Non-Equilibrium Universality in Many-Body Physics
September 29, 2021
KITP, Santa Barbara, USA (via zoom)

Institute for Theoretical Physics University of Cologne

Measurement Induced Phases and Phase Transitions in Fermion Chains

Sebastian Diehl

Institute for Theoretical Physics, University of Cologne

work with

Ori Alberton, Michael Buchhold, SD — PRL 126, 170602 (2021)

Michael Buchhold, Yuri Minoguchi, Alex Altland, SD — arXiv:2102.08381, PRX in press

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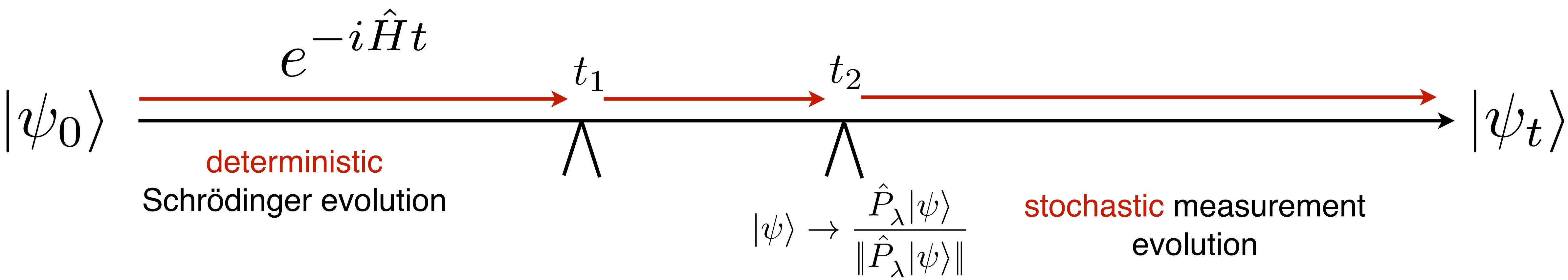
DFG Deutsche Forschungsgemeinschaft

erc European Research Council
Established by the European Commission

Introduction

Small quantum systems: Measurements

- two types of quantum dynamics



for measurement observable $\hat{M} = \sum_\lambda m_\lambda |\lambda\rangle\langle\lambda| \equiv \sum_\lambda m_\lambda \hat{P}_\lambda$

- dynamics non-trivial (eigenstates not shared) once $[\hat{H}, \hat{M}] \neq 0$

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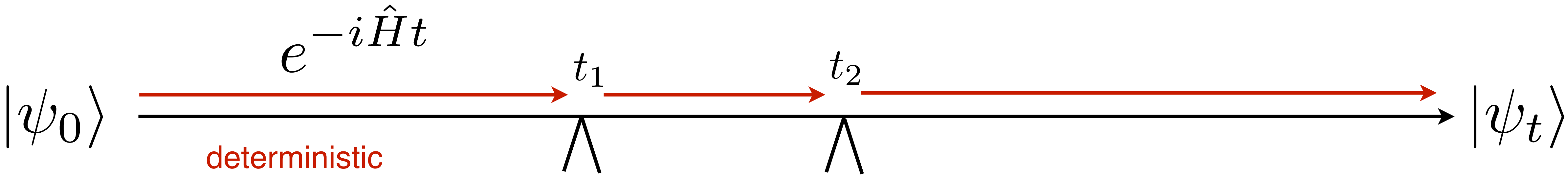

deterministic
Schrödinger evolution

$$|\psi\rangle \to \frac{\hat{P}_\lambda|\psi\rangle}{\|\hat{P}_\lambda|\psi\rangle\|}$$

stochastic measurement
evolution

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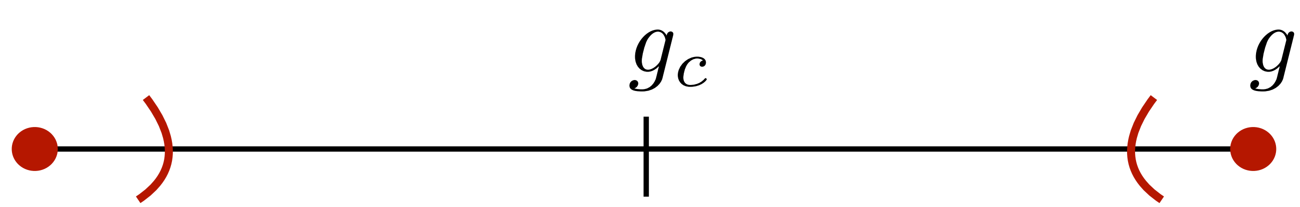
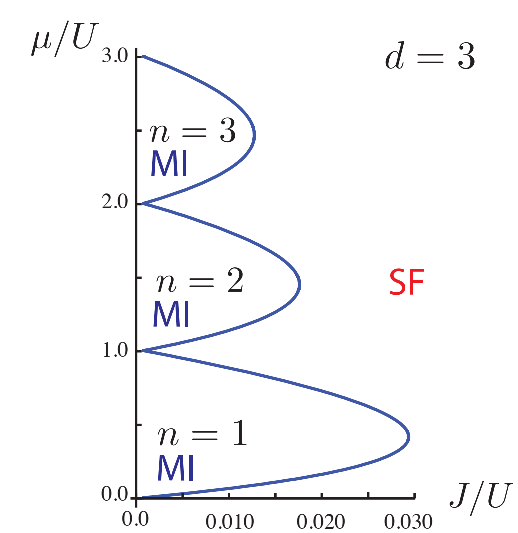
Many-body systems: Phase transitions

- non-commuting operators lead to (quantum) phase transitions

$$\hat{H} = \hat{H}_1 + g\hat{H}_2 \qquad [\hat{H}_1, \hat{H}_2] \neq 0$$





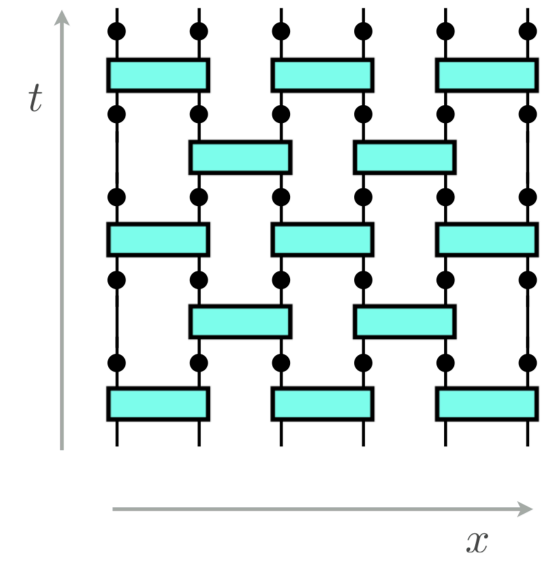
e.g. Mott-insulator to superfluid
transition in cold atoms

➡ combine measurement and many particles: similar scenario?

Entanglement Phase Transitions in Random Circuits

- model and key ingredients:
  - randomly chosen local entangling unitary gates
  - projective local measurement of non-commuting observables

Skinner, Ruhman, Nahum PRX (2019)



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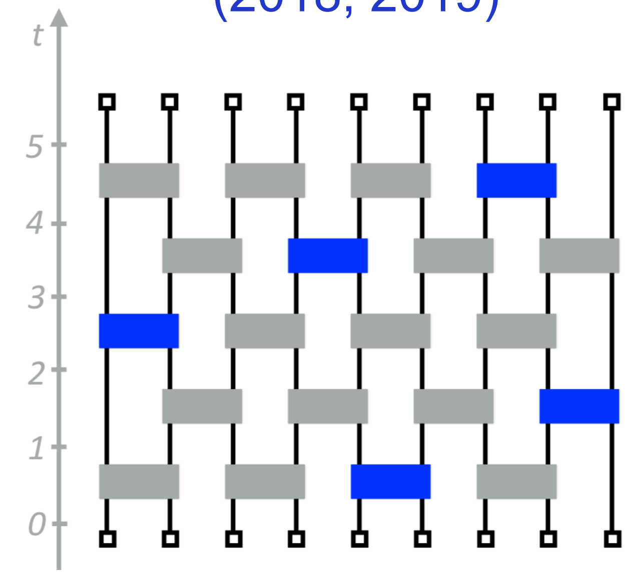

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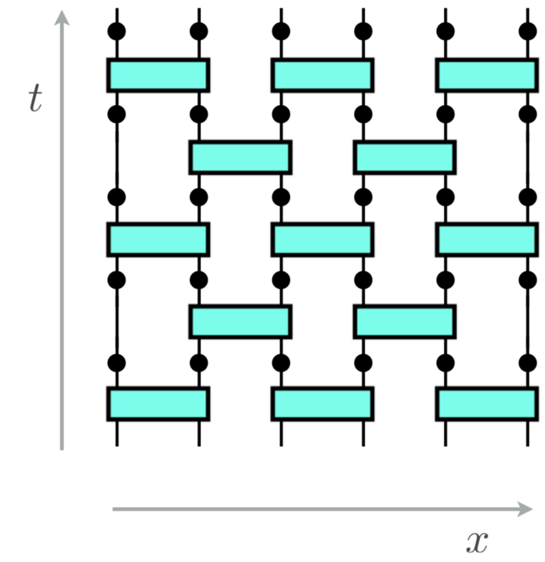

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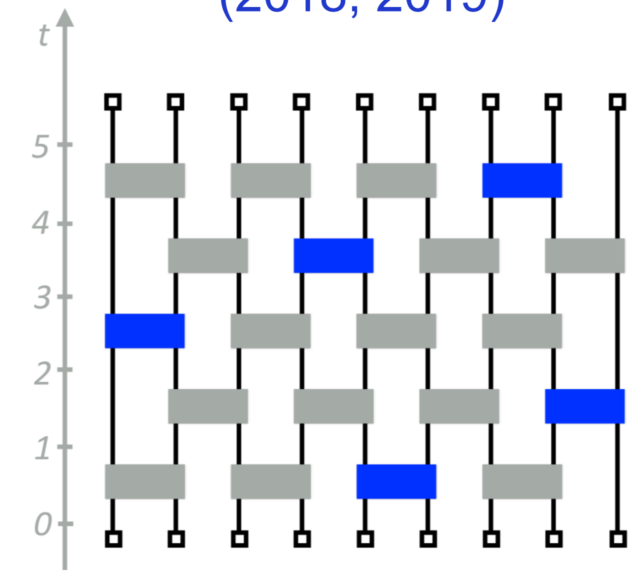

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- basic picture: competition in many-body context (measure σ_i^z)

$g = 0$ — chaotic dynamics ➡ volume law entanglement growth

$g^{-1} = 0$ — product state $\prod_i |\sigma_i\rangle \quad \sigma_i = \uparrow, \downarrow$ ➡ area law entanglement growth

$$g = \frac{\text{\# measurements/time}}{\text{\# unitaries/time}}$$

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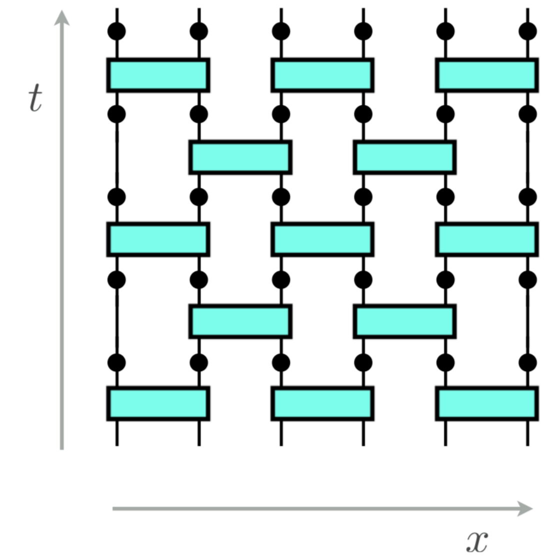

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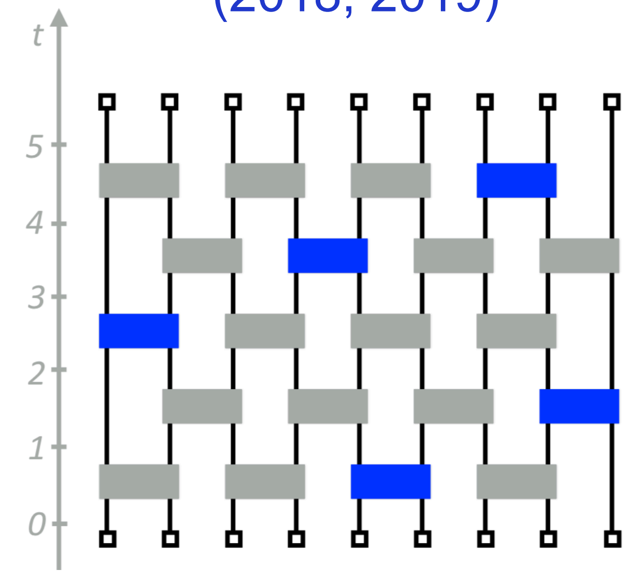

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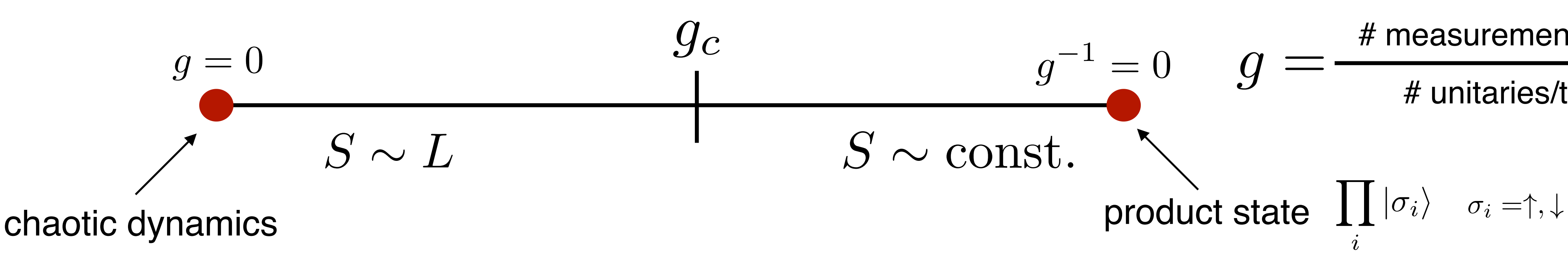

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➡ volume law entanglement growth

➡ area law entanglement growth

➡ Phase transition in entanglement growth at finite competition ratio g

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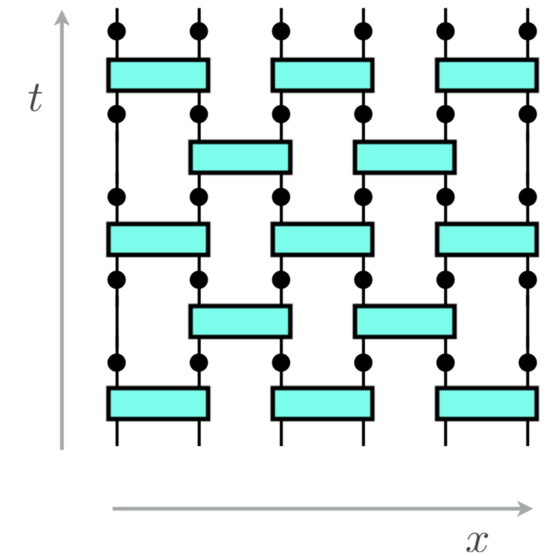

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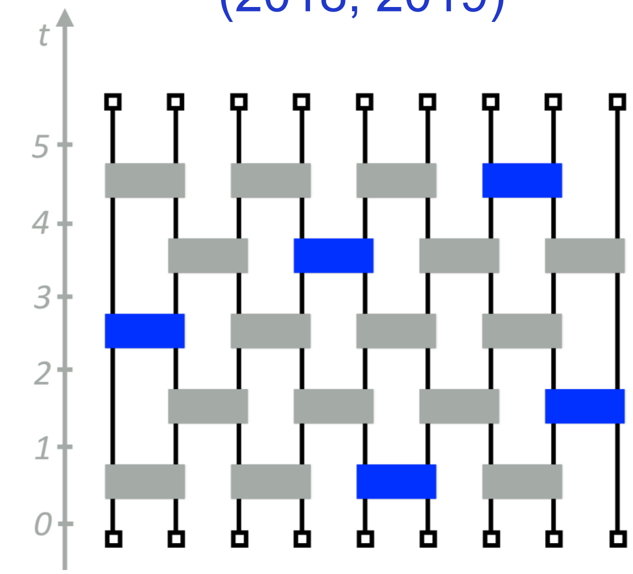

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$S \sim L$ | $S \sim \text{const.}$

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chaotic dynamics

➡ volume law entanglement growth

product state $\prod_i |\sigma_i\rangle \quad \sigma_i = \uparrow, \downarrow$

➡ area law entanglement growth

- Physical pictures
  - quantum error correction — Choi, Bao, Qi, Altman, PRL (2020); Fan et al. arxiv (2020); Li Fisher PRB (2021)
  - purification transition — Gullans, Huse, PRX (2020); Gopalakrishnan, Gullans PRL (2021)
  - mapping to stat mech models — Jian, You, Vasseur, Ludwig, PRB (2020), Nahum et al., PRX Quantum (2021)

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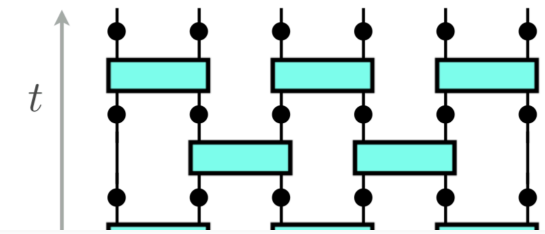

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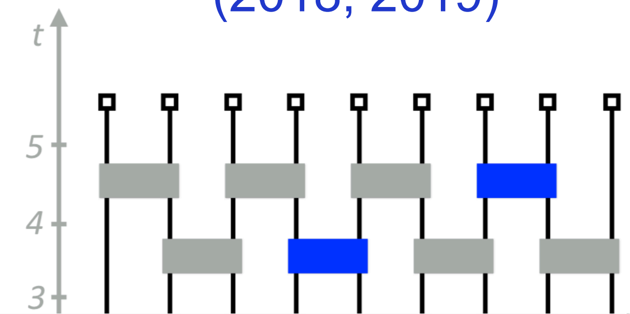

Outline

- Model: locally monitored fermion chain
- New entanglement transition: from critical scaling to area law
- Physical understanding: Effective field theory for measured Dirac fermions
- Outlook

- Physical pictures
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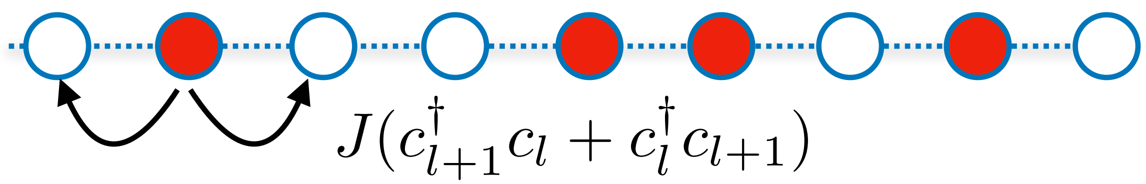
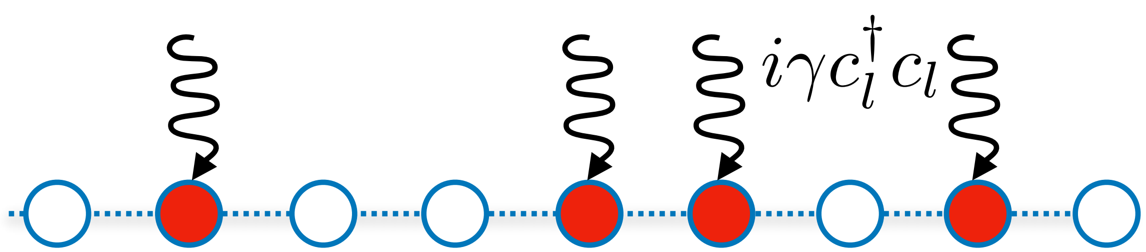
Entanglement Phase Transition in a Monitored Fermion Chain

model: see also
Cao, Tilloy, DeLuca SciPost (2019)

Hamiltonian:

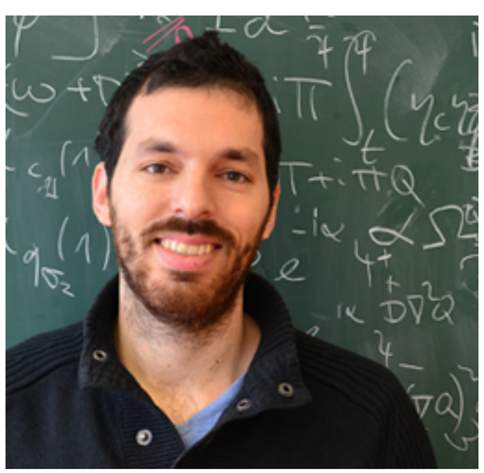


entanglement growth

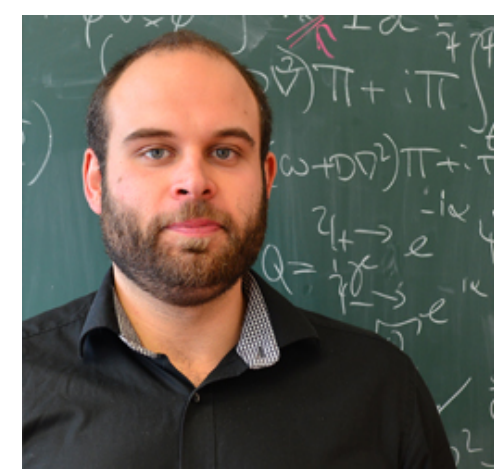


Monitoring:

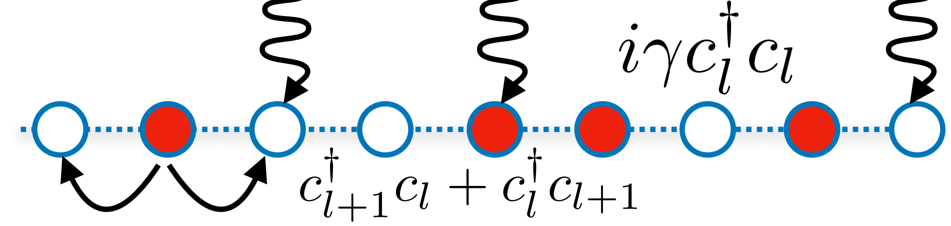
entanglement saturation



O. Alberton, M. Buchhold, SD,
PRL 126, 170602 (2021)



Monitored Fermion Dynamics



Belavkin, Phys. Lett A (1989); Gisin, Percival, JPA (1993)

Jacobs, Steck, Contemp. Phys. (2006)

- Weak continuous measurements: Quantum state diffusion

$$d|\psi_t\rangle = dt(-i\hat{H} - \tfrac{\gamma}{2}\sum_l \hat{M}_l^2|\psi_t\rangle + \sum_l \underbrace{dW_l}_{\text{Gaussian white noise}} \hat{M}_l|\psi_t\rangle$$

related formulations:
Gardiner, Parkins, Zoller, PRA (1992)
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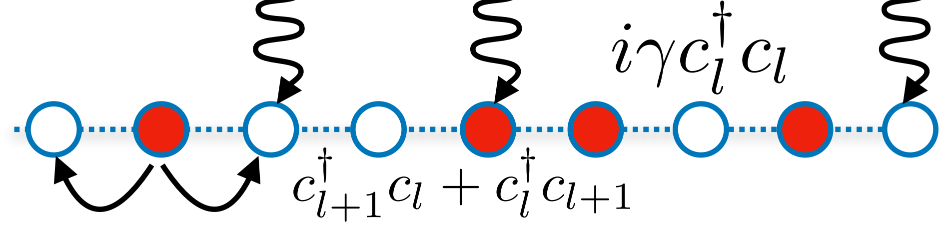

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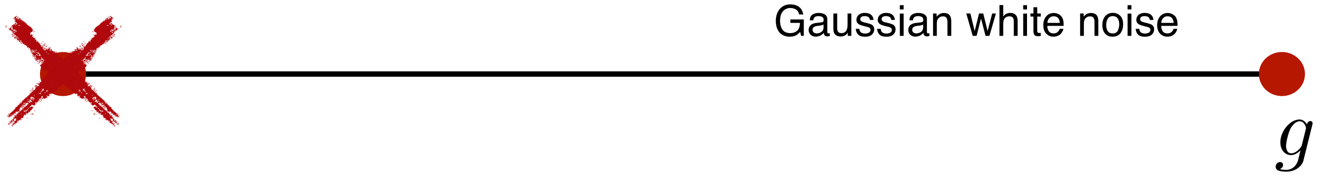
- competition: $g = \dfrac{\gamma}{J}$



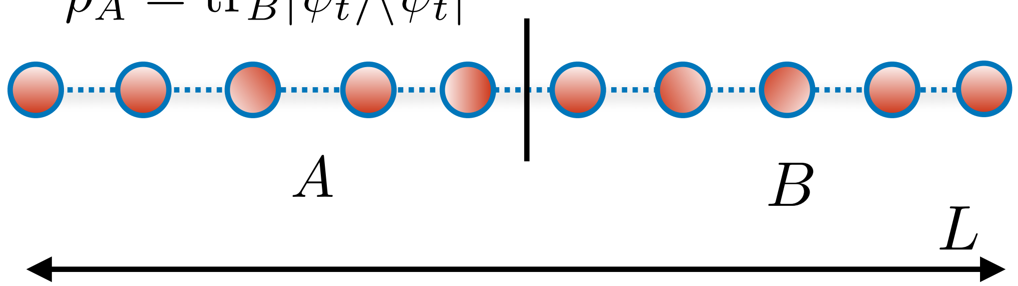
- unitary dynamics: hopping

$$H = -J\sum_l \left(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l\right)$$

➡ volume law entanglement entropy

$$S_{vN}(L/2, L) = \mathrm{tr}\rho_A \log(\rho_A) \overset{t\to\infty}{\sim} L$$

$$\rho_A = \mathrm{tr}_B|\psi_t\rangle\langle\psi_t|$$



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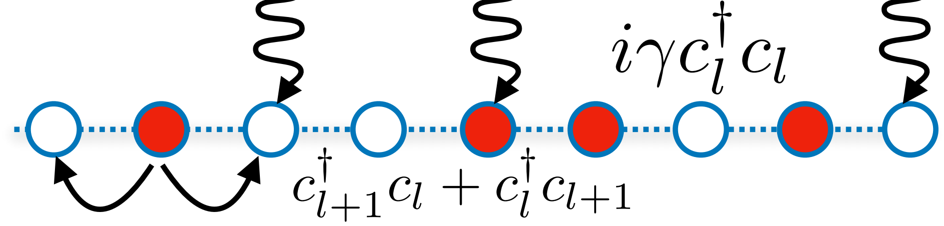

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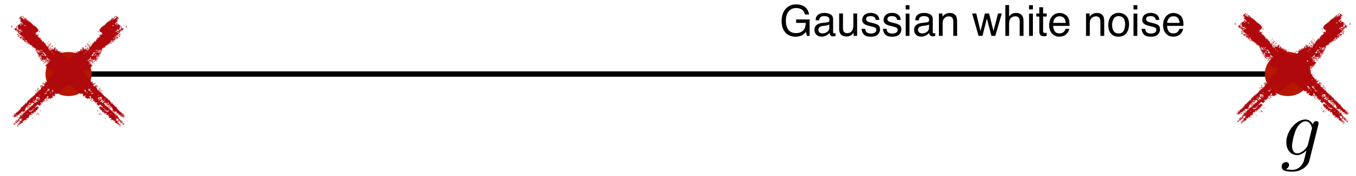

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- H = 0: continuous collapse into dark state

$$\hat{M}_l|\psi_t\rangle = 0 \quad \text{for} \quad \hat{n}_l|\psi_t\rangle = n_l|\psi_t\rangle$$

eigenstate of measurement operator

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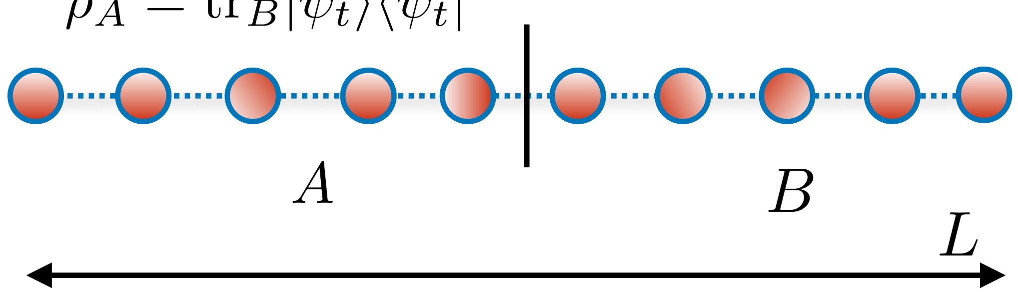
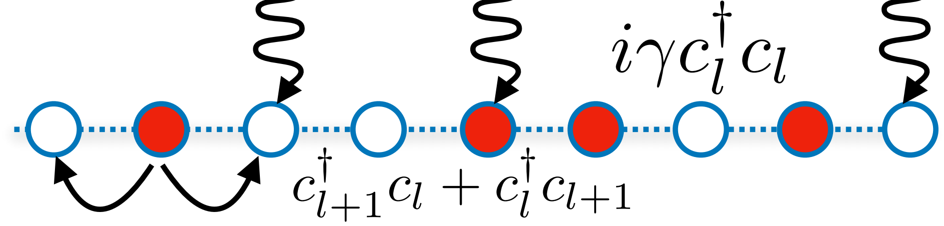


➡ area law entanglement entropy

$$S_{vN}(L/2, L) = s_0$$

Monitored Fermion Dynamics



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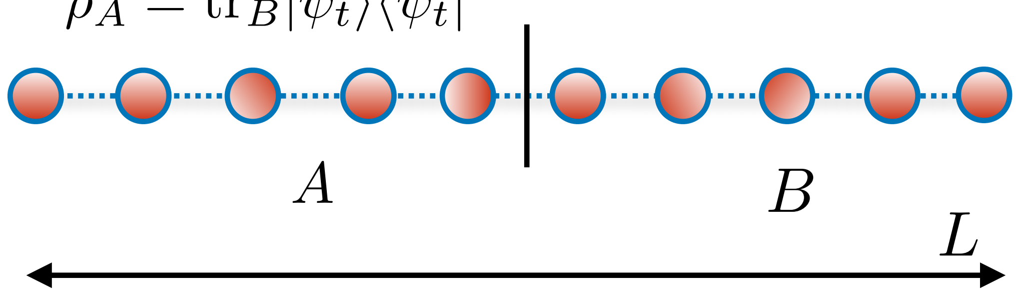

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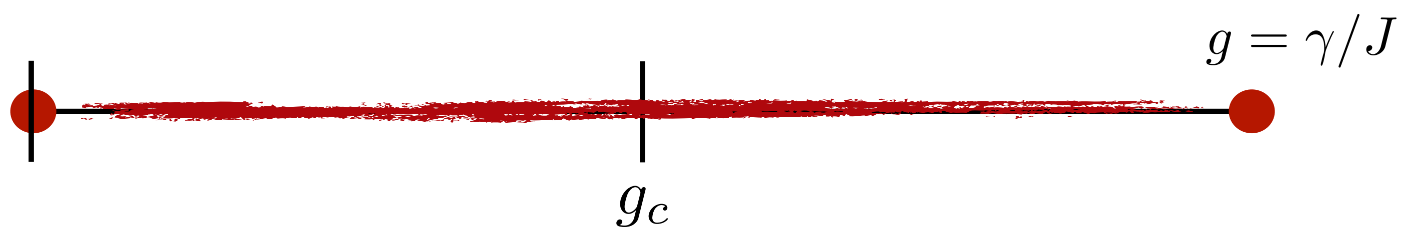
- caveat: $|\psi_{t\to\infty}\rangle$ is a random variable

➡ consider trajectory ensemble to extract information

Monitored Fermion Dynamics: Extracting Information



- usual observables:

$$\langle \hat{A} \rangle = \langle \psi_t | \hat{A} | \psi_t \rangle = \mathrm{tr}[\hat{A}\hat{\rho}_t]$$

quantum average

$$\hat{\rho}_t = |\psi_t\rangle\langle\psi_t|$$

state projector

$$\xrightarrow[\text{e.g. trajectories}]{\text{statistical average}} \quad \overline{\langle \hat{A} \rangle} = \mathrm{tr}[\hat{A}\overline{\hat{\rho}_t}]$$

- problem: Hermitian measurement operators => $\overline{\hat{\rho}_t} \sim \mathbf{1}$

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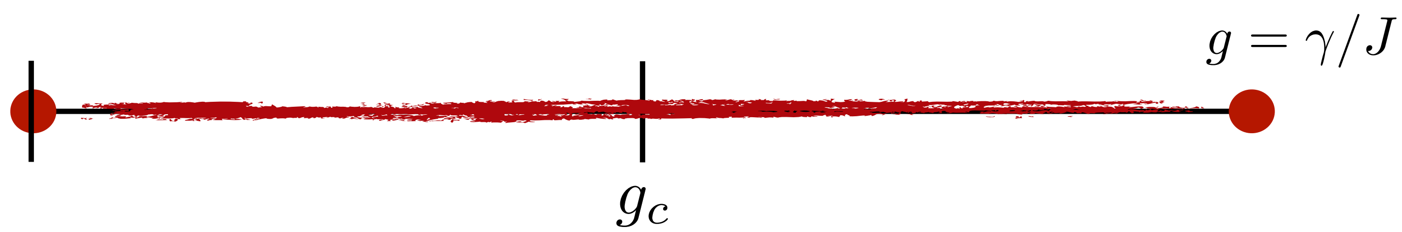

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more promising, because in general $F(\overline{\hat{\rho}}) \neq \overline{F[\hat{\rho}]}$

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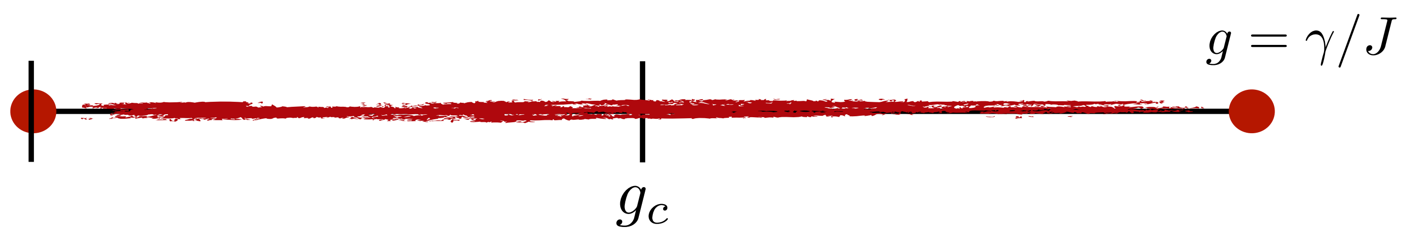

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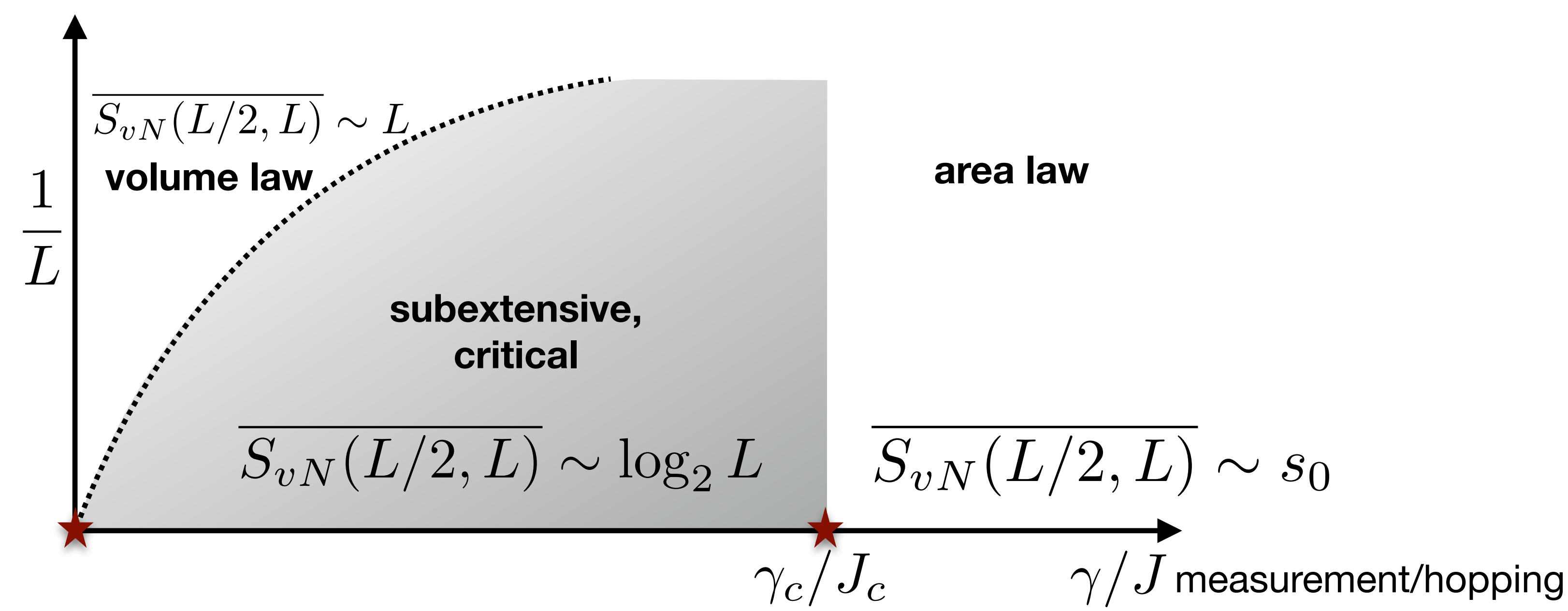
- examples:
  - von Neumann entropy $\overline{S_{vN}(l,L)} = \overline{\langle \log(\rho_A) \rangle}$ arbitrarily high power of state projector
  - correlation function $\overline{\langle \hat{n}_i \rangle \langle \hat{n}_j \rangle}$ quadratic in state projector

Trajectory Ensemble Phase Diagram

- weak monitoring
- strong monitoring



➡ extended subextensive, critical phase at intermediate monitoring

no volume to area law transition: Cao, Tilloy, DeLuca SciPost (2019)

critical phase but no transition for non-unitary circuits: Chen, Li, Fisher, Lucas PRR (2020)

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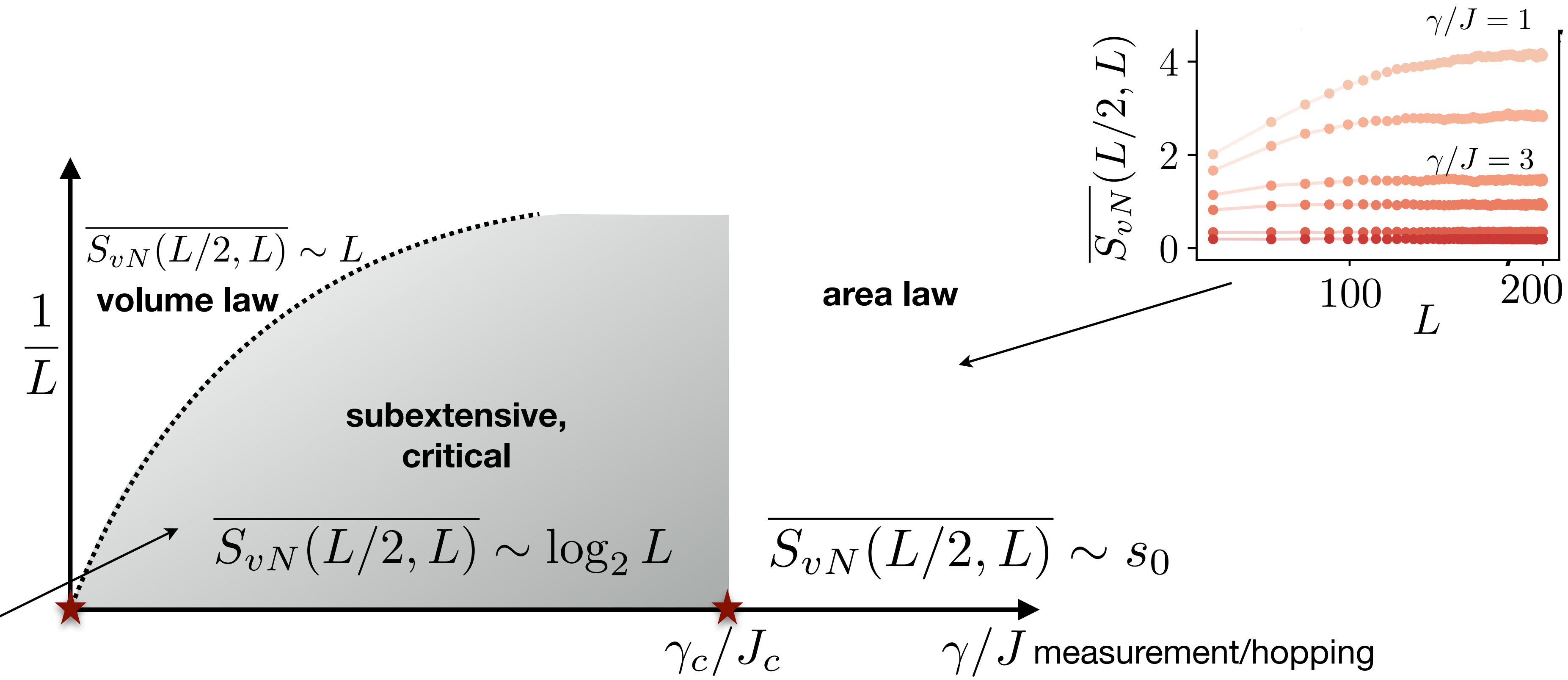

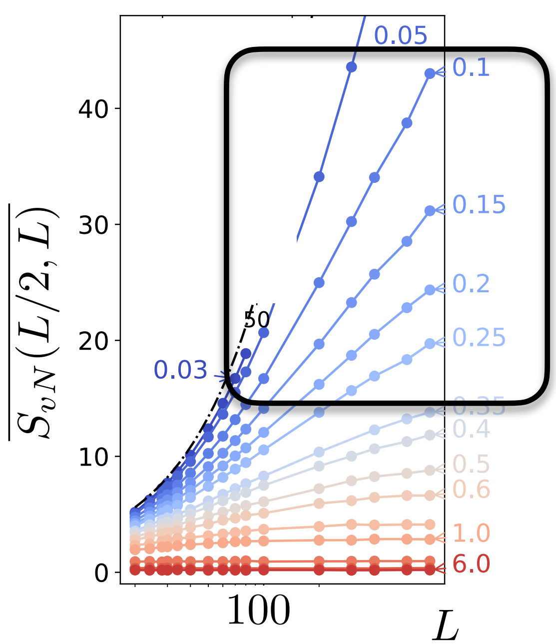

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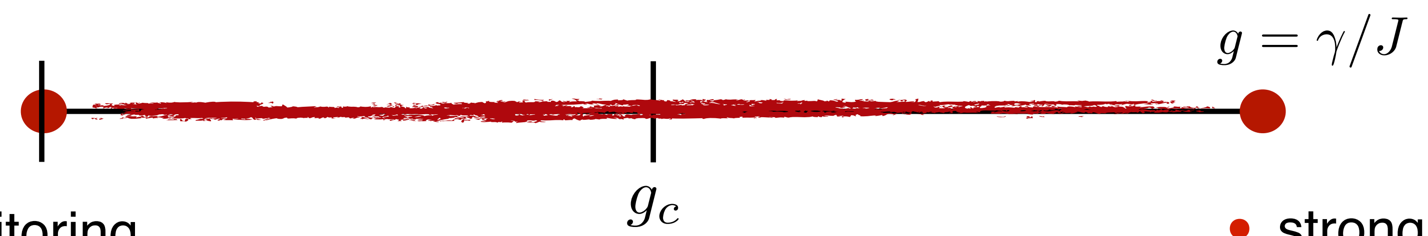
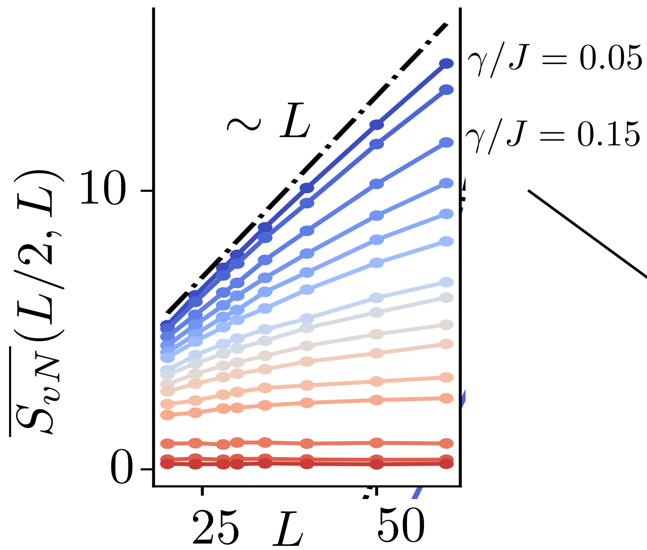
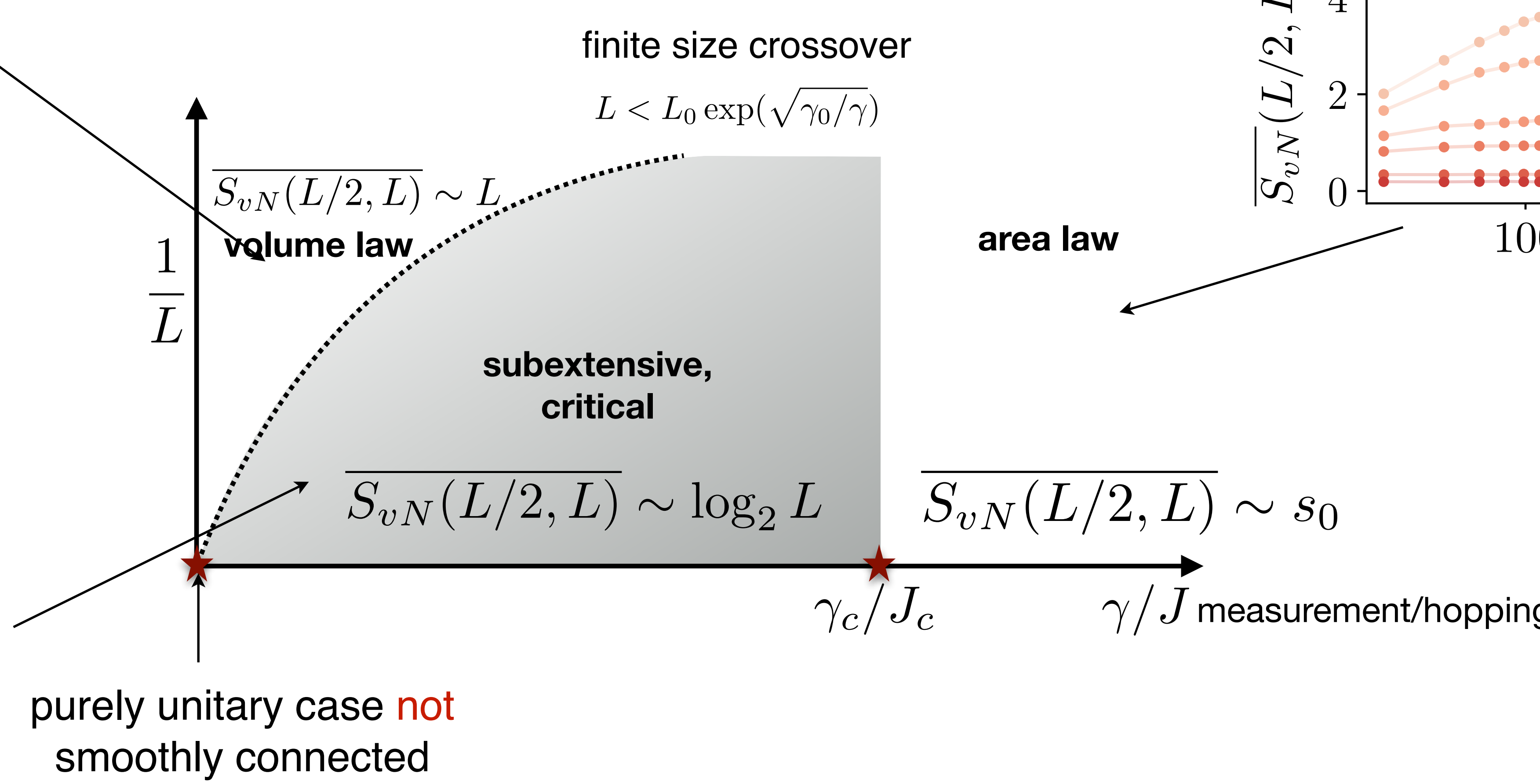
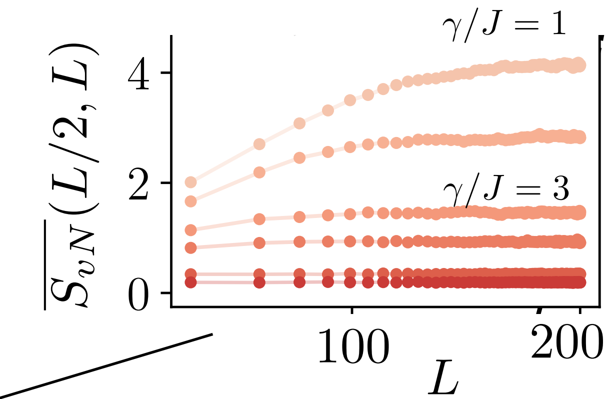
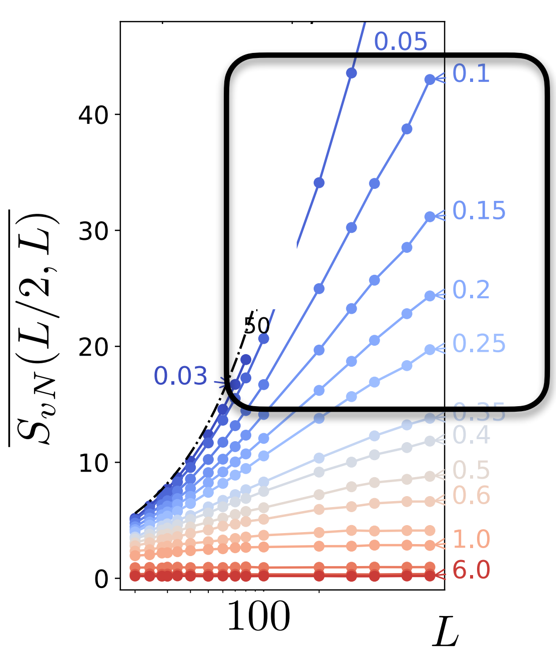


- weak monitoring
- strong monitoring









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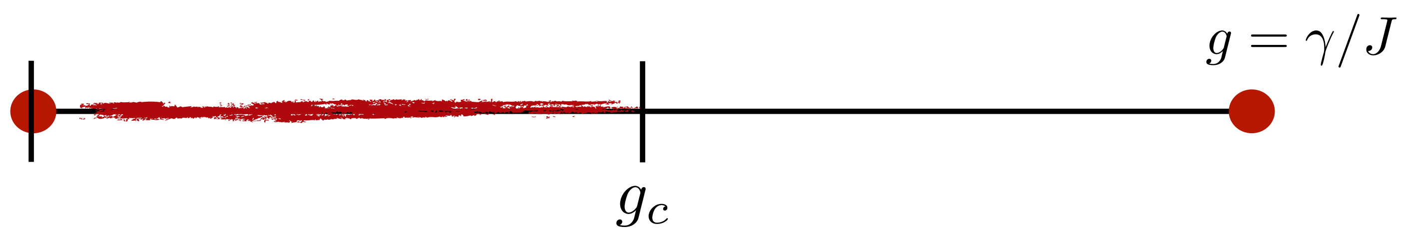
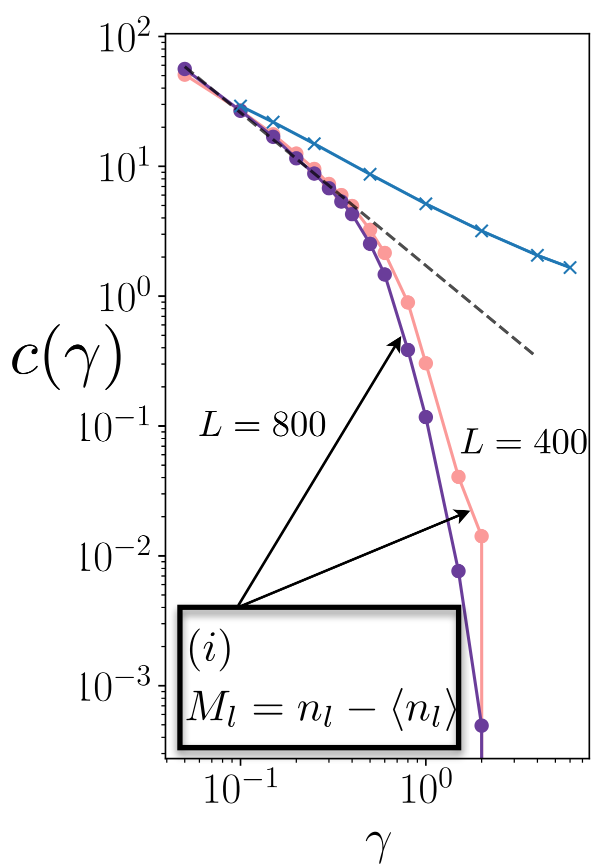
Characterizing the Weak Monitoring Phase & Phase Transition



- effective central charge $c(\gamma)$

$$\overline{S_{vN}(l,L)} = \frac{c(\gamma)}{3}\log_2\left[\frac{L}{\pi}\sin\left(\frac{\pi l}{L}\right)\right] + s(\gamma)$$



parameter dependent c

random systems: Cardy Jacobsen PRL (1997); Refael, Moore PRL (2004)

➡ sudden jump reminiscent of BKT

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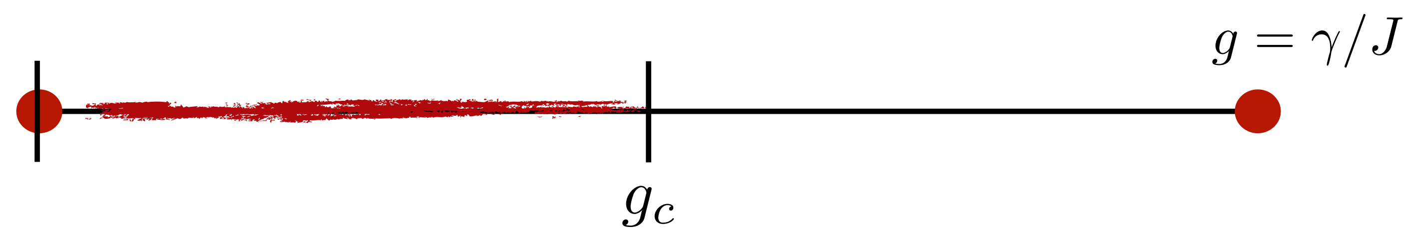

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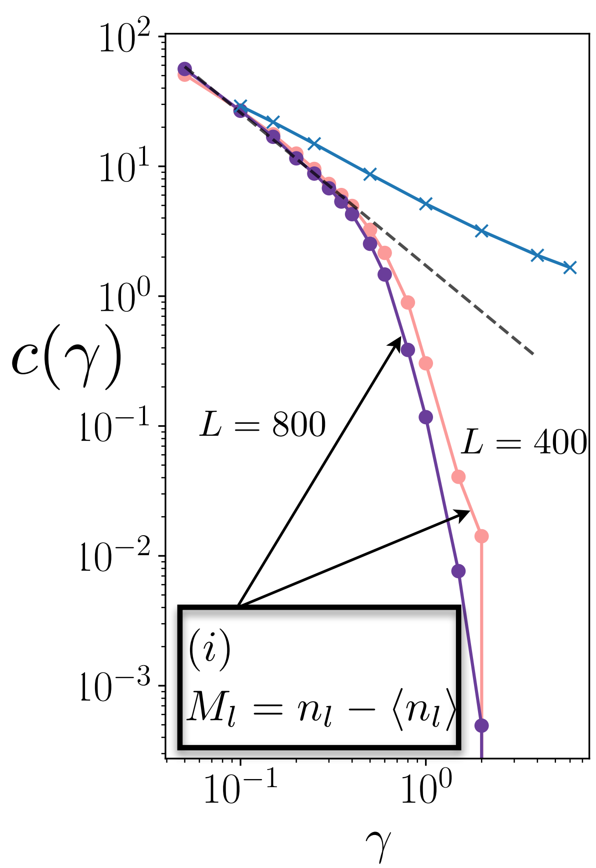

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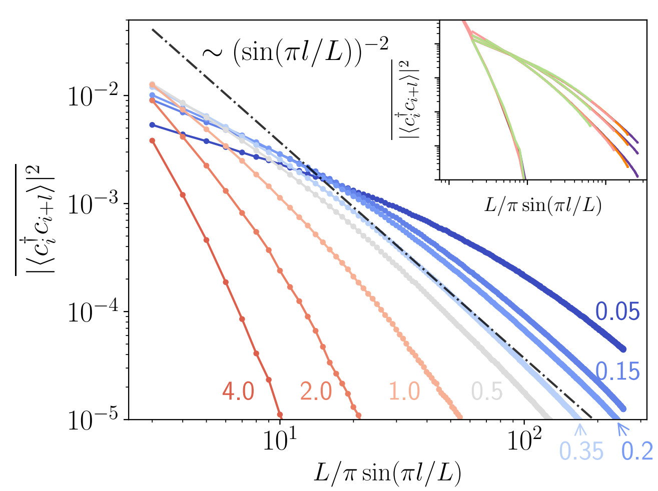
- extended criticality: Connected correlation function

$$C_{i,i+l} = \overline{\langle \hat{n}_i \rangle \langle \hat{n}_{i+l} \rangle} - \overline{\langle \hat{n}_i \hat{n}_{i+l} \rangle}$$

$$\sim l^{-2}$$



$$C_{i,i+l} \sim \begin{cases} 0 & \text{for } H = 0 \\ \exp(-l/\xi) & \text{for } \gamma \gg J \\ l^{-2} & \text{for } \gamma \ll J \\ l^{-1} & \text{for } \gamma = 0 \end{cases}$$

➡ correlation functions equally characterize the transition

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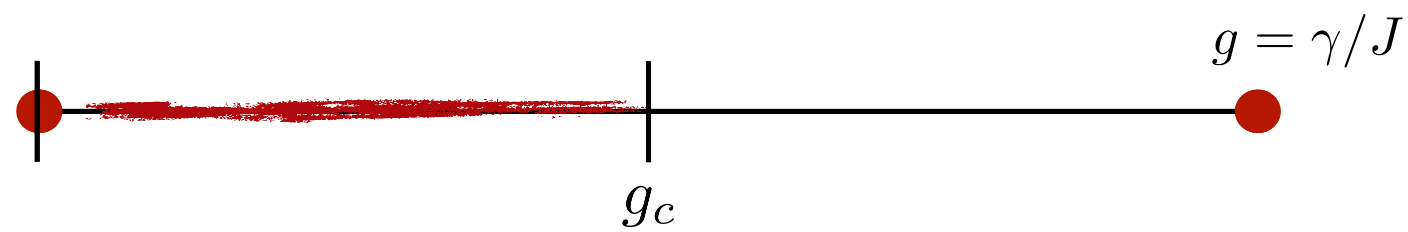

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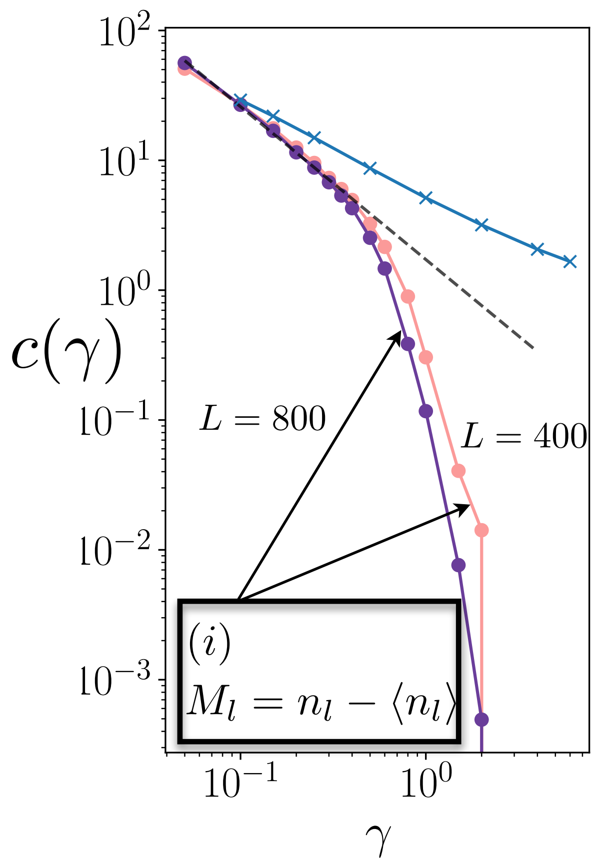

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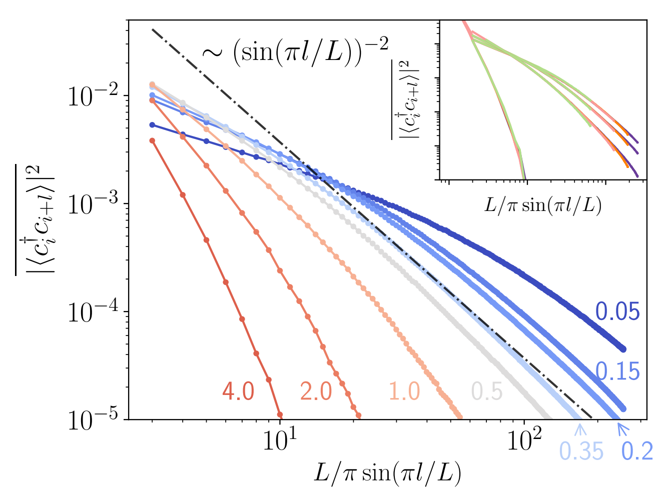

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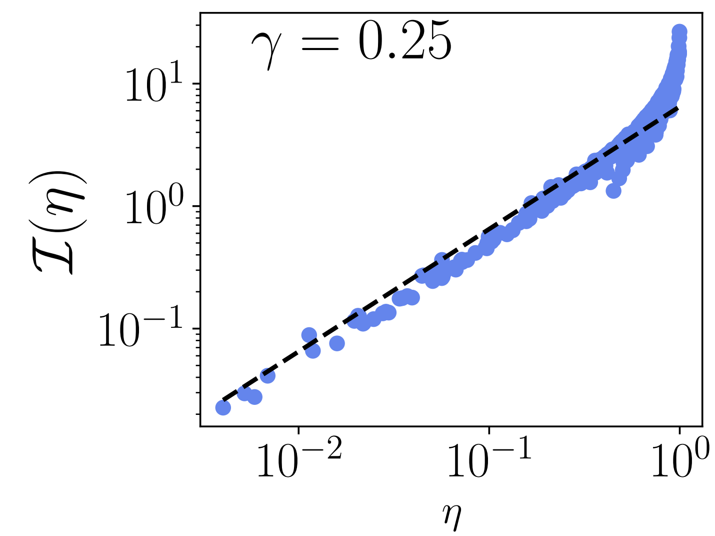
- Mutual information

$$\mathcal{I}(A,B) = \overline{S_{vN}(A)} + \overline{S_{vN}(A)} - \overline{S_{vN}(A \cup B)}$$

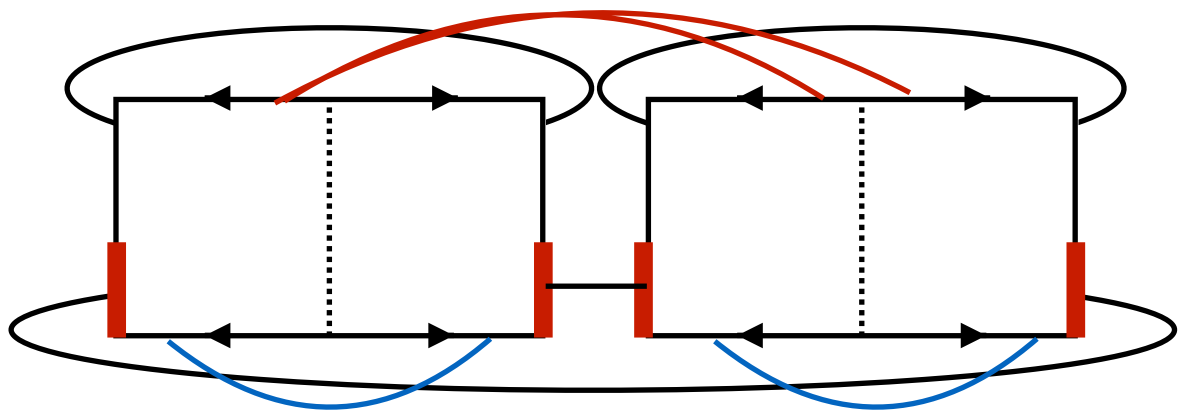


conformally invariant critical point: Nahum et al. PRX (2019); Li Chen Fisher PRB (2019); Jian et al. PRB (2020);

➡ emergent conformality

➡ further: measurement protocol dependence, trajectory entanglement distribution as probe of transition…

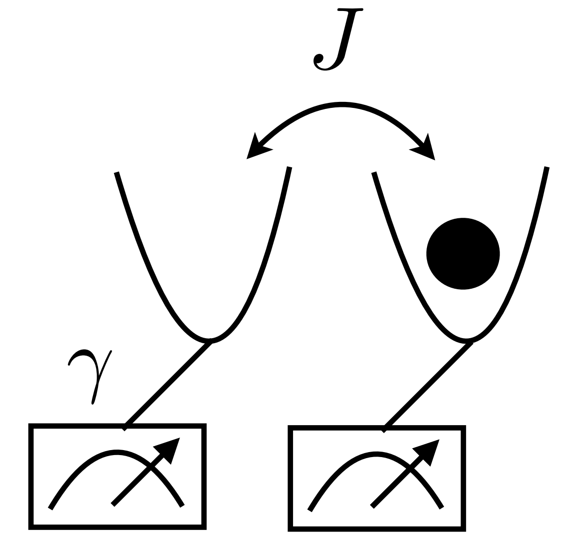
Effective Replica Field Theory
for
Measurement Induced Phase Transitions



M. Buchhold, Y. Minoguchi, A. Altland, SD, arxiv:2102.08381, PRX to appear

microphysics → macrophysics

Pinning picture: Toy model



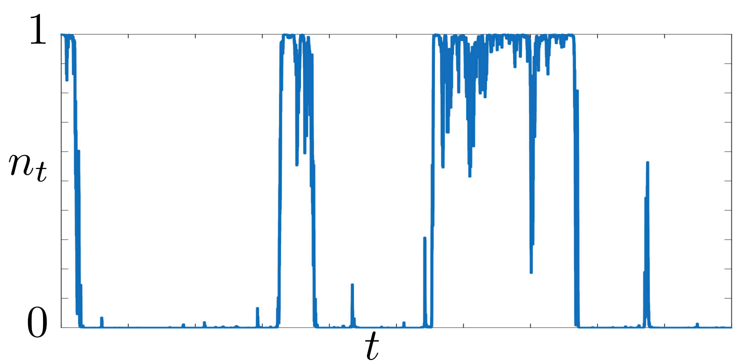
- toy model: trajectory evolution of single fermion on two sites

$$|\psi_{t+dt}\rangle = |\psi_t\rangle - idt\hat{H}_{\text{eff}}|\psi_t\rangle + \sum_{l=1}^{2} dW_l \left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)|\psi_t\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hat{K} \qquad \hat{H} = -J\left(c_1^\dagger c_2 + h.c.\right) \qquad \hat{K} = \frac{\gamma}{2}\sum_{l=1}^{2}\left(\hat{n}_l - \langle \hat{n}_l \rangle_t\right)^2$$

➡ H=0: collapse into dark state at long times $\hat{n}_l|\psi_t\rangle = \langle \hat{n}_l\rangle|\psi_t\rangle \Longrightarrow n_l = 0, 1$

- strong monitoring $J/\gamma \ll 1$



➡ pinning to measurement eigenstate

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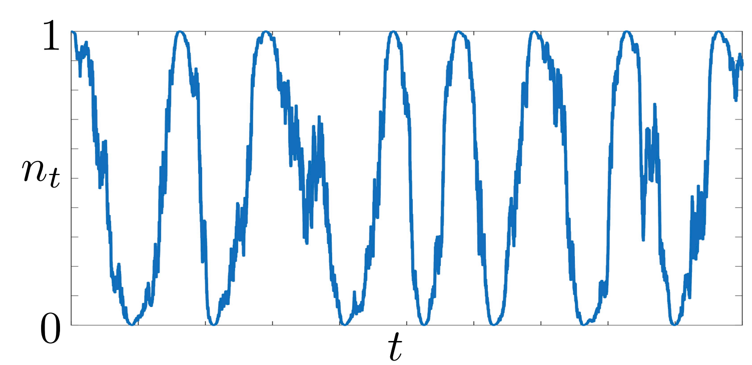

➡ vanishing time spent in eigenstate

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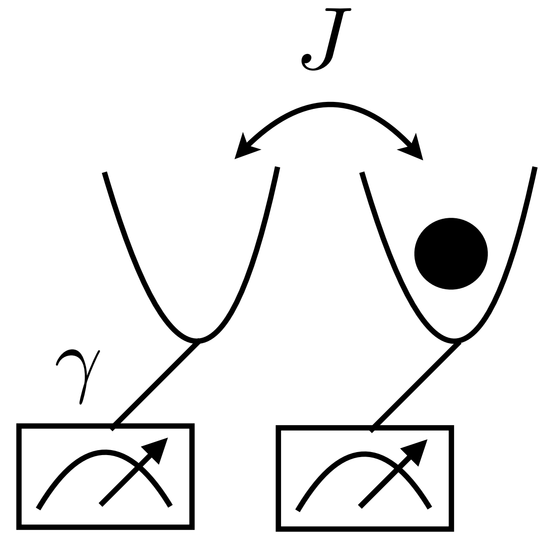

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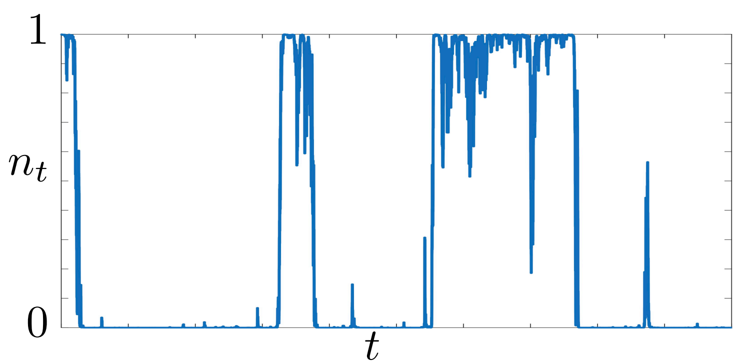

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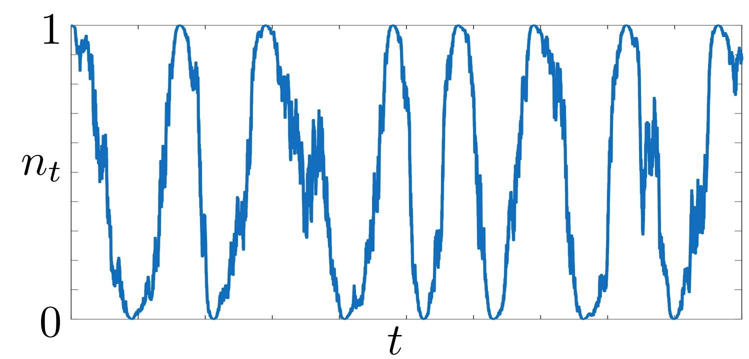
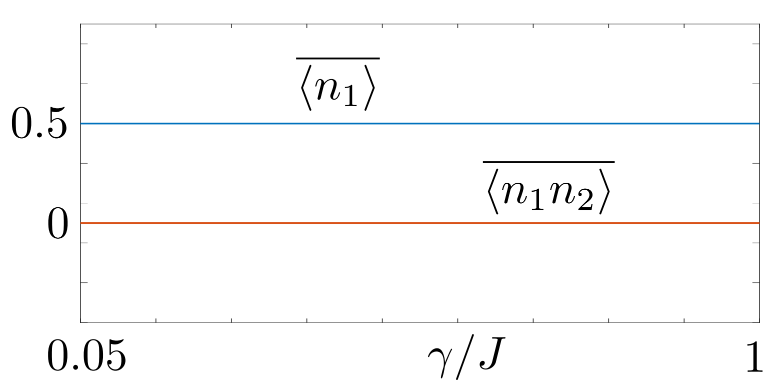
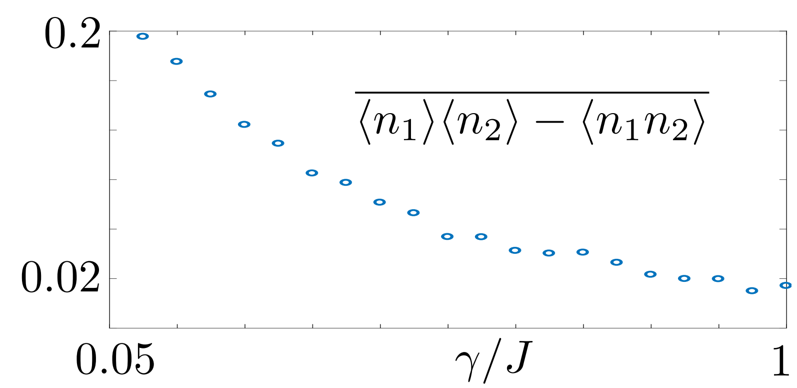


➡ vanishing time spent in eigenstate

- invisible in linear averages



- seen in averaged trajectory covariance matrix



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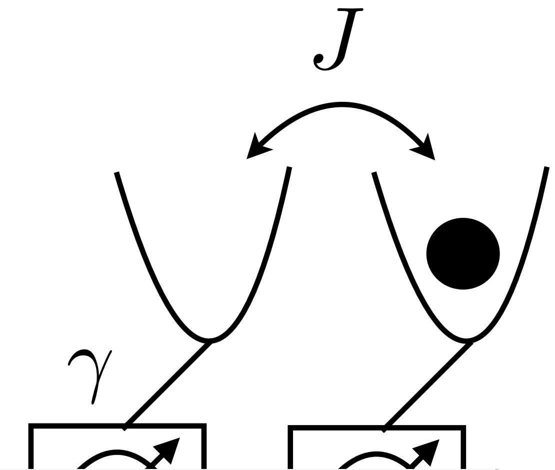

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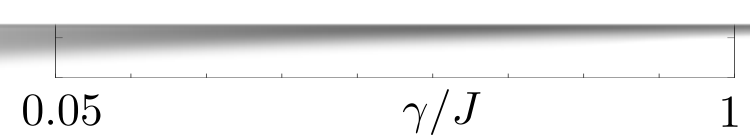
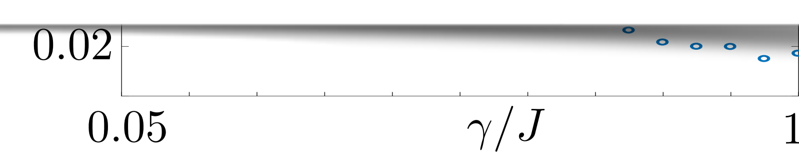
guiding picture and practical approach:

- thermodynamic limit: pinning quantum phase transition may happen at sharply defined point
  - ➡ Minimal continuum model in (1+1) dimensions)
- signalled in state dependent ‘observable’, like the covariance matrix
  - ➡ Replica construction

main insight:

➡ Replica degrees of freedom host non-Hermitian Sine-Gordon model => Pinning transition in BKT universality class





Continuum (1+1) dimensional Model

- preface: model obtains from naive continuum limit and bosonization of lattice fermion model

fermionic variant $\longrightarrow$ bosonized variant

- Hamiltonian: massless Dirac fermions $\hat{\Psi}_x = (\hat{\psi}_{R,x}, \hat{\psi}_{L,x})^T$

Luttinger liquid

$$\hat{H} = iv \int_x \hat{\Psi}_x^\dagger \sigma_z \partial_x \hat{\Psi}_x \longrightarrow \hat{H} = \frac{v}{2\pi} \int_x [\underbrace{(\partial_x \hat{\theta}_x)^2}_{\text{phase}} + \underbrace{(\partial_x \hat{\phi}_x)^2}_{\text{density}}]$$

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- measurement operators: current and vertex operators

rate γ_1 : $\hat{O}_{1,x} = \Psi_x^\dagger \Psi_x = \hat{J}_x^{(0)} \longrightarrow \hat{O}_{1,x} = -\frac{1}{\pi}\partial_x \hat{\phi}_x$ linear gapless

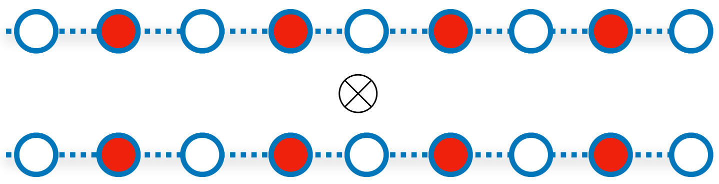
rate γ_2 : $\hat{O}_{2,x} = \Psi_x^\dagger \sigma_x \Psi_x \longrightarrow \hat{O}_{2,x} = \underbrace{m}_{\mathcal{O}(1)} \cos(2\hat{\phi}_x)$ nonlinear

common eigenstates: $\hat{\phi}_x |\Psi_D\rangle = \phi_x |\Psi_D\rangle$

- stabilize product dark states: exactly local
- realize competition: do not commute with H (phase fluctuations)

Replica Approach (n=2)

- Access state-dependent observables, e.g. covariance matrix $C_{xy} = \overline{\langle \hat{n}_x \hat{n}_y \rangle} - \overline{\langle \hat{n}_x \rangle \langle \hat{n}_y \rangle}$

- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$ 

- All quadratic-in-state observables encoded in

$$\rho^{2\mathrm{R}} = \overline{|\Psi_t\rangle\langle\Psi_t|}$$

➡ linear statistical average of replica density matrix

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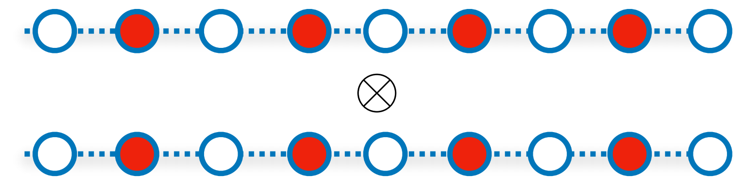

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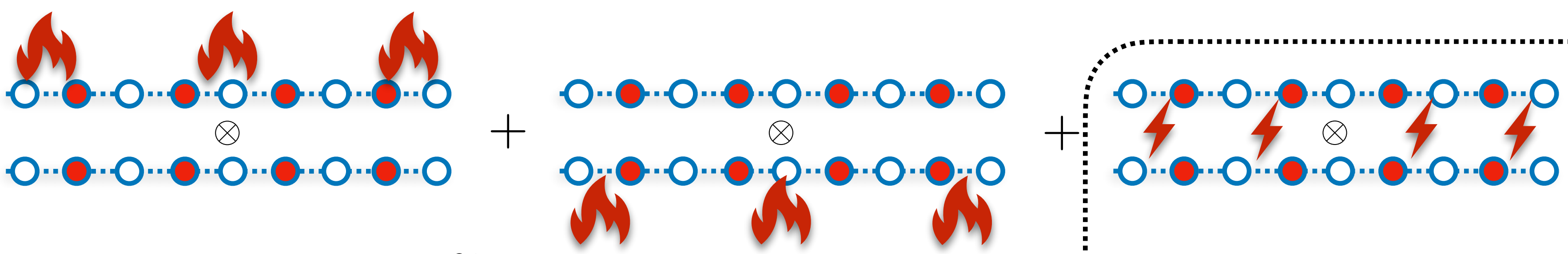
- Quantum master equation (truncate coupling to ρ^{3R})

$$\partial_t \rho^{2\mathrm{R}} =$$



$$i[\rho^{2R}, H^{(\alpha)}] - \frac{\gamma}{2}[\hat{M}_x^{(\alpha)}, [\hat{M}_x^{(\alpha)}, \rho^{2R}]]$$

individual heating Lindbladians

$$\gamma\{\hat{M}_x^{(1)}, \{\hat{M}_x^{(2)}, \rho^{2R}\}\}$$

replica coupling

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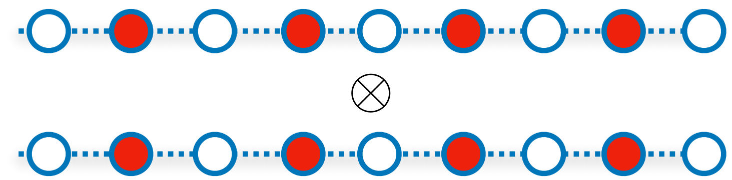
- Introduce replicas in Hilbert space $|\Psi_t\rangle = |\psi_t^{(1)}\rangle \otimes |\psi_t^{(2)}\rangle =$ 

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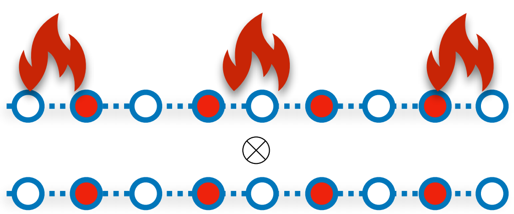
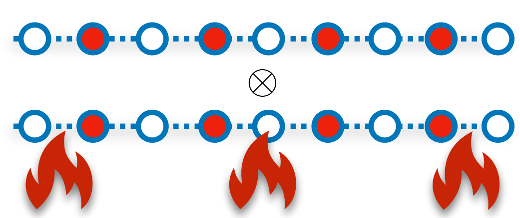
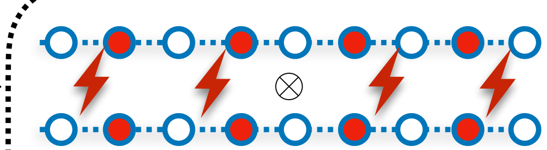
$\partial_t \rho^{2\mathrm{R}} =$  $+$  $+$ 

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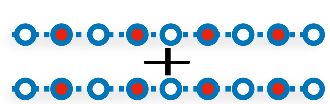
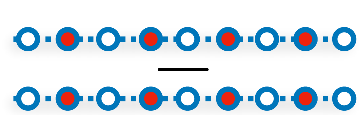
replica coupling

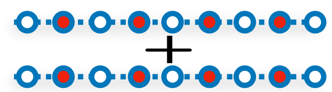
- New degrees of freedom

 : $\hat{\phi}^{(a)} = \hat{\phi}^{(1)} + \hat{\phi}^{(2)}$ average coordinate

 : $\hat{\phi}^{(r)} = \hat{\phi}^{(1)} - \hat{\phi}^{(2)}$ replica fluctuations

Boson Replica Quantum Master Equation

- New degrees of freedom



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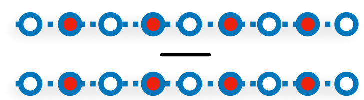

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➡ Master equation becomes separable (exact for Gaussian dynamics, useful more generally)

- Average coordinate: heating to infinite temperature

$$\partial_t \rho^{(a)} = i[\rho^{(a)}, H^{(a)}] + \frac{2\gamma}{\pi}\sum_l \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right) \rho^{(a)} \left(\partial_x \hat{\phi}^{(a)} - \overline{\langle \partial_x \hat{\phi}^{(a)} \rangle}\right) \longleftarrow \text{only jump term!}$$

- Relative coordinate: cooling/damping into dark state

$$\partial_t \rho^{(r)} = i[\rho^{(r)}, H^{(r)}] - \frac{\gamma}{\pi}\sum_l \left\{ (\partial_x \hat{\phi}^{(r)})^2, \rho^{(r)} \right\} \longleftarrow \text{no jump term!}$$

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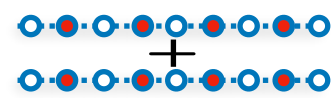

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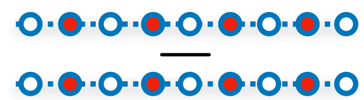

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⟵ no jump term!

- Non-Hermitian Schrödinger equation for relative coordinate

$$\partial_t |\psi_t^{(r)}\rangle = -i H_{\text{eff}} |\psi_t^{(r)}\rangle$$

➡ cooling into dark state

$$H_{\text{eff}} = \frac{\nu}{2\pi} \int_x (\partial_x \hat{\theta})^2 + (1 - i\eta^2)(\partial_x \hat{\phi})^2 \; - i\frac{\gamma m}{\pi} \int_x [1 - \cos(\sqrt{8}\hat{\phi}_x)]$$

depinning

pinning: effect of non-linearity

➡ non-Hermitian Sine-Gordon model

➡ extract physics in path integral approach

Phase diagram

➡ Sine-Gordon action with complex coefficients

Fendley, Saleur, Zamolodchikov,
International Journal of Modern Physics (1993)

$$\boxed{S = \int_{t,x} \left\{ \frac{K}{16\pi} \left[\frac{1}{\eta} (\partial_t \phi)^2 - \eta (\partial_x \phi)^2 \right] - i\lambda \cos(\phi) \right\}}$$

➡ RG flow: standard KT flow with complex K, λ

- flow modified at short distance

➡ shift of phase border

- standard BKT flow at long distance

➡ same long wavelength properties

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- resulting phase diagram:

$g = \gamma/J$

g_c

$$C_{xy} = \frac{C(\gamma)}{|x-y|^2}$$

➡ algebraic decay (generalized CFT)

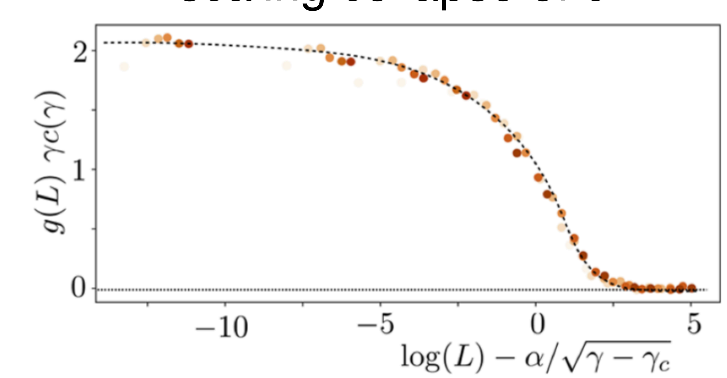
➡ depinned phase

$$C_{xy} \sim e^{-\frac{|x-y|}{\xi}} \qquad \xi^{-1} = \log(2\pi\gamma/v)$$

➡ exponential decay

➡ pinned phase

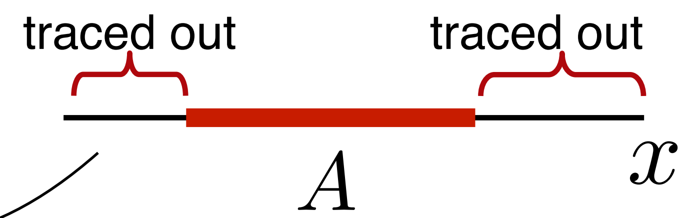
scaling collapse of c



➡ phase transition in the BKT universality class

➡ all in line with numerics for lattice fermions

Entanglement Entropies: n-Replica Keldysh approach



- Rényi entropy $S_n(L) = \frac{1}{1-n}\overline{\log Z_A(n,\{dW\})}, \; Z_A(n,\{dW\}) \equiv \mathrm{tr}[(\hat{\rho}_A^{(c)})^n]$

Calabrese, Cardy, JPA (2009)

- von Neumann entropy: $n \to 1$

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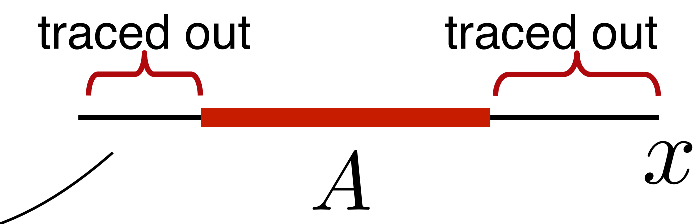

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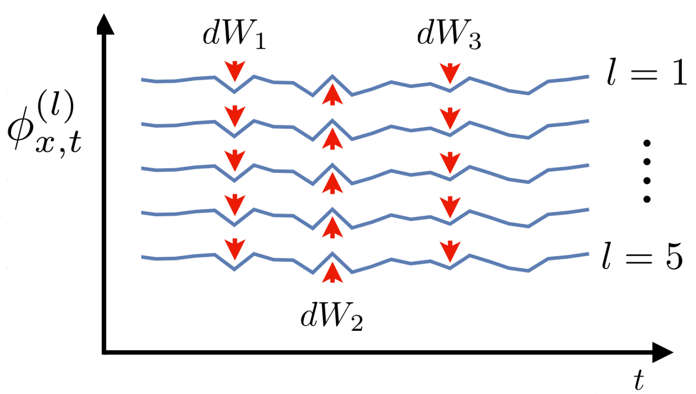
- approach: Keldysh replica field theory for n replicas (entropies via modified boundary conditions)

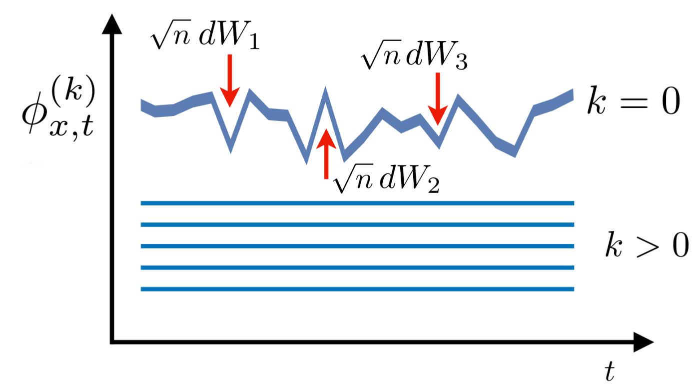
ground states: Casini, Fosco, Huerta, J. Stat. Mech. (2005)

- decoupling of center-of-mass and relative modes

replica space picture



reciprocal replica space picture



- 1 mode heats up (noisy)
- n-1 modes cool down (noiseless)

$\longrightarrow$

- noisy contribution A independent
- all A dependence in noiseless modes!

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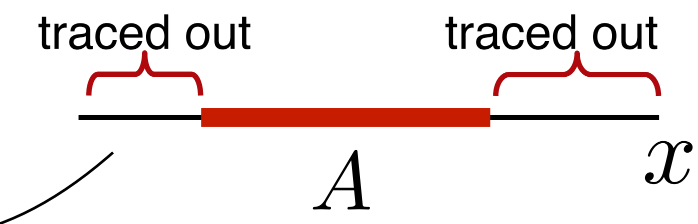

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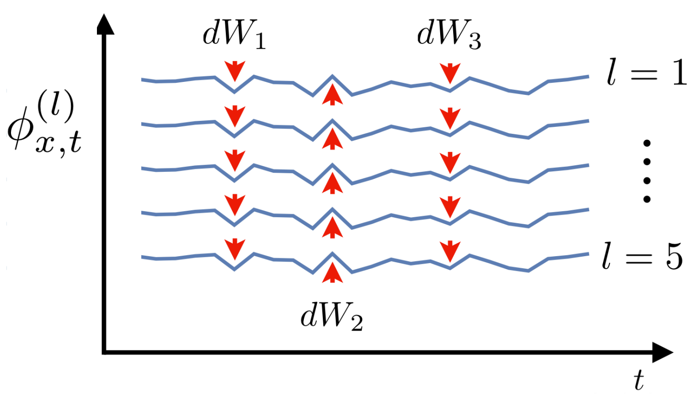

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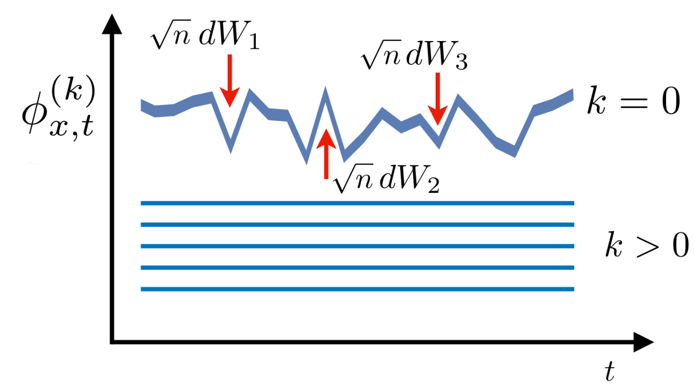

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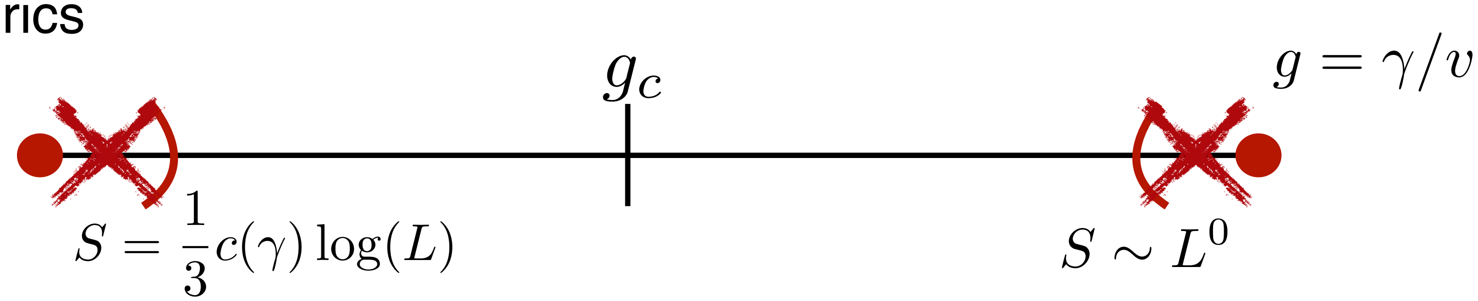
➡ Rényi entropy calculation as for ground states

➡ results in line with numerics

massless



massive

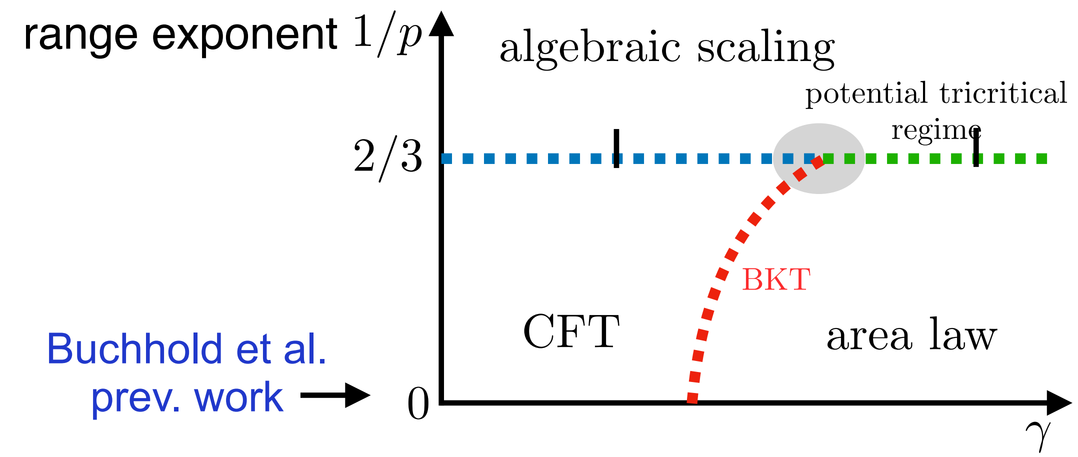
Other directions

T. Mueller, SD, M. Buchhold arxiv:2105.08076

see also Minato et al, arXiv:2104.09118; Block et al. arxiv:2104.13372

- different entropy scaling?

  ➡ new phase transition to algebraic subvolume scaling in long range models



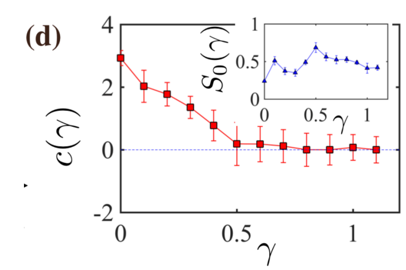
Buchhold et al. prev. work →

- different competition patterns?

  ➡ dual log-to-area transition:

  - critical phase stabilized by engineered dissipation
  - area law phase stabilized by Hamiltonian

Botzung, SD, Mueller arxiv 2106.10092

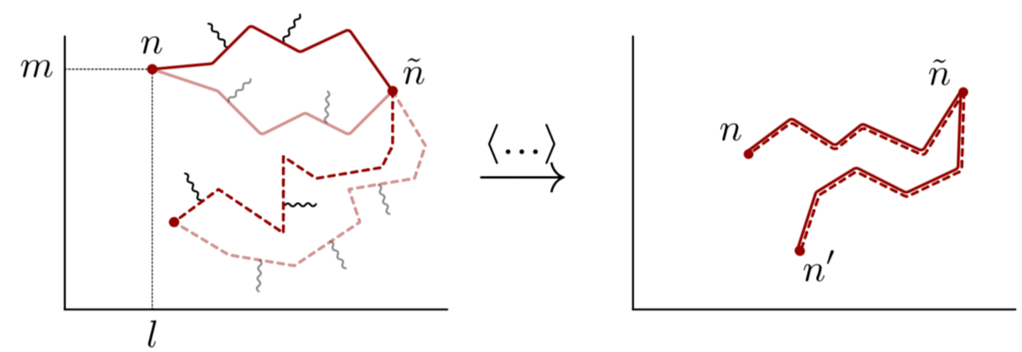


effective central charge

- measurement induced phase transitions with many-body chaotic Hamiltonians

  ➡ ordering principle due to strong decoherence

  with A. Altland, M. Buchhold, D. Huse, T. Micklitz

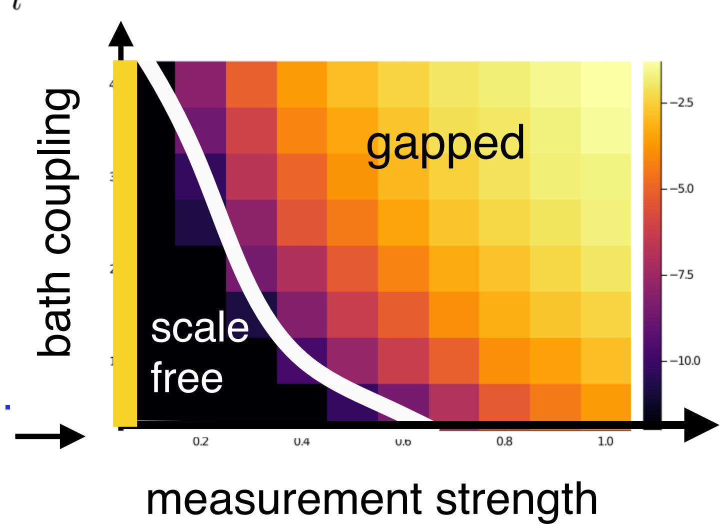


- robustness of measurement induced phases in the presence of decoherence

  ➡ gapless phase stable against e.g. imperfect measurements

  with M. Buchhold, B. Ladewig

Buchhold et al. prev. work →



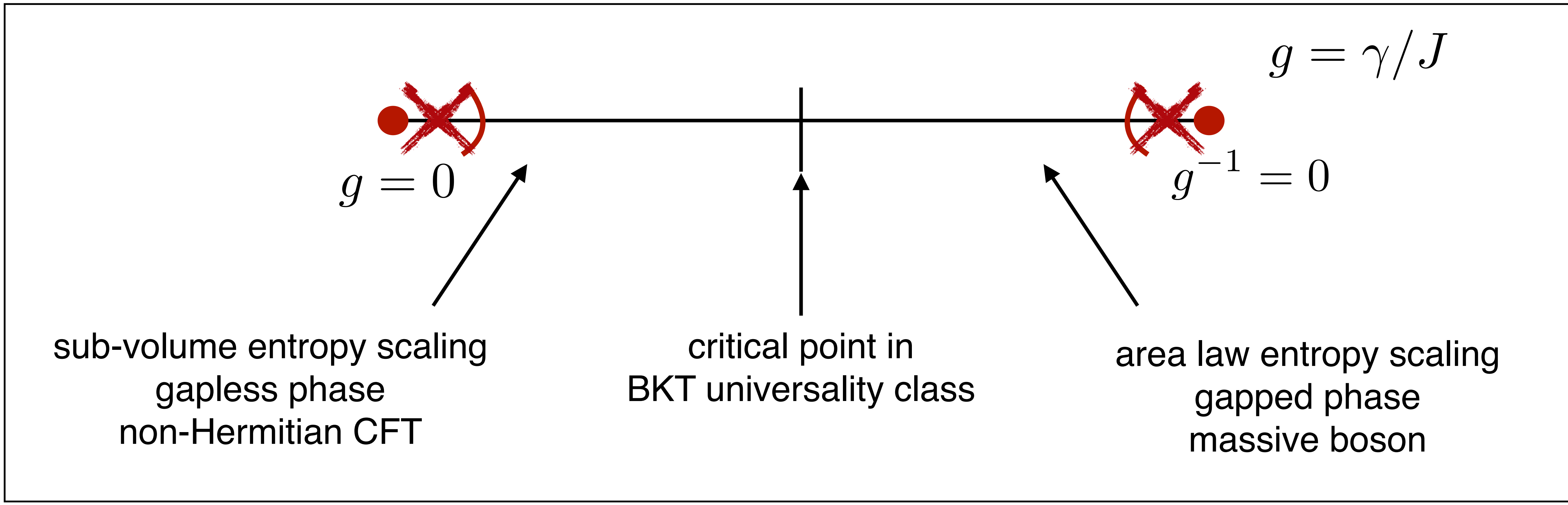
see also Y. Li, M.P.A. Fisher, arxiv: 2108.04274

talk by E. Altman

Conclusions & Outlook

O.Alberton, M.Buchhold, SD, PRL 126, 170602 (2021)
M. Buchhold, Y. Minoguchi, A. Altland, SD, arXiv:2102.08381, PRX in press

- monitored fermions: new type of measurement induced phase transition



- quantum phase transition in trajectory wavefunction witnessed by state-dependent ‘observables’ beyond entanglement entropy
- ‘hot’ and ‘cold’ modes as relevant degrees of freedom for the transition
- physical picture: transition induced by pinning/localization into measurement operator eigenstates

Directions:

- observability for many-body experiments?
- area-to-volume law transitions as incomplete decoupling of ‘hot’ and ‘cold’ modes?

integrability vs. non-integrability:
O. Lunt, A. Pal, PRR (2020)

- relation to no-click evolutions?

Biella, Schiro, arxiv:2011.11620 (2020); Gopalakrishnan Gullans, PRL (2021)
Turkeshi et al. arxiv:2103.09138 (2021)