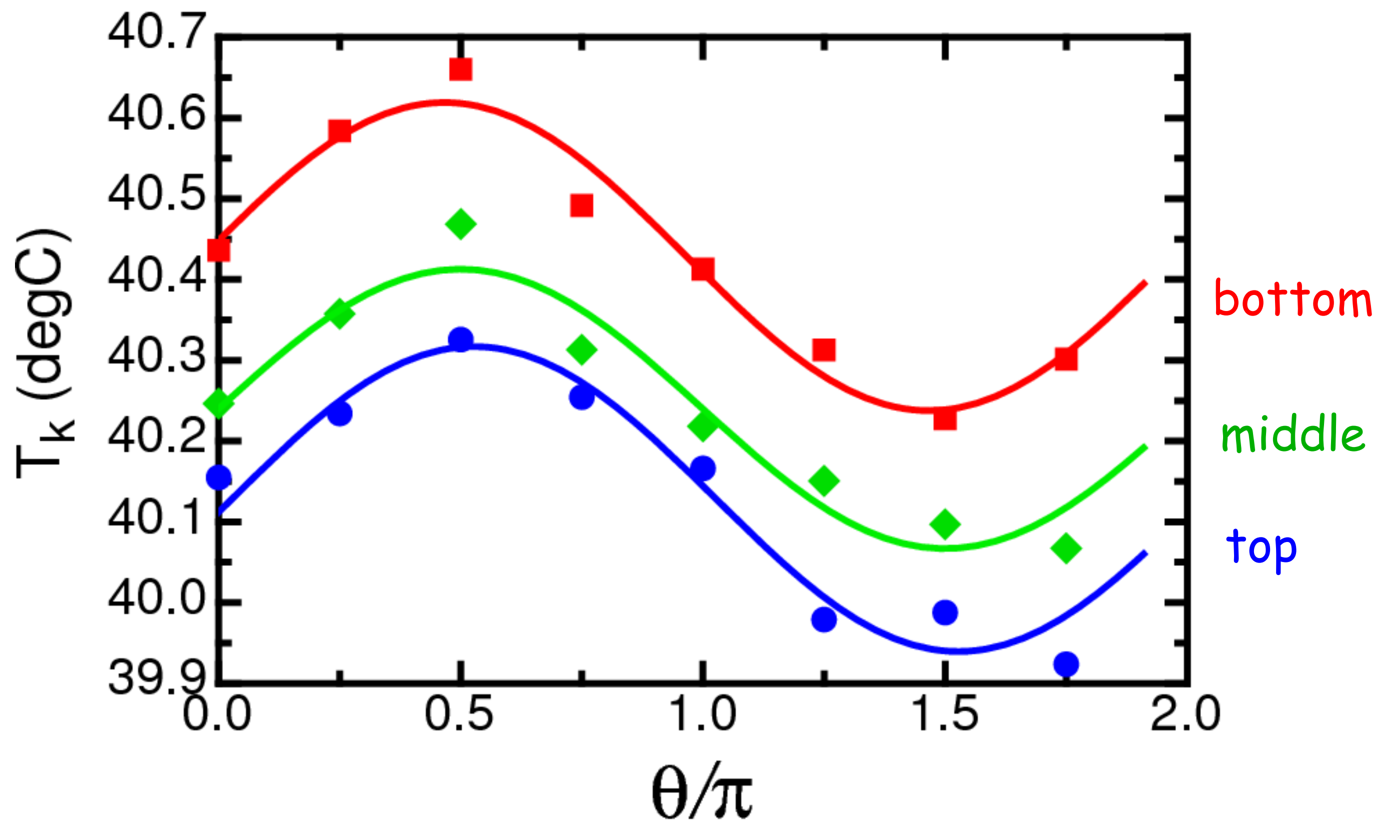
i = 0 ... 7

$T_k$ … average temperature

$\delta_k$ … amplitude (strength)

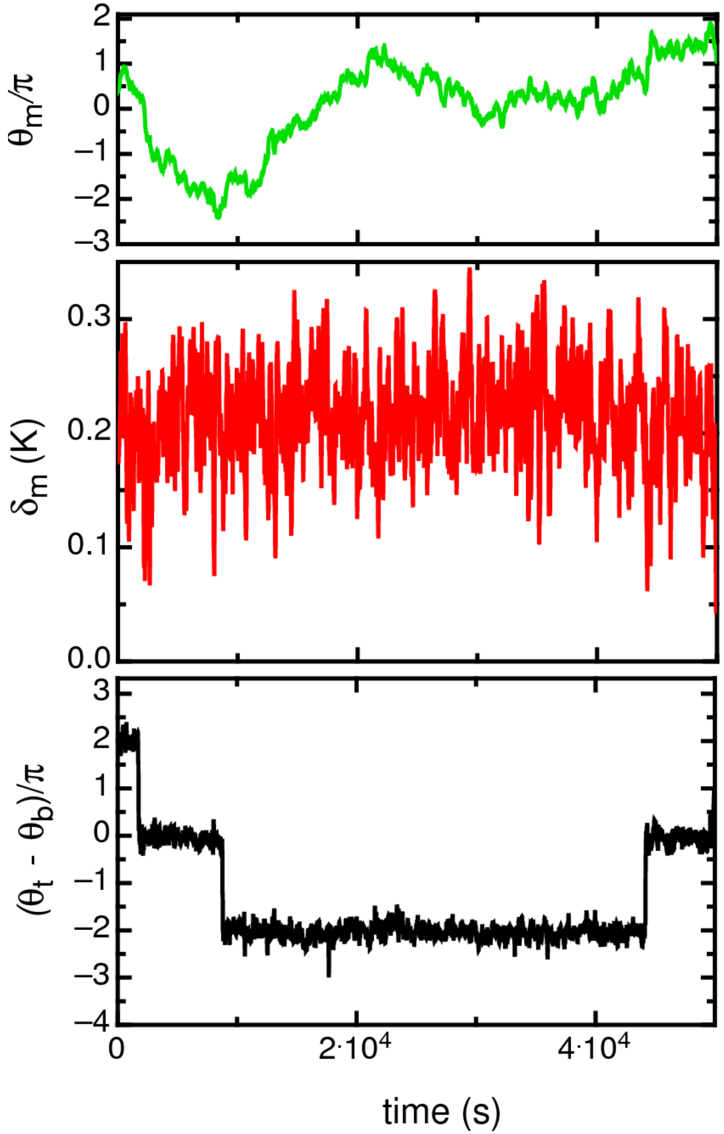
$\theta_k$ … phase (orientation)

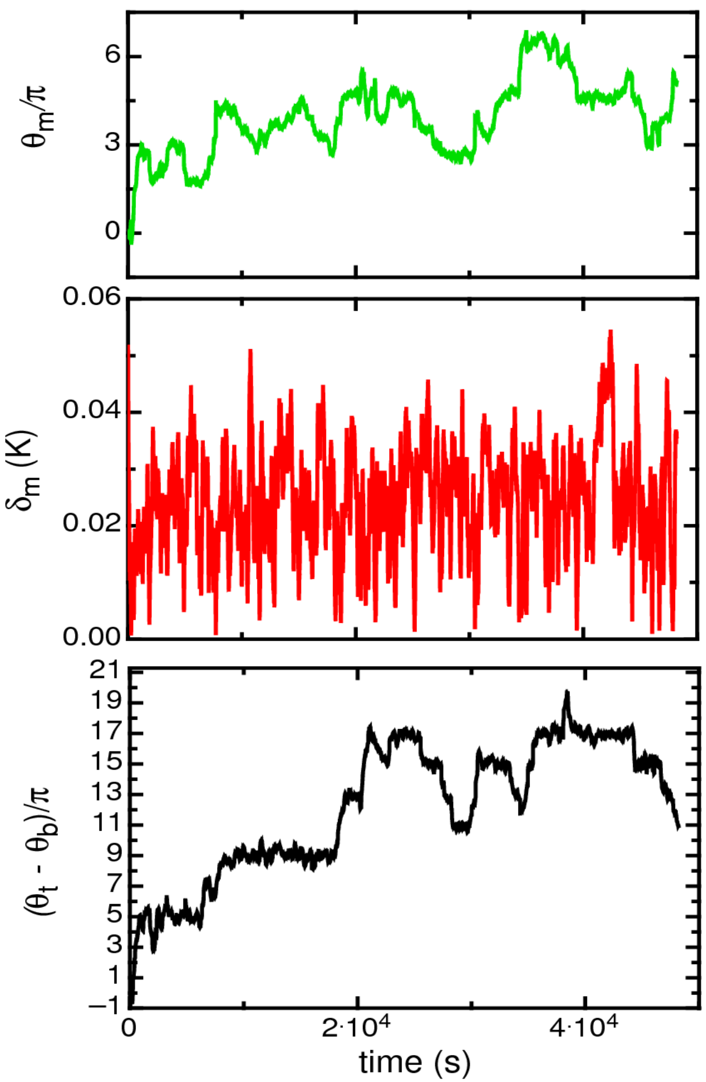


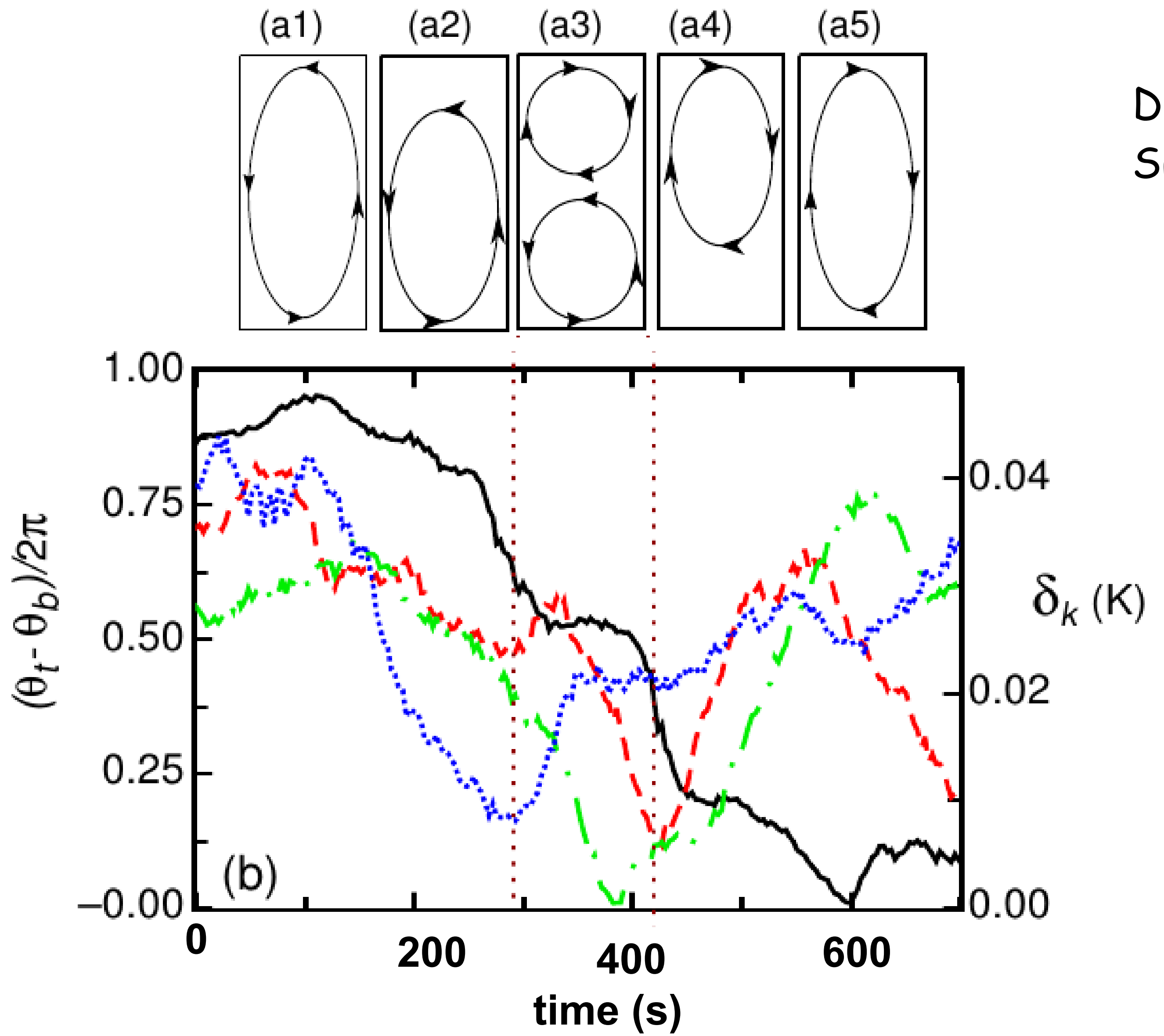
# LSC dynamics

Γ = 1 Ra = 9.0e9 Γ = 0.5



# Flow Mode Transition in Γ = 1/2




DRS … Double roll state

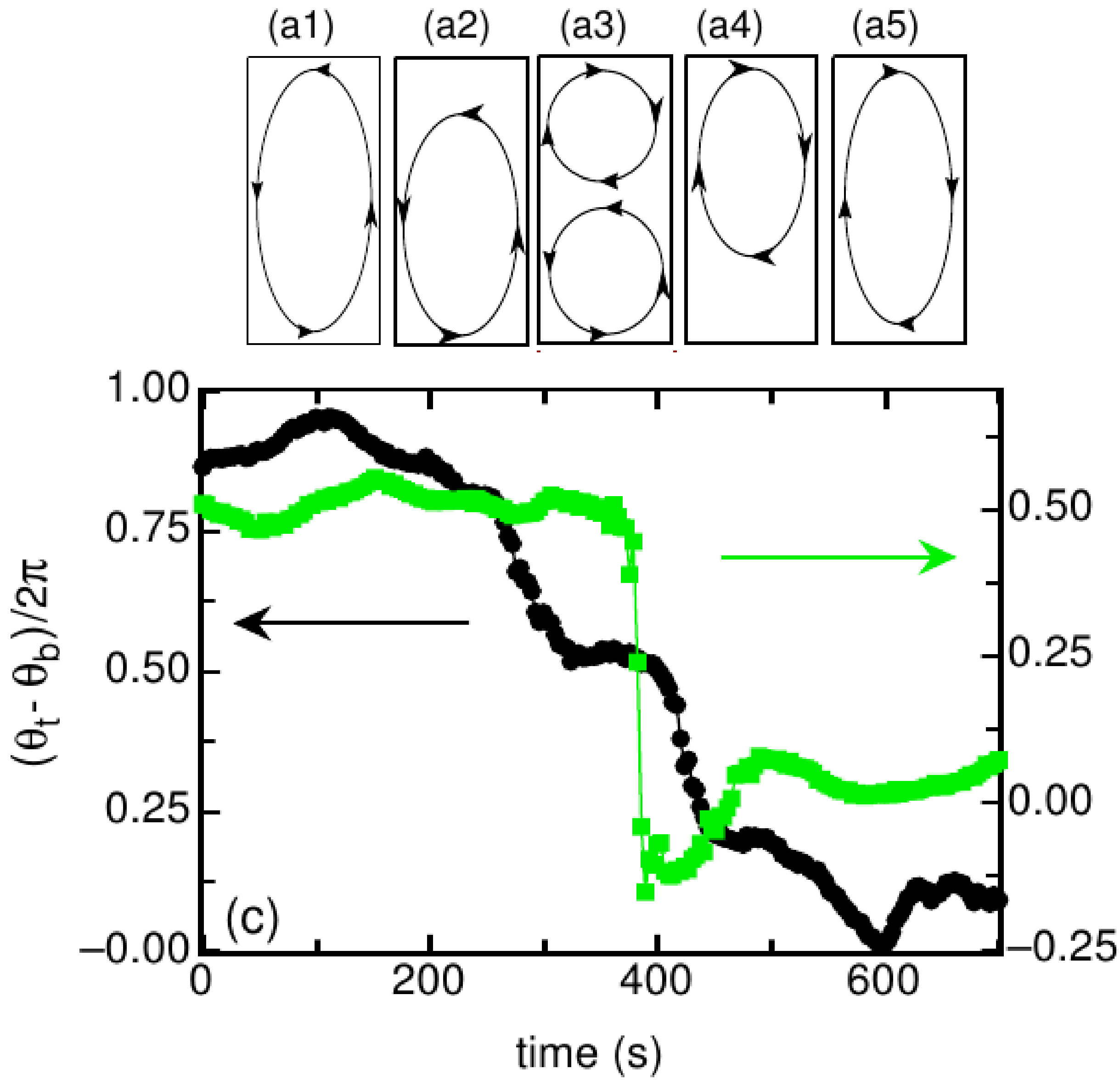
SRS … Single roll state

$\delta_t$, $\delta_m$, $\delta_b$ } right axis

$\theta_t - \theta_b$ } left axis

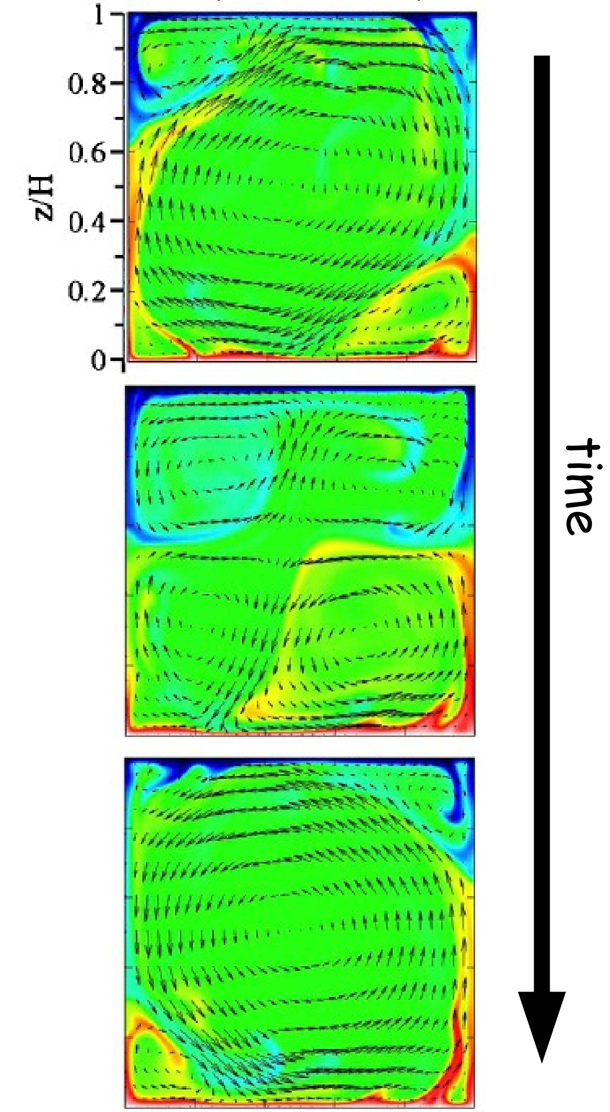
First reported by: Xi and Xia, *Phys. Fluids*. **20**, 055104 (2008)

# Flow Mode Transition in $\Gamma = 1/2$




2D convection, $\Gamma$=1

DNS, Pr=4.38, Ra =$10^8$



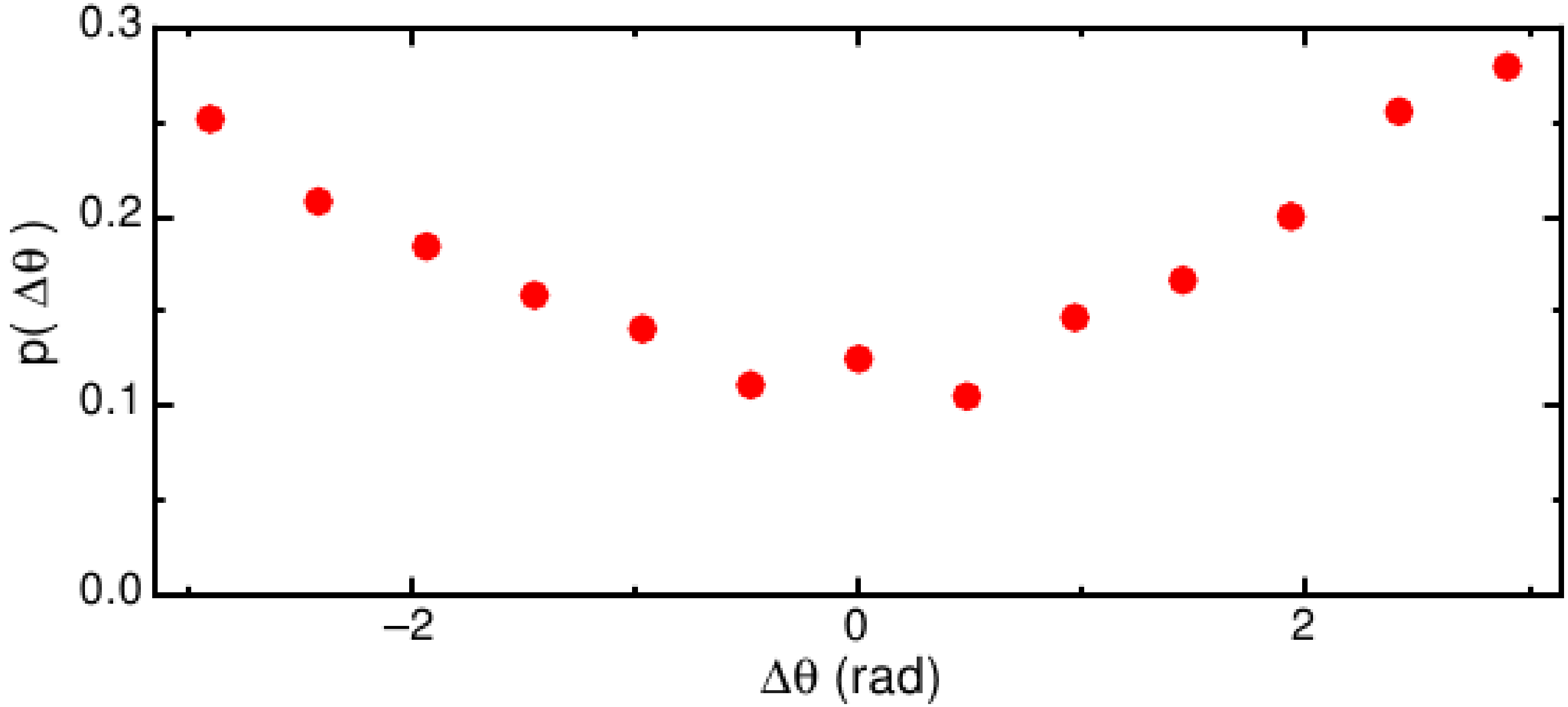

Sugiyama et al. ,PRL, **105**, 034503 (2010)

# Change of Orientation during Flow Mode transition

$$\Delta\theta = \theta_m(t_0 + \Delta t) - \theta_m(t_0)$$

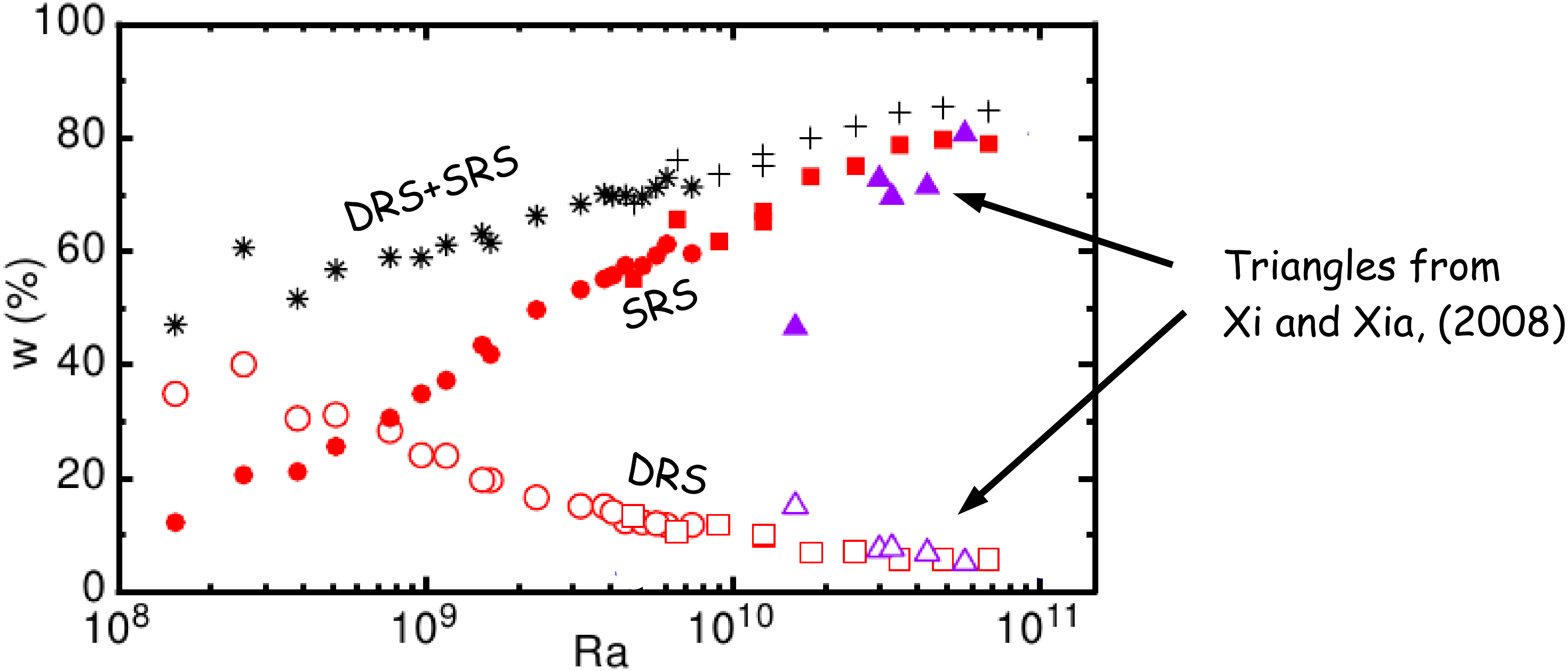
$t_0$ … beginning of flow mode transition

$\Delta t$ … duration of flow mode transition



# Flow Mode transitions as function of Ra

w … the ratio of time the system spends in DRS and SRS



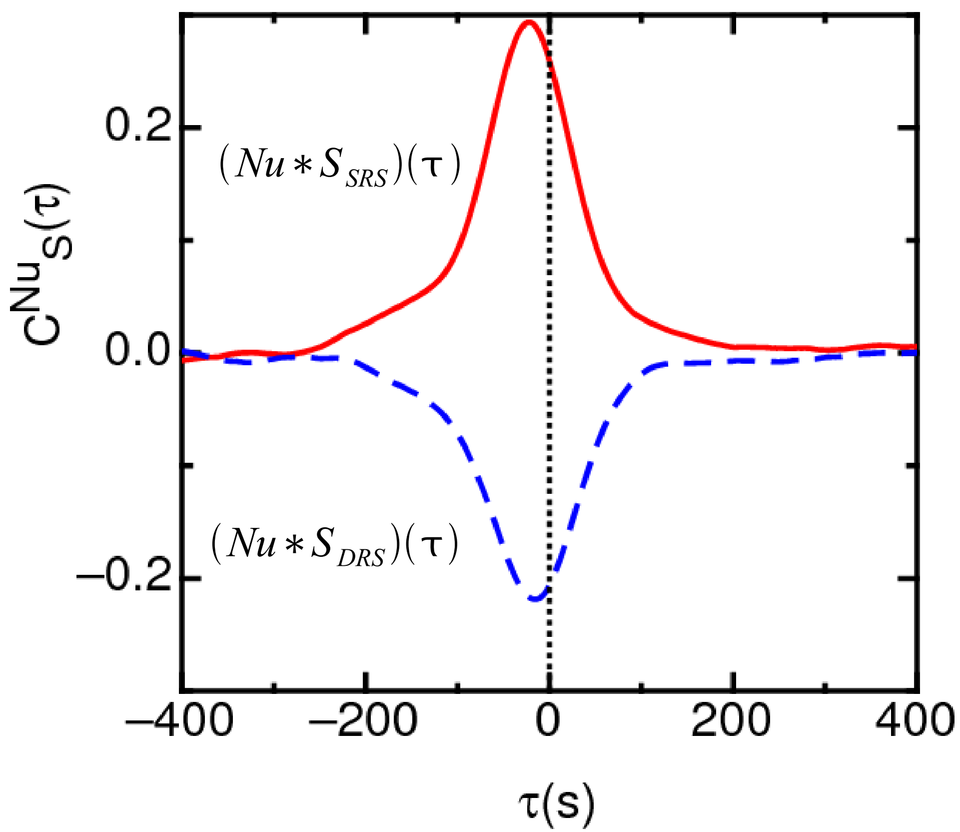

LSC and SRS become more stable with increasing Ra

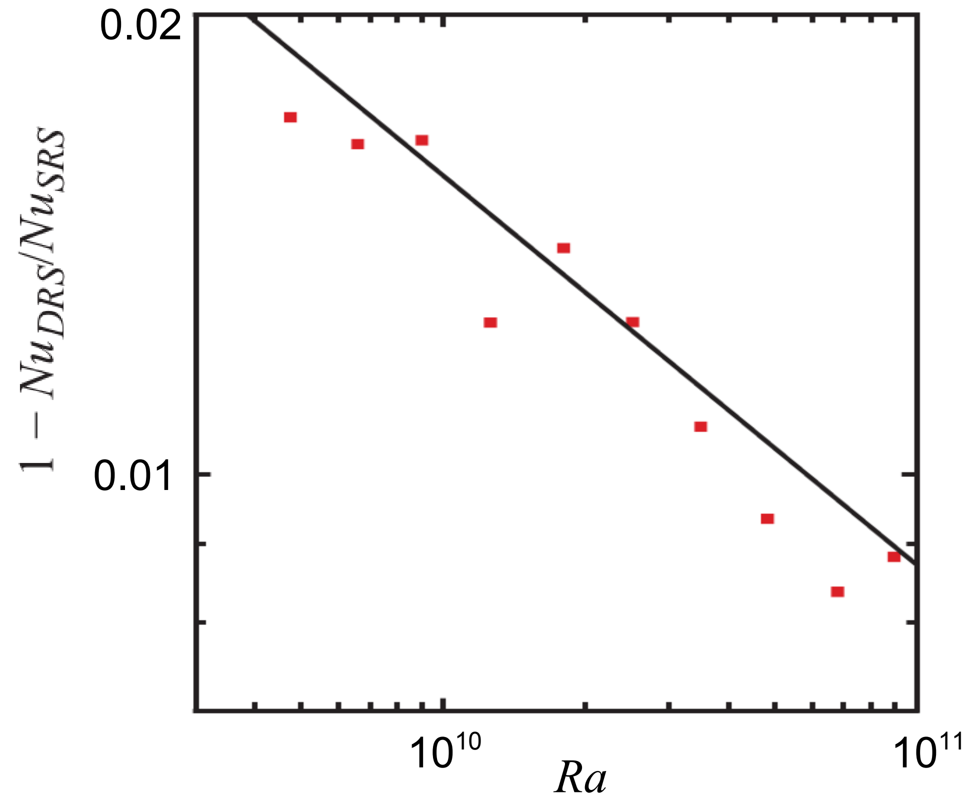
# Flow Mode Transition and Heat transport

$$C_S^{Nu}(\tau)=\frac{\langle (Nu(t)-\langle Nu\rangle)\cdot S(t+\tau)\rangle_t}{\sqrt{\langle Nu^2\rangle_t \langle S^2\rangle_t}}$$

$$S_{SRS}=\begin{cases}1 & \text{... when system in SRS}\\ -1 & \text{... otherwise}\end{cases}$$

$$S_{DRS}=\begin{cases}1 & \text{... when system in DRS}\\ -1 & \text{... otherwise}\end{cases}$$




Double roll state corresponds to lower heat transport !!

# Short Summary

- Different dynamics of the LSC in $\Gamma=1$ and $\Gamma=0.5$ convection cylinders
- LSC more erratic in $\Gamma=0.5$ as in comparision to $\Gamma=1$
- Flow mode transitions in $\Gamma=0.5$ from a single to a double roll state
- Heat transfer in double roll state approximatly 1% lower as in single roll state

# Effect of Rotation on the LSC

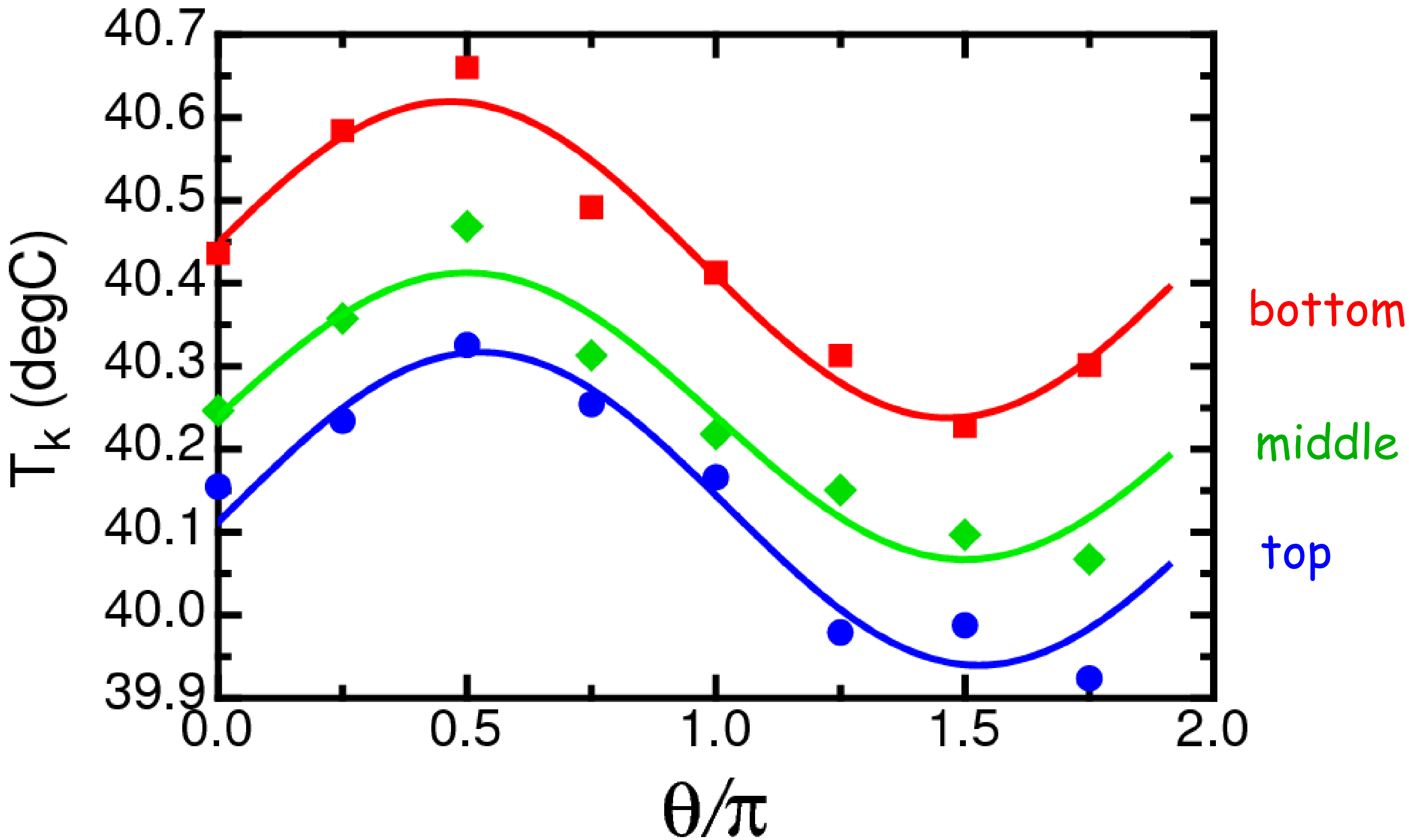
# Does the large scale circulation still exist ?

Stevens, Kunnen , Overkamp, Sun , v. Heijst , Clercx, JFM (submitted):

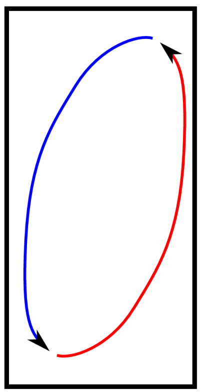
$$S_{LSC} = max\left[0, \left(\frac{E_1}{E_{tot}} - \frac{1}{N}\right) / \left(1 - \frac{1}{N}\right)\right]$$

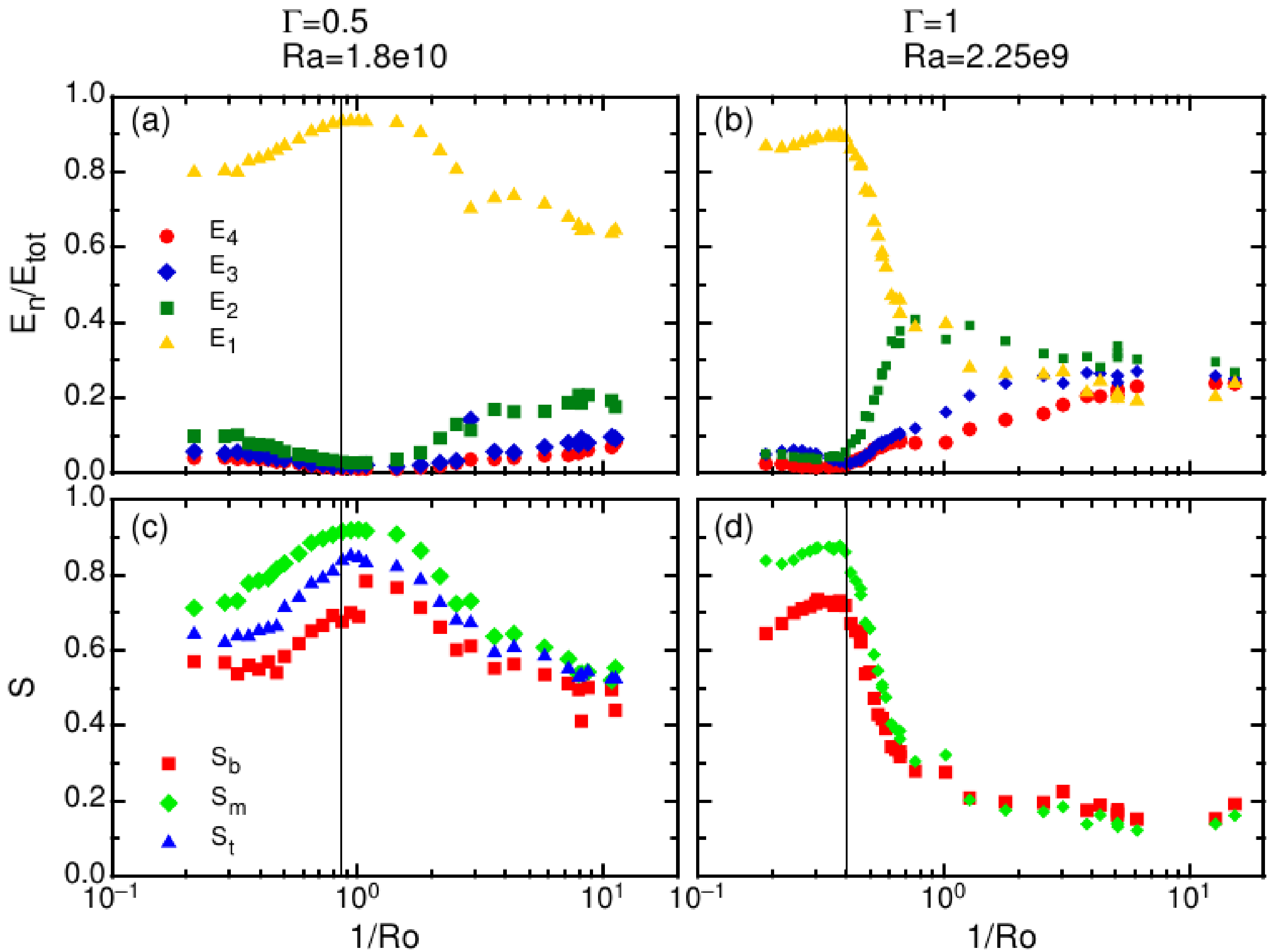
$E_k$ ... Energy of the k-th Fourier mode

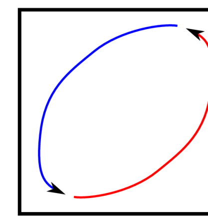
$$E_{tot} = \sum_{k=1}^{4} E_k$$



# Does the large scale shear flow still exist ?

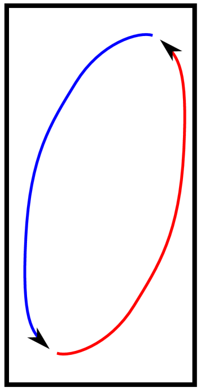


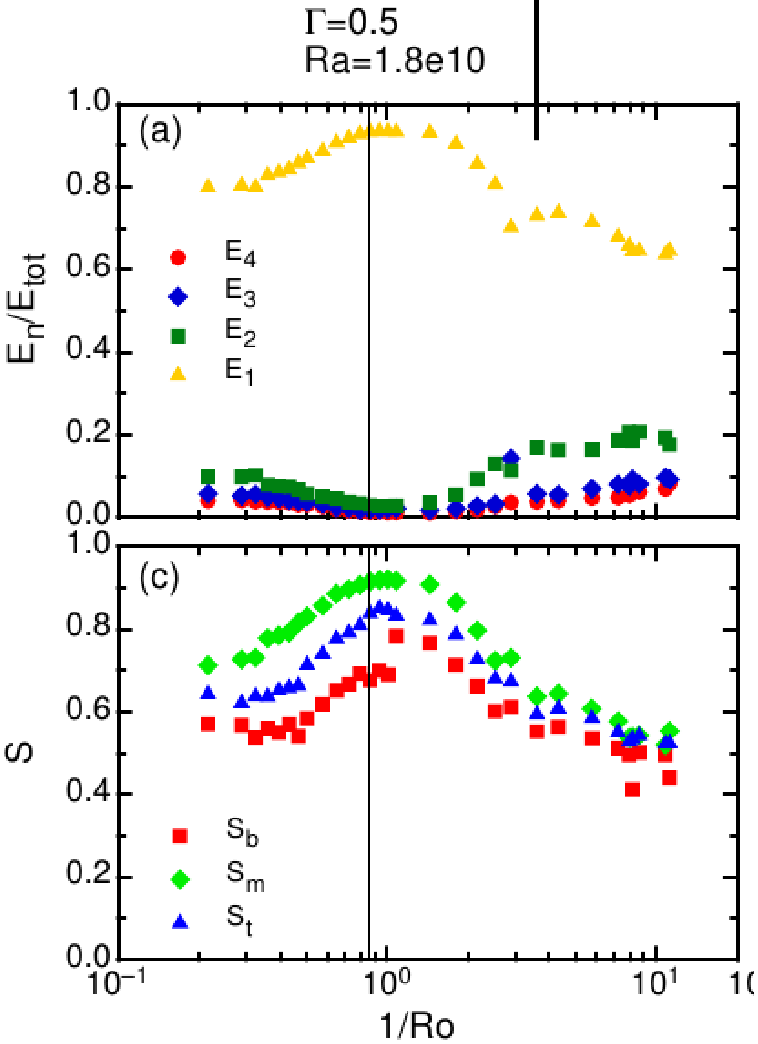


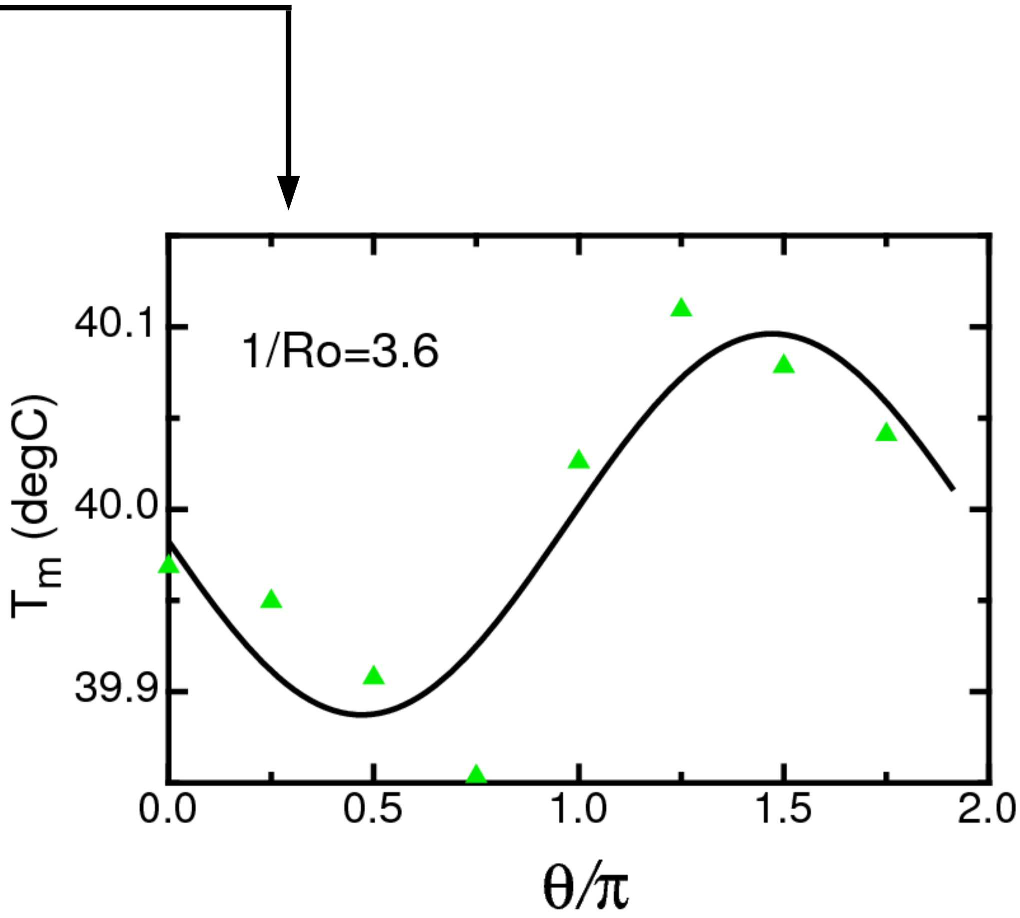



Γ =1 results agree with: Kunnen , Clercx and Geurts, *EPL* **84**, 24001 (2008)

# Does the large scale shear flow still exist ?

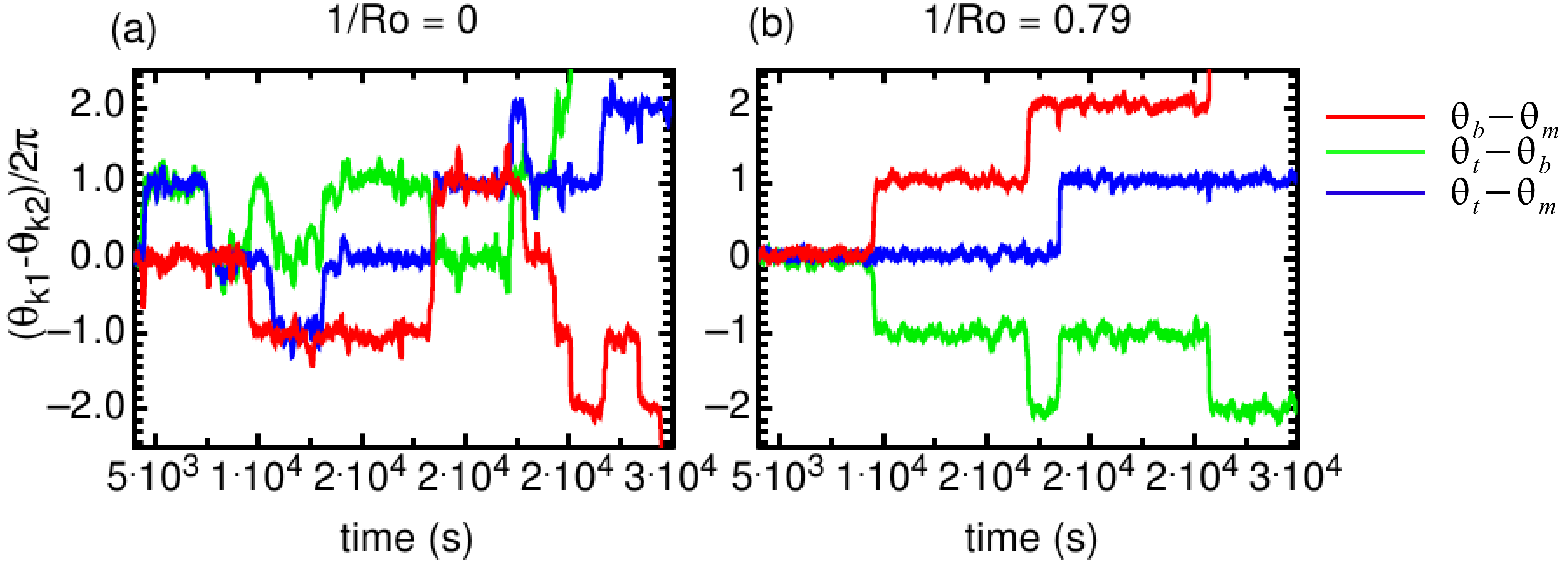




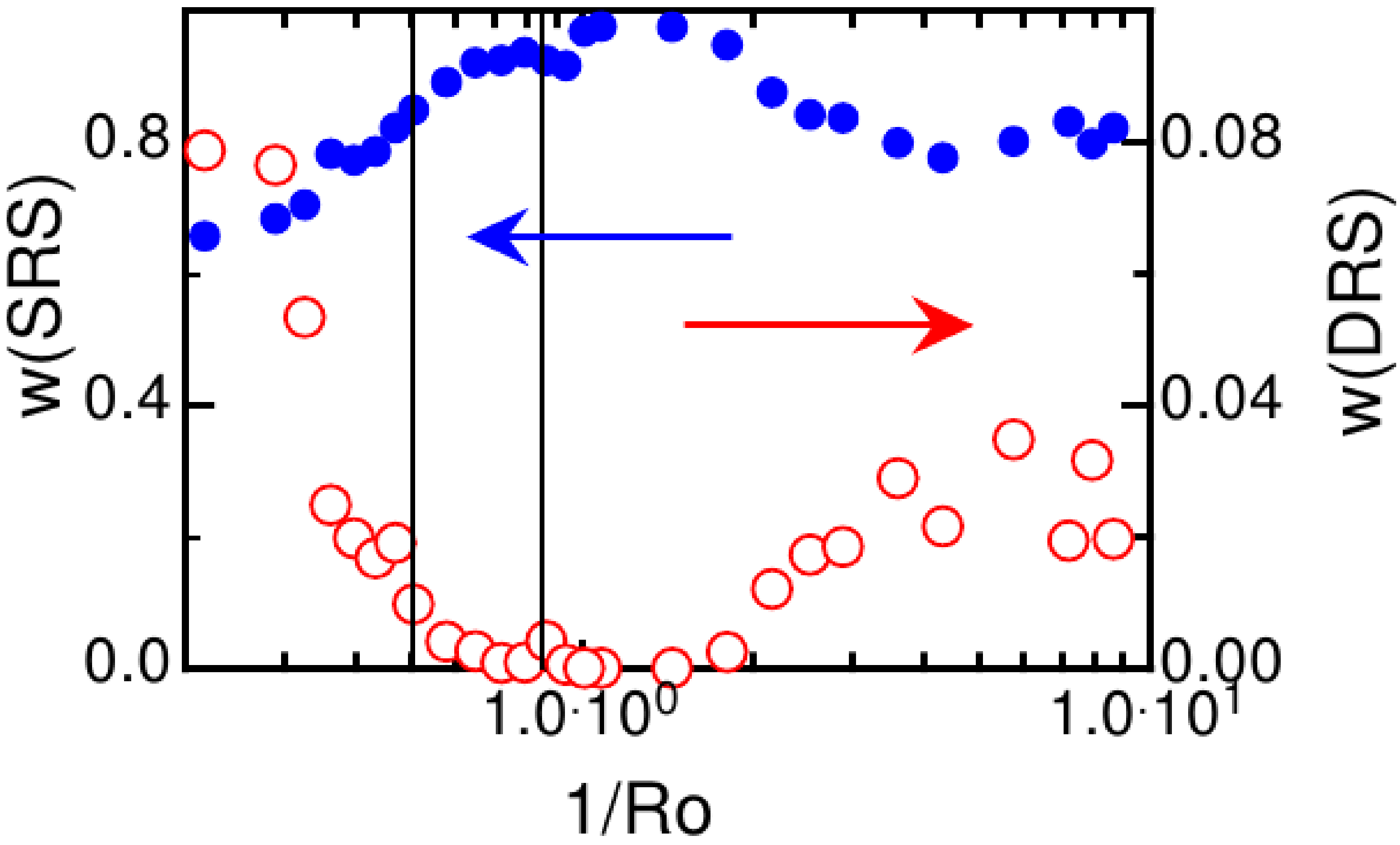
# Slow rotation stabilises the LSC

Ra = 1.8e10
Γ = 1/2



# Slow rotation stabilises the SRS

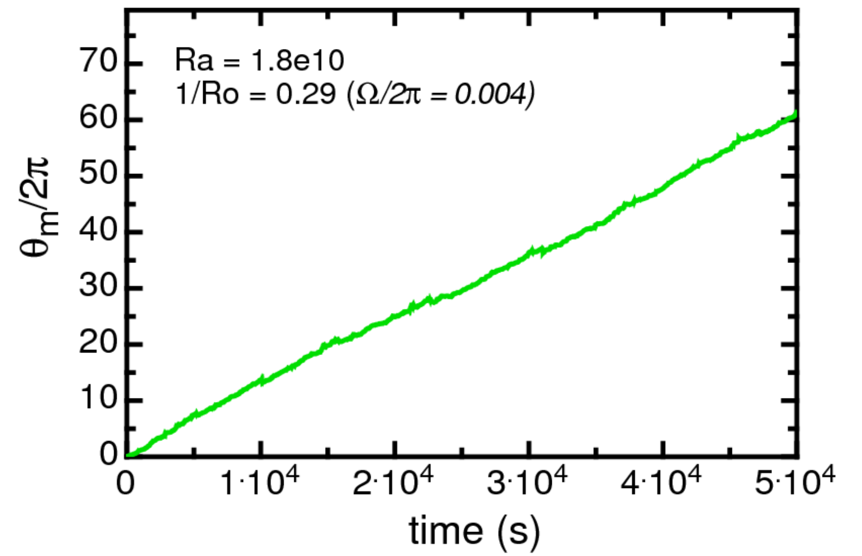
Ra = 1.8e10
Γ = 1/2

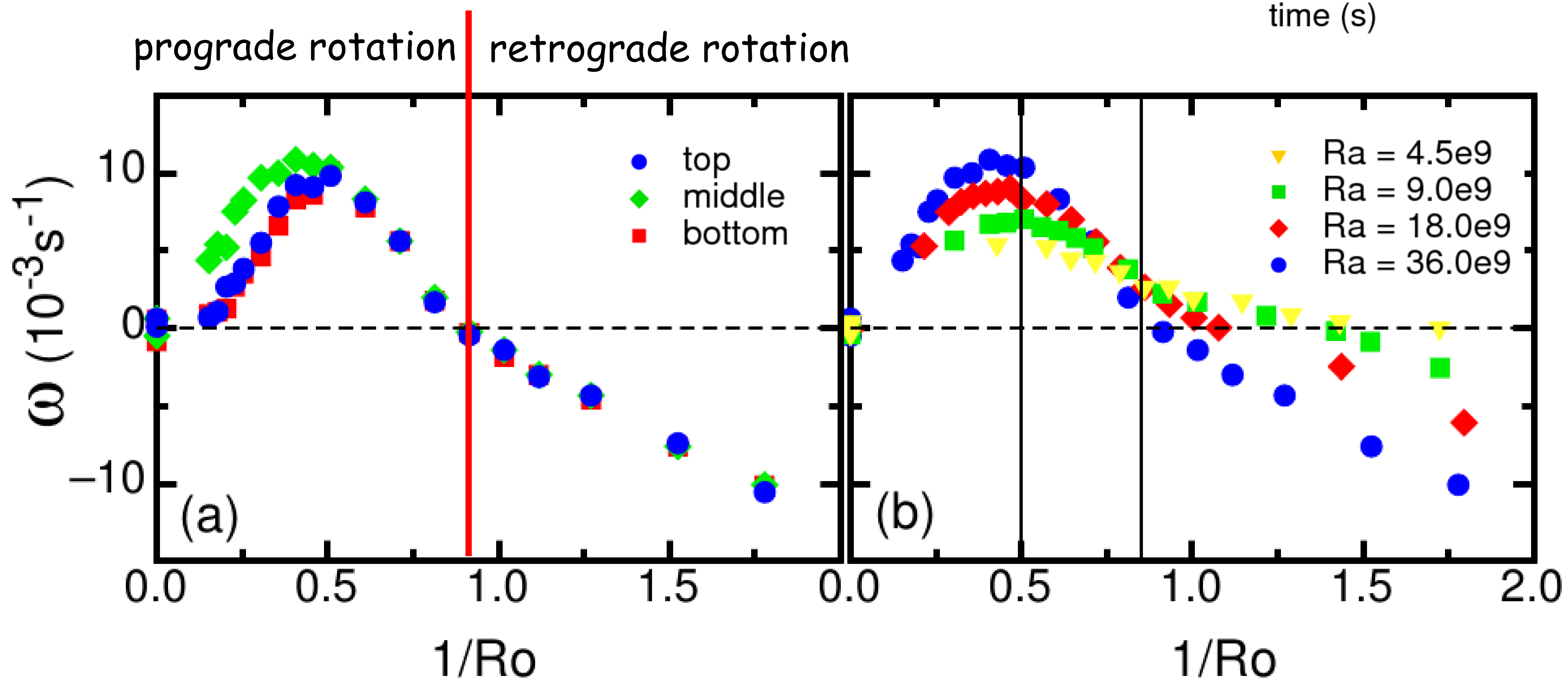


# Rotation of the LSC

Ra = 1.8e10
Γ = 1/2

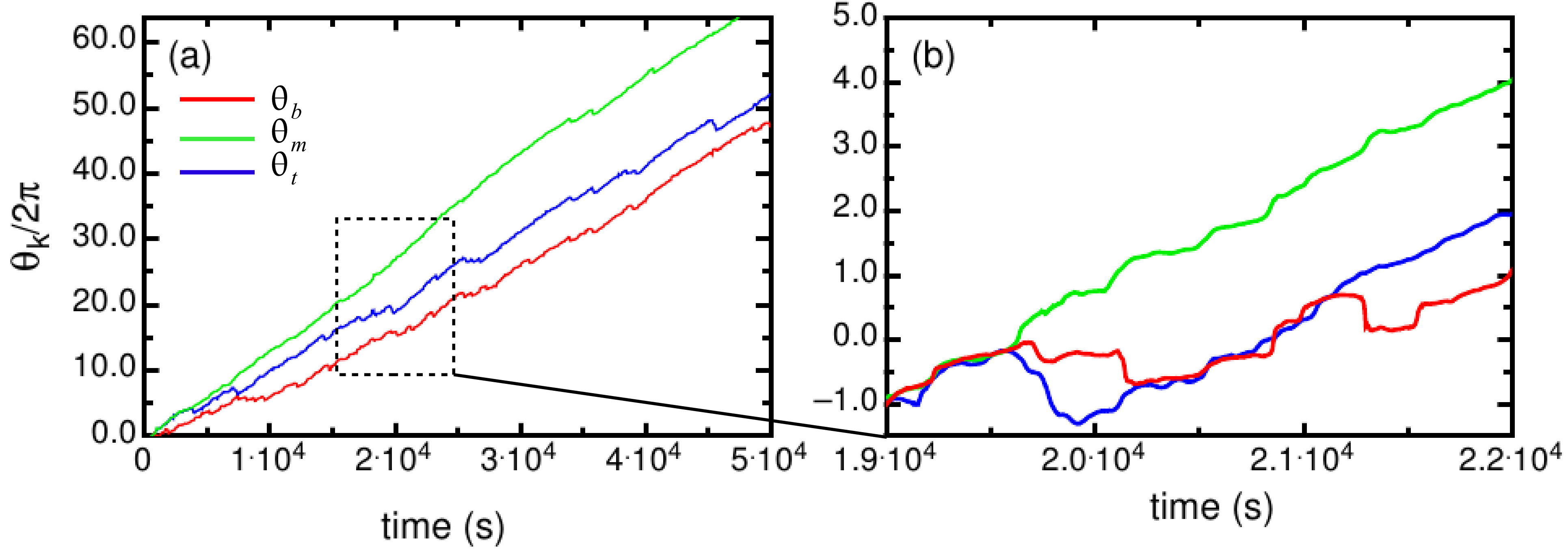
Rotation rate: $\omega = \frac{\Delta \theta}{\Delta t}$



# Rotation of the LSC

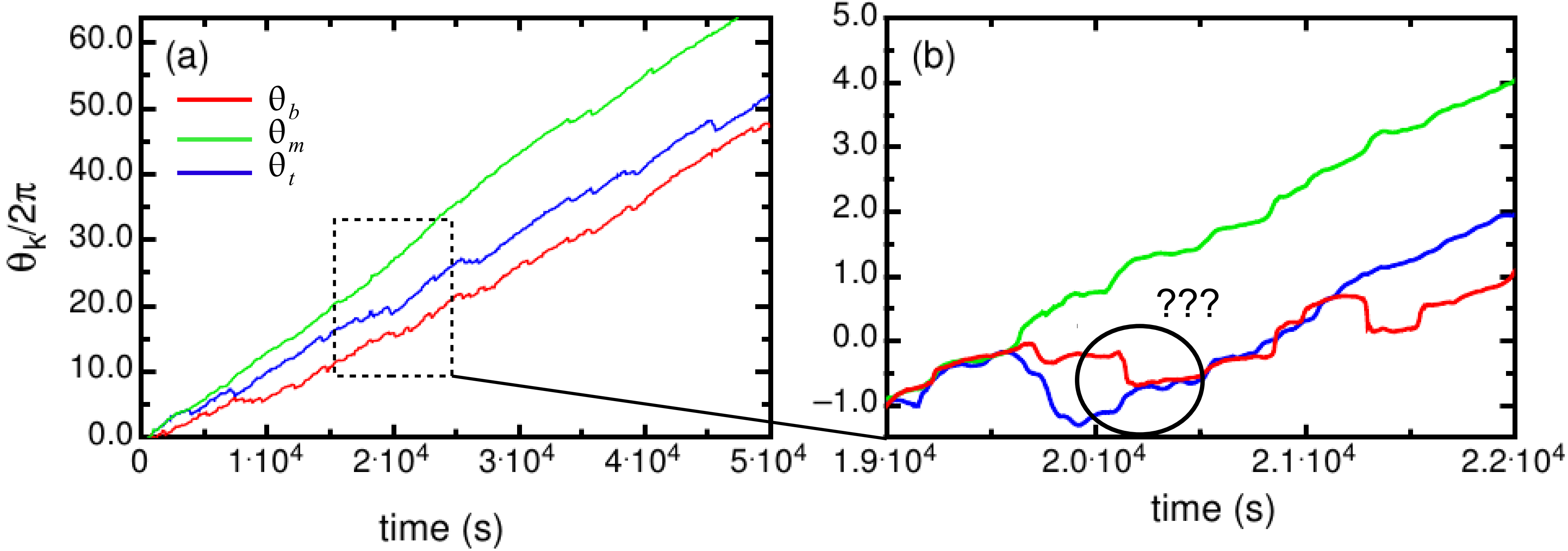
Ra = 1.8e10     1/Ro = 0.4




Steps of $2\pi$ lead to different rotation rate of the orientation $\theta_k$

# Rotation of the LSC

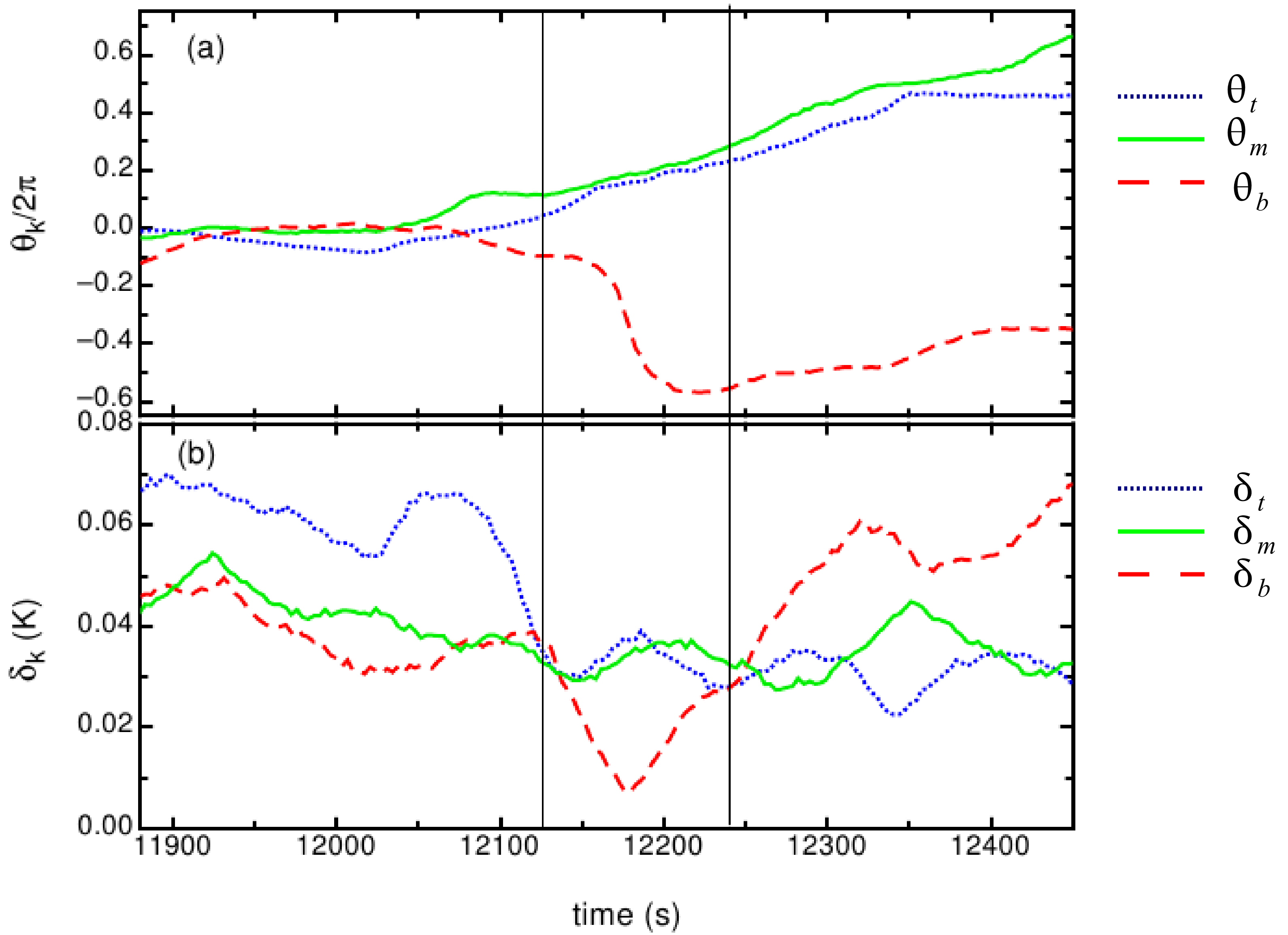
Ra = 1.8e10     1/Ro = 0.4



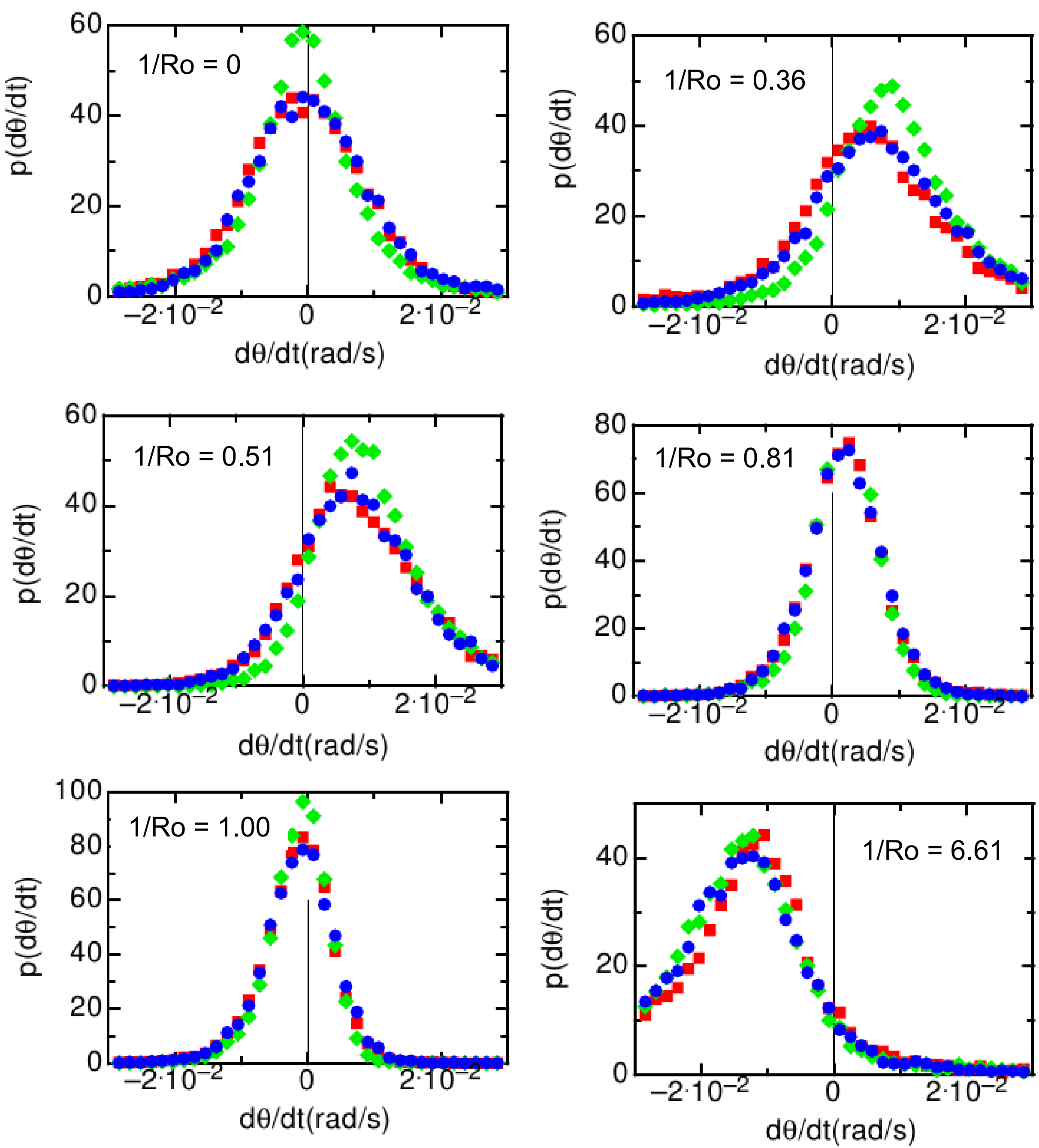

Steps of $2\pi$ lead to different rotation rate of the orientation $\theta_k$

# Partial rotation events

Ra = 1.8e10 1/Ro = 0.4



# The Momentary rotation of the LSC

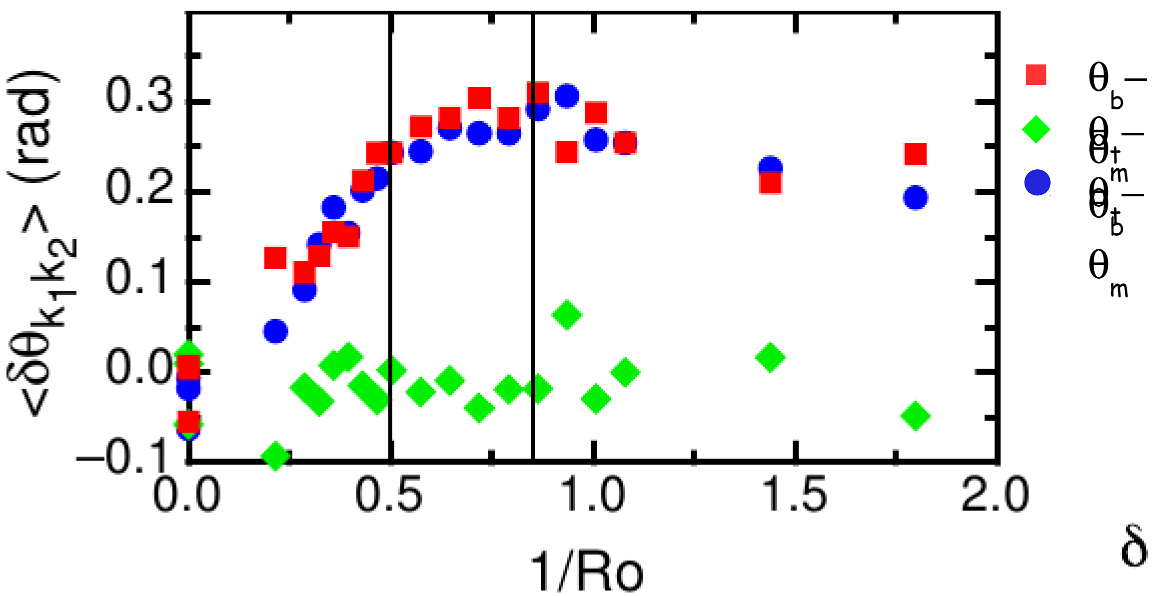


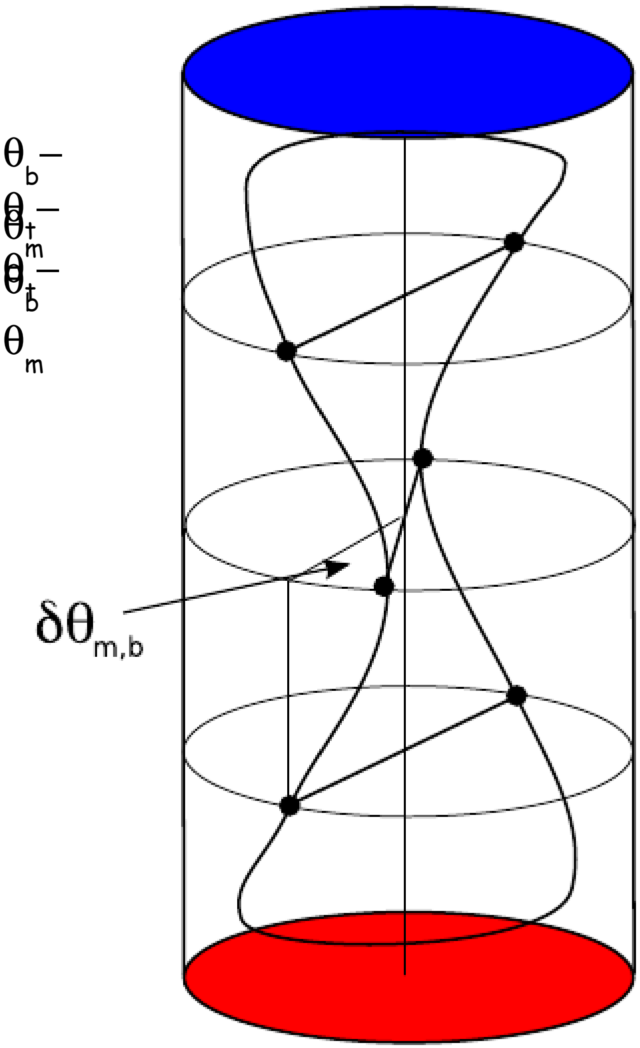

Ra = 1.8e10

Γ = 1/2

# The bend of the LSC

$$\delta\theta_{k_1 k_2} = \theta_{k_1} - \theta_{k_2}$$



# Short Summary

- Slow rotation stabilises LSC
- Slow rotation favors single roll state
- Prograde rotation of the LSC for slow cylinder rotaions
  retrograd rotation of the LSC for moderate
  cylinder rotations
- Partial break of the LSC for slow rotations
- Bend of the LSC for slow rotations

Thank you for your attention !