Bulk Turbulence of Dilute Polymer Solutions: Lagrangian Particle Tracking Measurements

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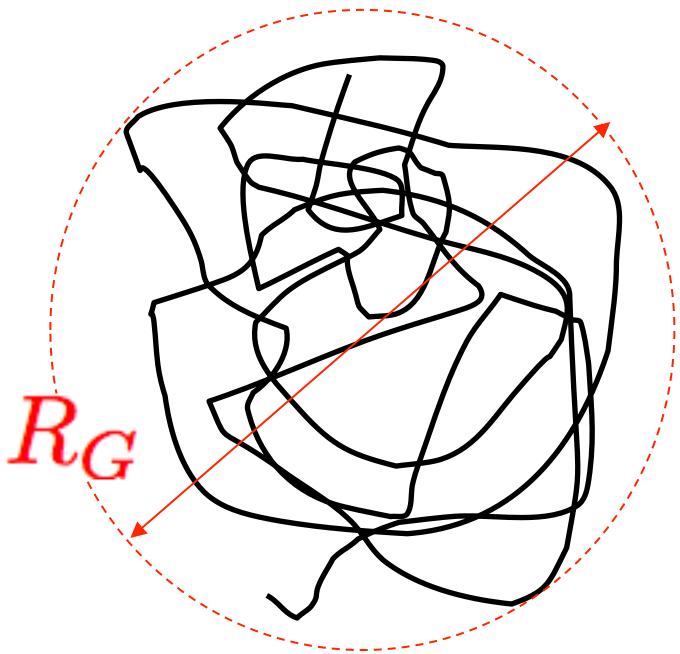
Alexander von Humboldt Stiftung/Foundation

June 1, 2011, KITP, UCSB, Santa Barbara, USA

Outline

- Introduction
- Acceleration Statistics
- Velocity Structure Functions

Flexible Polymers in Flow Field



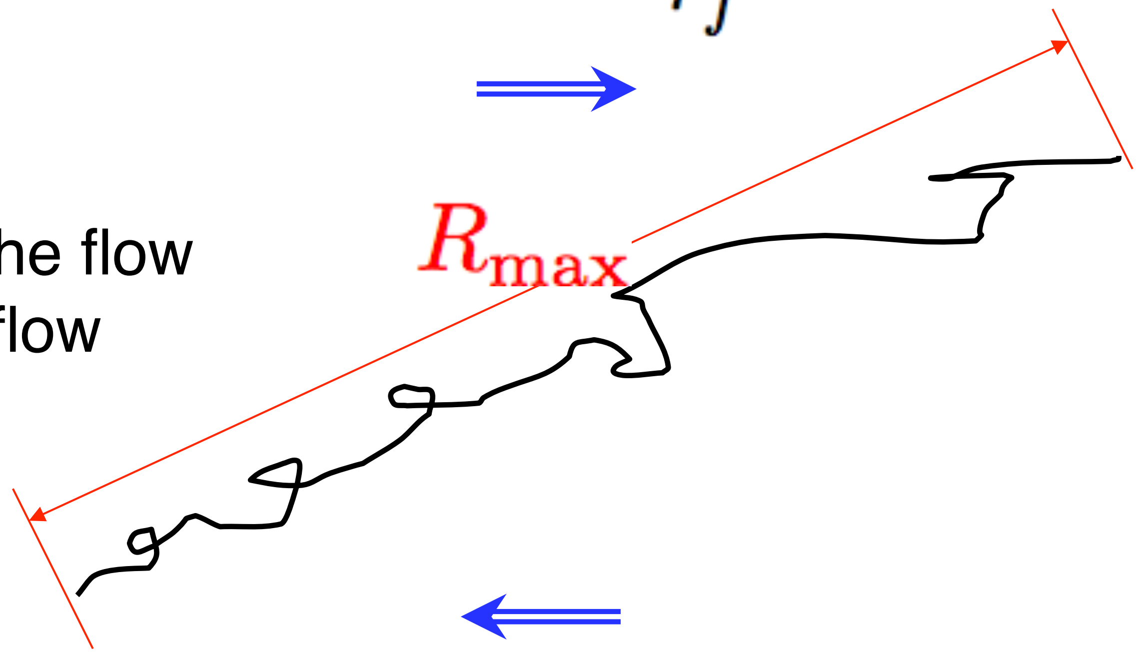
Radius of gyration R_G

Polymer relaxation τ_p

Flow time scale τ_f

$$Wi = \frac{\tau_p}{\tau_f} > 1$$

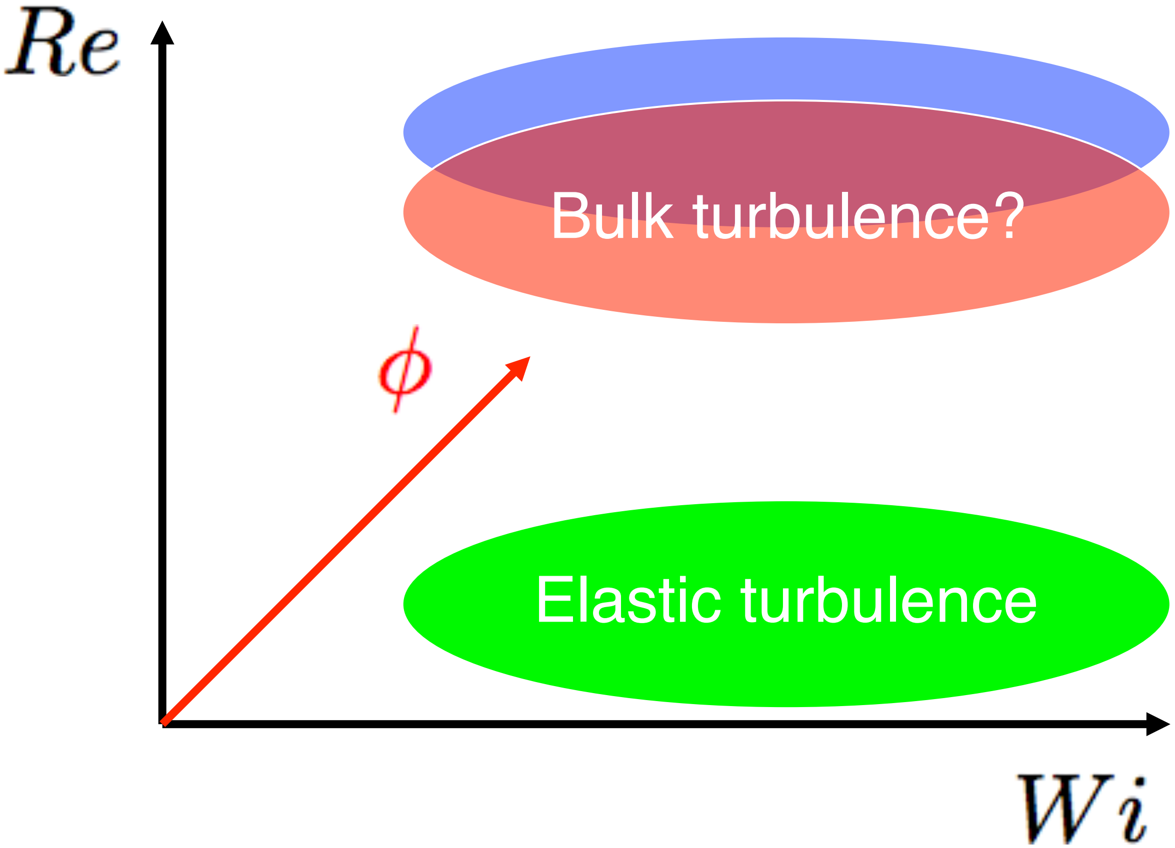
Draw energy away from the flow
Feed energy back to the flow
Dissipate energy



Control Parameters

Reynolds number Re, R_λ

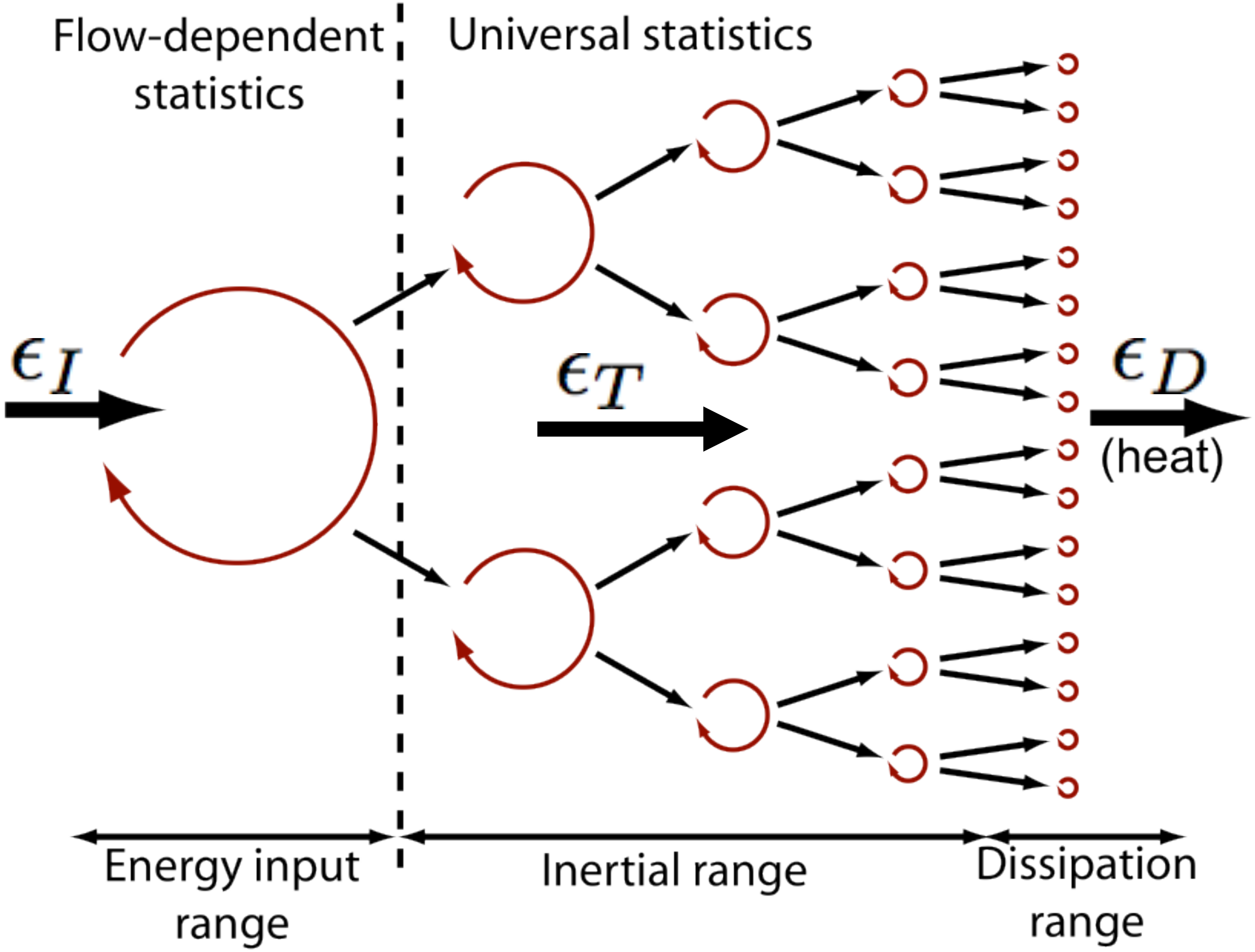
Weissenberg number Wi



Polymer concentration

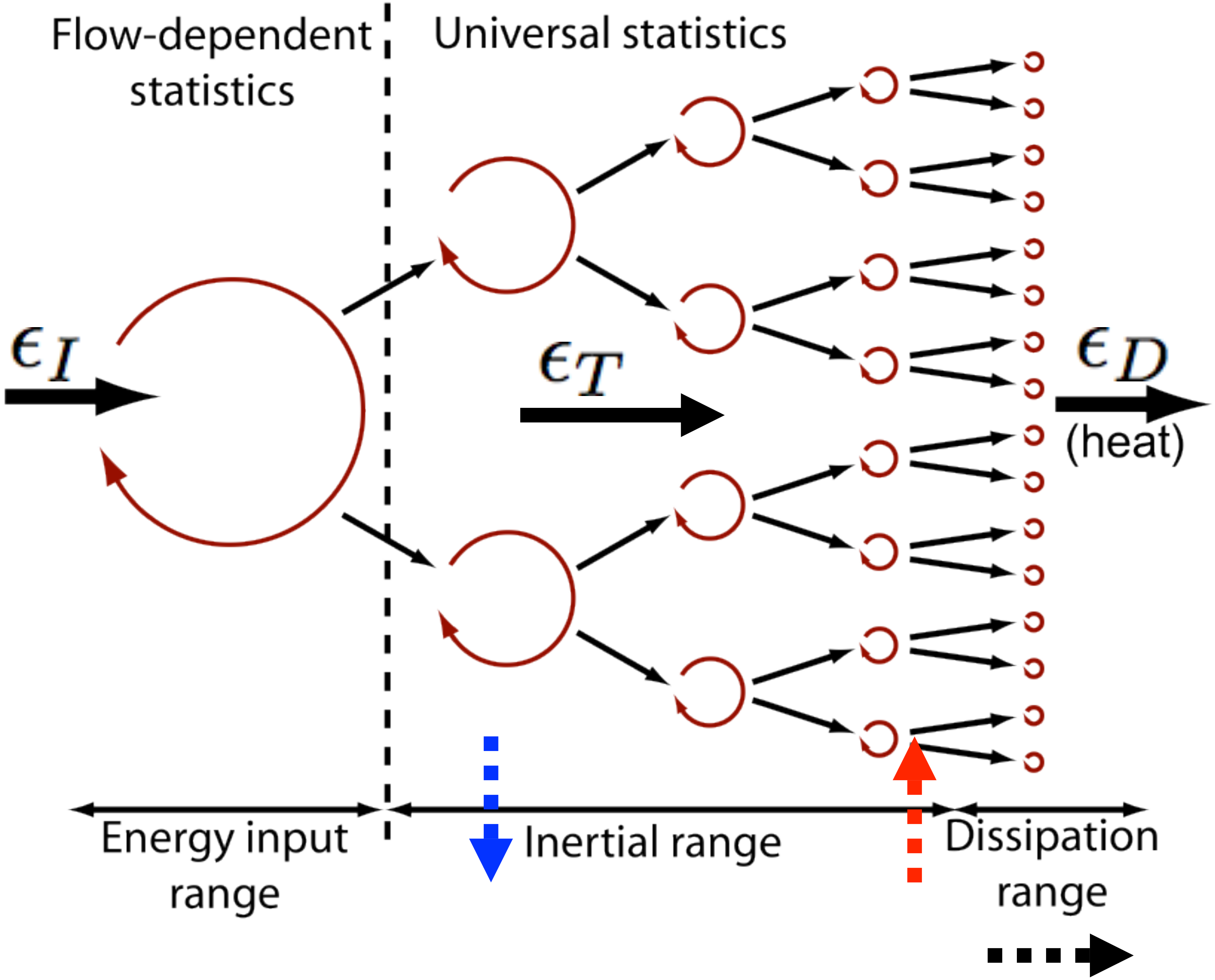
ϕ

Turbulence Energy Cascade



$$\epsilon_I = \epsilon_T = \epsilon_D$$

Turbulence Energy Cascade



$$\epsilon_I \overset{?}{=} \epsilon_T \overset{?}{=} \epsilon_D$$

Polymers

Polyacrylamide (PAM): 18 x 10^6 amu

$$R_G = 0.5\mu m, \quad R_{\max} = 77\mu m, \quad \tau_p = 43ms$$

Turbulence property at $R_\lambda = 350$:

$$\eta = 84\mu m, \quad \tau_\eta = 7.1ms$$

$$Wi = \frac{\tau_p}{\tau_\eta} = 6$$

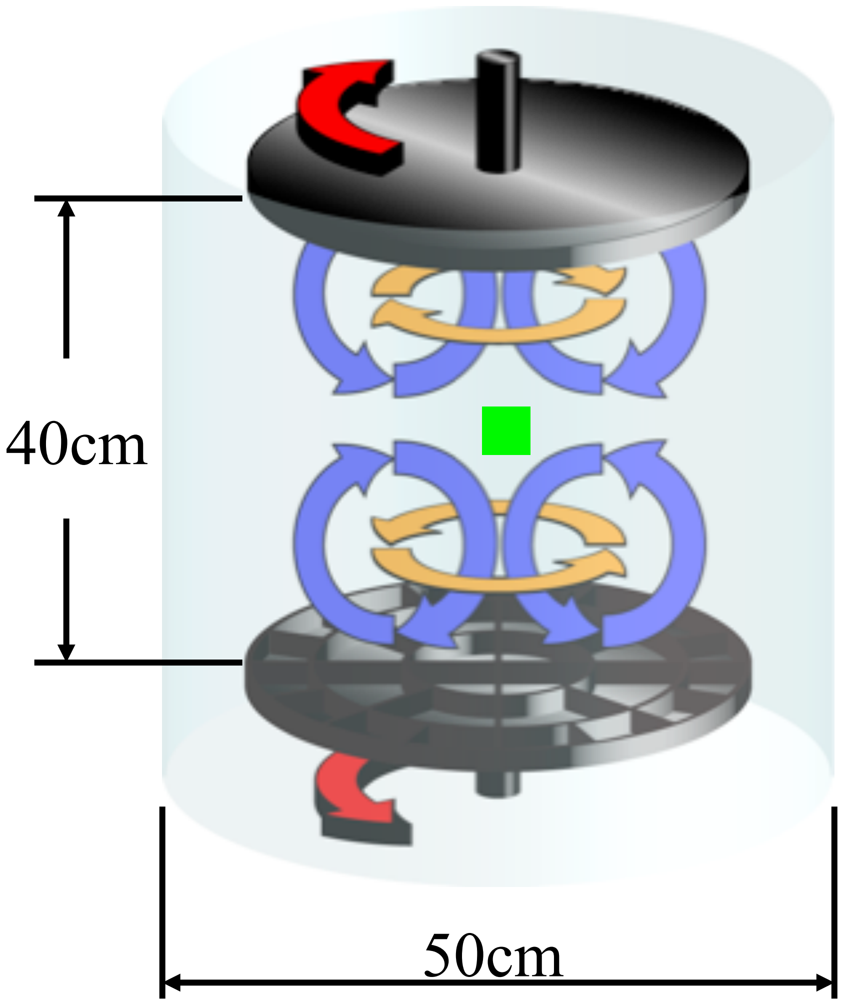
Change concentration from 0 to 20 wppm.

Acceleration Statistics: Small Scales

A. M. Crawford, N. Mordant, HX, E. Bodenschatz (NJP, 2008)

Parameters of Experiments

von Karman swirling flow between counter-rotating disks



$$L \approx 7 \text{ cm}$$

$$R_\lambda = 140 - 485$$

$$\eta = 320 - 50 \ \mu\text{m}$$

$$\tau_\eta = 105 - 2.6 \text{ ms}$$

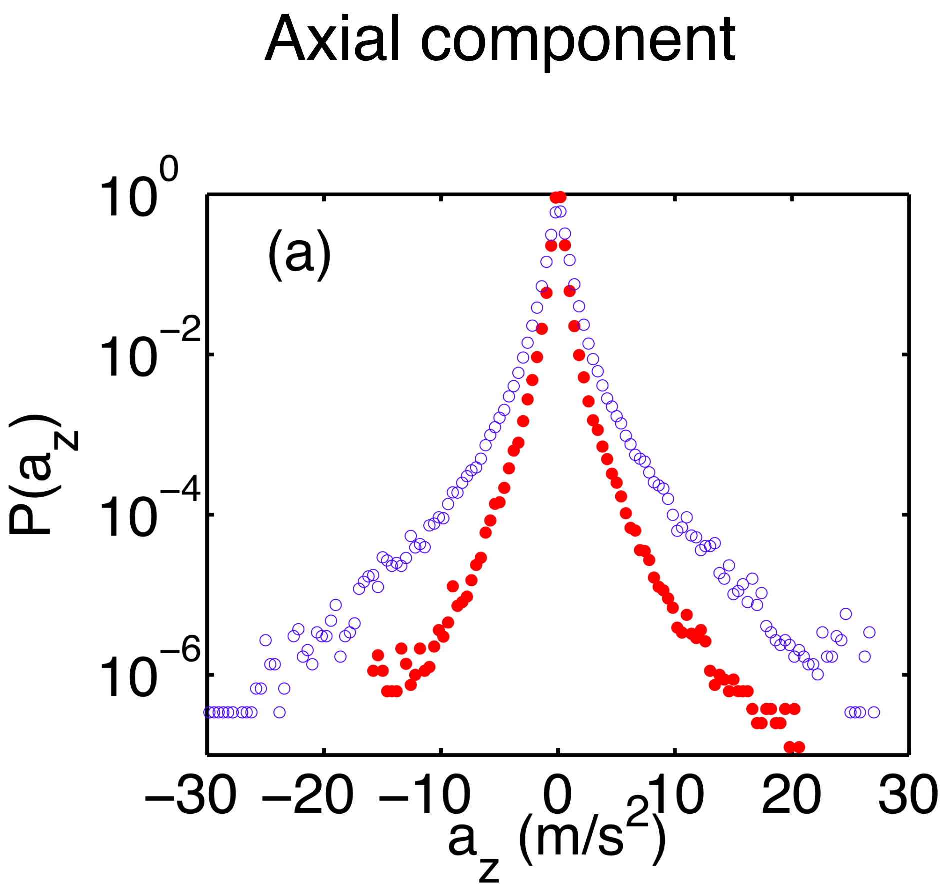
Size of the meas. vol. $(4\text{mm})^3$

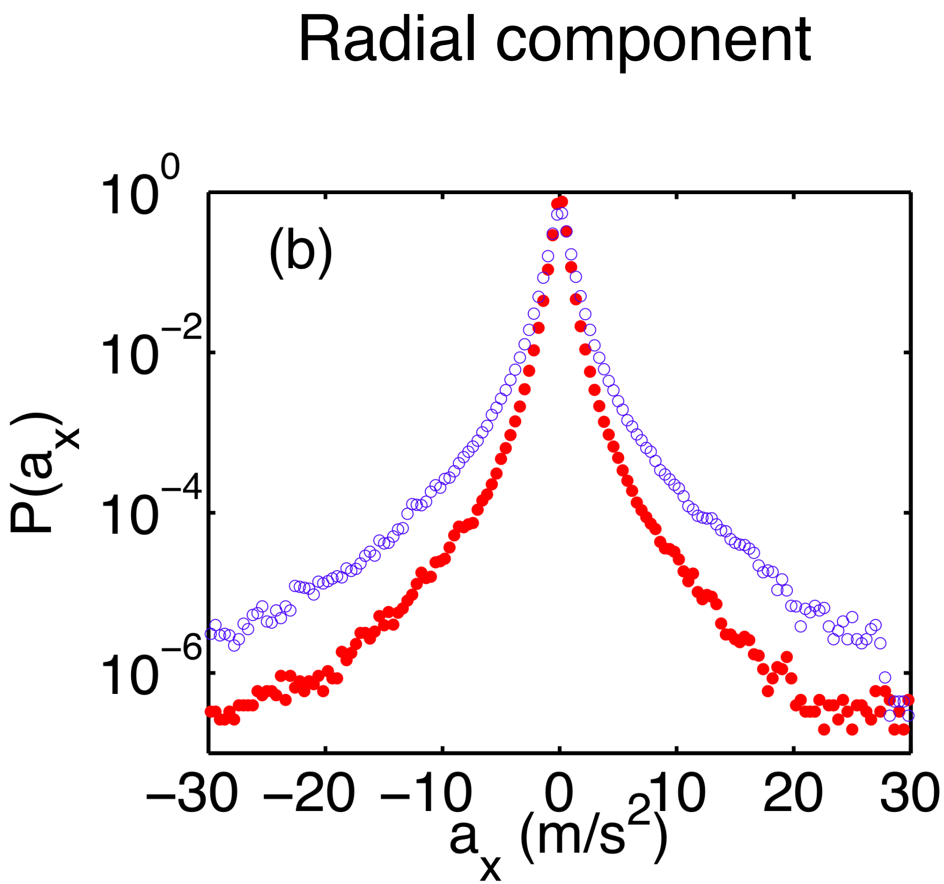
Tracer particles: neutrally buoyant polystyrene spheres.

$$d_p = 26\mu\text{m}, \ \ \rho_p/\rho_w = 1.06$$

Acceleration PDF

$$R_\lambda = 285,\ \phi = 5\ \text{ppm}$$

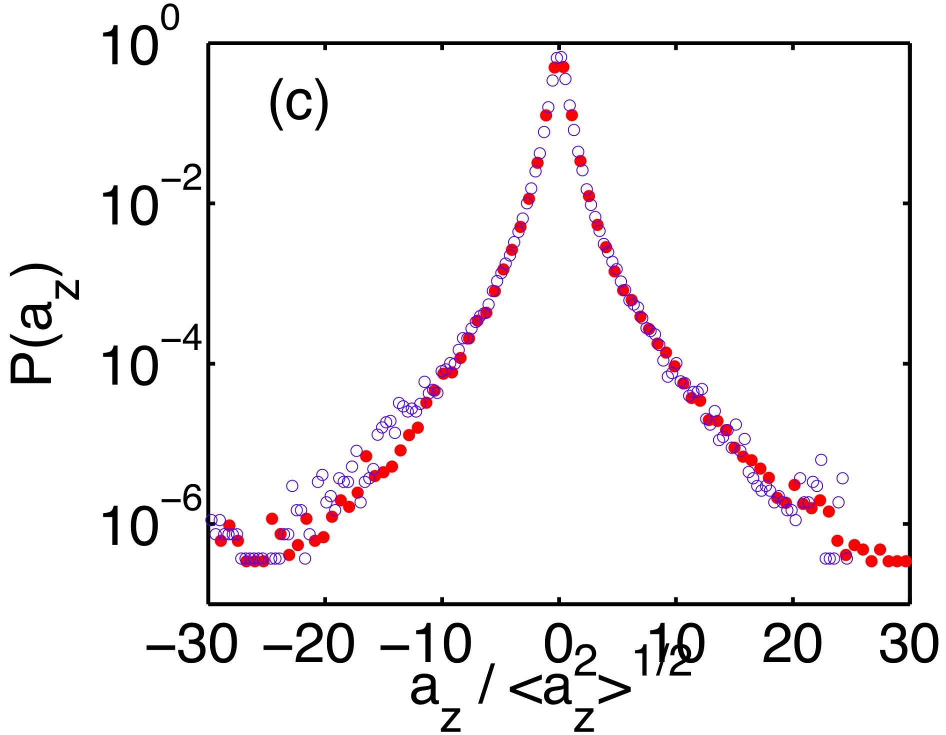




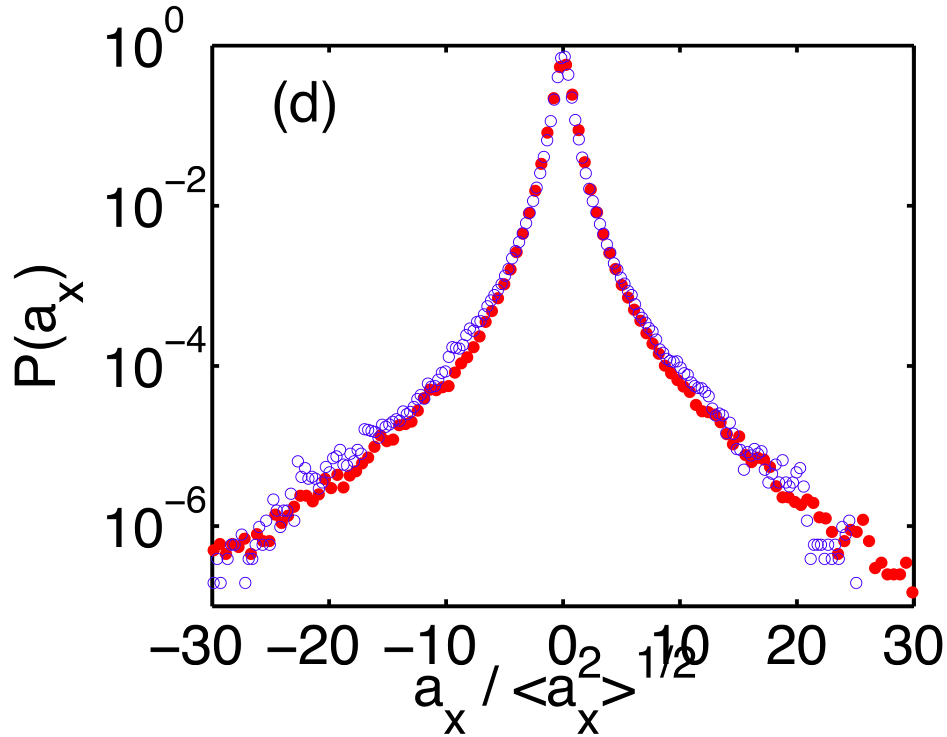
Acceleration PDF, normalized

$$R_\lambda = 285, \ \phi = 5 \text{ ppm}$$

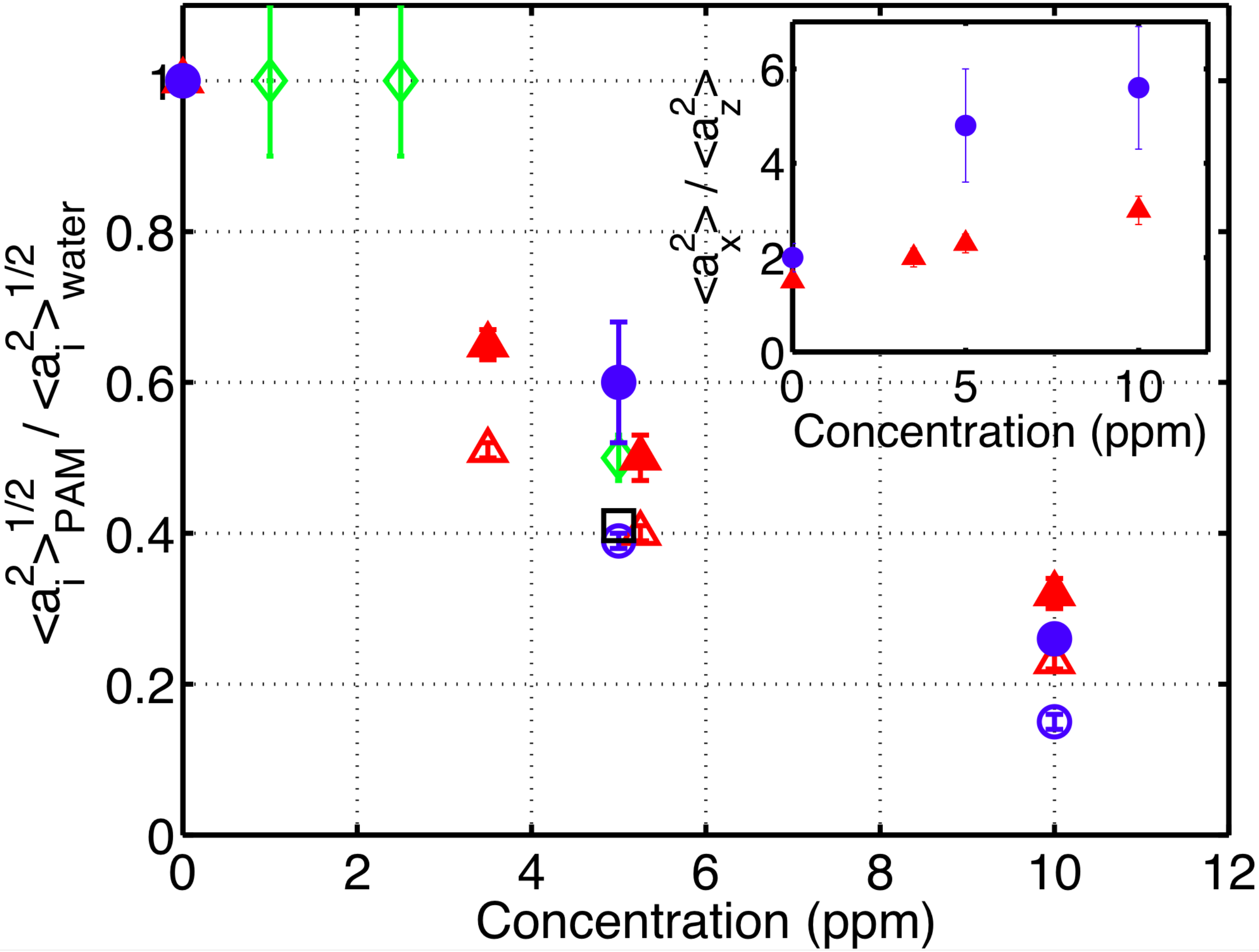
Axial component



Radial component



Acceleration Variance



$$\langle a^2\rangle \sim \frac{\varepsilon^{3/2}}{\nu^{1/2}}$$

$$\varepsilon_I \approx \frac{u^3}{L}$$ ✗

ε_D ?

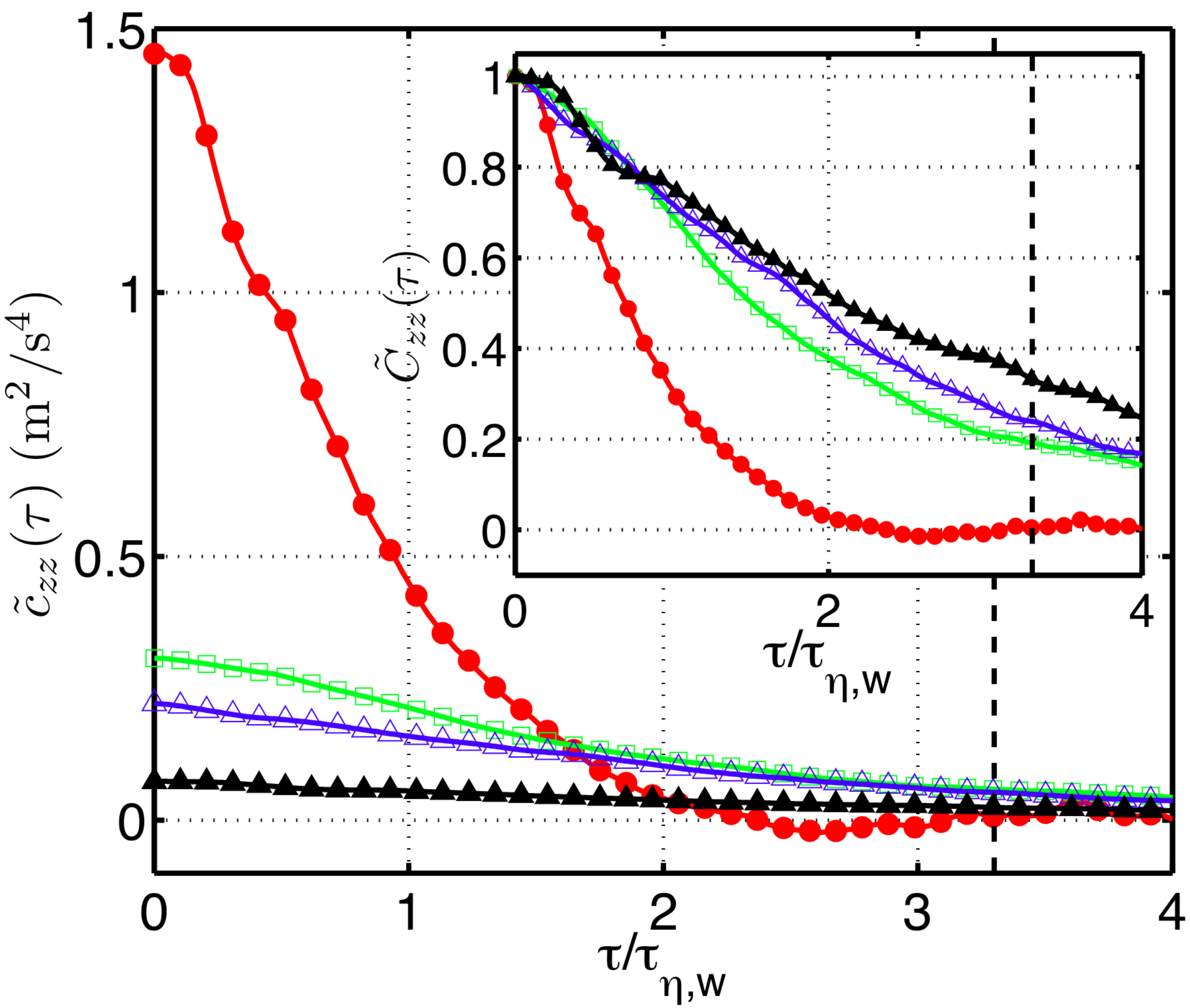
$$R_\lambda = 140(\square), 200(\circ), 285(\triangle), 485(\diamond);\ \phi = 0 - 10 \text{ ppm}$$

Acceleration Autocorrelation

$$\tilde{c}_{ij}(\tau) = \frac{\langle a_i(t) a_j(t+\tau) \rangle_{t\&\tau}}{\left(\langle a_i(t)^2 \rangle_{t\&\tau} \langle a_j(t+\tau)^2 \rangle_{t\&\tau} \right)^{1/2}} \left\langle a(t)^2 \right\rangle_t$$

$$\tilde{C}_{ij}(\tau) = \frac{\tilde{c}_{ij}(\tau)}{\langle a^2 \rangle}$$

Acceleration Autocorrelation



e-folding time $\tau_{1/e}$:

$$\tilde{C}(\tau_{1/e}) = 1/e$$

$$R_\lambda = 285,\ \phi = 0, 3.5, 5, 10\ \mathrm{ppm}$$

What is Changed at the Small Scale?

$$\langle a^2 \rangle \sim \frac{\varepsilon_D^{3/2}}{\nu^{1/2}} \qquad \tau_{1/e} \sim \frac{\nu^{1/2}}{\varepsilon_D^{1/2}}$$

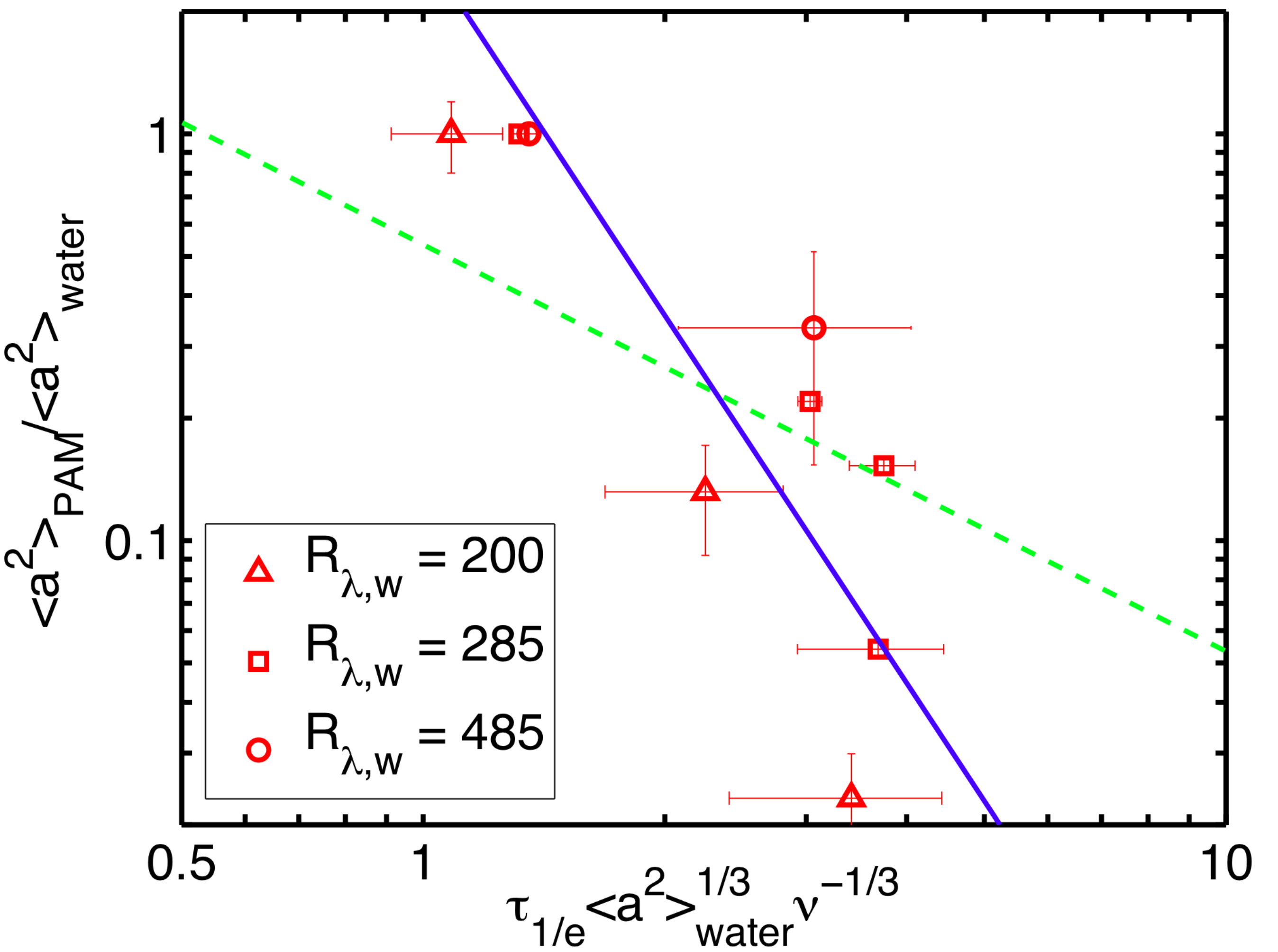
If ν is increased, but ε_D stays the same:

$$\langle a^2 \rangle \sim \frac{\varepsilon_D}{\tau_{1/e}}$$

If ε_D stays the same, but ν is decreased:

$$\langle a^2 \rangle \sim \frac{\nu}{\tau_{1/e}^3}$$

What is Changed at the Small Scale?



$$\langle a^2 \rangle \sim \frac{\varepsilon_D}{\tau_{1/e}}$$

$$\langle a^2 \rangle \sim \frac{\nu}{\tau_{1/e}^3}$$

Conclusions

- Polymers suppress the magnitude of fluid acceleration, but the shape of the acceleration PDF.

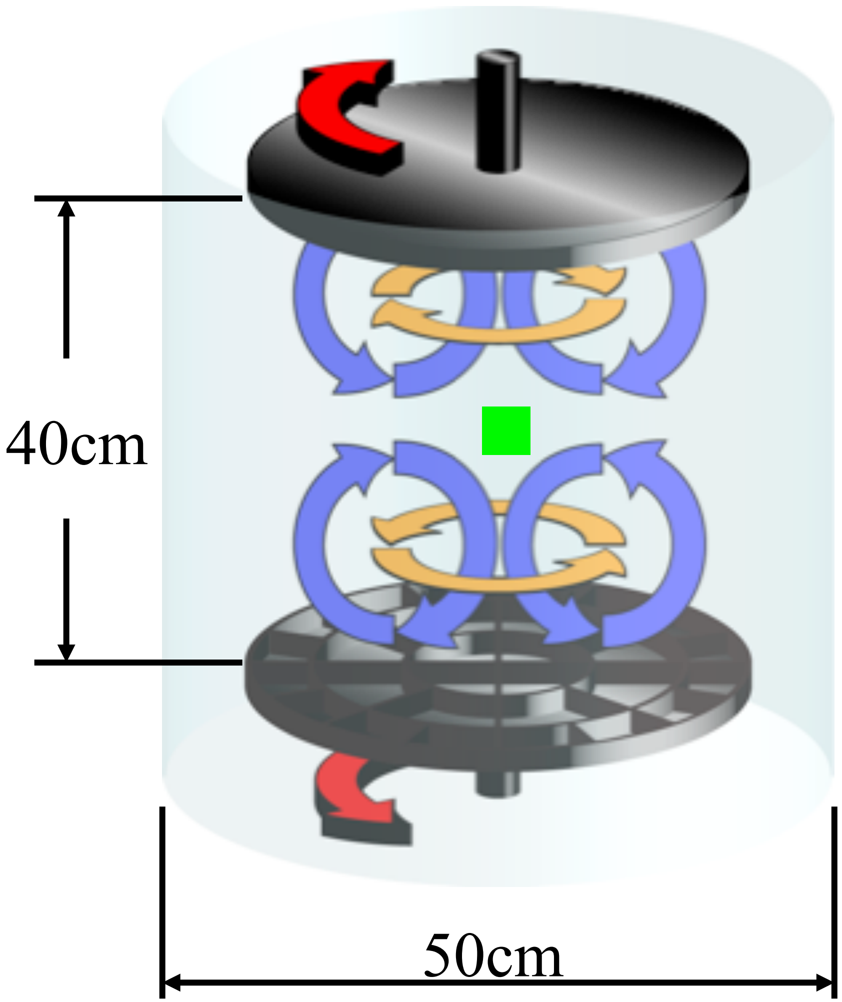
- The decrease of acceleration variance and the increase of acceleration correlation time suggest that the dissipation rate at the small scales is decreased by the presence of polymers.

- These effects depend on polymer concentration.

Acceleration Statistics: “Inertial Range”

Parameters of Experiments

von Karman swirling flow between counter-rotating disks



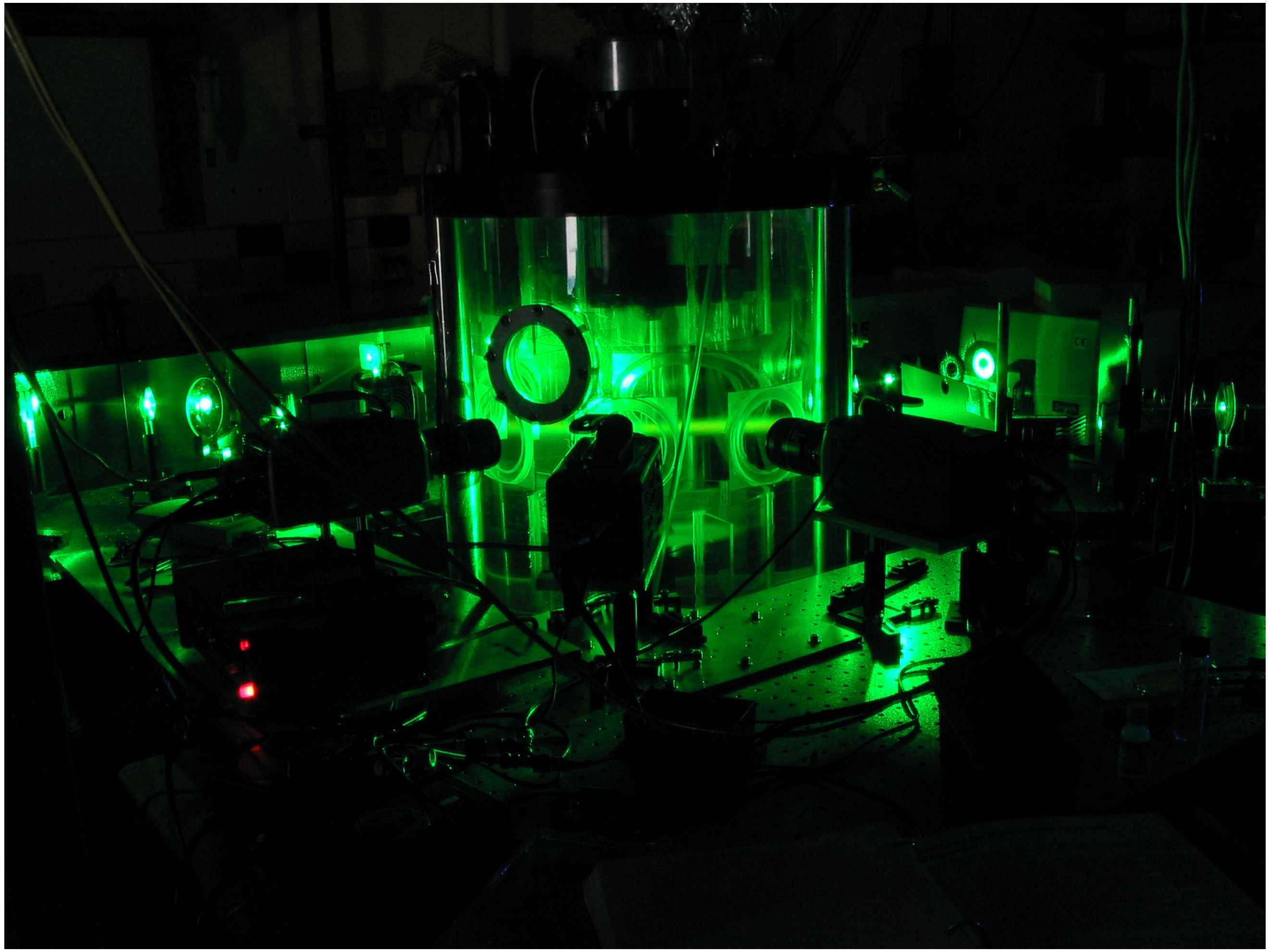
$$R_\lambda = 350$$
$$\eta = 84\mu m$$
$$\tau_\eta = 7.1ms$$
$$L = 7cm$$

Size of the meas. vol. $(2cm)^3$

Tracer particles: neutrally buoyant fluorescent polystyrene spheres.

$$d_p = 33\mu m,\ \rho_p/\rho_w = 1.06$$

Experiment Setup

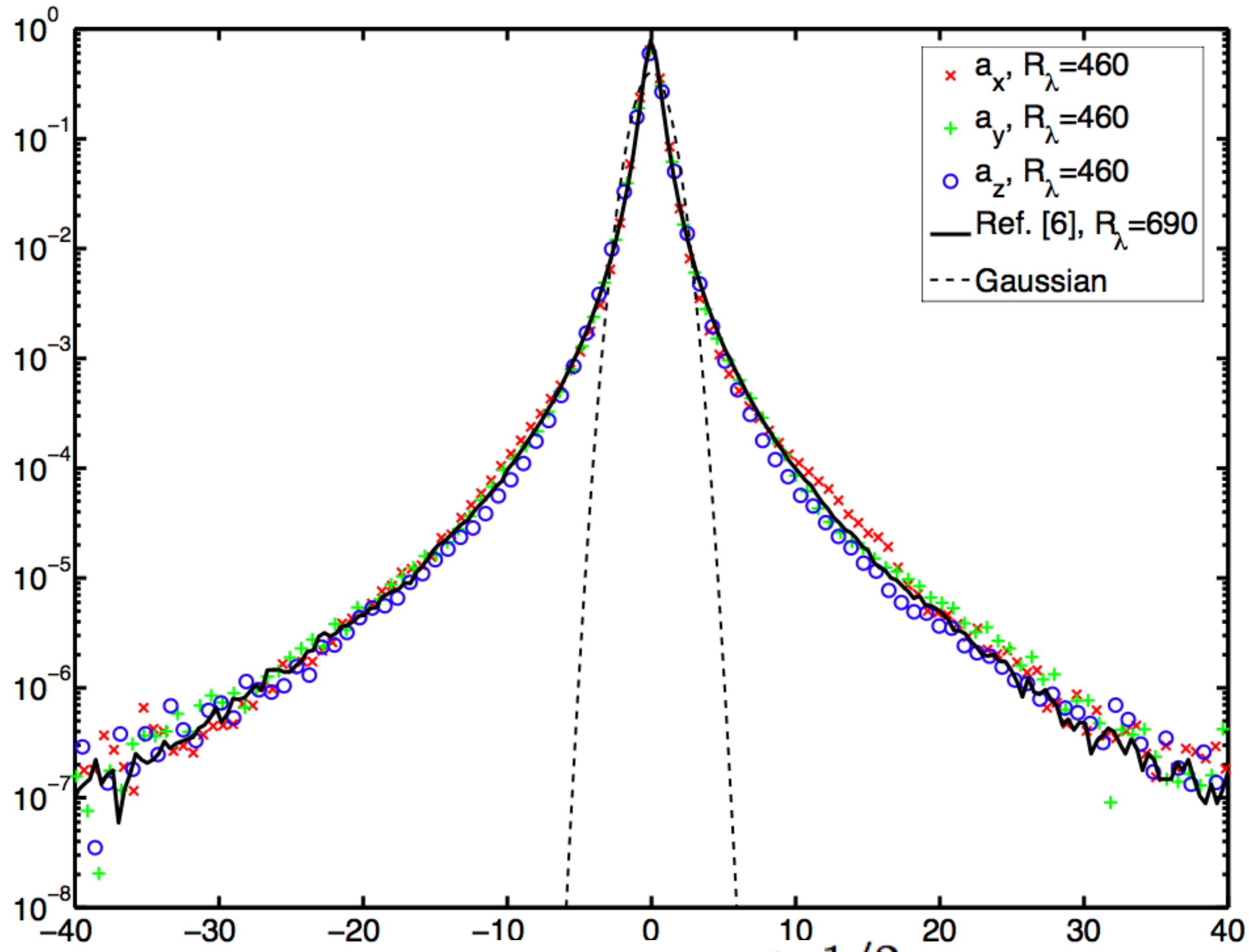


- Phantom v7.1 CMOS cameras: 5000 fps at 512 x 512 pixels
- Q-switched Nd:YAG laser: 60W, up to 120 kHz

Acceleration PDF

$$R_\lambda = 460$$

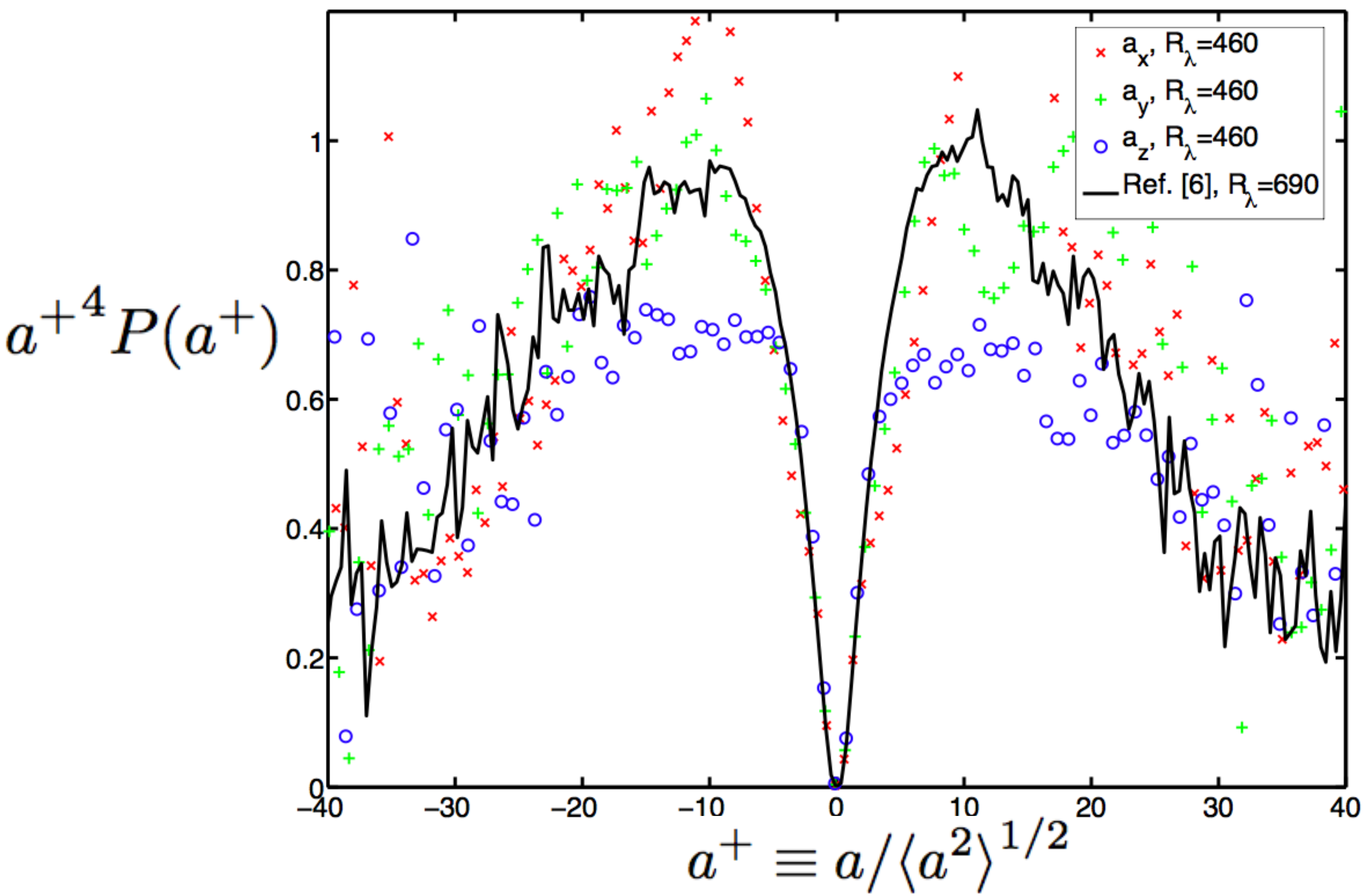
$P(a^+)$



$$a^+ \equiv a/\langle a^2 \rangle^{1/2}$$

Acceleration PDF

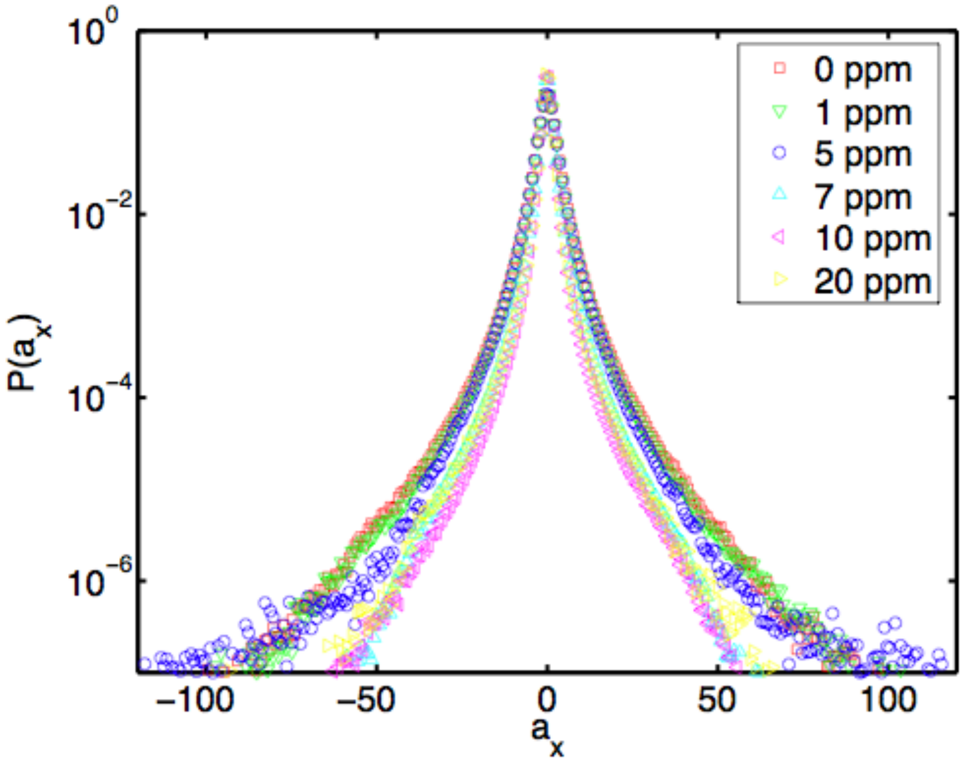
$$R_\lambda = 460$$



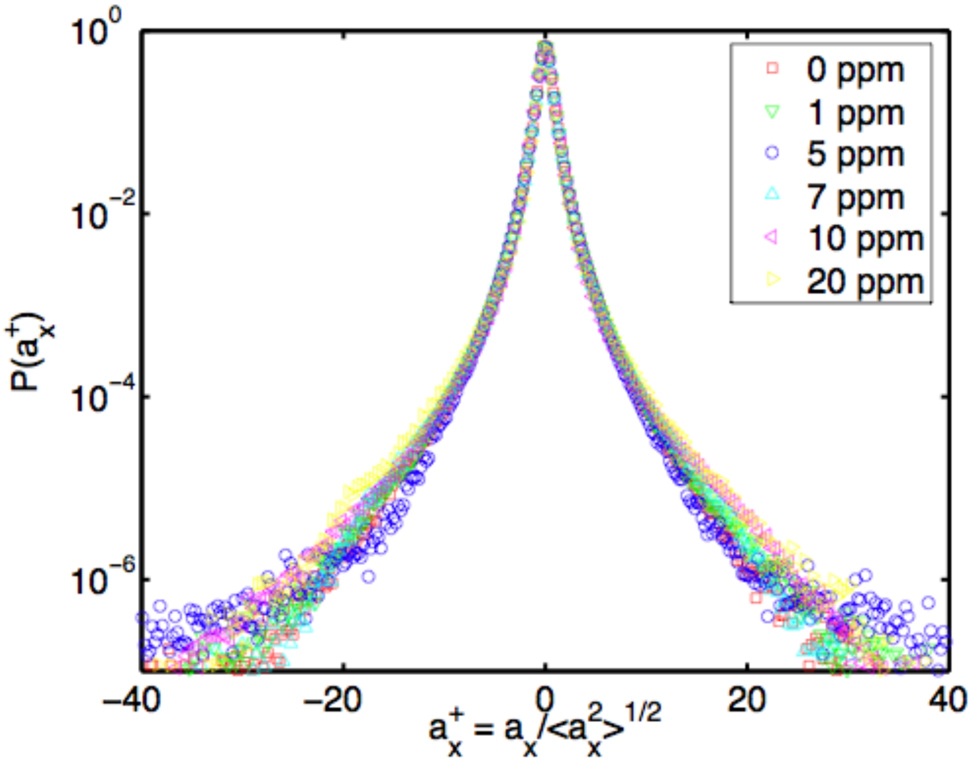
Acceleration PDF

$$R_\lambda = 350$$

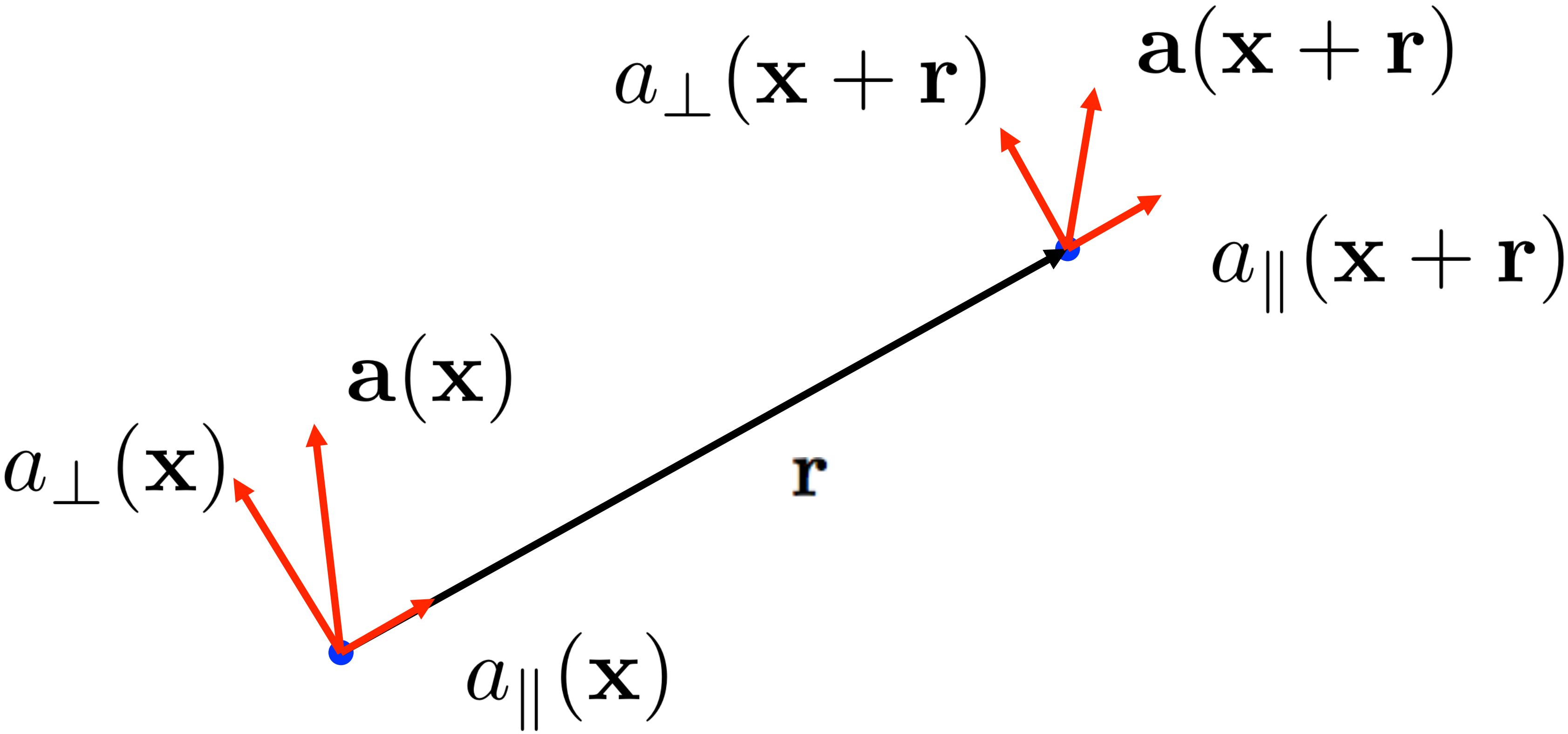
Un-normalized



normalized



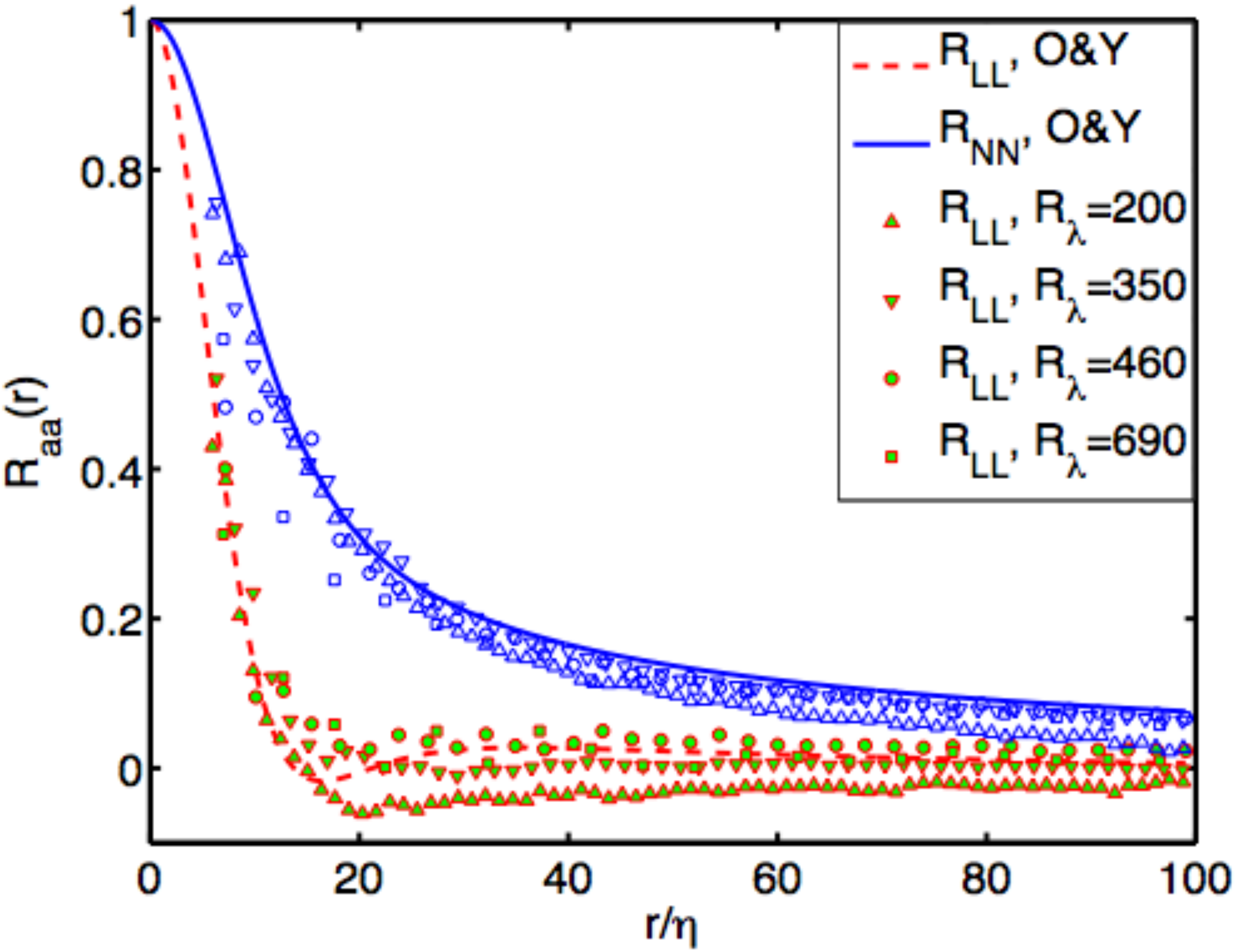
Spatial Correlations



$$R_{LL}(r) = \langle a_{||}(\mathbf{x}) a_{||}(\mathbf{x}+\mathbf{r}) \rangle$$

$$R_{NN}(r) = \langle a_\perp(\mathbf{x}) a_\perp(\mathbf{x}+\mathbf{r}) \rangle$$

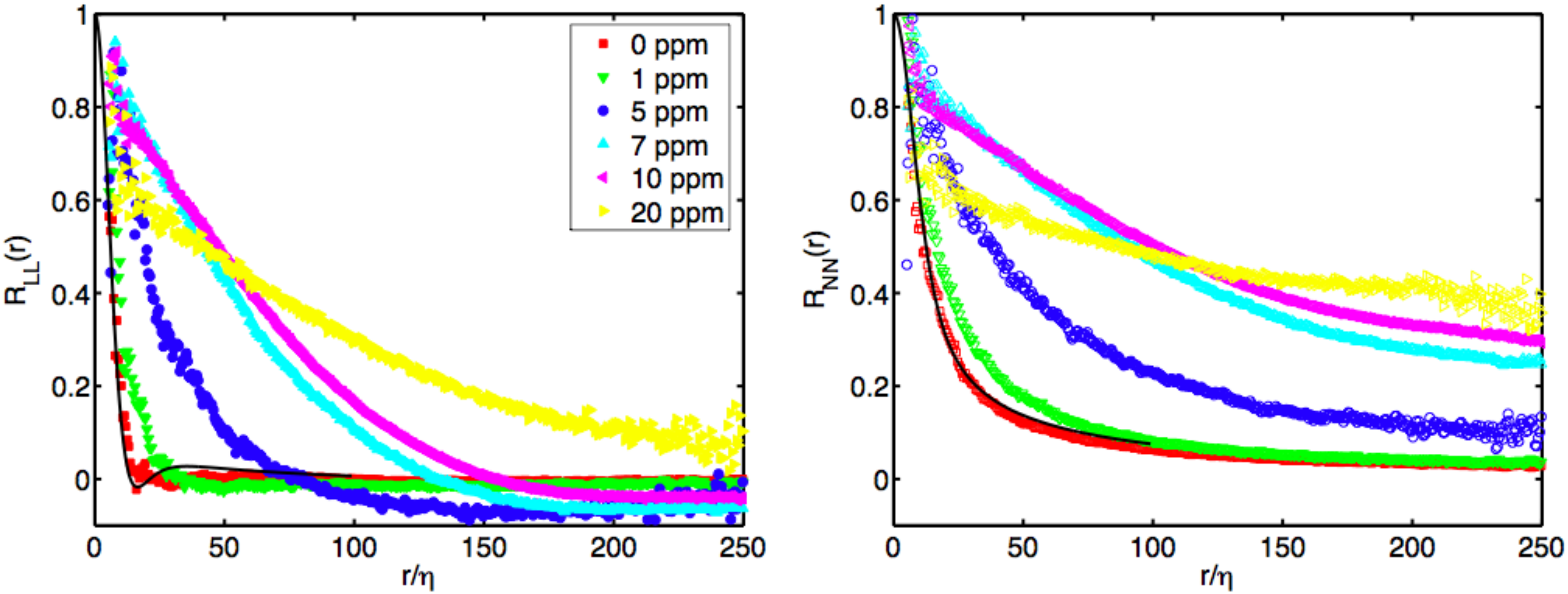
Spatial Correlations of Acceleration



HX, Ouellette, Vincenzi, Bodenschatz (PRL, 2007)

Spatial Correlations of Acceleration

$$R_\lambda = 350$$

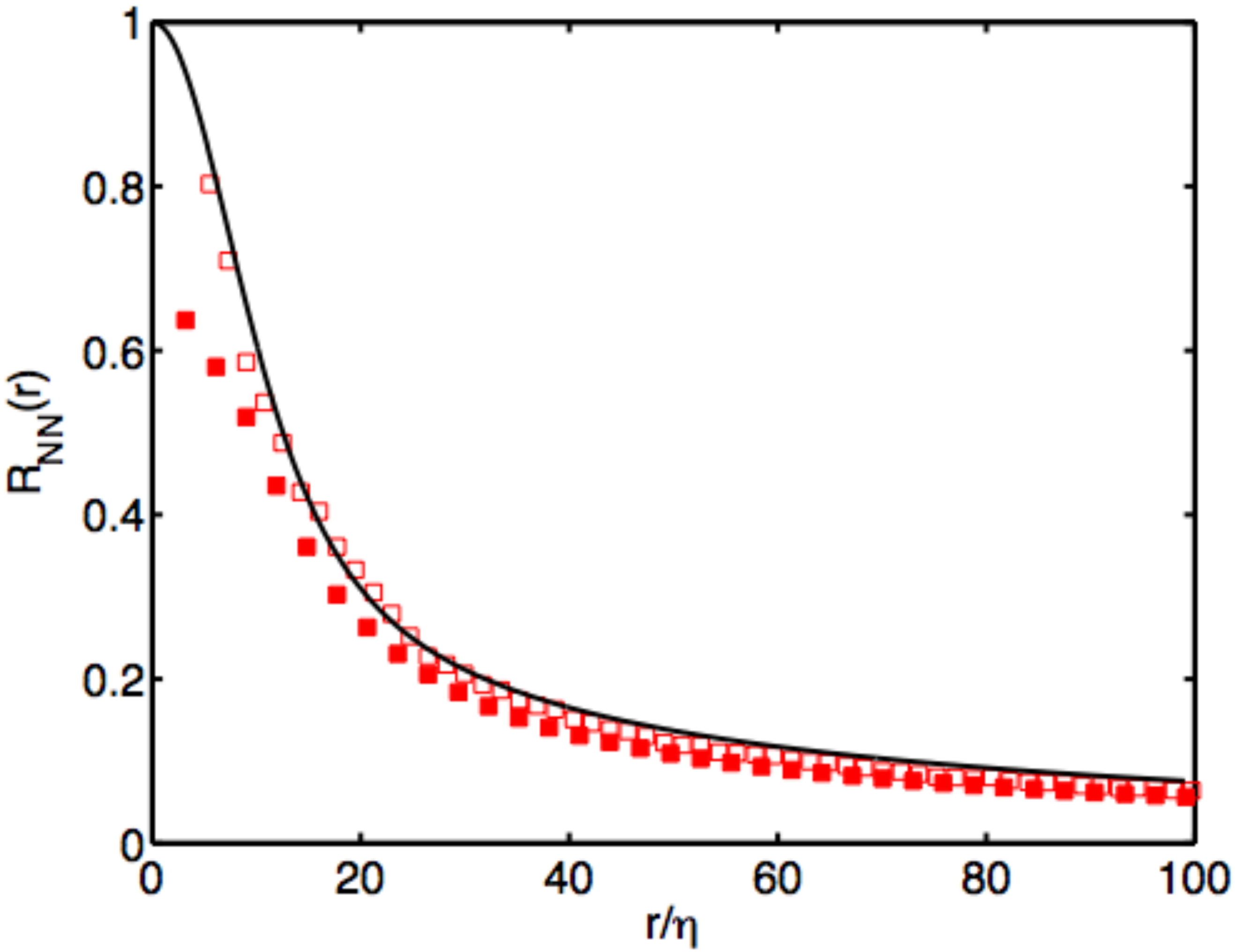


Isotropic Relation

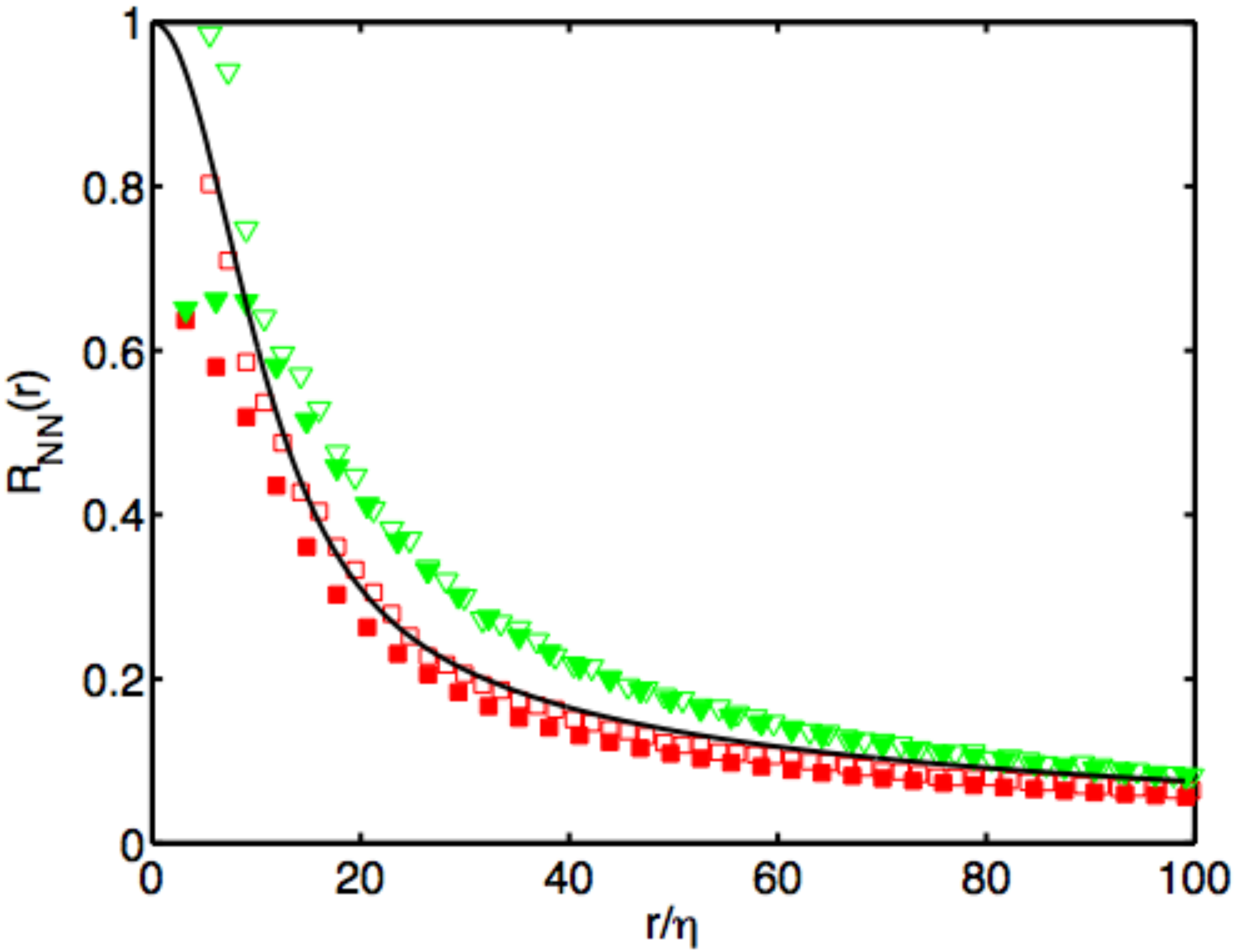
$$\mathbf{a} = -\nabla p + \nabla \cdot \mathbf{T}^p$$

$$\Rightarrow \quad R_{NN}(r) \stackrel{?}{=} \frac{1}{r}\int_0^r R_{LL}(r)dr$$

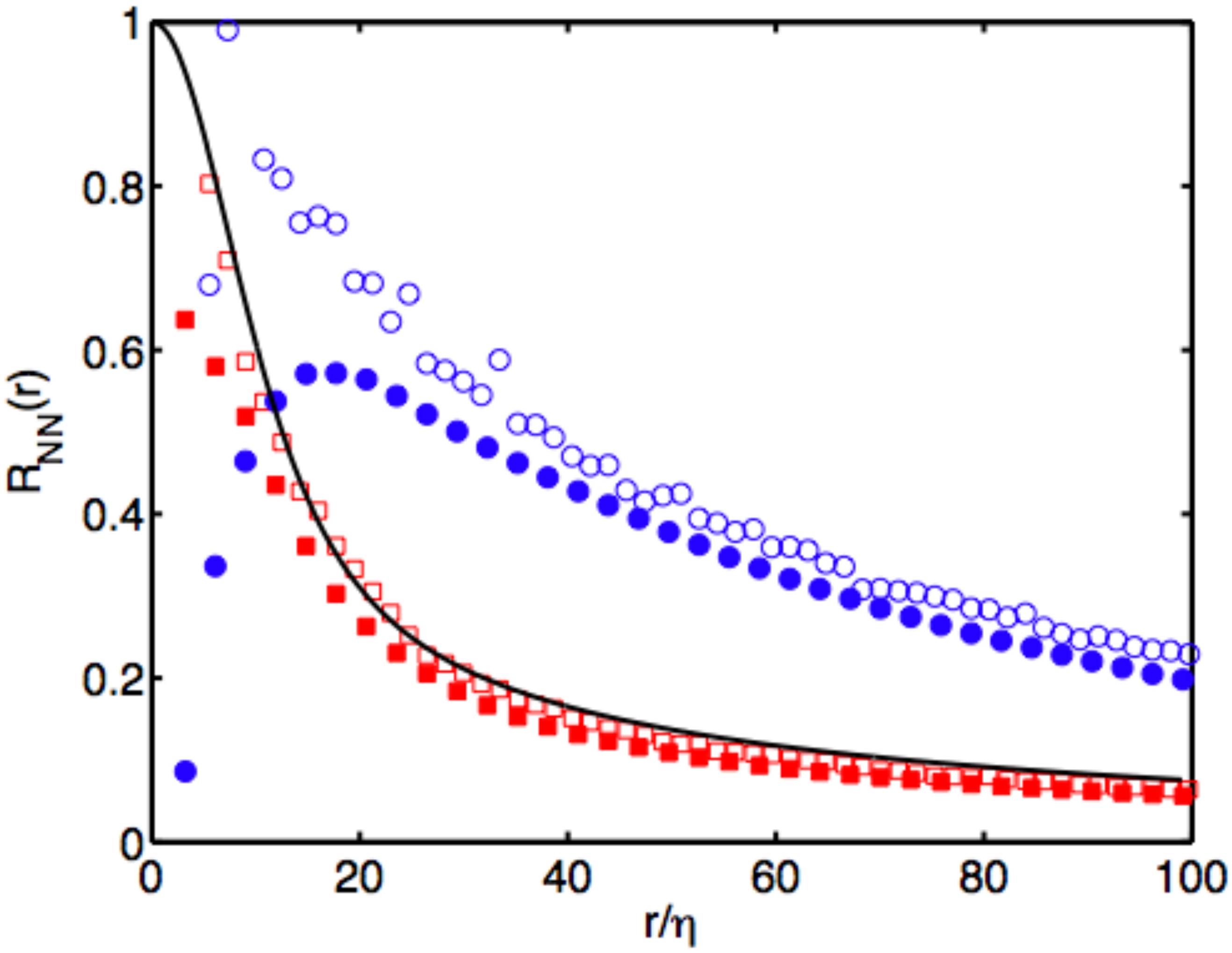
Isotropic Relation



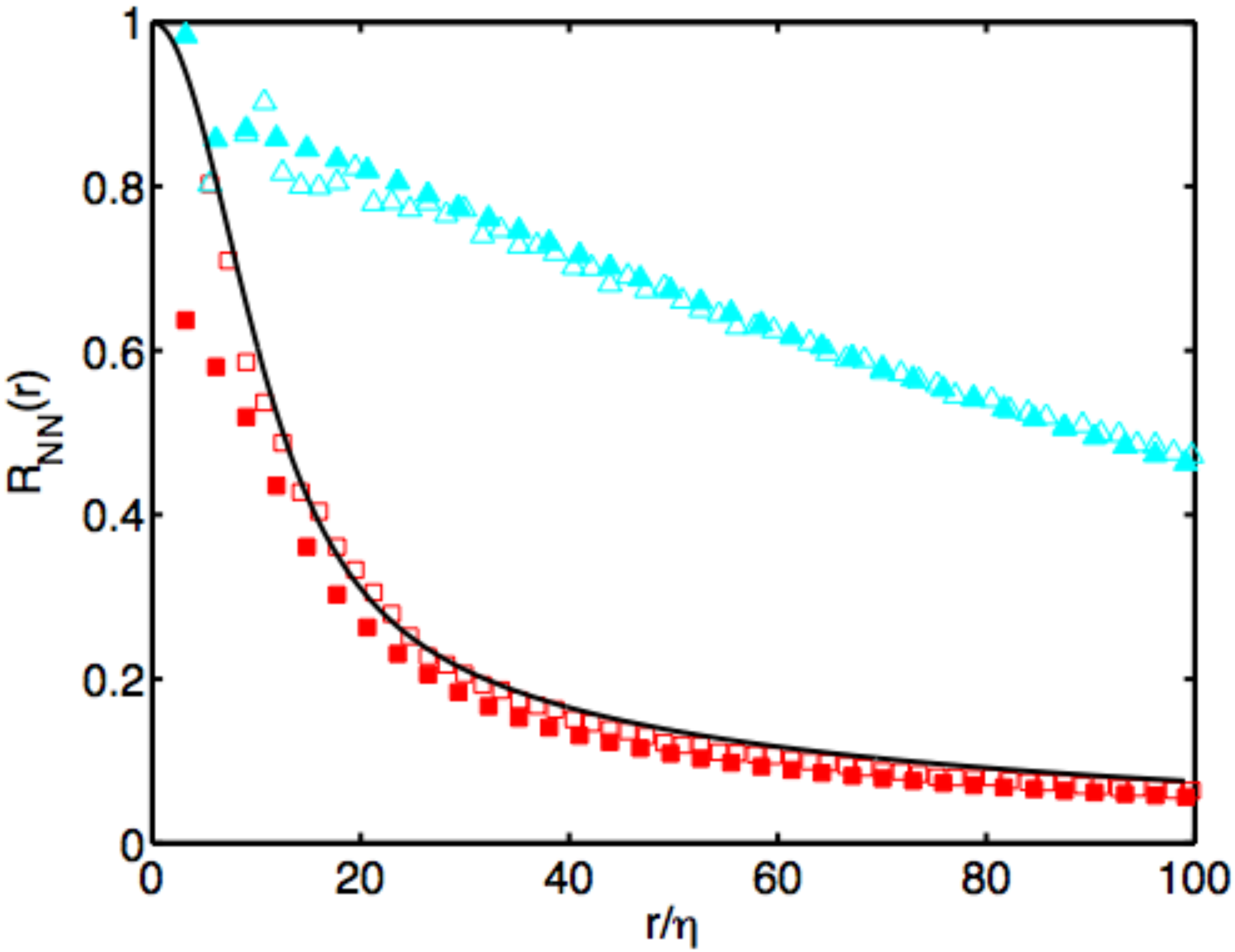
Isotropic Relation



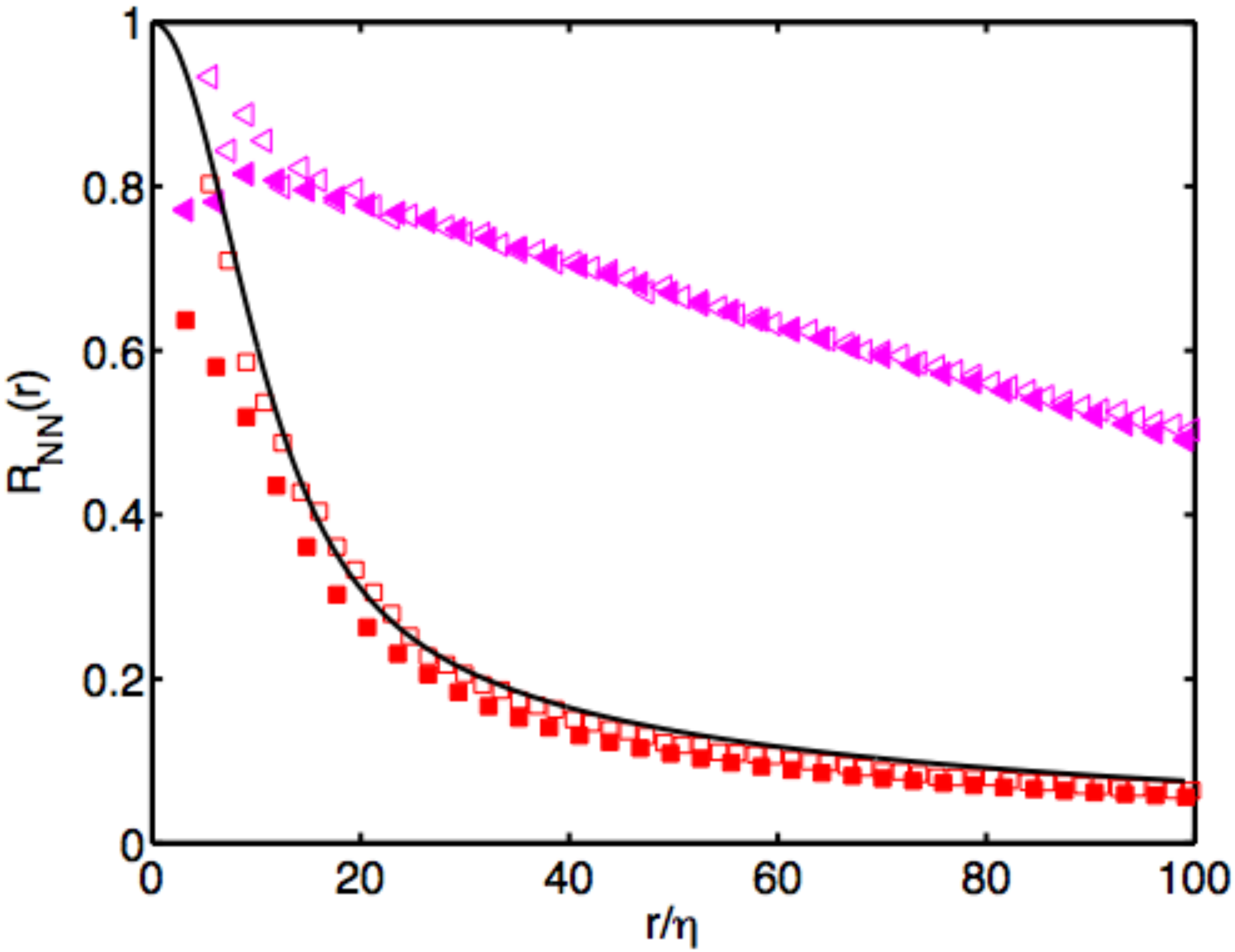
Isotropic Relation



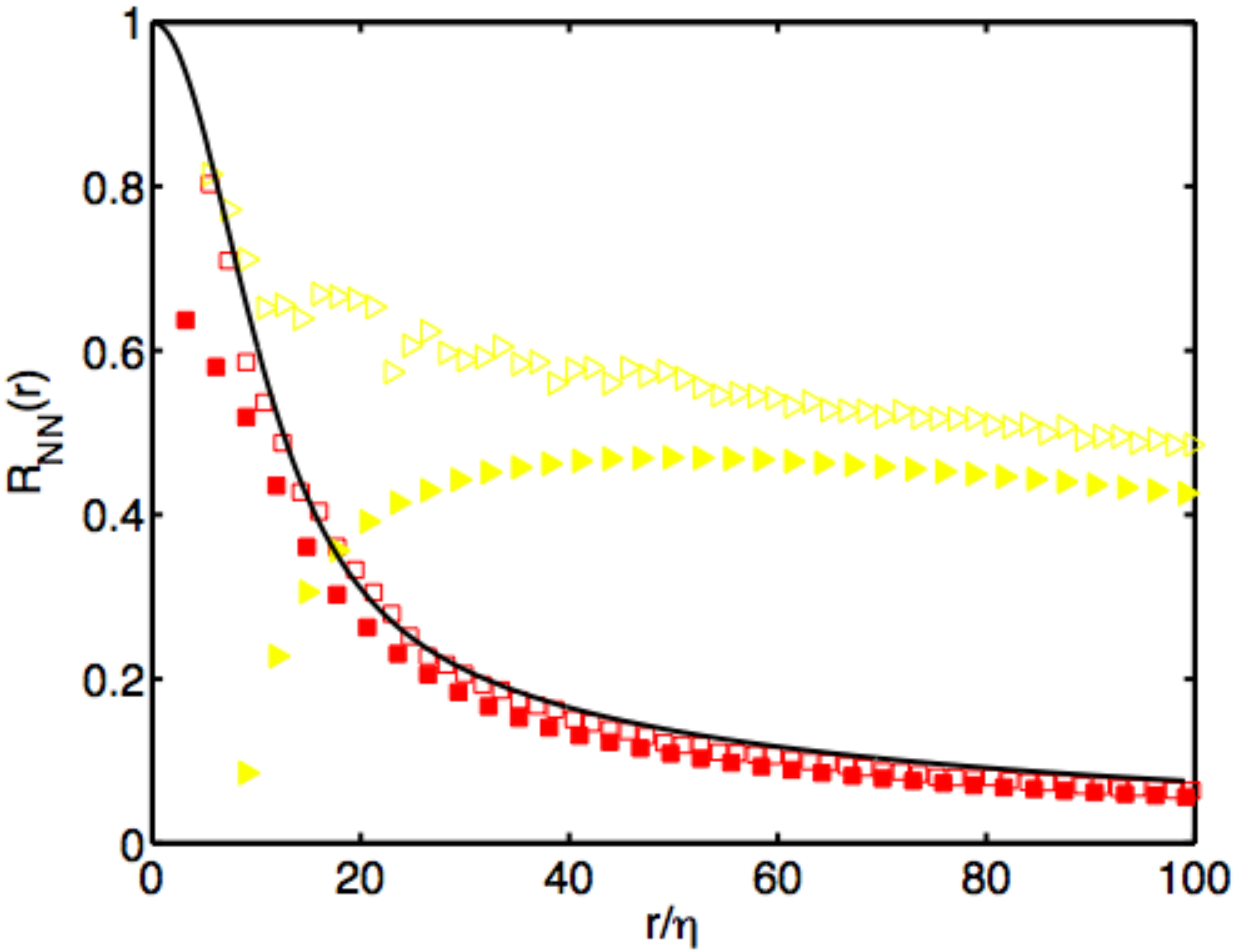
Isotropic Relation



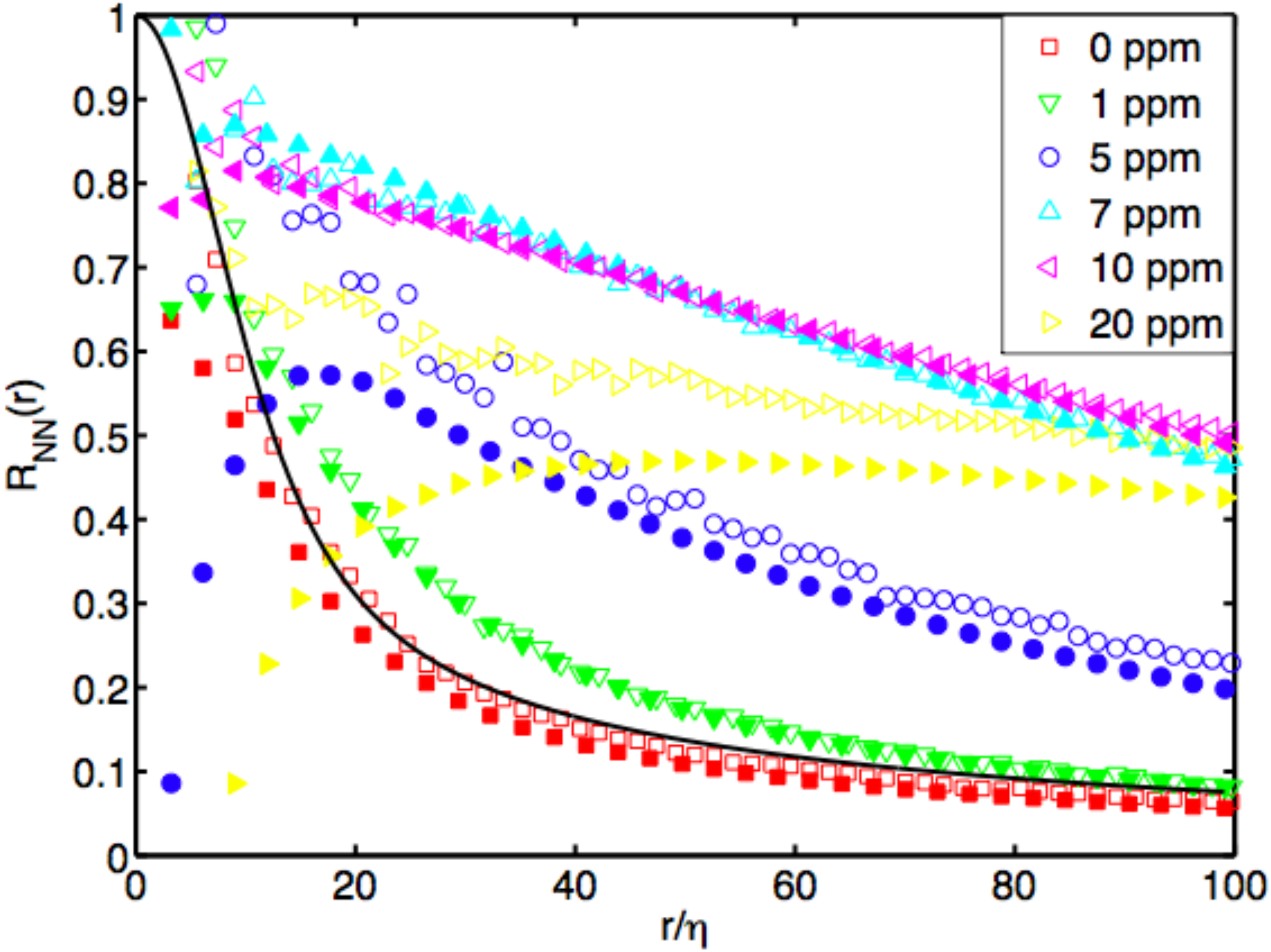
Isotropic Relation



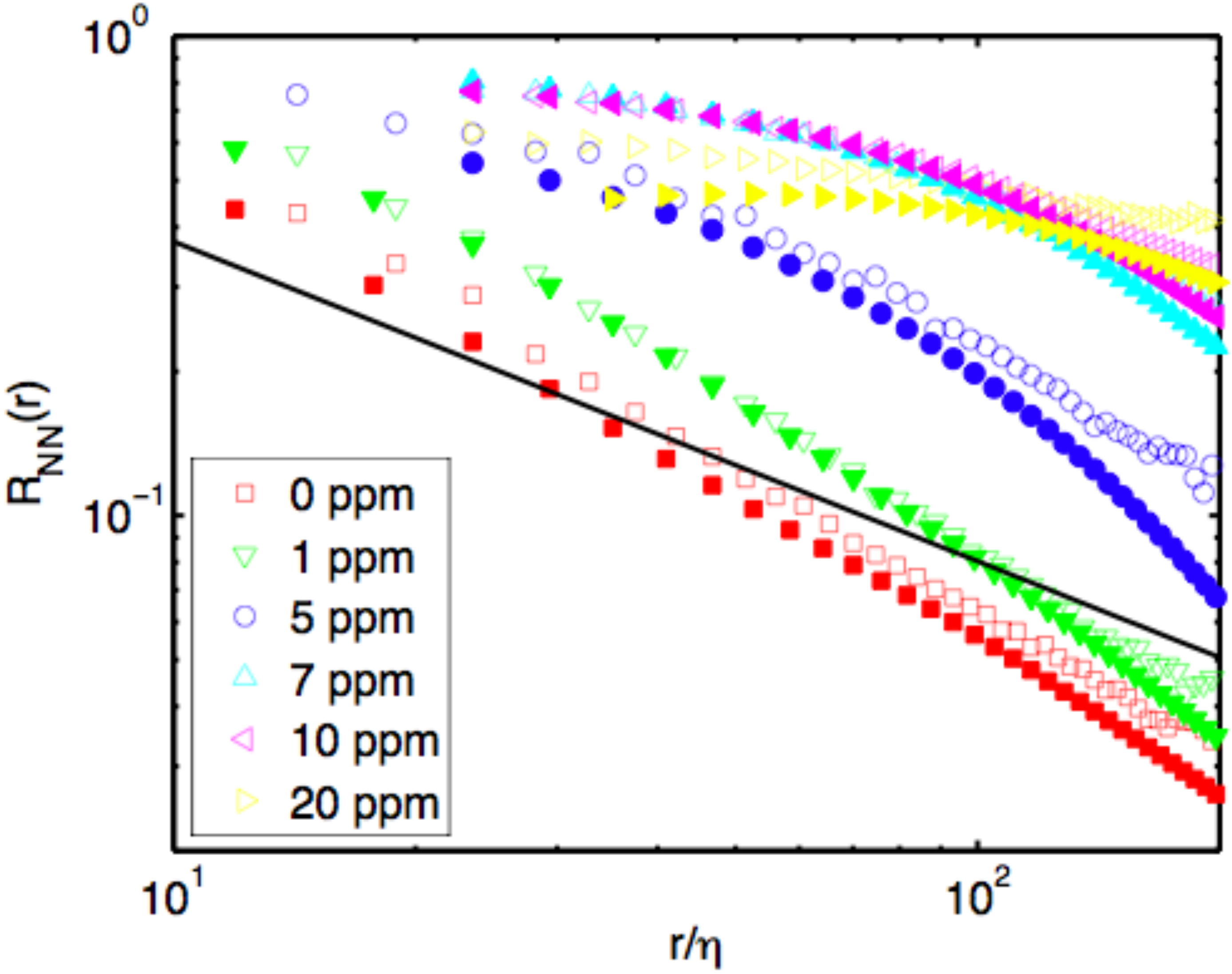
Isotropic Relation



Isotropic Relation



Isotropic Relation: Large r



Conclusions/Question

- Fluid acceleration in polymer solutions correlation over much larger range.

- At large separations, the correlations seem to still satisfy the isotropic relation.

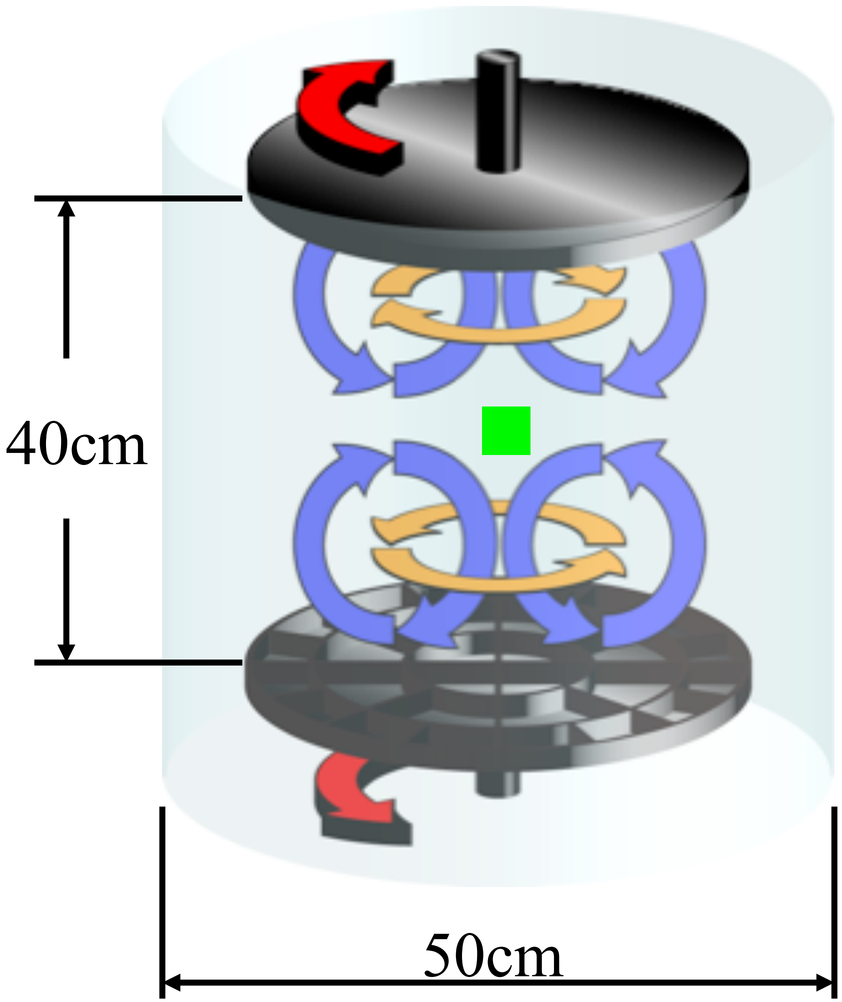
- What constraints would this observation impose on the constitutive equations for the polymer stress tensor?

Velocity Structure Functions

N. T. Ouellette, HX, E. Bodenschatz (JFM, 2009)

Parameters of Experiments

von Karman swirling flow between counter-rotating disks



$$R_\lambda = 200 — 415$$

$$\eta = 190 — 64\ \mu\text{m}$$

$$\tau_\eta = 37 — 4.1\ \text{ms}$$

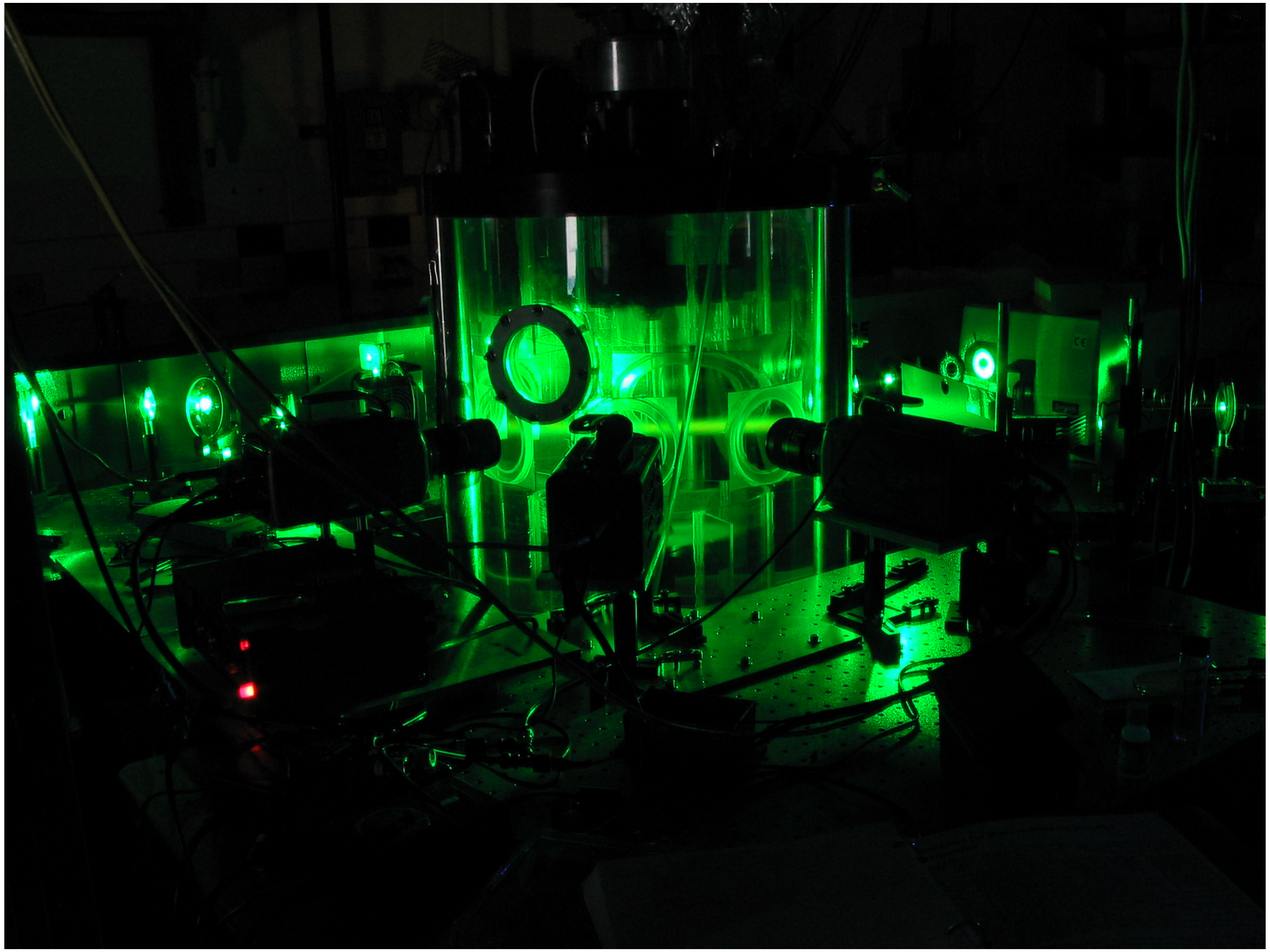
$$L \equiv \frac{u'^3}{\epsilon} \approx 7\ \text{cm}$$

Size of the meas. vol. $(5\text{cm})^3$

Tracer particles: nearly neutrally buoyant fluorescent polystyrene spheres.

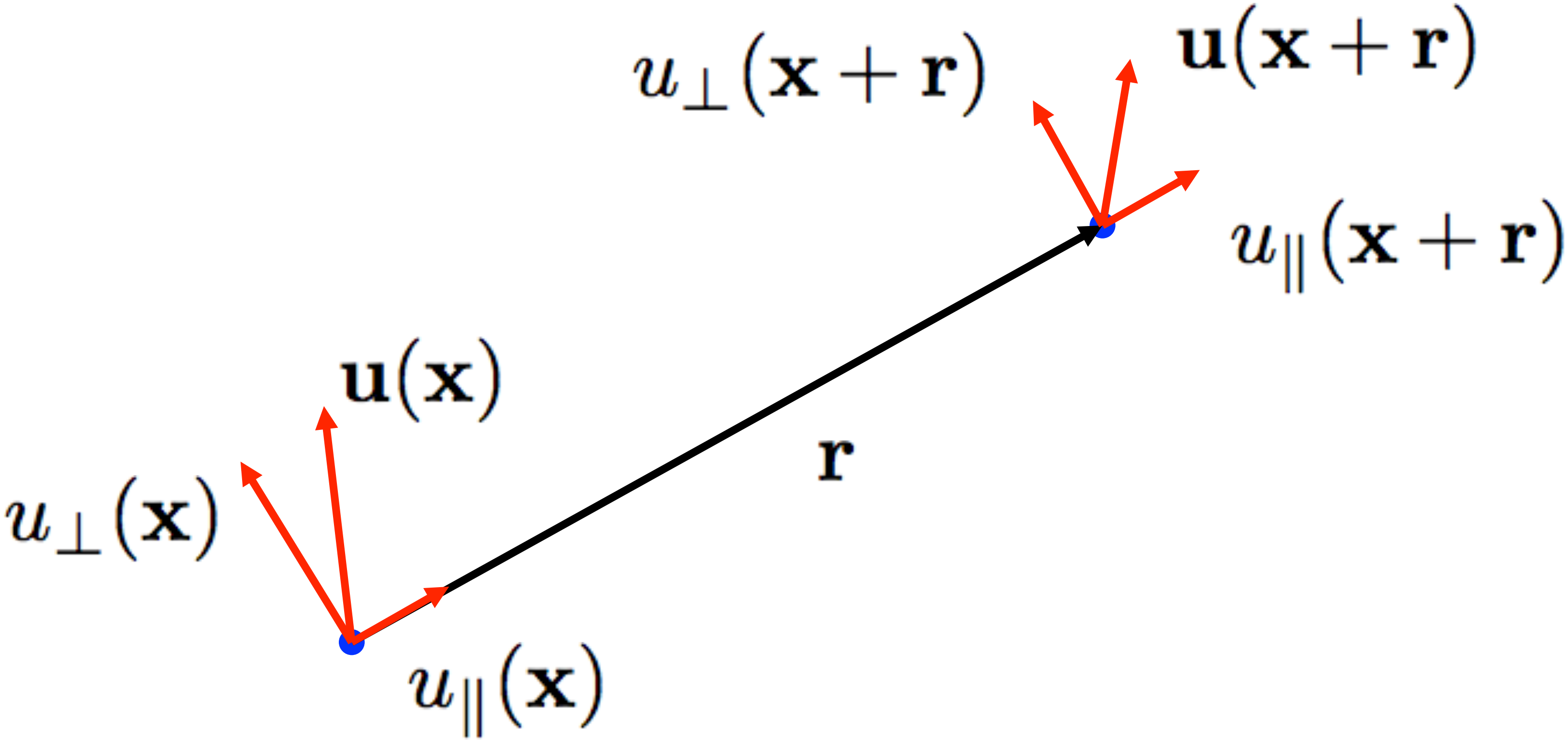
$$d_p = 33\mu m,\ \rho_p/\rho_w = 1.06$$

Experiment Setup



- Phantom v7.1 CMOS cameras: 5000 fps at 512 x 512 pixels
- Q-switched Nd:YAG laser: 60W, up to 120 kHz

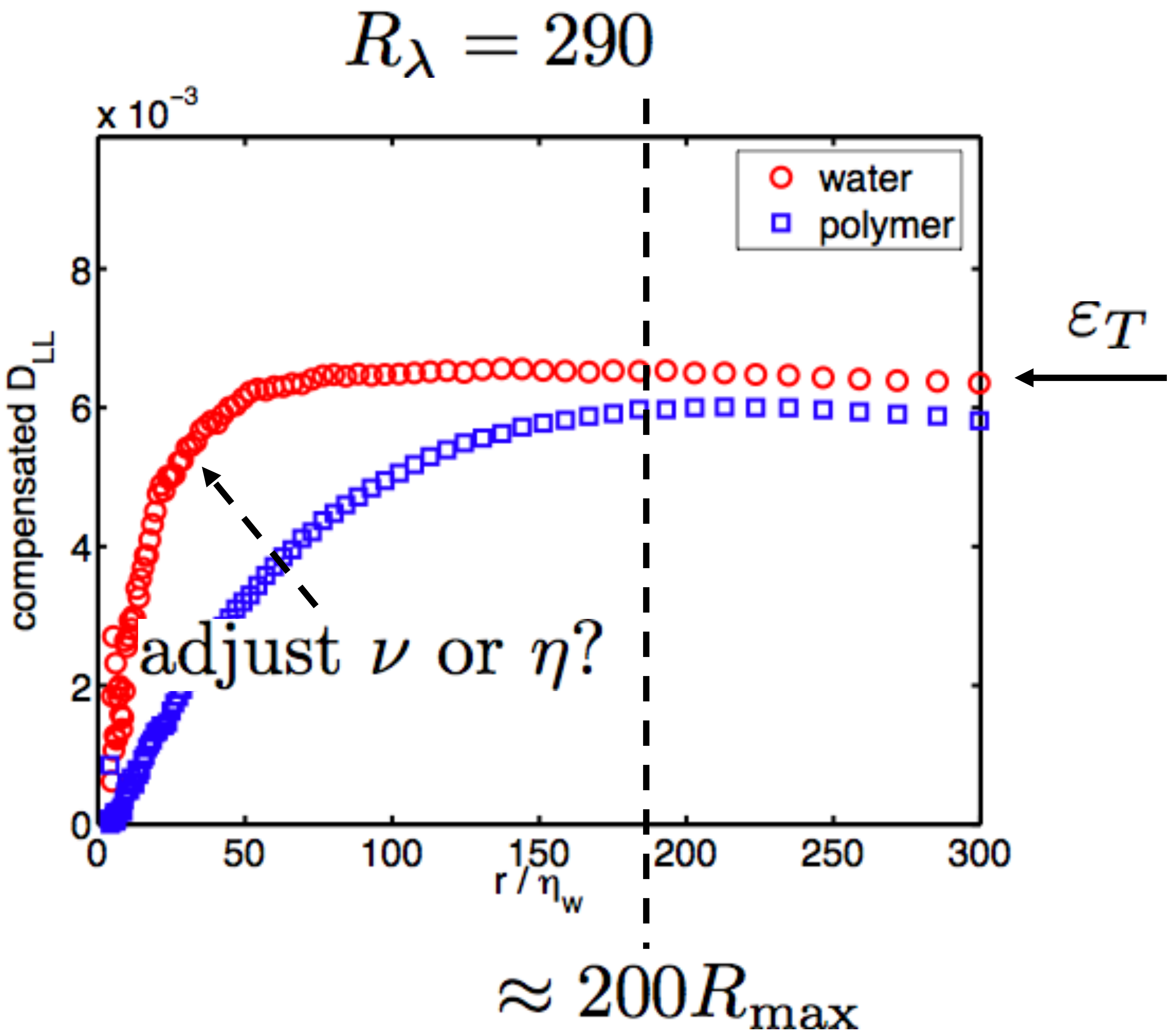
Eulerian Structure Functions



$$D_{NN}(r) = \langle |u_\perp(\mathbf{x}+\mathbf{r}) - u_\perp(\mathbf{x})|^2 \rangle$$

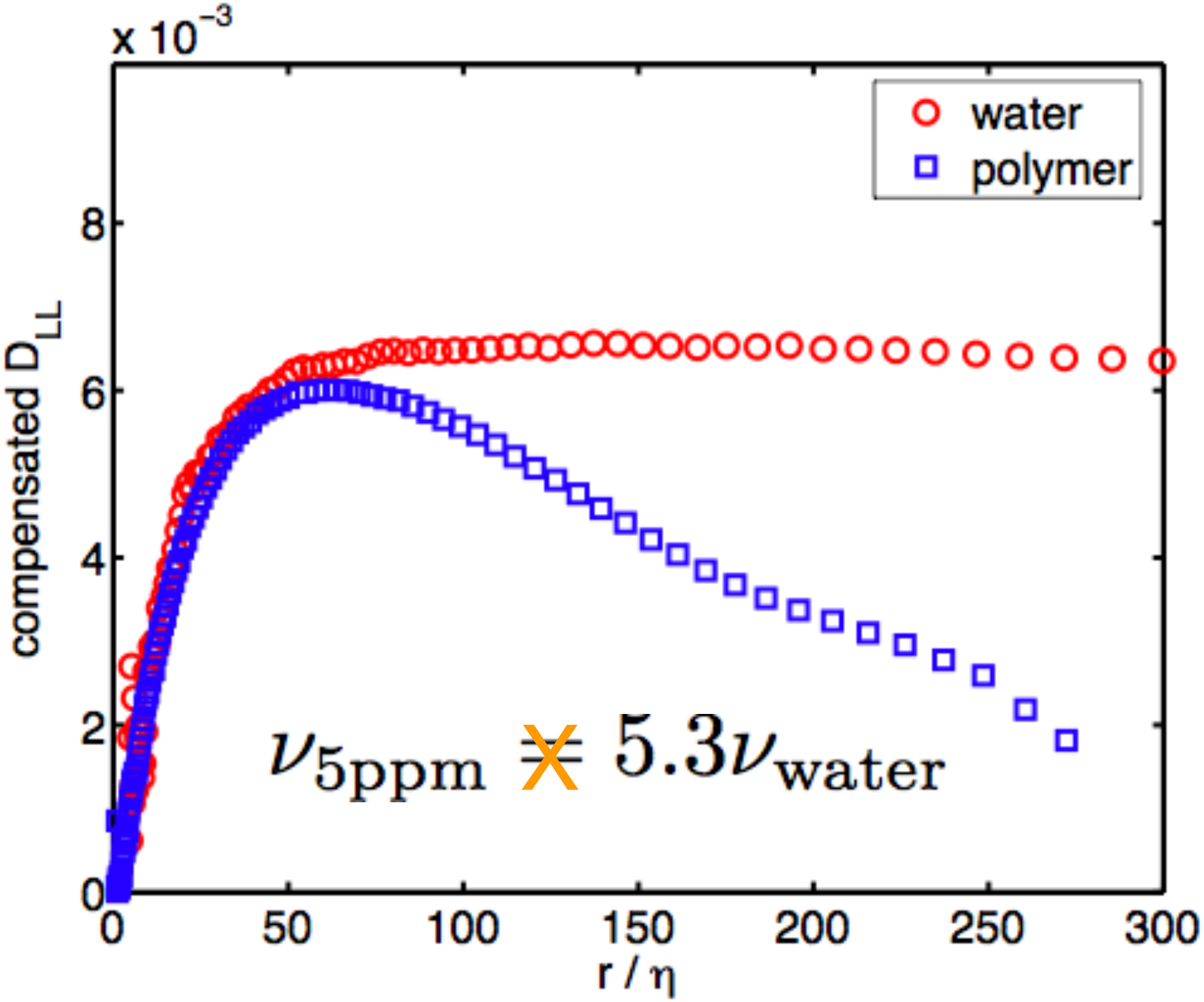
$$D_{LL}(r) = \langle |u_{||}(\mathbf{x}+\mathbf{r}) - u_{||}(\mathbf{x})|^2 \rangle$$

Polymer Effect on Structure Function

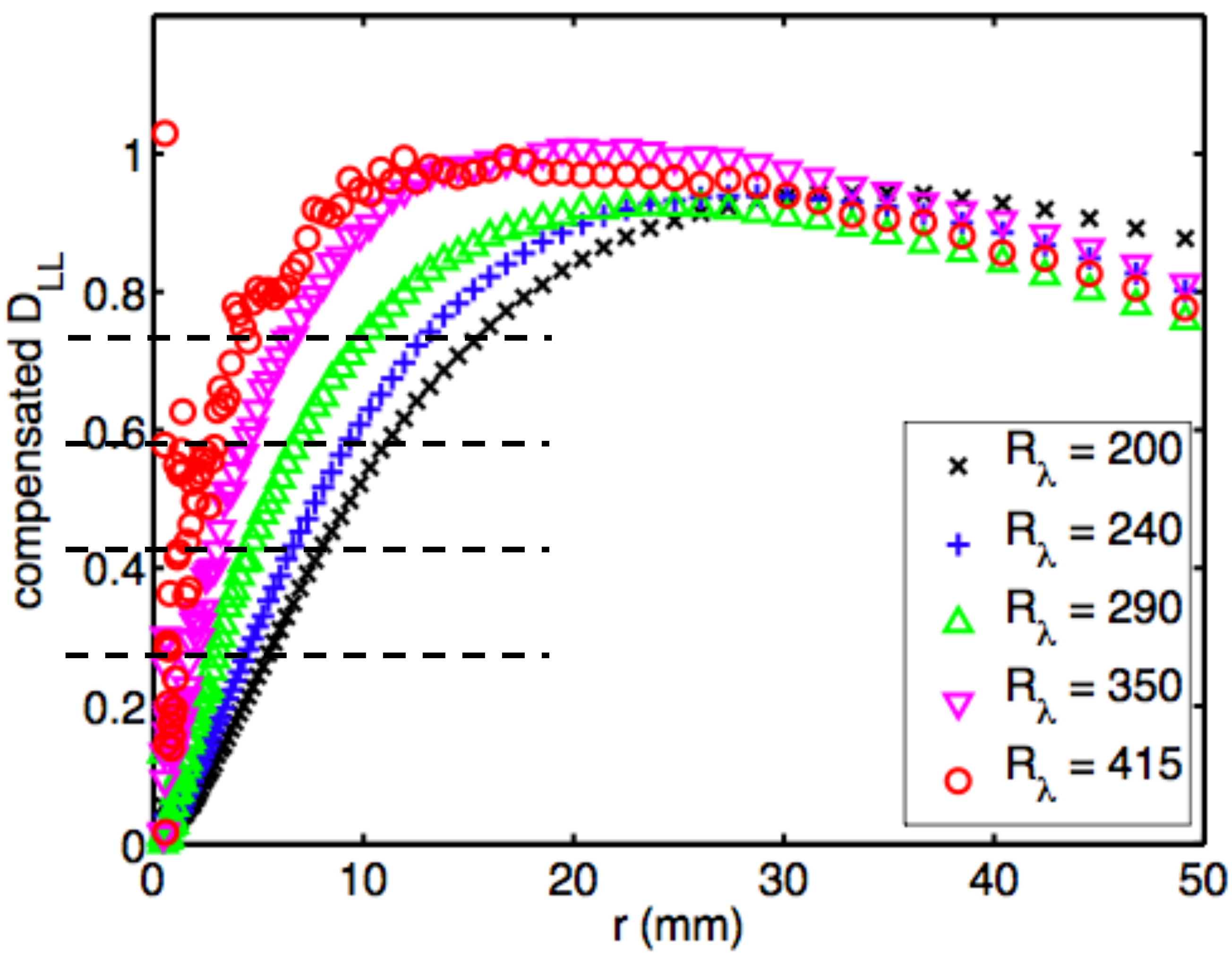


Viscosity Effect?

$$R_\lambda = 290$$



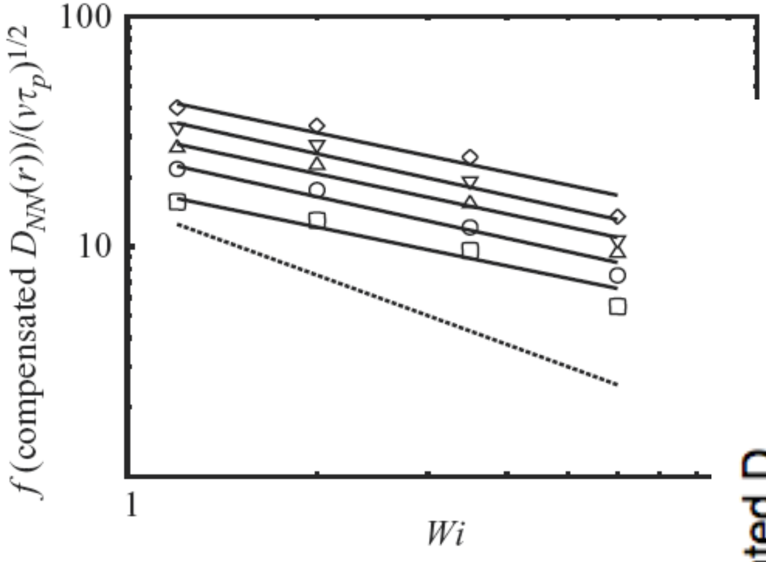
Wi Dependence



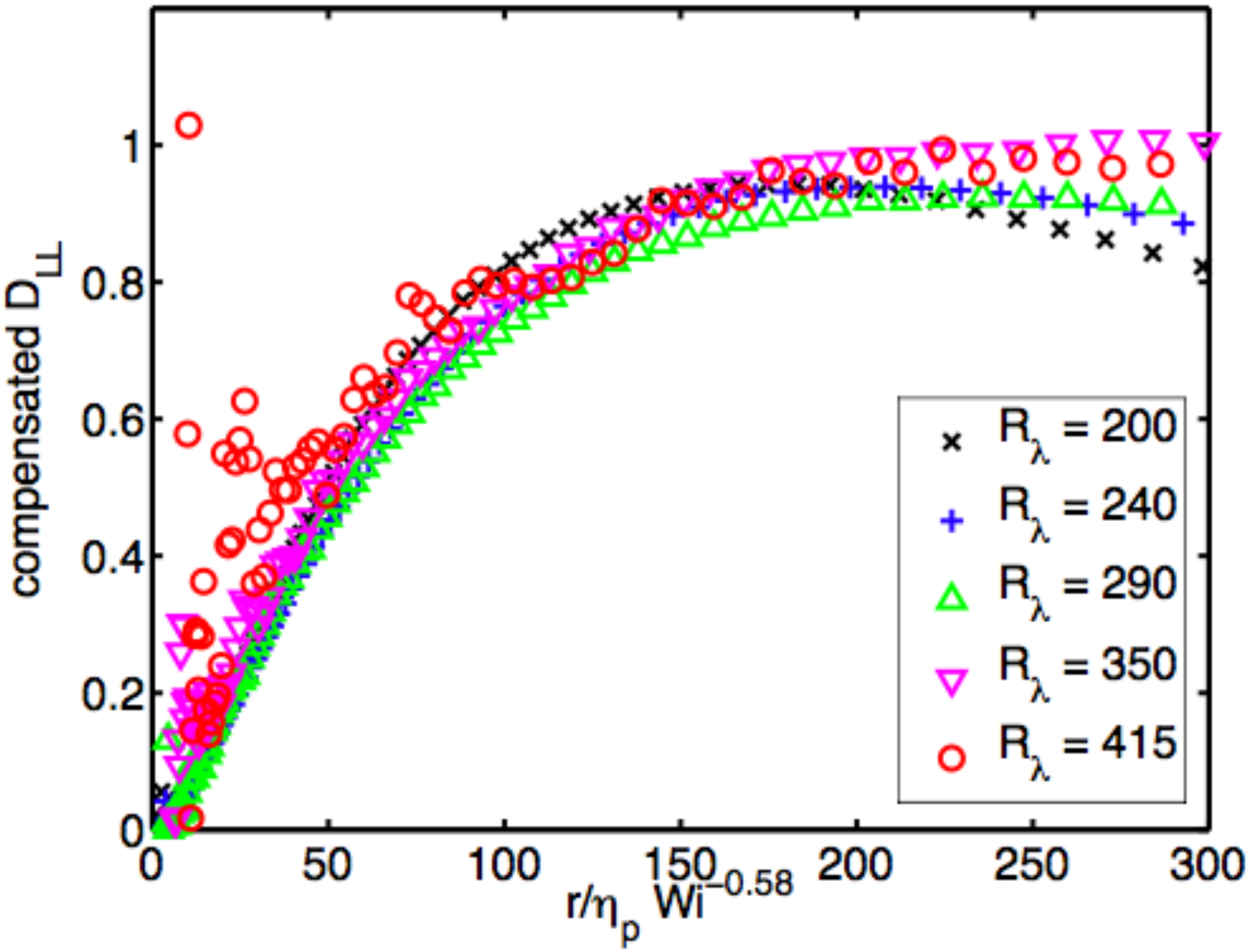
$$r_p \sim \eta_p Wi^n$$

$$\eta_p \equiv (\nu \tau_p)^{1/2}$$

Length Scale

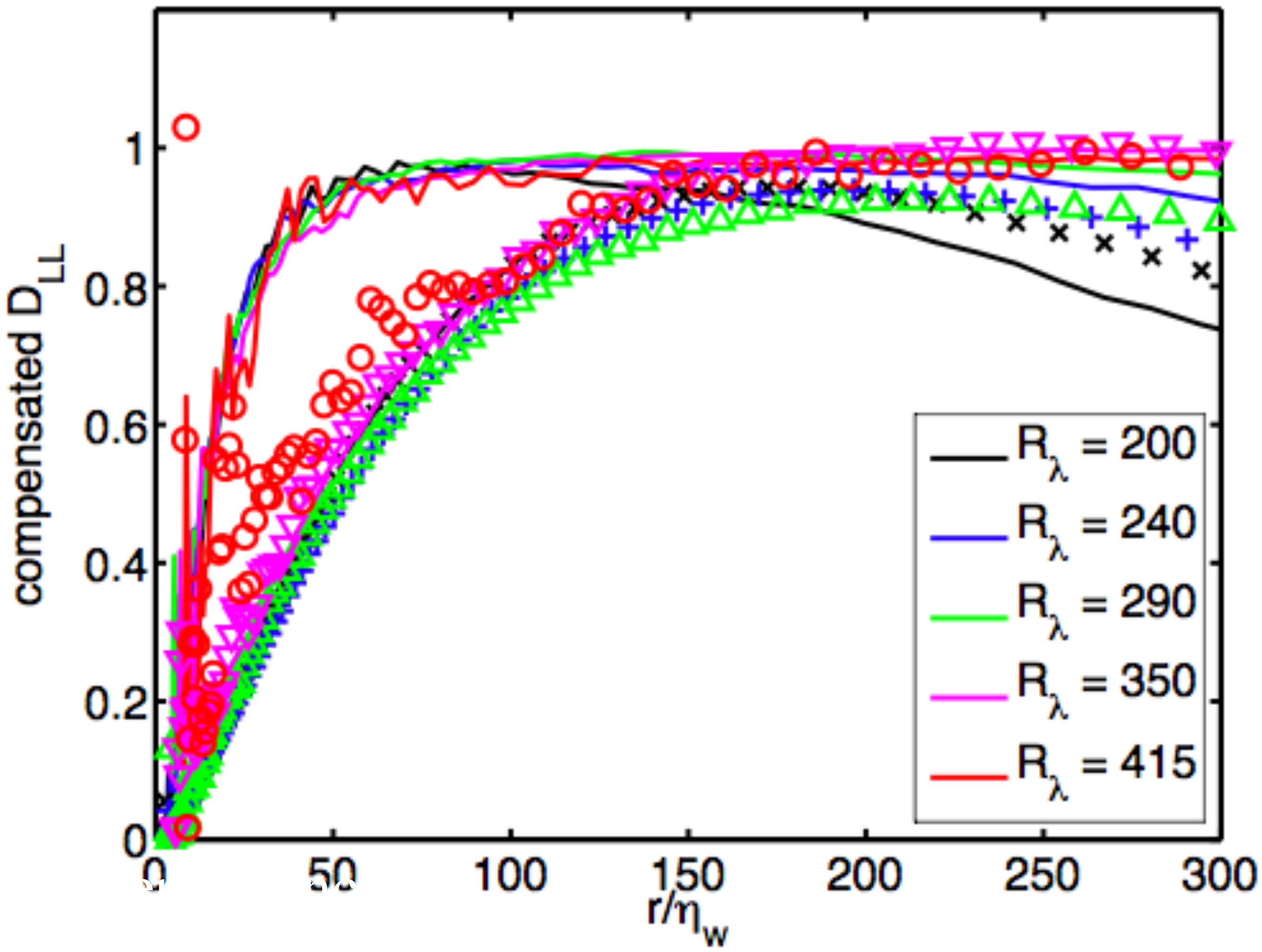


$$r_p \sim \eta_p Wi^{-0.58}$$



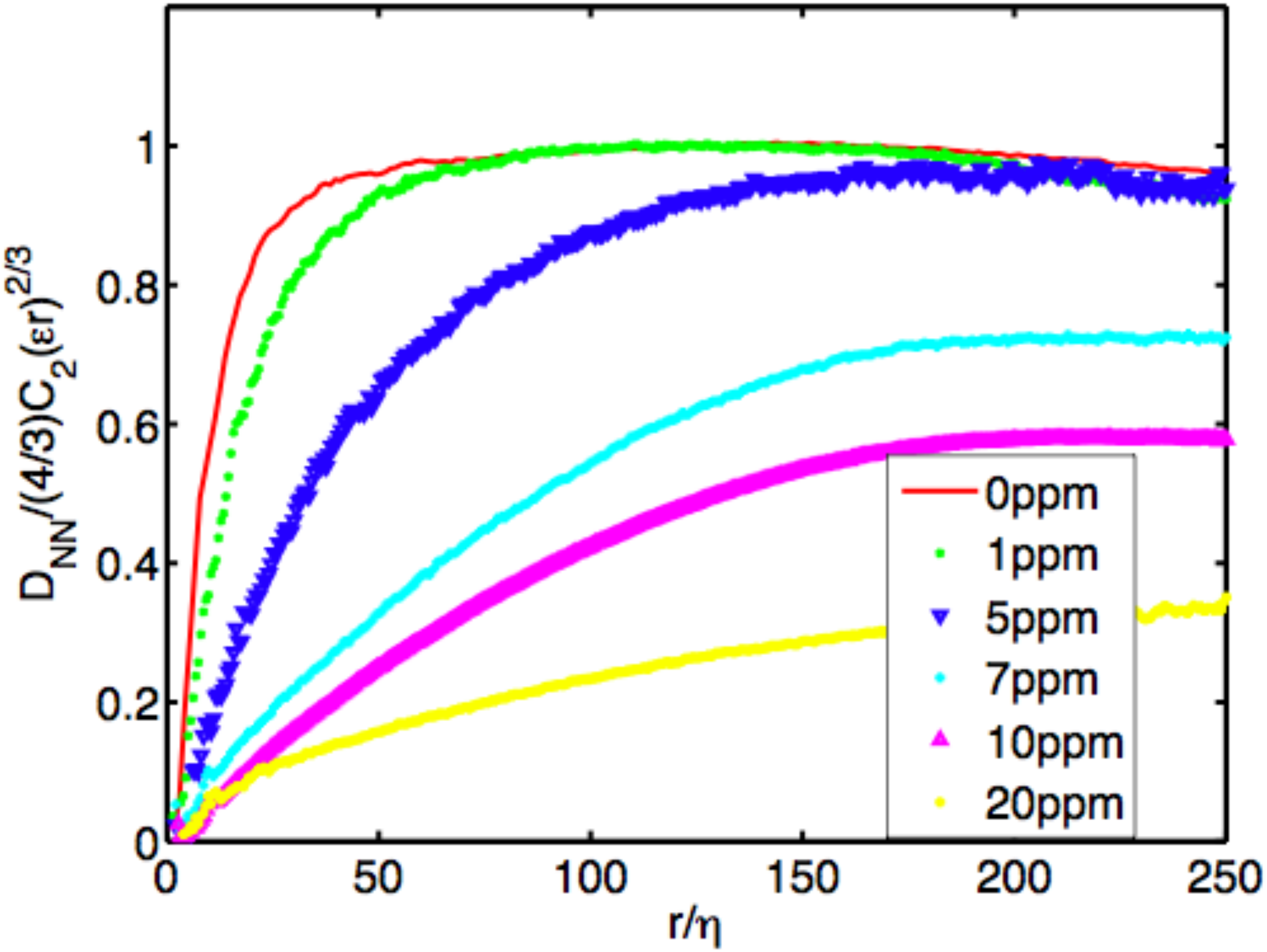
Length Scale

$$\eta_p Wi^{-0.58} = (\nu\tau_p)^{1/2} Wi^{-0.58} = \eta_w Wi^{-0.08}$$



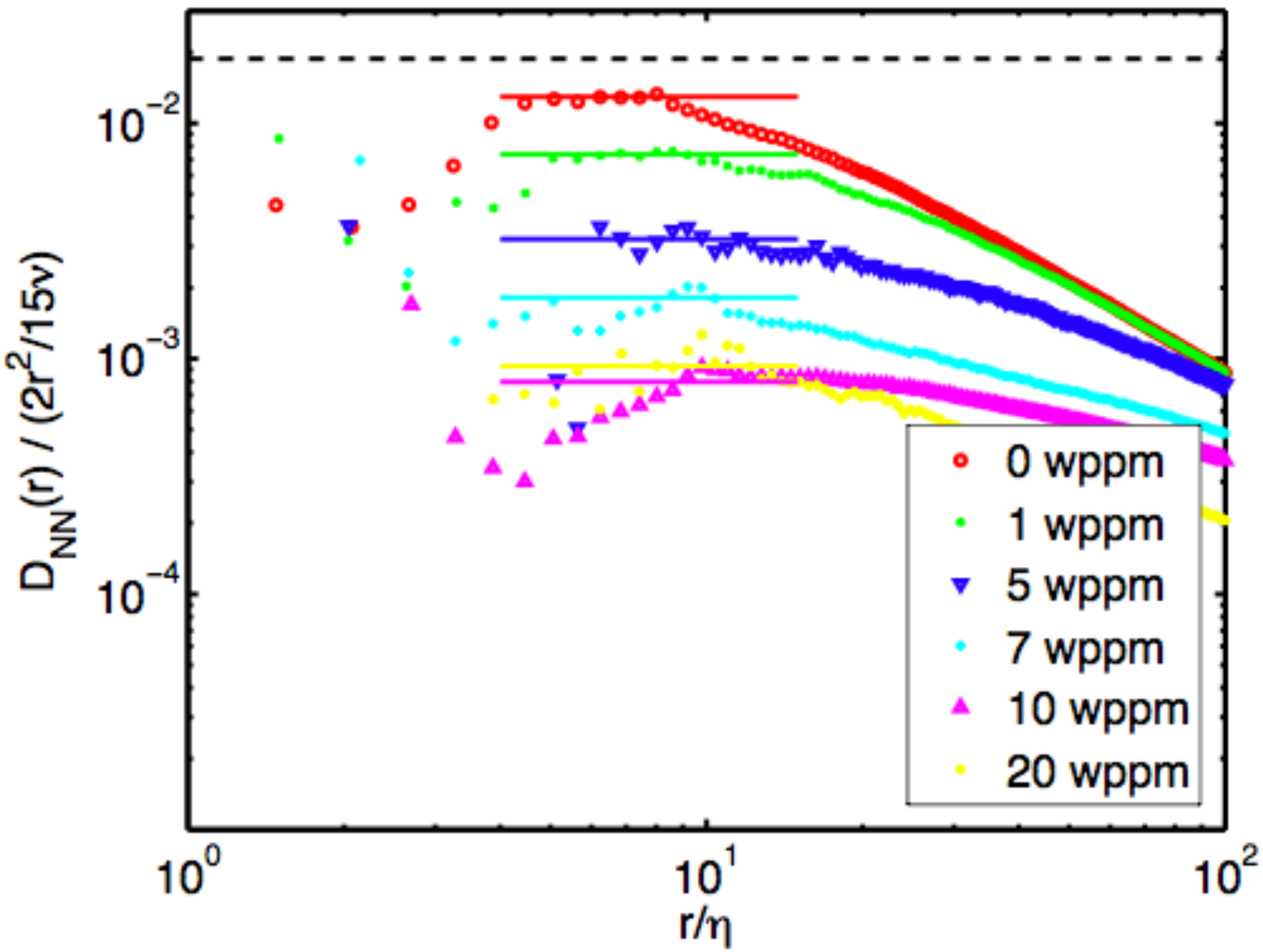
Concentration Effect

Eulerian Structure Functions



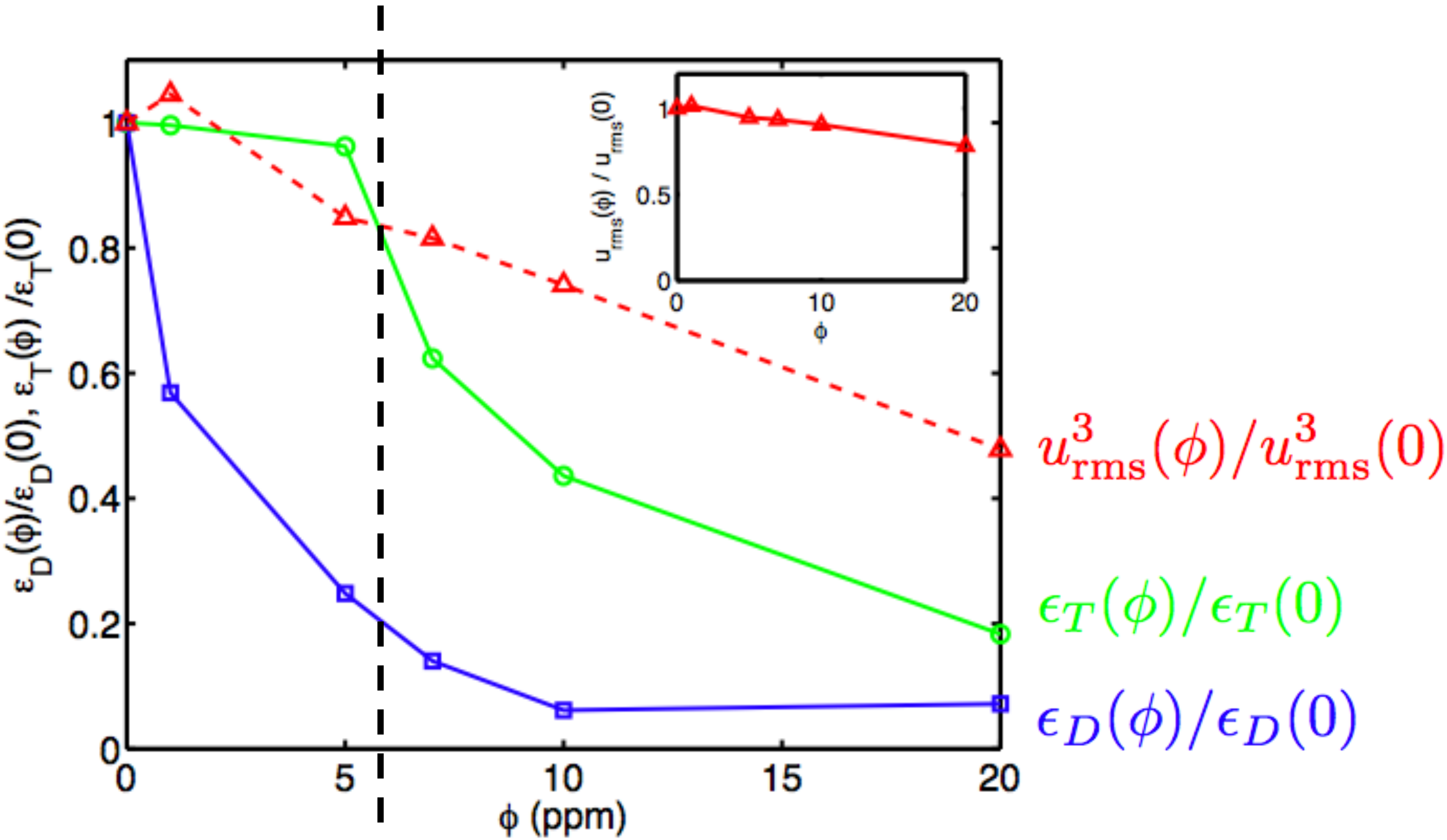
$$D_{NN}(r) = \frac{4}{3}C_2(\epsilon_T r)^{2/3}; \quad (\eta \ll r \ll L)$$

Eulerian Structure Functions



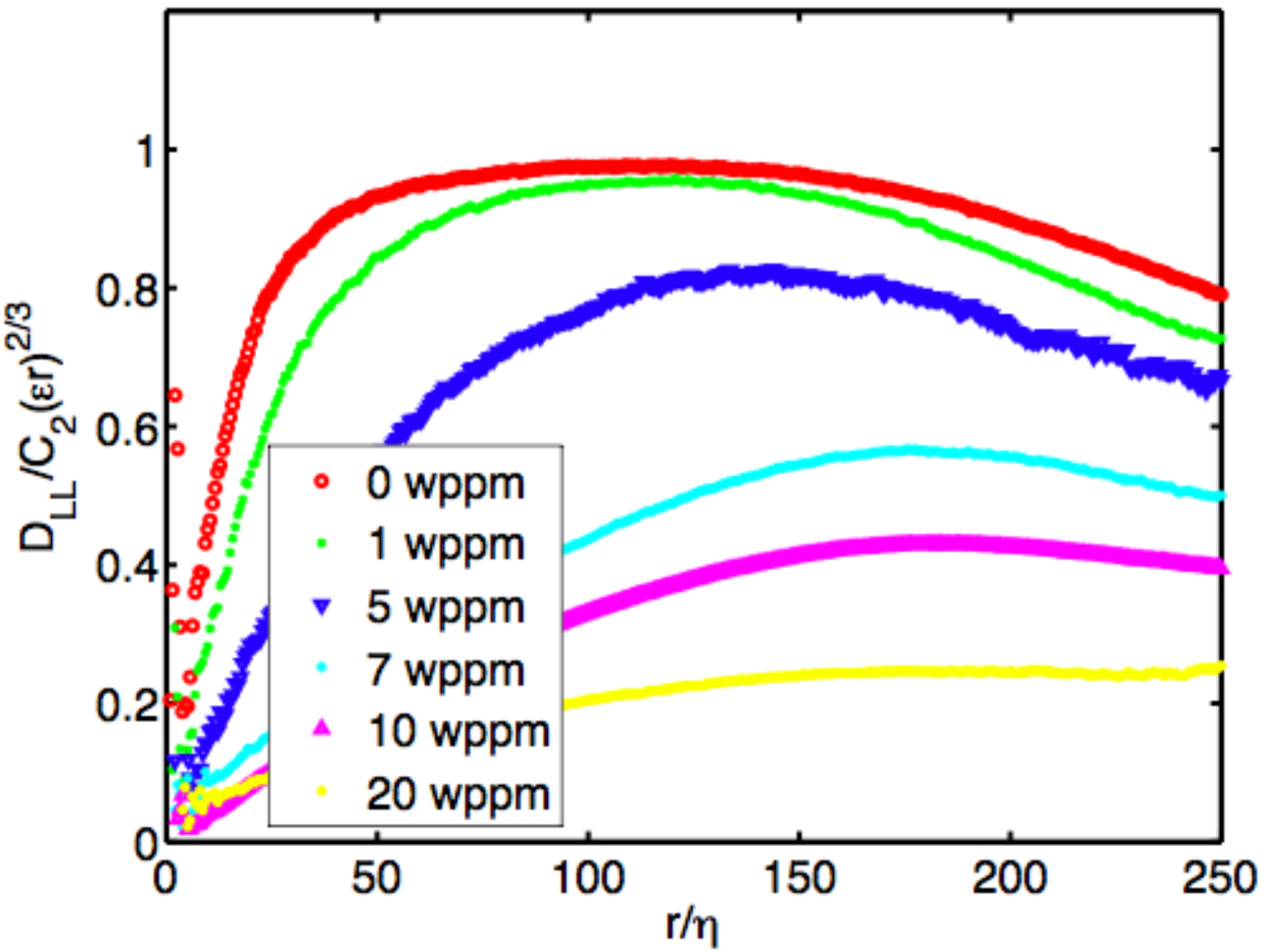
$$D_{NN}(r) = \frac{2\epsilon_D}{15\nu} r^2; \quad (r \ll \eta)$$

Energy Transfer and Dissipation Rates



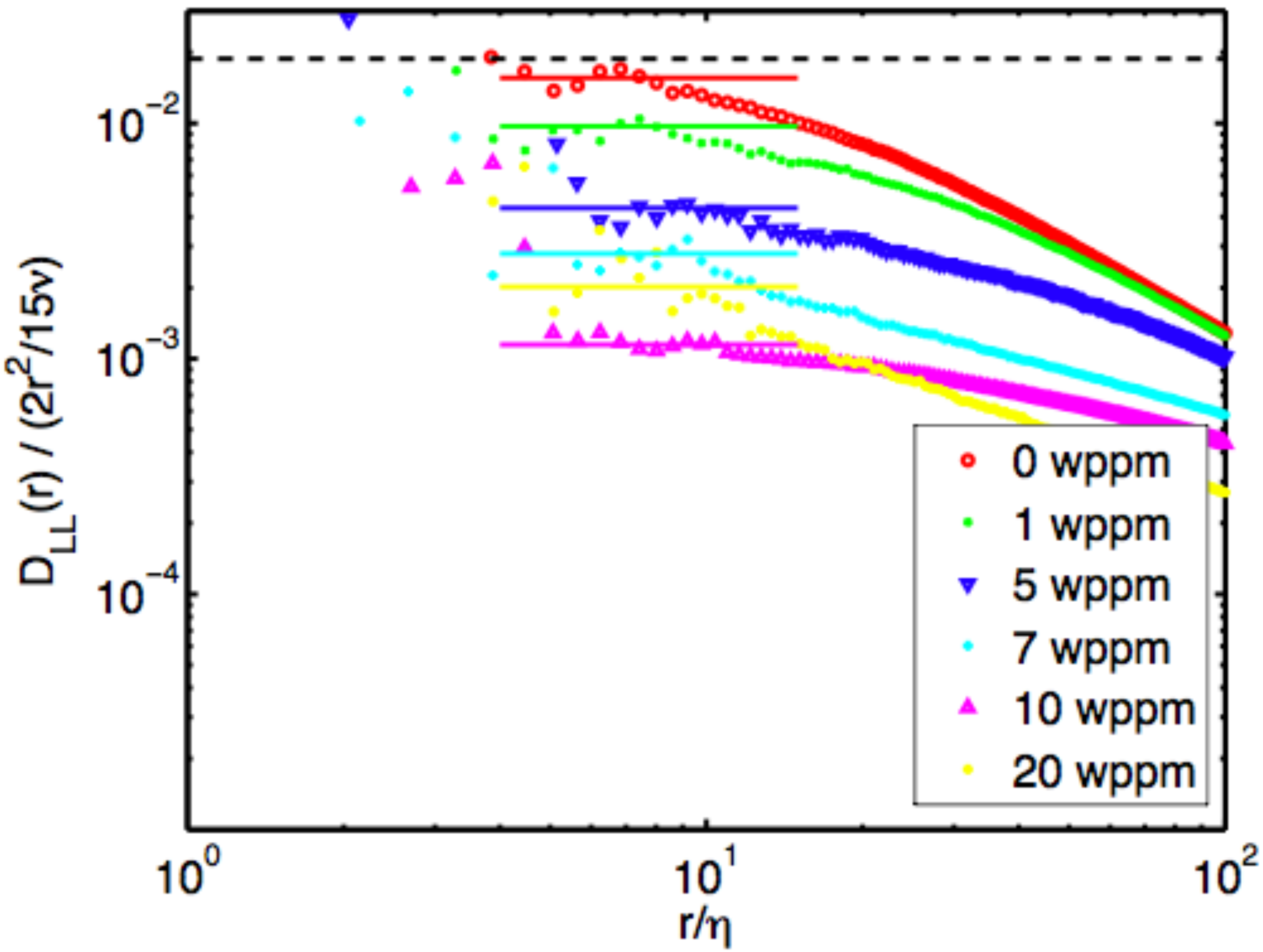
Change of ϵ_T can not be explained by u_{rms} alone.

Eulerian Structure Functions



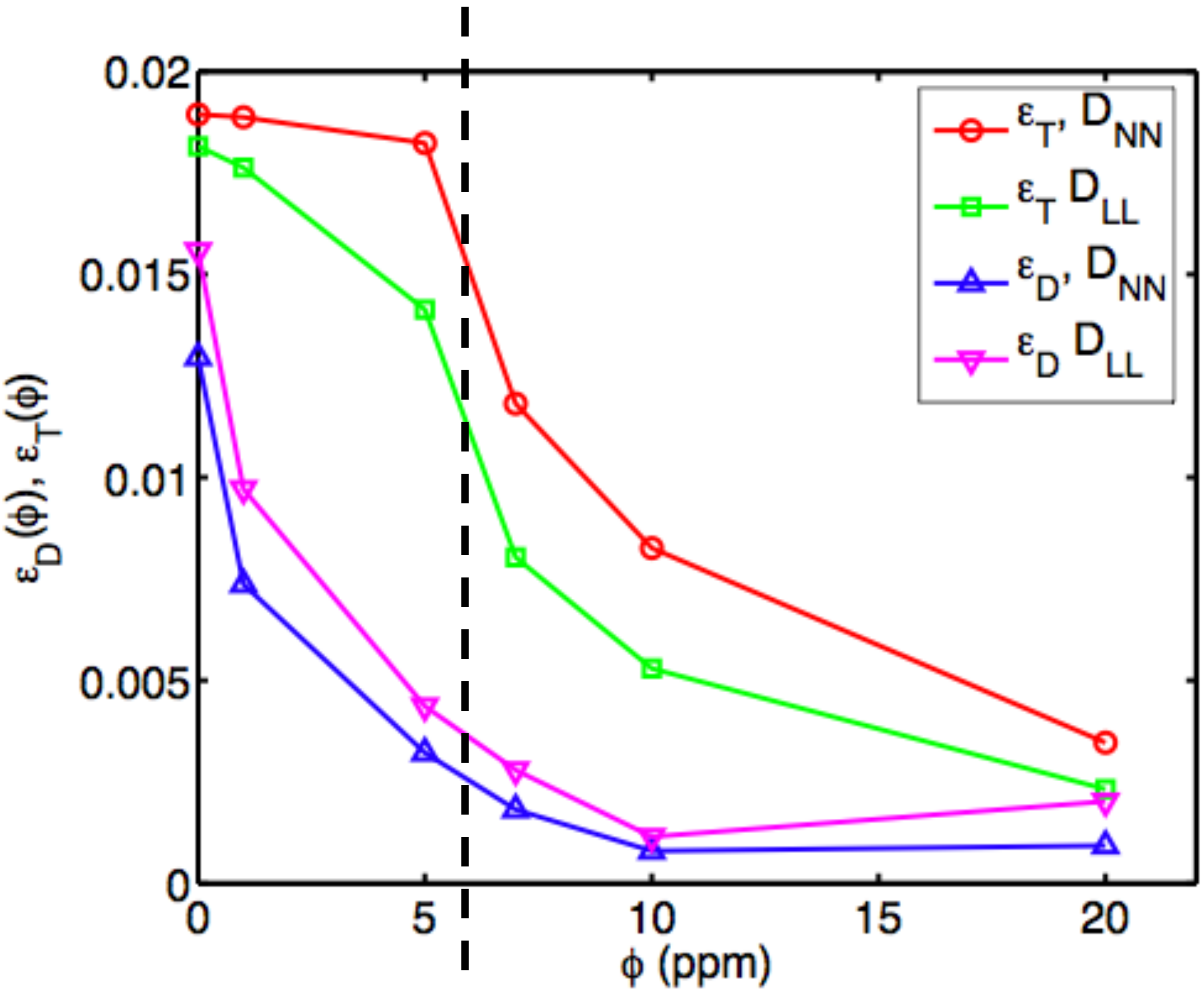
$$D_{LL}(r) = C_2(\epsilon_T r)^{2/3}; \quad (\eta \ll r \ll L)$$

Eulerian Structure Functions



$$D_{LL}(r) = \frac{\epsilon_D}{15\nu} r^2; \quad (r \ll \eta)$$

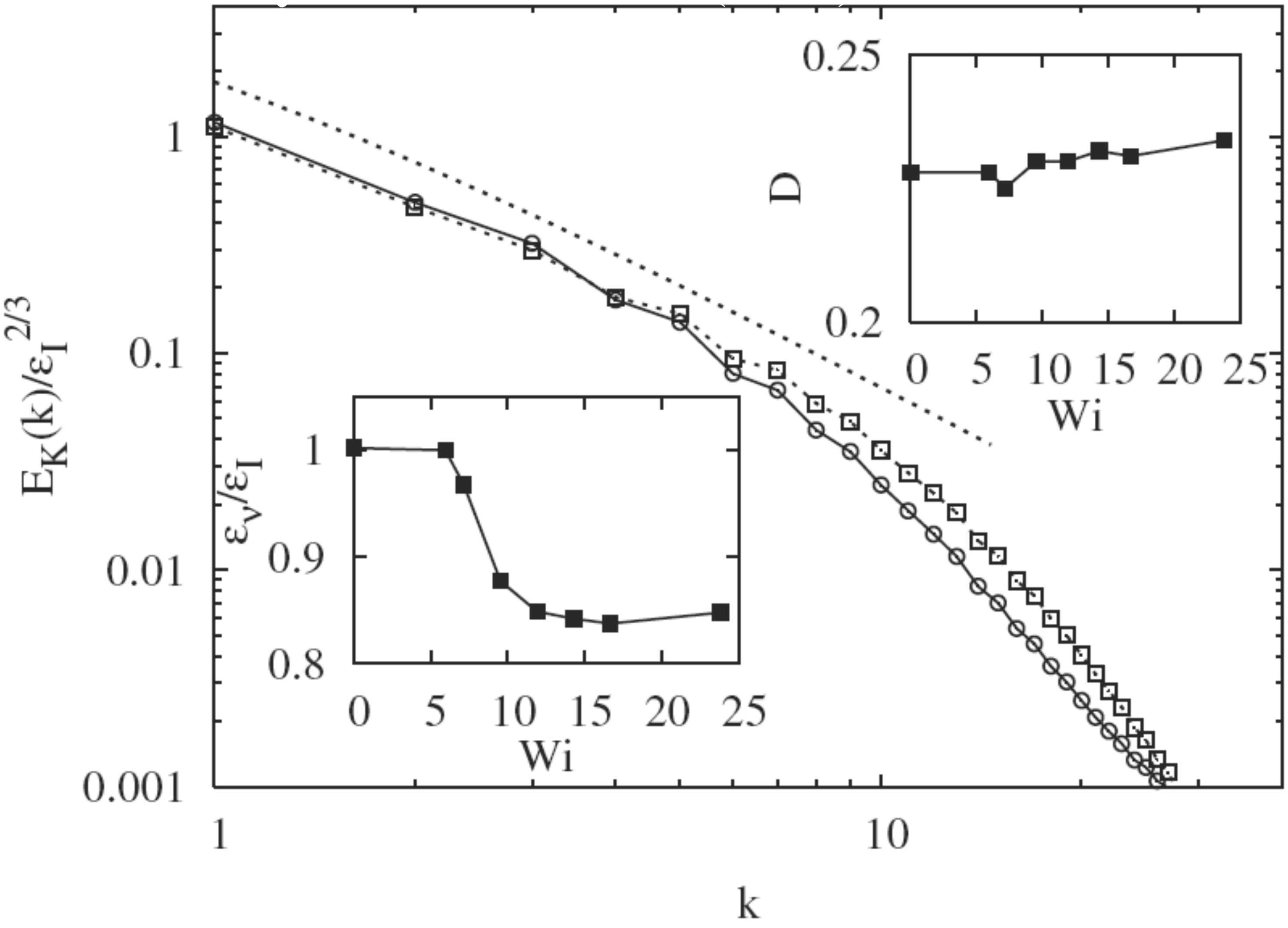
Energy Transfer and Dissipation Rates



Sharp transition in ϵ_T between 5 and 7 wppm.

Wi Dependence?

$R_\lambda = 87$



Berti et al. (EPL, 2006)

Conclusions

- Effect of polymers at varying Wi (by varying R_λ) on Eulerian velocity structure functions may be normalized with re-scaled separation. The length-scale has a (weak?) Wi dependence.

- The effect of polymer concentration on apparent energy transfer rate and energy dissipation rate is different. There exist a critical concentration on energy transfer rate

Velocity Structure Functions: New Experiments

with H.-D. Xi and E. Bodenschatz

How to Isolate Wi Effect?

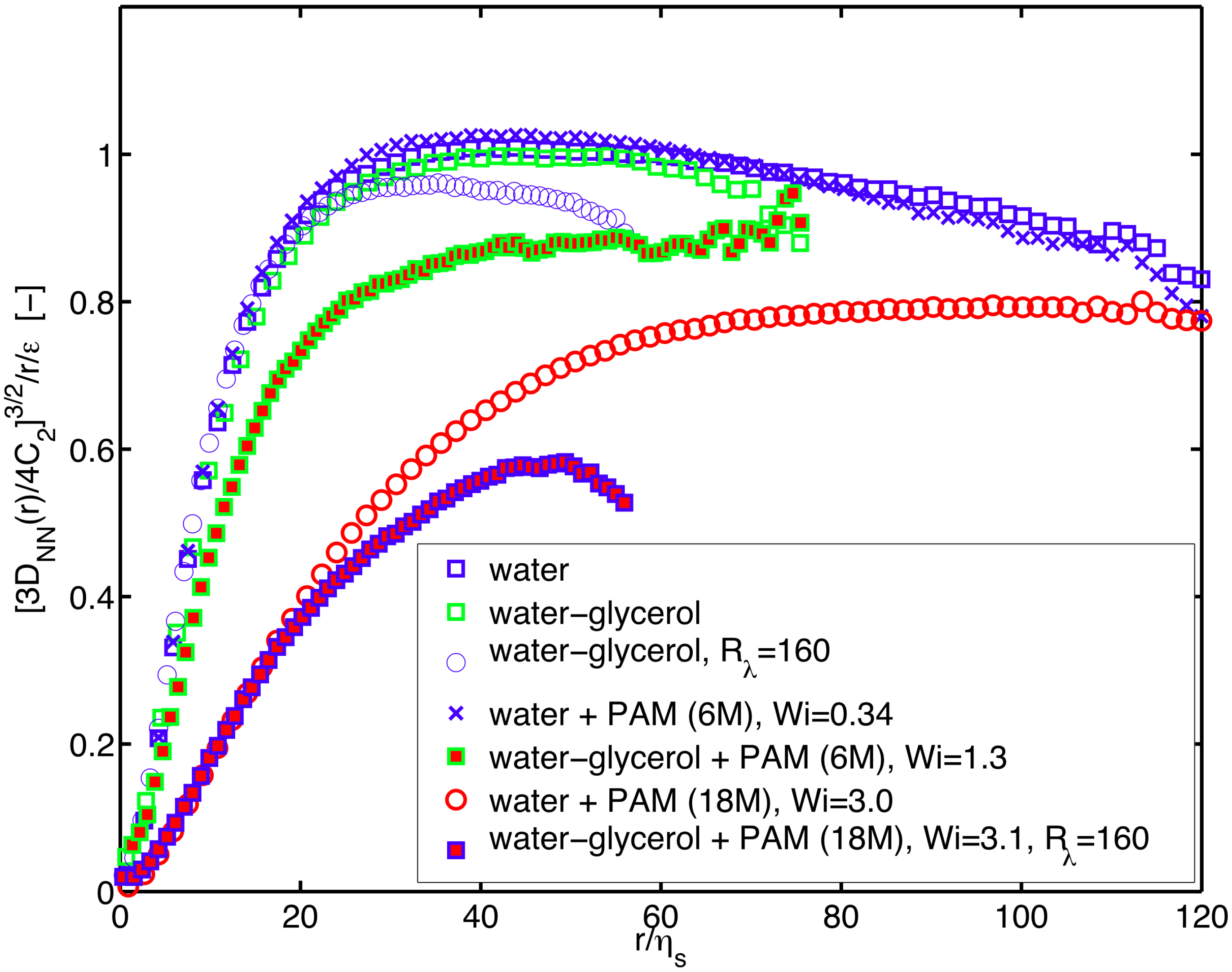
$$L \approx \text{const.} \qquad \eta = \frac{\nu^3}{\varepsilon}$$

$$R_\lambda \propto (L/\eta)^{2/3}$$

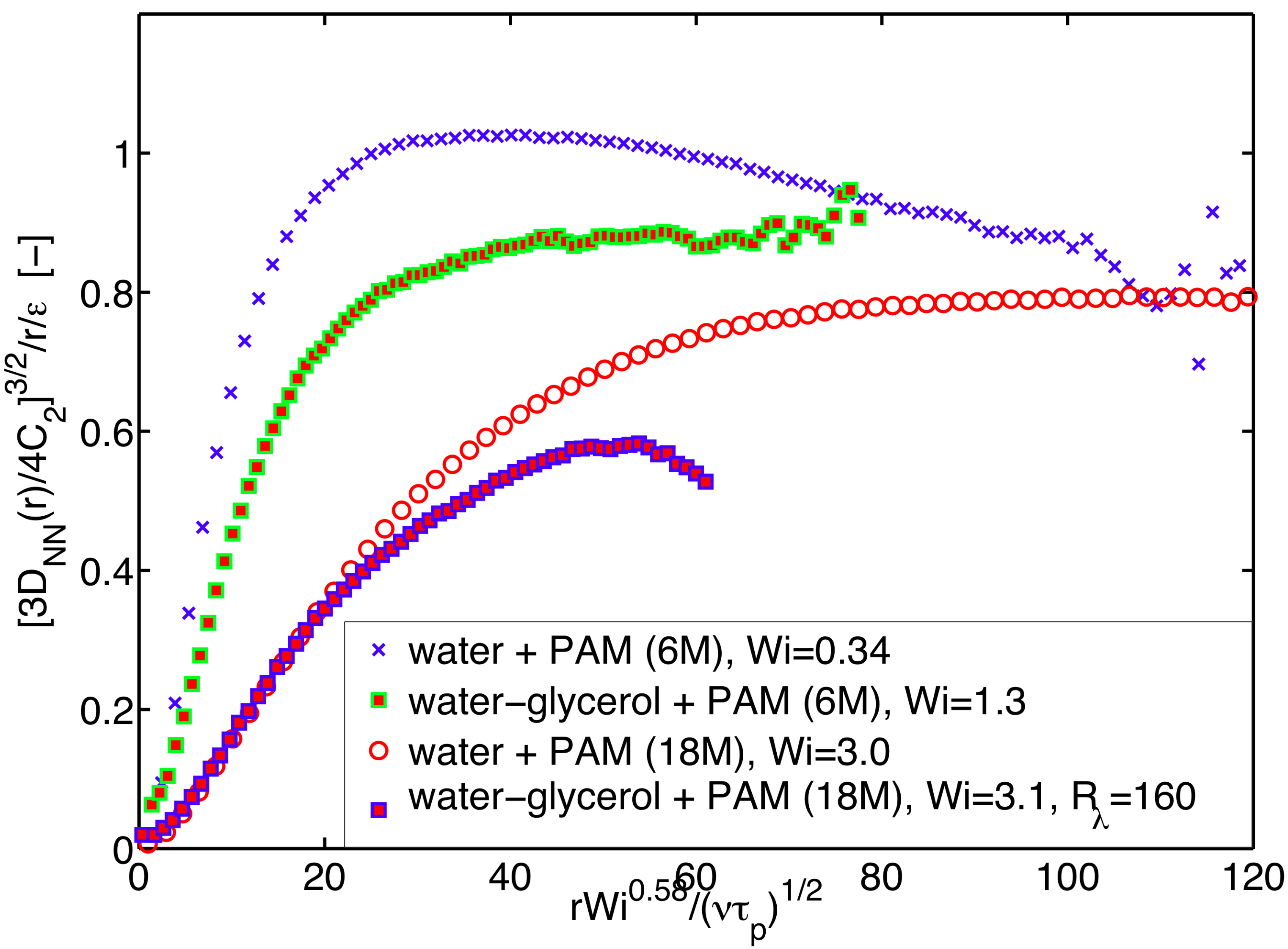
$$Wi = \frac{\tau_p}{\tau_\eta} \propto \frac{\mu/k_b T}{(\nu/\varepsilon)^{1/2}} \propto \nu^2$$

$$R_\lambda = \sqrt{\frac{15u^4}{\nu\varepsilon}} \qquad \tau_\eta = \sqrt{\frac{\nu}{\varepsilon}} \propto \frac{1}{\nu}$$

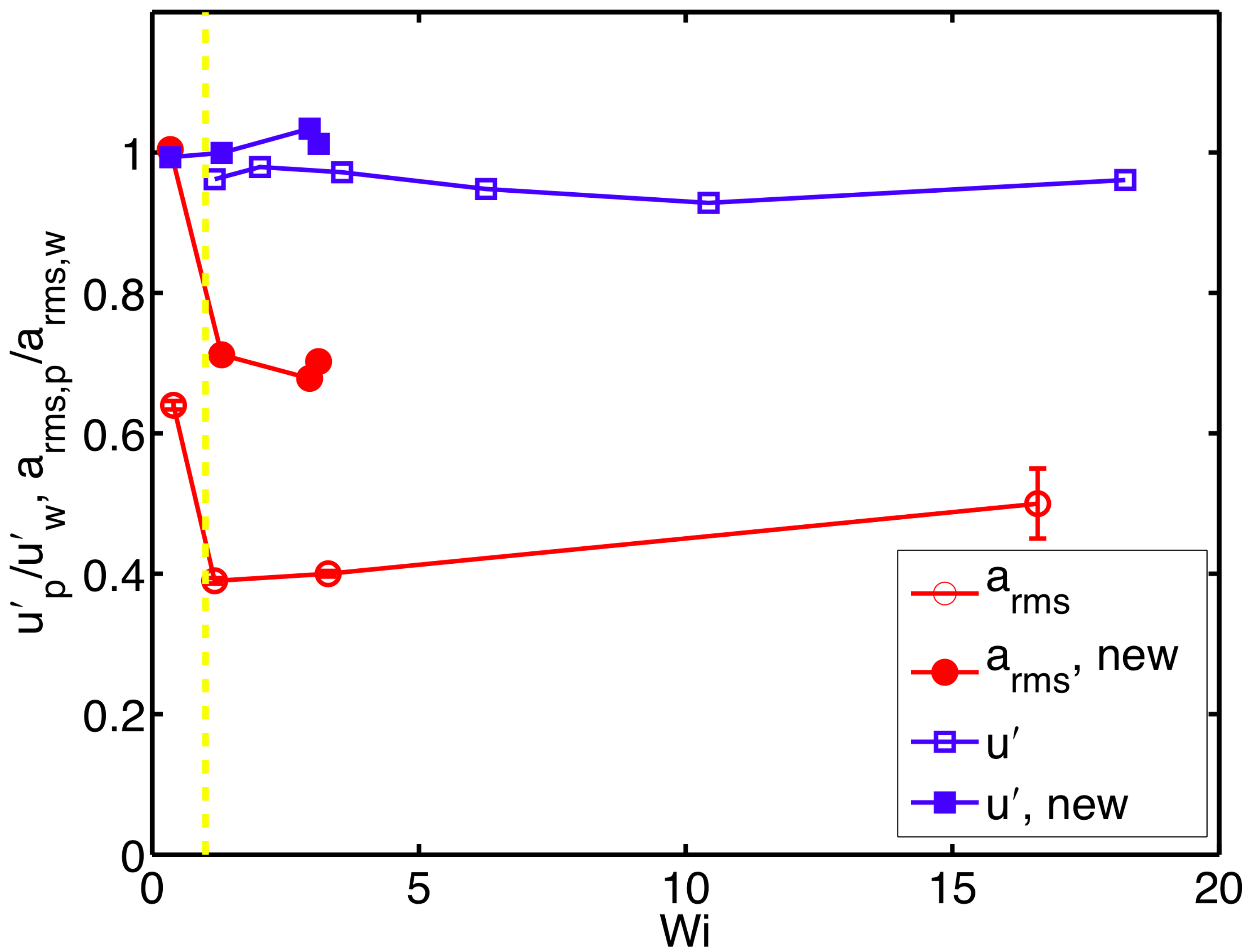
Preliminary Results



Preliminary Results



Preliminary Results



Thanks!