A test-tube model for rainfall

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Overview

- Some background on clouds and rainfall.
- Goettingen experiment on rain in a test-tube.
- Its analysis uses the Lifshitz-Slezov theory of Ostwald ripening.
- There are general lessons, applicable to real clouds.

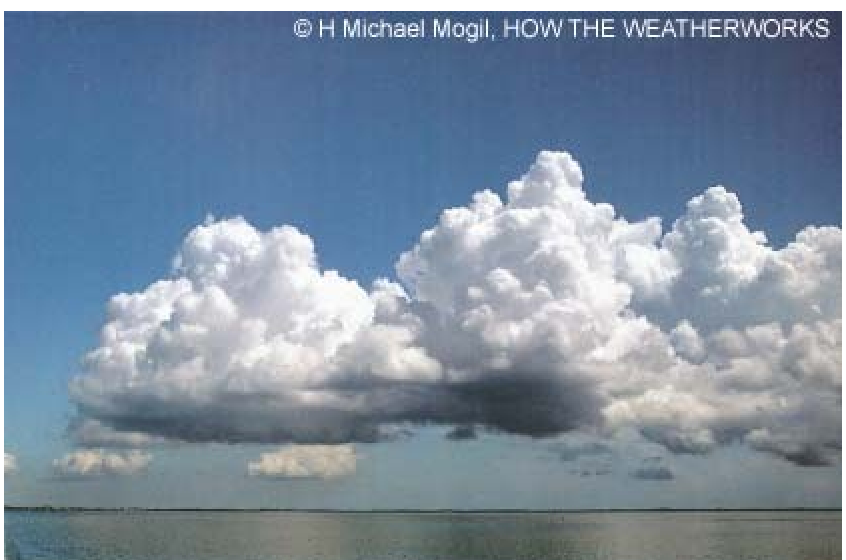
The rain cycle

- Evaporation of water into an air mass.
- Cooling of the air mass causes supersaturation.
- Supersaturation leads to nucleation of water droplets onto dust particles, creating clouds.
- The microscopic water droplets in clouds grow by collisions or other processes, until they are heavy enough to fall at a significant rate.
- Falling droplets can sweep up smaller ones and experience a runaway growth. They become rain drops.
- The rain drops sweep the excess moisture our of the air mass.

Types of cloud

There are two major clouds types: *stratus* and *cumulus*:

Stratus clouds: layers in an air mass which is stable against convection. Associated with slow onset of rain from warm fronts. No turbulence.



Cumulus clouds: moisture in clumps of rising air in an atmosphere which is unstable against convection. Associated with showers or thunderstorms from cold fronts. Turbulence.

Kinetics of droplet growth

Droplets may grow by

- Condensation of supersaturated vapour.
- Collisions, leading to droplet coalsecence. The relative velocity may be due to

  a) Brownian diffusion of droplets

  b) Gravitational settling

  c) Consequences of turbulence in the cloud.

It is argued that there is a 'condensation-collision bottleneck' which is a barrier to droplet growth in the range $15 - 50\,\mu\mathrm{m}$, and that turbulence is required to bridge the gap.

Is turbulence significant?

http://www.usatoday.com/weather/wcumulus.htm

Physical parameters:

R.A.Shaw, *Ann. Rev. Fluid Mech*., **35**, 183, (2003)

$$a \approx 10\mu\mathrm{m} \qquad n \approx 10^8\mathrm{m}^{-3}$$

$$\nu \approx 10^{-5}\mathrm{m}^2\mathrm{s}^{-1} \quad \mathcal{E} = 0.1\mathrm{m}^3\mathrm{s}^{-2}$$

From these values we find:

$$\eta \approx 3 \times 10^{-4}\mathrm{m} \qquad \tau \approx 10^{-2}\mathrm{s}$$

$$\gamma \approx 500\,\mathrm{s}^{-1}$$

The Stokes numbers may approach unity: here $\mathrm{St} \approx 0.2$

Stokes numbers of order unity imply clustering and other effects, but this only happens for extreme conditions.

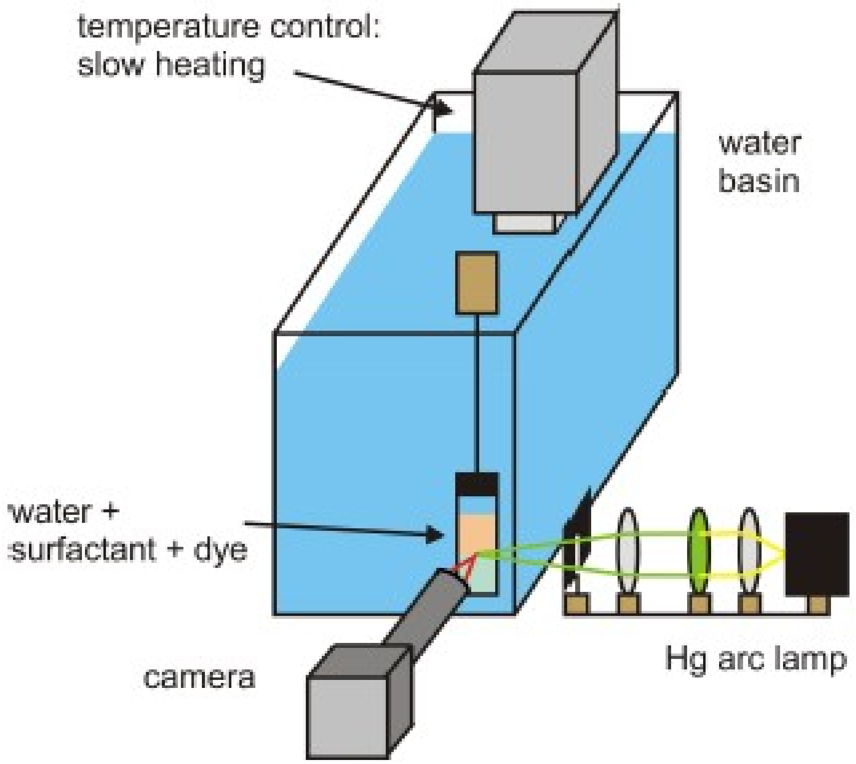
Laboratory models

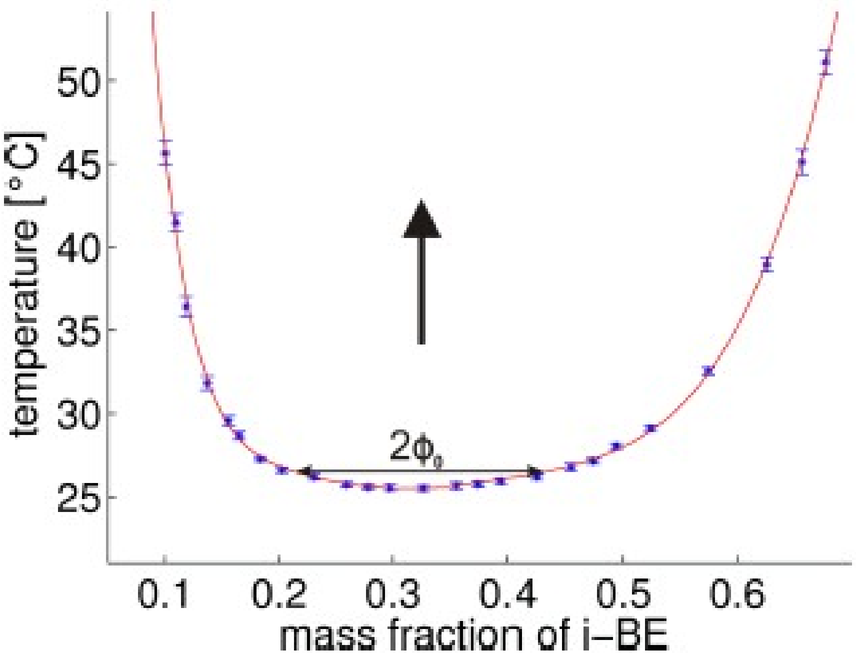
- There are studies of water vapour in large tanks. These are too small to produce rainfall.
- There are reports of rain cycles in liquid sodium reactor cooling circuits.
- An alternative approach, pursued by Juergen Vollmer and co-workers, uses two partly miscible liquids in a test-tube. The temperature is varied smoothly to decrease miscibility. Material comes out of solution in the form of small droplets. These grow and move towards the interface as 'rain'.

  The rain events are seen as in increase in turbidity of the mixture. They are roughly periodic in time, with a period which increases as the rate of cooling decreases.

The Goettingen experiment

Uses water and isobutoxyethanol, which are not fully miscible above $T_{\mathrm{c}} = 25.51°\mathrm{C}$

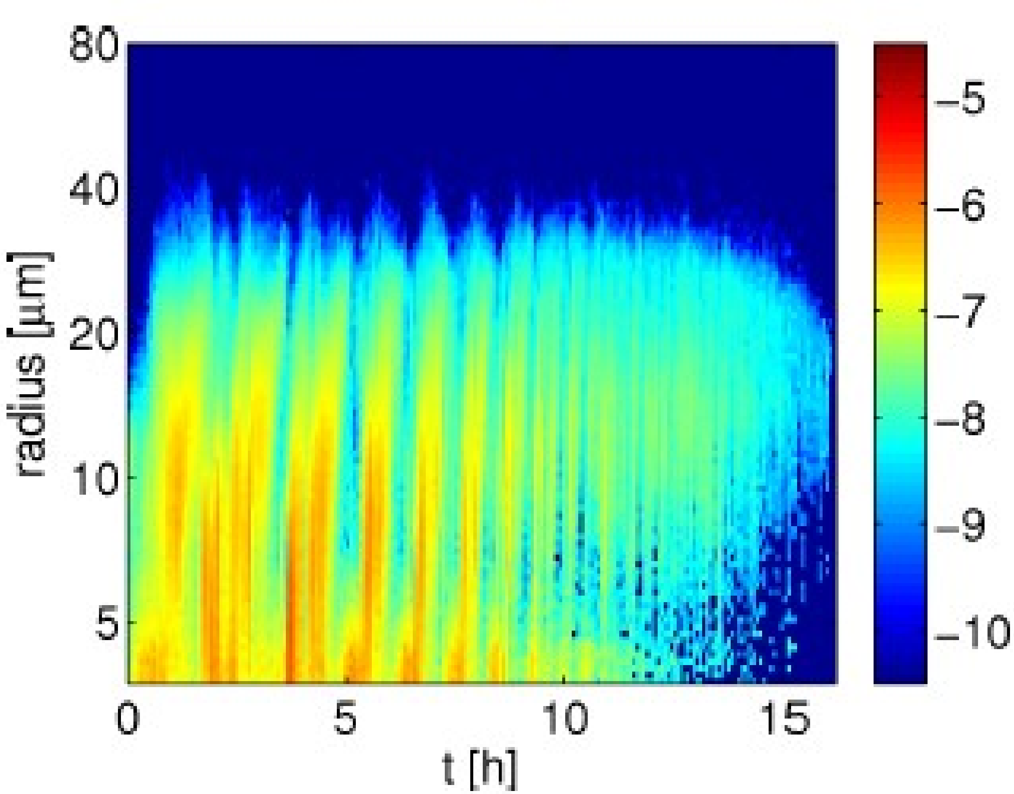




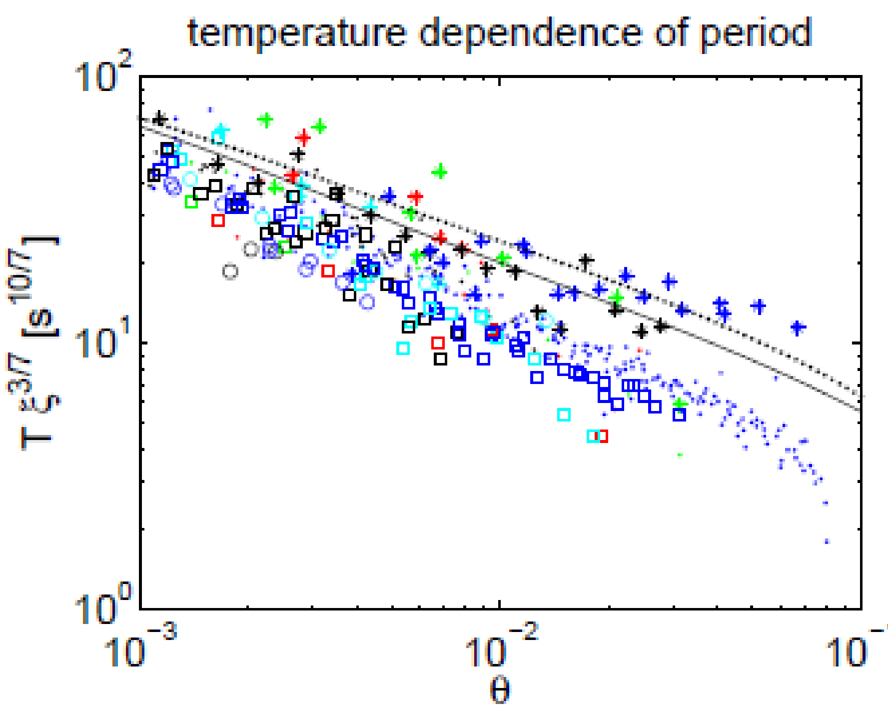
A camera and image-processing software determines the distribution of droplet sizes. Wide range of heating rates and temperatures, allow discriminating test of theories.

Goettingen experiment - results

Time variation of droplet size distribution:



Period as a function of cooling rate:



$$\xi_{\mathrm{b}} \equiv \frac{1}{V_{\mathrm{b}}}\frac{\delta V_{\mathrm{d}}}{\delta T}\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\Phi_{\mathrm{b}} - \Phi_{\mathrm{t}}}\frac{\mathrm{d}\Phi_{\mathrm{b}}}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}t}$$

$$\theta = \frac{T - T_{\mathrm{c}}}{T_{\mathrm{c}}}$$

$$\Delta t \sim \xi^{-3/7}$$

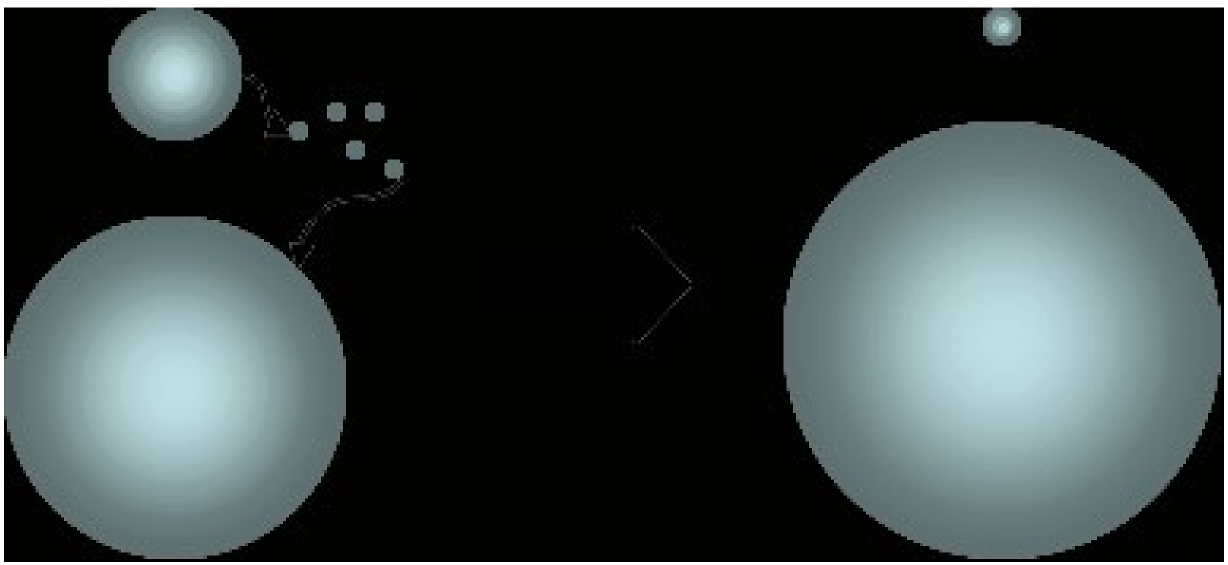
A model for test-tube rain

- After a 'rain' event, negligible supersaturation.
- Heating causes supersaturation, nucleation on dust.
- The microscopic droplets grow by *Ostwald ripening*.
- When droplets are large enough, they drift towards the interface at a significant rate.
- Larger droplets catch up with smaller ones, coalesce, then move faster. There is a finite-time singularity.
- The 'rainfall' sweeps away the excess material, cycle can start again.

  \- *All this is similar to 'drizzle' from stratus clouds.*

Ostwald ripening

Consider water droplets in air. The smallest droplets have the highest Laplace pressure. They need a higher level of supersaturation to prevent evaporation. Water vapour from the smallest drops condenses onto the larger ones.



Laplace pressure:

$$\Delta p = \frac{2\gamma}{a}$$

The mechanism is more rapid than collisions due to Brownian motion of droplets, because molecules diffuse faster than droplets.

Lifshitz-Slezov-Wagner theory

The surface of a droplet is in contact with vapour at a higher concentration:

$$\Phi(a) = \Phi_\mathrm{e} + \frac{\sigma}{a} \qquad \sigma = \frac{2\gamma v \Phi_\mathrm{e}}{kT} = \frac{2\gamma V_\mathrm{m}}{RT}\Phi_\mathrm{e}$$

The concentration obeys Laplace's equation. The concentration gradient at the surface leads to diffusive deposition, which increases the radius:

$$\Phi(r) = \Phi_\mathrm{e} + s + \frac{\sigma - sa}{r}$$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -D\frac{\partial \Phi}{\partial r}\bigg|_{r=a} = \frac{D\sigma}{a}\left(\frac{1}{a} - \frac{1}{a_0}\right)$$

Droplets smaller than $a_0 = \sigma/s$ shrink and evaporate.

Simplified model for droplet growth

The largest droplets are the important actors. Neglect the evaporation term, add a term for growth by sweeping up smaller particles. Vertical velocity due to buoyancy is:

$$u = \frac{2}{9}\frac{\Delta\rho g}{\rho\nu}a^2 \equiv \kappa a^2$$

Sweeping up material leads to additional term in equation for growth of radius:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{D\sigma}{a^2} + \frac{\epsilon}{4}\kappa a^2 \xi t$$

ϵ =collision efficiency=unity

Initial growth is by Ostwald ripening, leading to

$$\langle a(t)\rangle = \left(\frac{4}{9}D\sigma t\right)^{1/3}$$

Prediction of period

Growth by Ostwald ripening, collisional term is comparable, at time t_1 .

$$a^3 \sim D\sigma t$$

Then a runaway growth with finite-time singularity, time t_2 .

$$\Delta t = t_1 + t_2$$

Growth equation:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{D\sigma}{a^2} + \frac{\epsilon}{4}\kappa a^2 \xi t$$

Terms are comparable when:

$$\frac{D\sigma}{\kappa\xi} = a_1^4 t_1 \qquad a_1^3 \sim D\sigma t_1$$

$$t_1 = \left(\frac{1}{D\sigma(\kappa\xi)^3}\right)^{1/7} \qquad t_1 \sim t_2$$

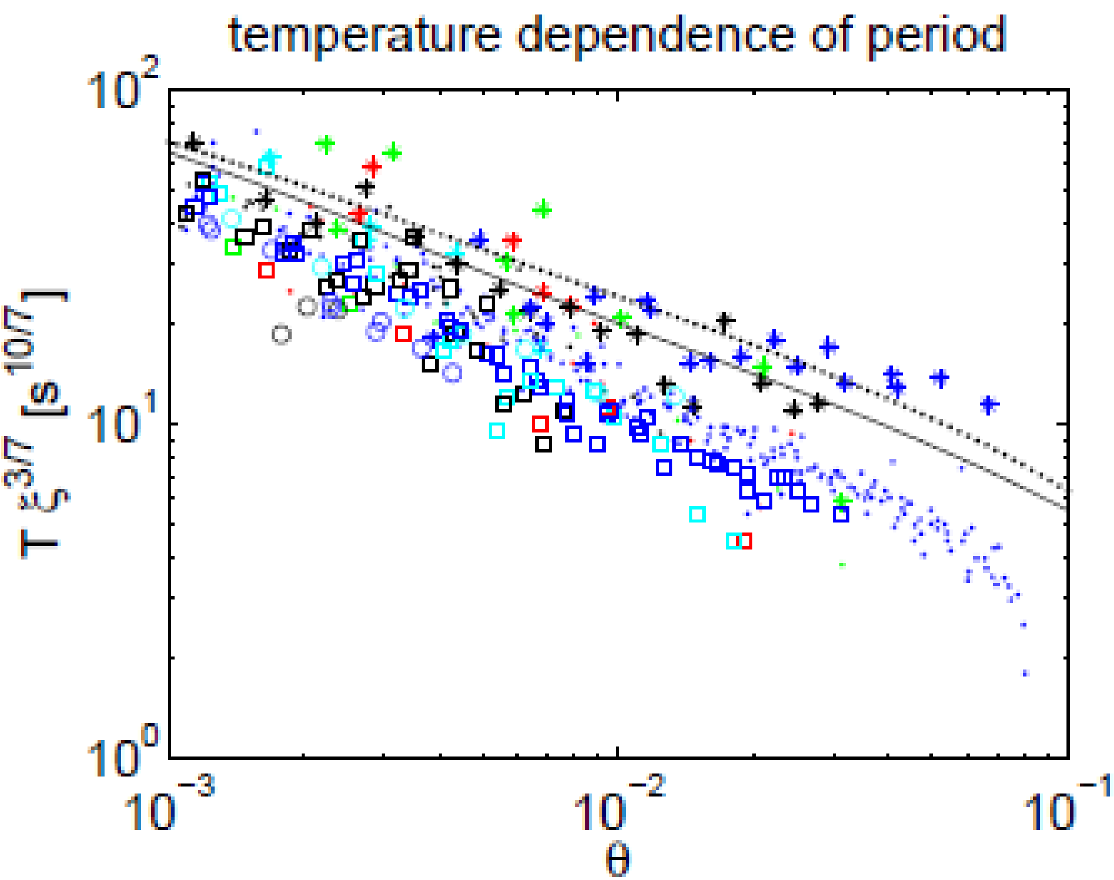
Comparison with experiment

Data on diffusion coefficients, densities, viscosities, phase diagram of the water/i-BE system are available.

$$\Delta t \approx \alpha \left(\frac{1}{D \sigma \kappa^3} \right)^{1/7} \xi^{-3/7}$$

$$u = \frac{2}{9} \frac{\Delta \rho g}{\rho \nu} a^2 \equiv \kappa a^2$$

$$\sigma = \frac{2 \gamma v \Phi_e}{kT} = \frac{2 \gamma V_m}{RT} \Phi_e$$



upper layer : stars

lower layer : dots : $h \approx 10\,\mathrm{mm}$ squares : $h \approx 5\,\mathrm{mm}$ circles : $h \approx 2.5\,\mathrm{mm}$

black : 0.1/hr, blue : 0.2/hr cyan : 0.4/hr green : 0.6 – 1.0/hr red : > 1/hr

Any lessons for real clouds?

Considering Ostwald ripening appears to be a new approach. The growth of droplets by Ostwald ripening is independent of their concentration (within very broad limits):

$$\langle a(t)\rangle = \left(\frac{4}{9}D\sigma t\right)^{1/3}$$

What does this imply for real clouds (at $10^\circ\mathrm{C}$)?

$$D = 2.4\times 10^{-5}\,\mathrm{m^2s^{-1}} \qquad \gamma = 7.4\times 10^{-2}\,\mathrm{Nm^{-1}}$$

$$V_\mathrm{m} = 1.8\times 10^{-5}\,\mathrm{m^3} \qquad \Phi_\mathrm{e} = 1.2\times 10^{-2}$$

These values give:

$$\sigma = 1.4\times 10^{-11}\,\mathrm{m} \qquad a(t)\approx 6\times 10^{-6}(t/\mathrm{s})^{1/3}$$

Growth to $50\,\mu\mathrm{m}$ in a quarter of an hour by Ostwald ripening. No turbulence is required.

Summary

We have the first quantitative description of a precipitation cycle. There is a surprising scaling with the cooling rate:

$$\Delta t \approx \alpha \left(\frac{1}{D\sigma\kappa^3} \right)^{1/7} \xi^{-3/7}$$

The theory involves Ostwald ripening, which allows competitive growth of droplets without collisions.

In clouds, the Ostwald ripening gives a base-line growth rate, present whether or not there is turbulence. There is no significant droplet growth bottleneck.

Unless, perhaps, the nucleating dust particles are hygroscopic (e.g. salt spray)?