review: V. Bulchandani, SG, E. Ilievski, JSTAT **2021**, 084001 (2021)

J. De Nardis, SG, R. Vasseur, B. Ware, PRL **127**, 057201 (2021)
J. De Nardis, SG, R. Vasseur, B. Ware, arxiv:2109.13251

jacopo de nardis

romain vasseur

brayden ware

superdiffusion, subdiffusion, integrability

Sarang Gopalakrishnan (Penn State)

quasiparticle picture of integrable systems

One-dimensional elastic scattering

constraints (momentum + k.e.)

If particles have equal mass:

$$v_1^f = v_2^i, v_2^f = v_1^i$$

Simply exchange velocities

Set of velocities $\{v\}$ preserved

Three-body collisions relax $\{v\}$ *unless* they factorize (Hubbard/Heisenberg)

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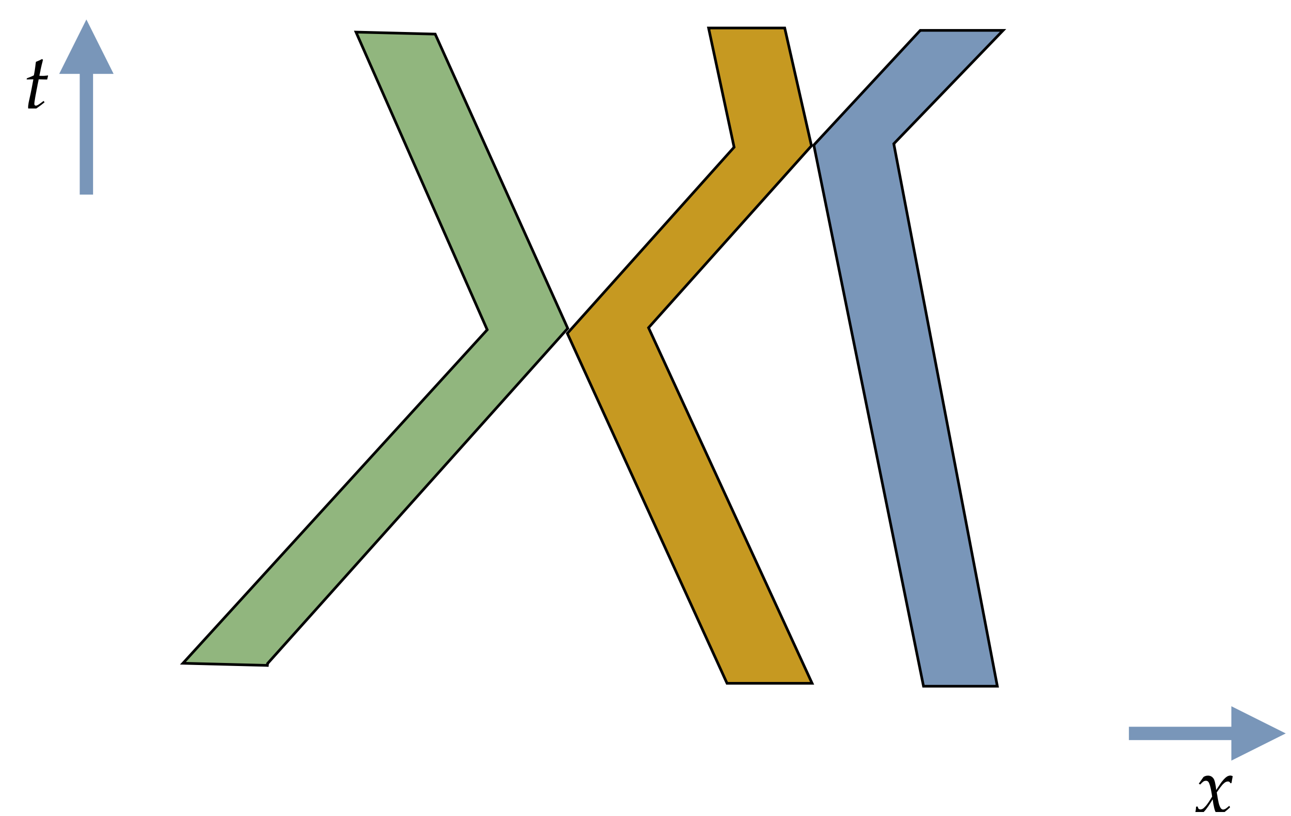
Quasiparticle picture

direct calculations of dynamics in integrable systems are hard

workaround: quasiparticle picture/ generalized hydrodynamics

[Castro-Alvaredo et al., Bertini et al. (2017), cf. Sachdev (1990s)]



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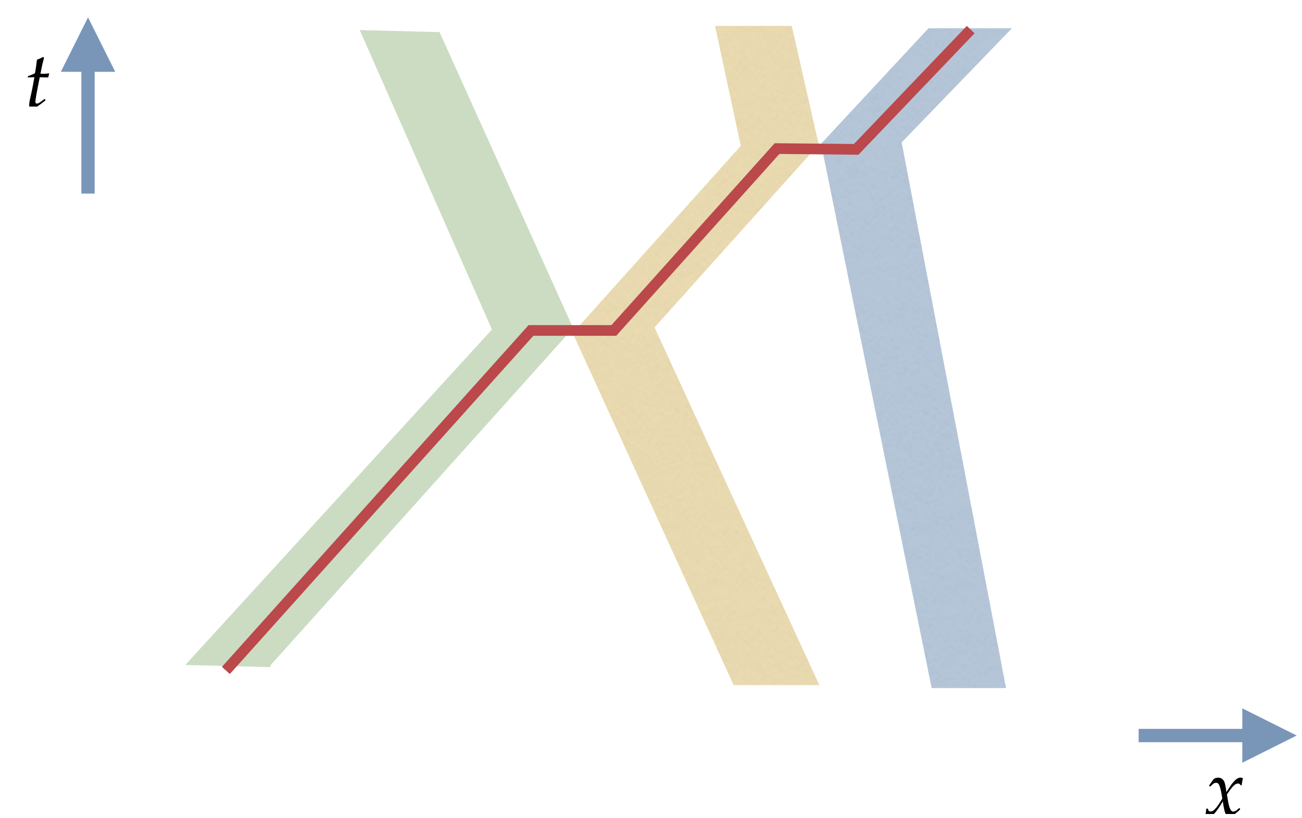
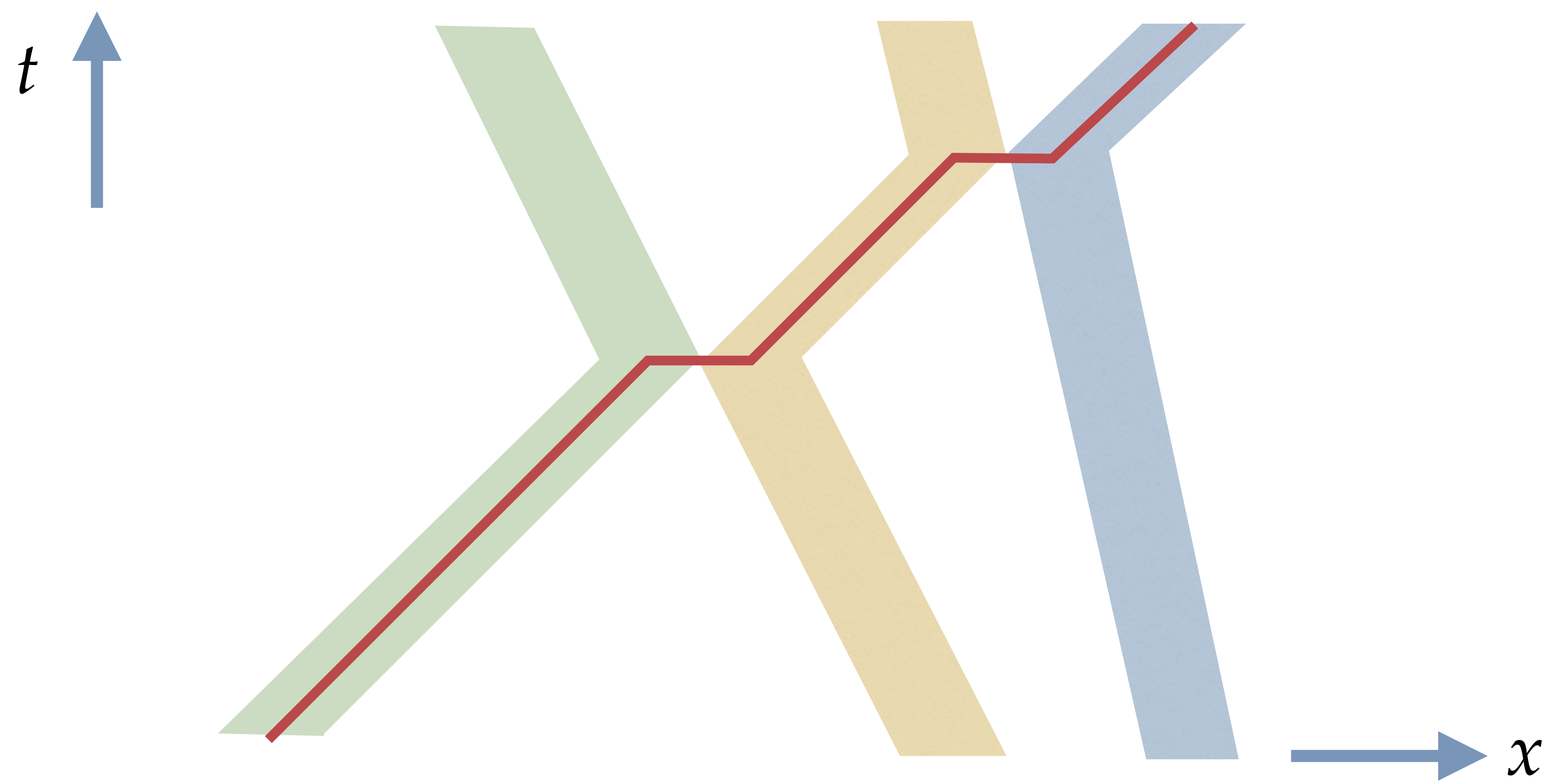


information spreads ballistically…

- Changing the velocity of one trolley modifies all downstream trajectories
- Quasiparticle picture captures entanglement growth [Alba, Calabrese, PNAS (2017)]
- In an initial thermal state, collisions give rise to diffusive broadening of operator fronts [SG, Huse, Khemani, Vasseur, PRB (2018)]



…but transport is more complicated

the case of the xxz spin chain as $T \to \infty$

$$H_{\mathrm{XXZ}} = \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z)$$

Zotos (1998)…Ljubotina et al. (2019)

the case of the xxz spin chain as $T \to \infty$

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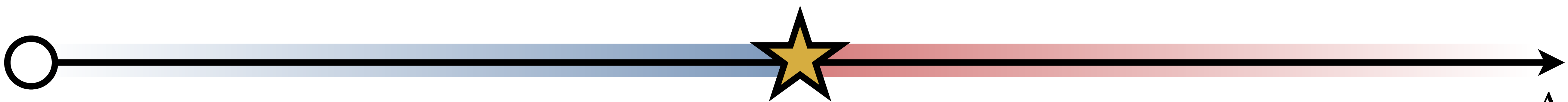
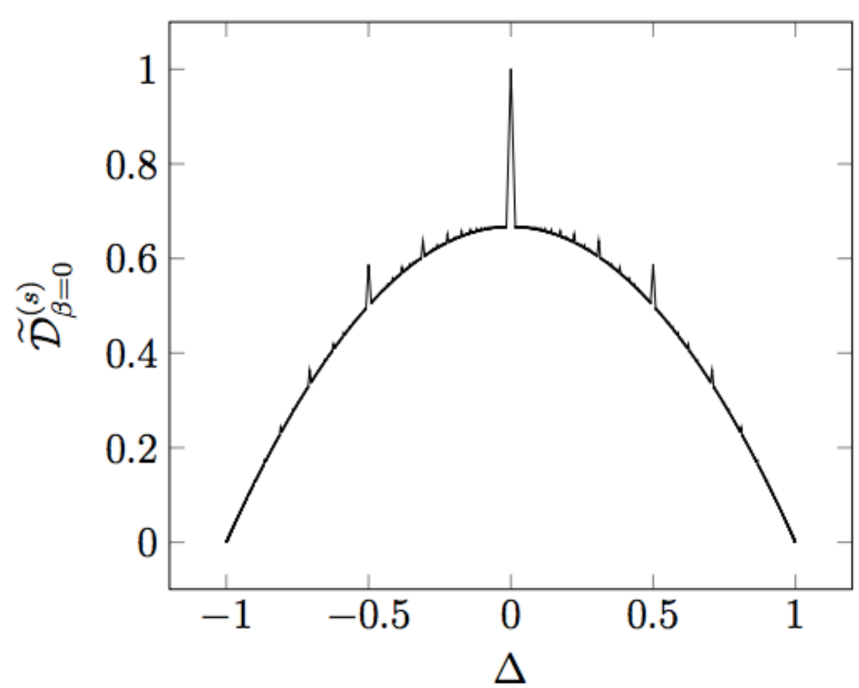


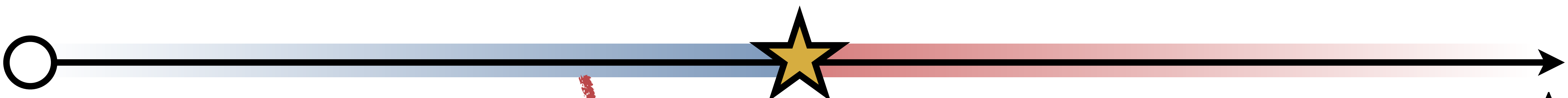
	free	*easy-plane*	*Heisenberg*	*easy-axis*
		ballistic spin transport	superdiffusion	diffusive spin transport
$\sigma_s(\omega)$:		$\mathcal{D}(\Delta)\delta(\omega)+??$	$\omega^{-1/3}$	$\sigma_0(\Delta)$



- Infinite-temperature dynamical *phase transition*
  - Only affects spin dynamics
  - Energy transport is always ballistic: energy current commutes with H
- Never mind the transition, what about the *phases*?

the case of the xxz spin chain as $T \to \infty$

$$H_{\mathrm{XXZ}} = \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z)$$



free | *easy-plane* | *Heisenberg* | *easy-axis* | Δ

	easy-plane	*Heisenberg*	*easy-axis*
	ballistic spin transport	superdiffusion	diffusive spin transport
$\sigma_s(\omega)$:	$\mathcal{D}(\Delta)\delta(\omega) + c\omega^{-1/2}$ *+ commensurability effects*	$\omega^{-1/3}$	$\sigma_0(\Delta)$

Agrawal, SG, Vasseur, Ware, PRB (2020)

Ilievski, De Nardis, Medenjak, Prosen, PRL (2018)
SG, Vasseur, PRL (2019)
De Nardis, Bernard, Doyon, SciPost (2019)
De Nardis, Karrasch, Medenjak, Ilievski, PRL (2019)
Bulchandani, PRB (2020)

*General principle: elementary excitations in integrable systems remain **stable** (but highly **renormalized**) in thermal states*

xxz away from integrability

$$H_{\mathrm{XXZ}} = \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z) + \gamma H'$$

	easy-plane	Heisenberg	easy-axis
	ballistic spin transport	superdiffusion	diffusive spin transport
$\sigma_s(\omega)$:	$\mathcal{D}(\Delta)\delta(\omega) + c\omega^{-1/2}$	$\omega^{-1/3}$	$\sigma_0(\Delta)$

???

$$\mathcal{D}\frac{\gamma}{\omega^2 + \gamma^2}$$

Friedman, Gopalakrishnan, Vasseur, PRB (2020)
review: Bastianello, De Luca, Vasseur, arXiv:2103.11997

studying integrability-breaking

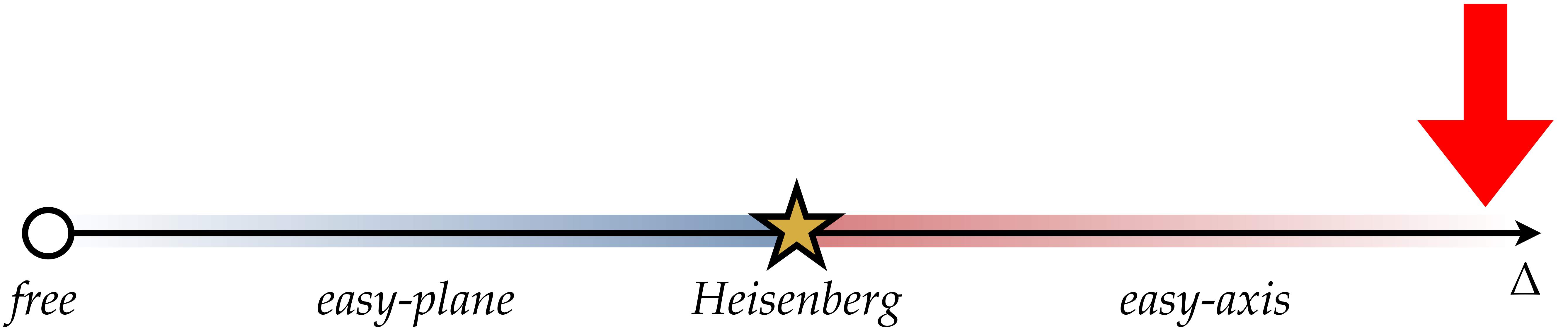
- How we break integrability:

$$H = H_0 + \sum_i \eta_i(t) O_i, \quad \langle \eta_i(t) \eta_j(0) \rangle = \gamma \delta_{ij} \delta(t)$$

- Why?
  - Noise-averaged dynamics is given by a quantum channel with dephasing
  - Operator entanglement growth stays under control at all times
  - Theoretically simple: less kinematics to worry about

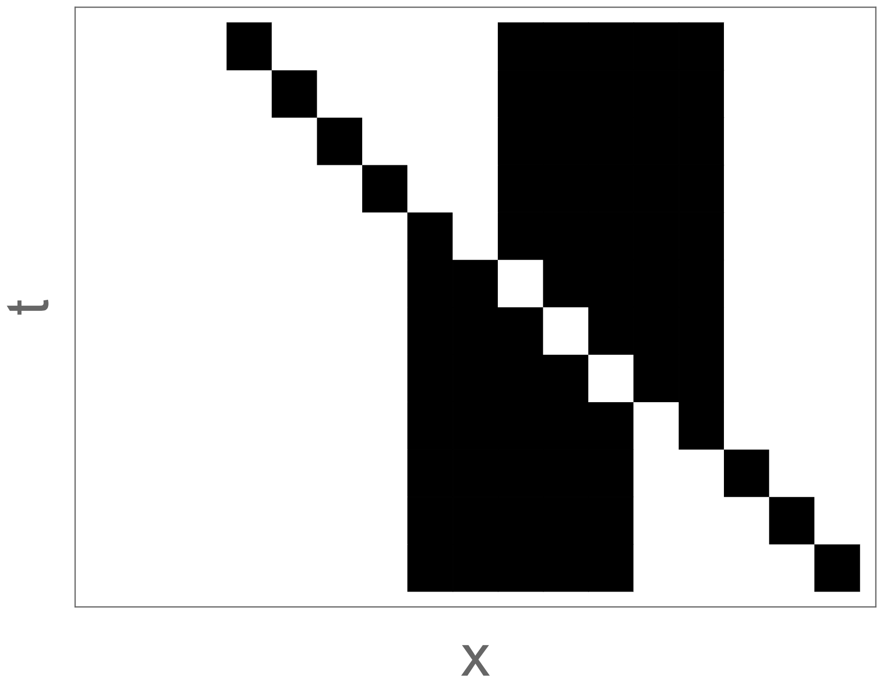
- Results consistent with nonintegrable Hamiltonian simulations with TEBD (but these become very expensive very fast)

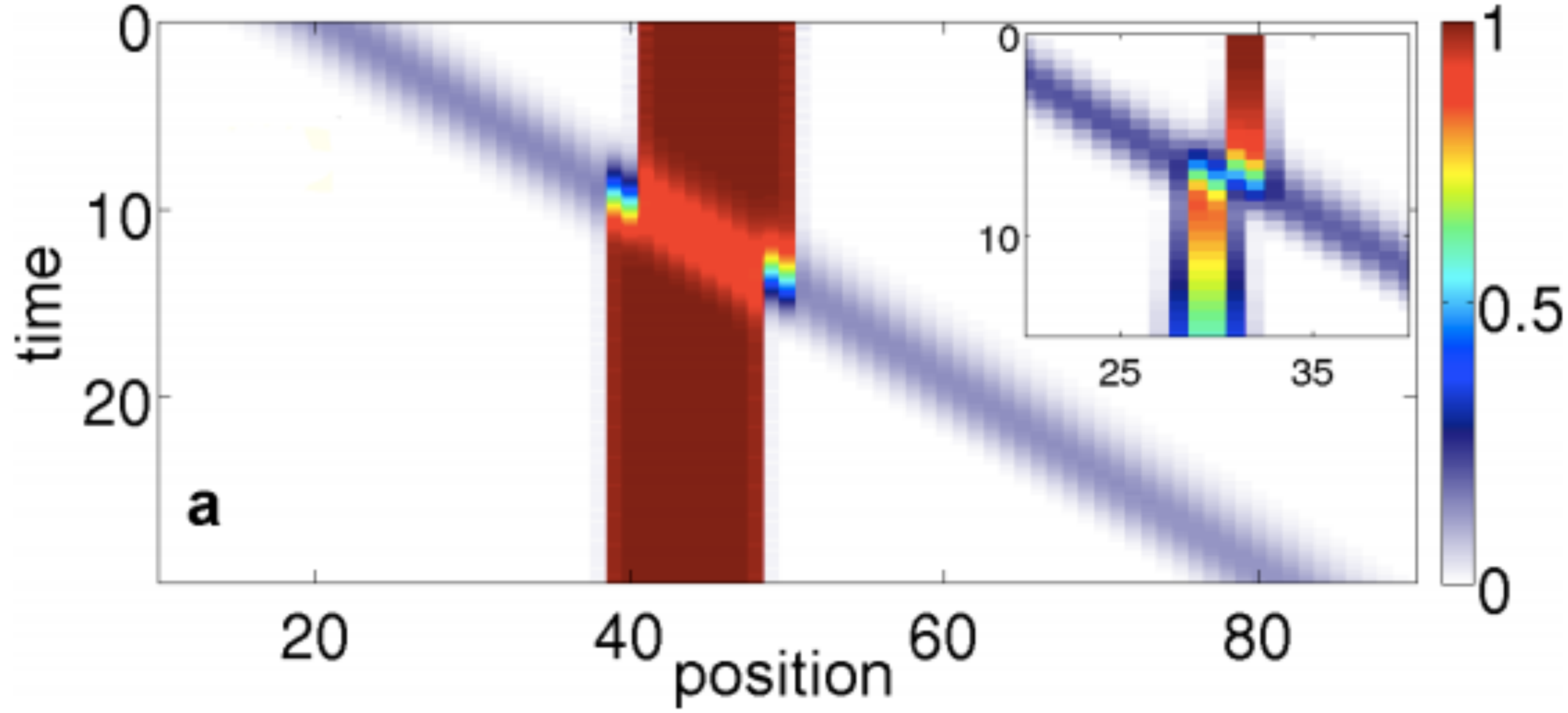
easy-axis phase



why is there no ballistic transport?

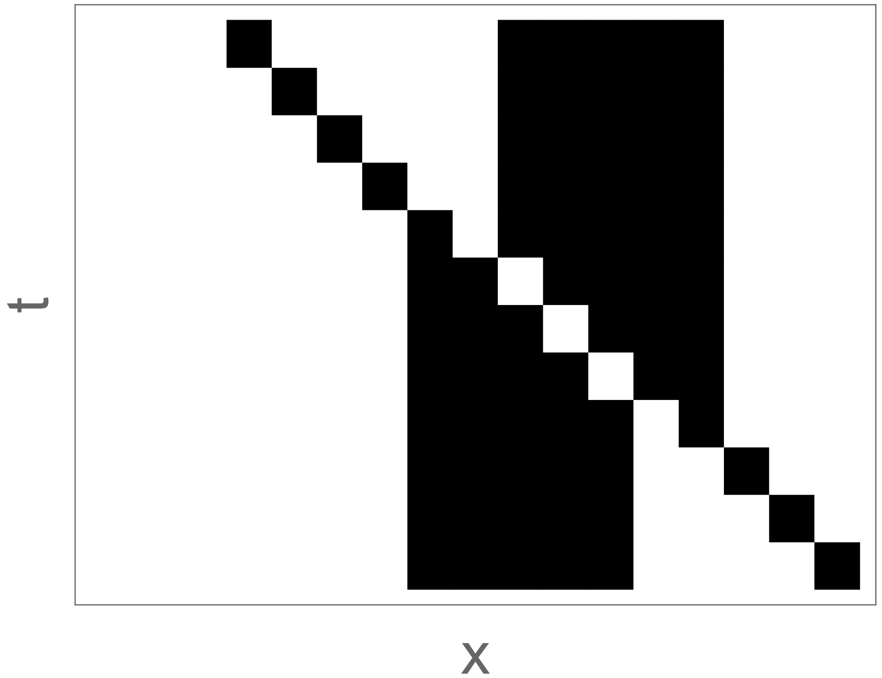
Ganahl et al., 2013



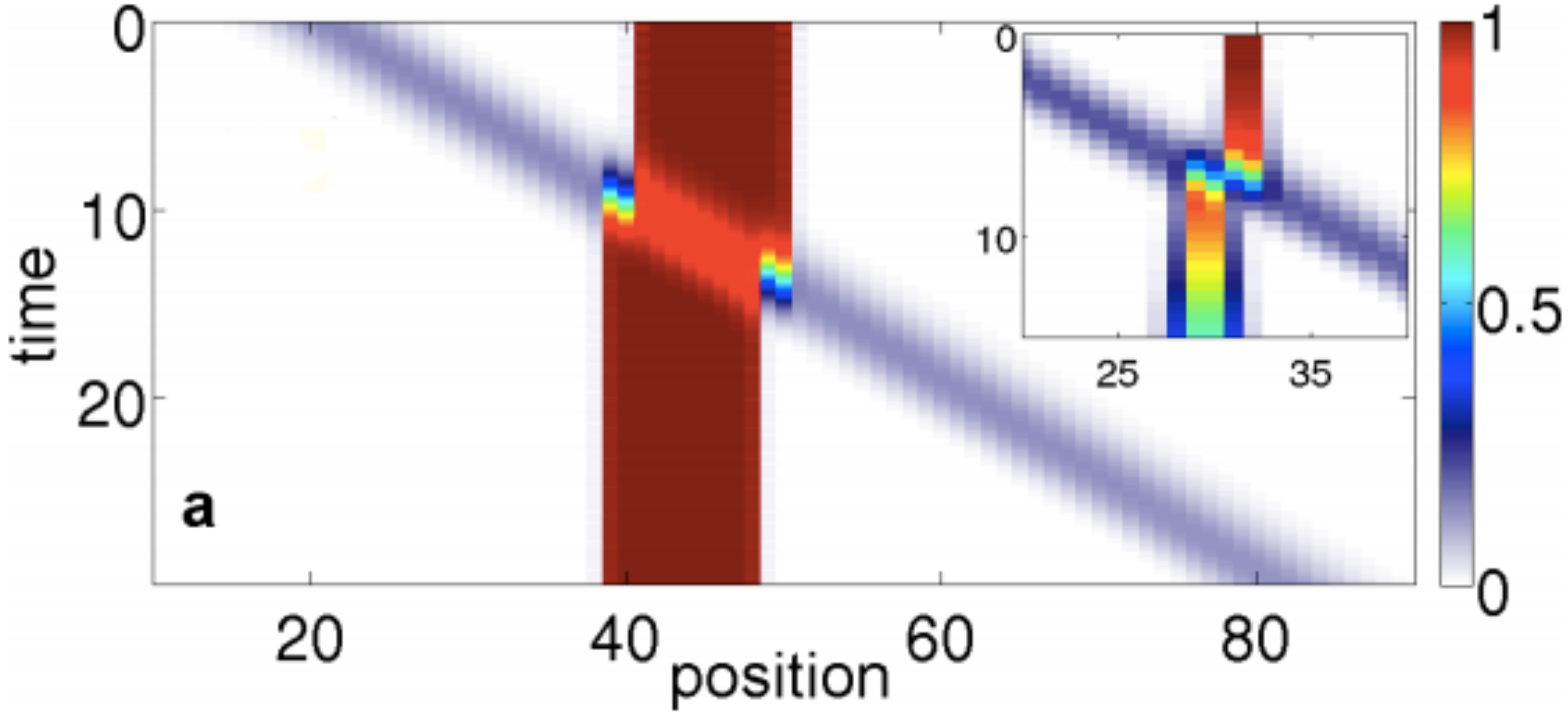


- Low-temperature state in the ferromagnet: domains of various sizes
- Because of U(1), domain can only move all at once
- Small domains can move, large domains are immobile (exponentially suppressed v)
- What happens when a magnon crosses a large domain?
  - **Diffusion.** Large domain shifts
  - **Dressing.** Magnon stripped of its spin (but only precisely at half filling)

intuitive argument for diffusion



Ganahl et al., 2013



- Magnon moves a distance vt in time t
- Dressed/screened magnetization of magnon over this scale:

$$m^{\mathrm{dr}} \sim 1/\sqrt{L} \sim 1/\sqrt{vt}$$

- Amount of charge transported [cf. Medenjak, Prosen, Karrasch, PRL (2017)]:

$$\langle \delta(m^{\mathrm{dr}} x)^2 \rangle \sim \frac{v^2 t^2}{vt} \sim vt$$

away from $\Delta \gg 1$

- Quasiparticles labeled by index s; also $\Delta = \cosh(\eta)$

$$\rho_s \sim \exp(-\mu s)/s^3, \quad v_s \sim \exp(-\eta s)/s, \quad m_s \sim \min(\mu s^2, s)$$

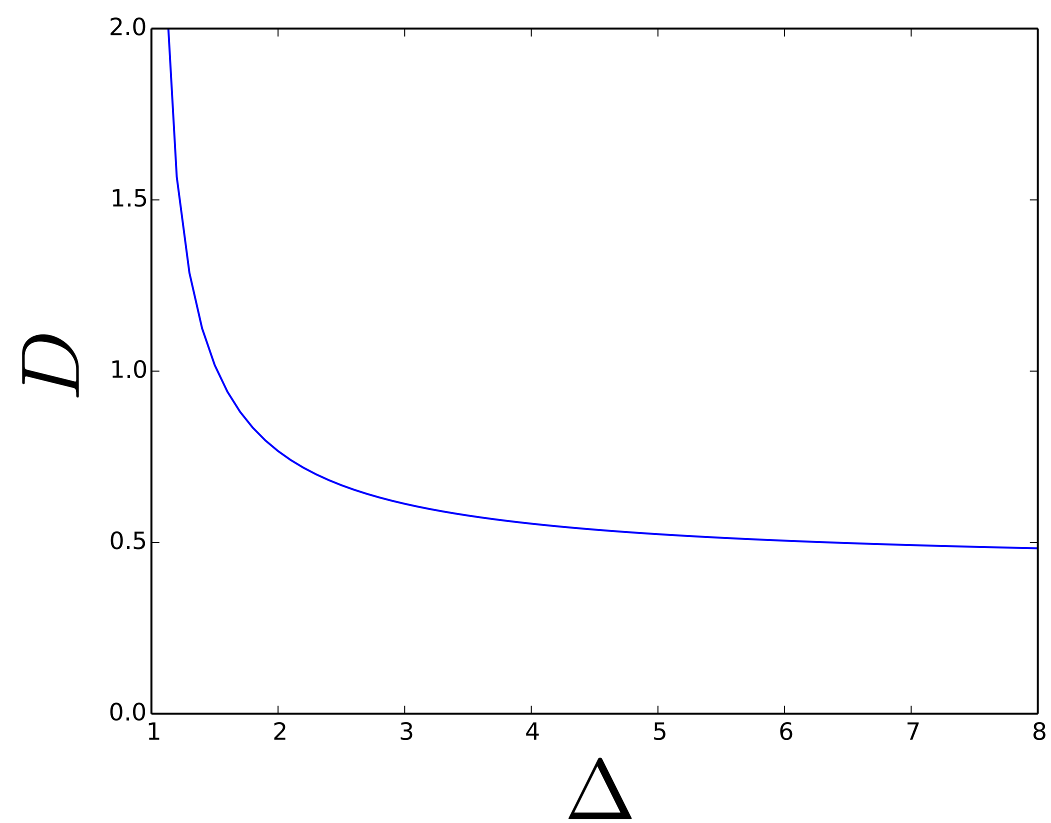
Ilievski, De Nardis, Medenjak, Prosen, PRL (2018)
Medenjak, Karrasch, Prosen, PRL (2017)

- Key idea:

$$\langle \delta(q_{\text{eff}} x)^2 \rangle \sim \sum\nolimits_s \rho_s (v_s t)^2 \langle m_s^2 \rangle_{\mu=1/\sqrt{v_s t}}$$

- Closed-form expression with prefactors:

$$D = \frac{2\sinh\eta}{9\pi} \sum_{s=1}^{\infty} (1+s) \left[\frac{s+2}{\sinh \eta s} - \frac{s}{\sinh \eta(s+2)} \right].$$



SG, Vasseur, PRL (2019)
De Nardis et al., PRL (2019)

away from integrability

- Naive guess 1: already diffusive, integrability-breaking should do nothing
- Naive guess 2: $vt \to \sqrt{Dt}$ so

$$\langle \delta (m^{\mathrm{dr}} x)^2 \rangle \sim \frac{v^2 t^2}{vt} \sim vt \quad \Longrightarrow \quad \langle \delta (m^{\mathrm{dr}} x)^2 \rangle \sim \sqrt{Dt}$$

$$x \sim t^{1/4}$$

subdiffusion!

De Nardis, SG, Vasseur, Ware, arxiv:2109.13251

away from integrability

- Naive guess 1: already diffusive, integrability-breaking should do nothing
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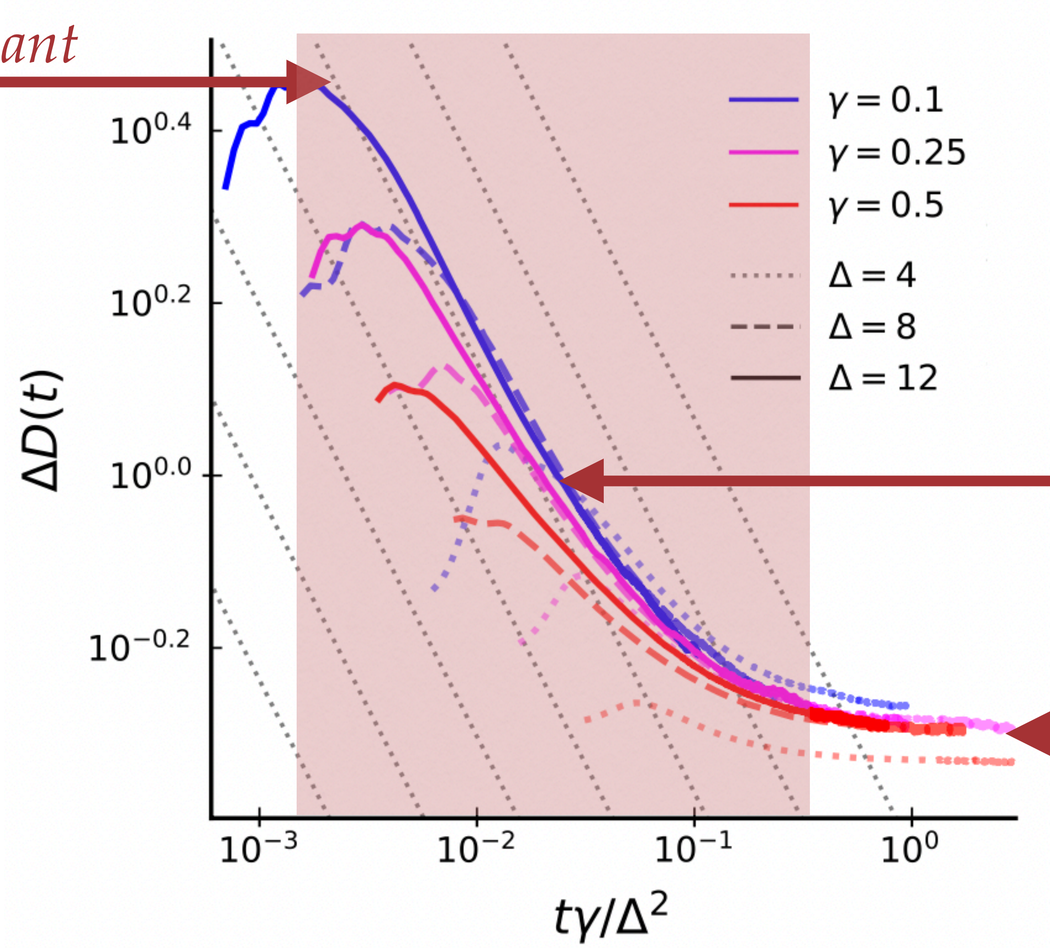
$$\langle \delta(m^{\mathrm{dr}} x)^2 \rangle \sim \frac{v^2 t^2}{vt} \sim vt \quad \Rightarrow \quad \langle \delta(m^{\mathrm{dr}} x)^2 \rangle \sim \sqrt{Dt}$$

subdiffusion!

integrable diffusion constant independent of Δ

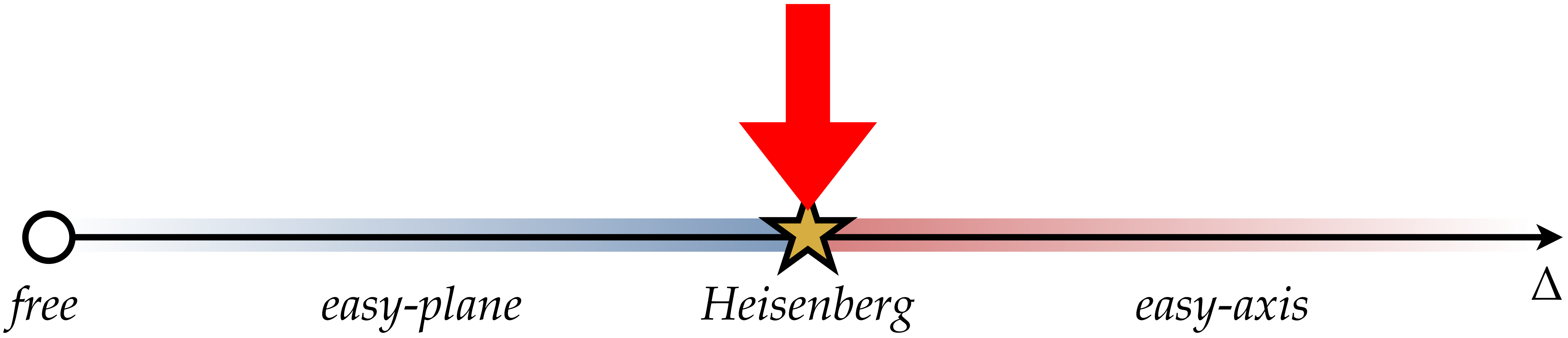


subdiffusive window $D(t) \sim 1/\sqrt{\gamma t}$

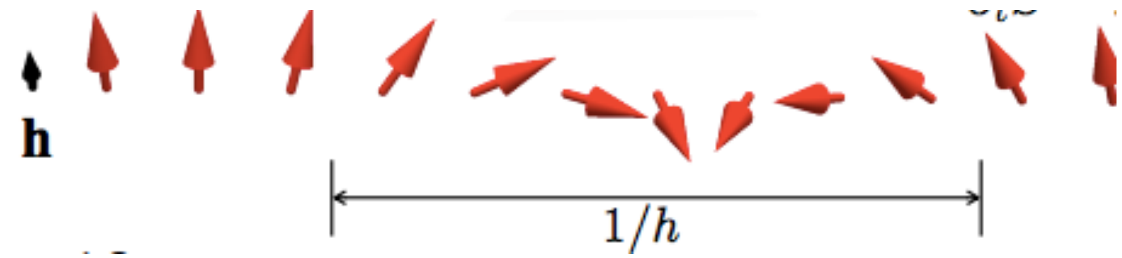
asymptotic diffusion constant, $\propto 1/\Delta$, independent of γ

- Limit $\gamma \to 0$ is subtle / discontinuous for all $\Delta > 1$
- For $\Delta = \infty$, subdiffusion at all γ [Singh et al., arXiv:2108.02205]

isotropic point



superdiffusion at the isotropic point



- Ultimate origin:
  large-amplitude Goldstone modes
  above a ferromagnet
- Critical exponent for Goldstone modes $z = 2$, speed ~ 1 / size
- Integrability means these modes survive but get thermally “dressed”

superdiffusion at the isotropic point

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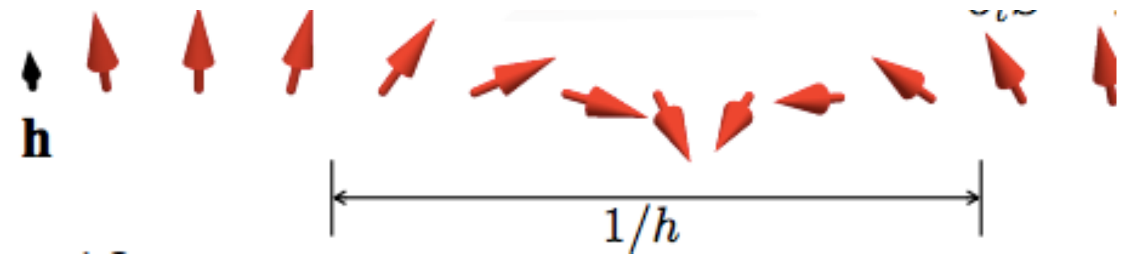


- Critical exponent for Goldstone modes $z = 2$, speed ~ 1/size
- Integrability means these modes survive but get thermally "dressed"

- General scaling form for correlation functions $C_{\text{anom.}} \sim \mu^{\delta} C(x\mu^{\nu}, t\mu^{\nu z})$
- Crossover length set by "resolving" magnetization: $\mu \sim 1/\sqrt{\xi}, \nu = 2$
- Largest Goldstone has size $1/\mu$
- Speed of Goldstone modes at that scale:

$$v \sim \mu \Rightarrow t = \mu^{-3} \Rightarrow z = 3/2$$

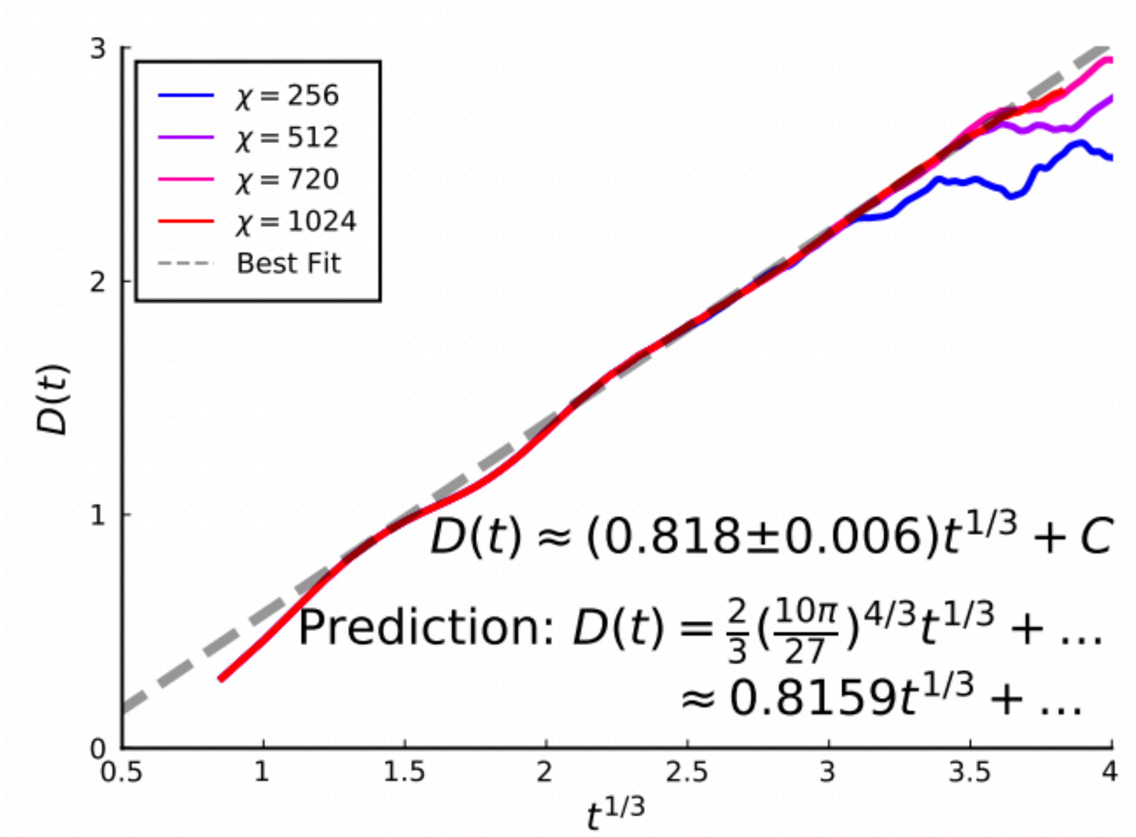
another framing

- Density of size-s Goldstones: $\rho_s \sim 1/s^3$ [forced by susceptibility sum rule]

- Each Goldstone is screened when it sees a bigger Goldstone

  Mean free path against screening: $\ell_s^{-1} \sim \sum_{s'>s} \rho_{s'} \sim 1/s^2$

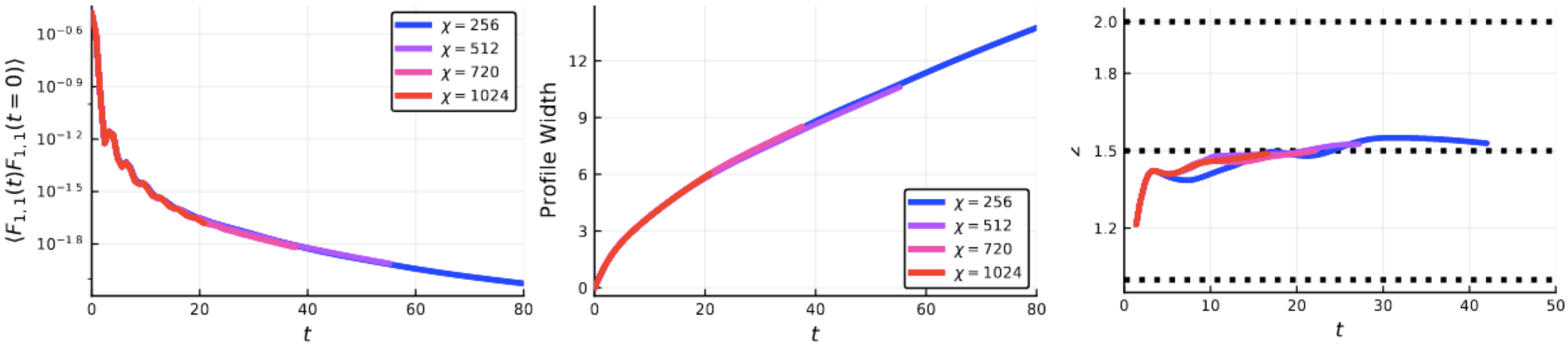
  Screening time: $\tau_s = \ell_s / v_s \sim s^3$



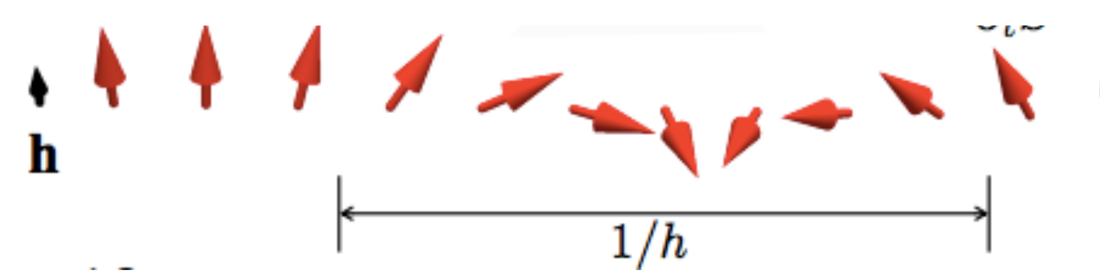
- a.c. conductivity from Kubo

$$T\sigma(\omega) = \int dt \langle J(t) j(0) \rangle \sim \sum_s \int dt \rho_s v_s m_s e^{-t/s^3} \sim \omega^{-1/3}$$

superuniversality of superdiffusion

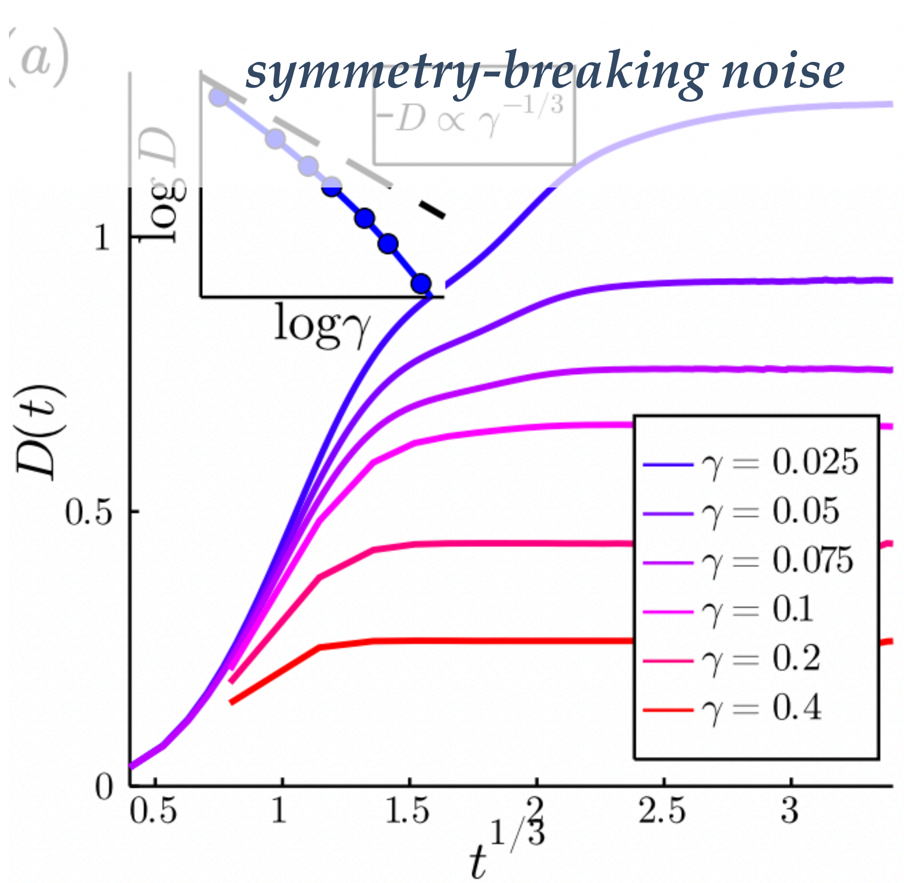


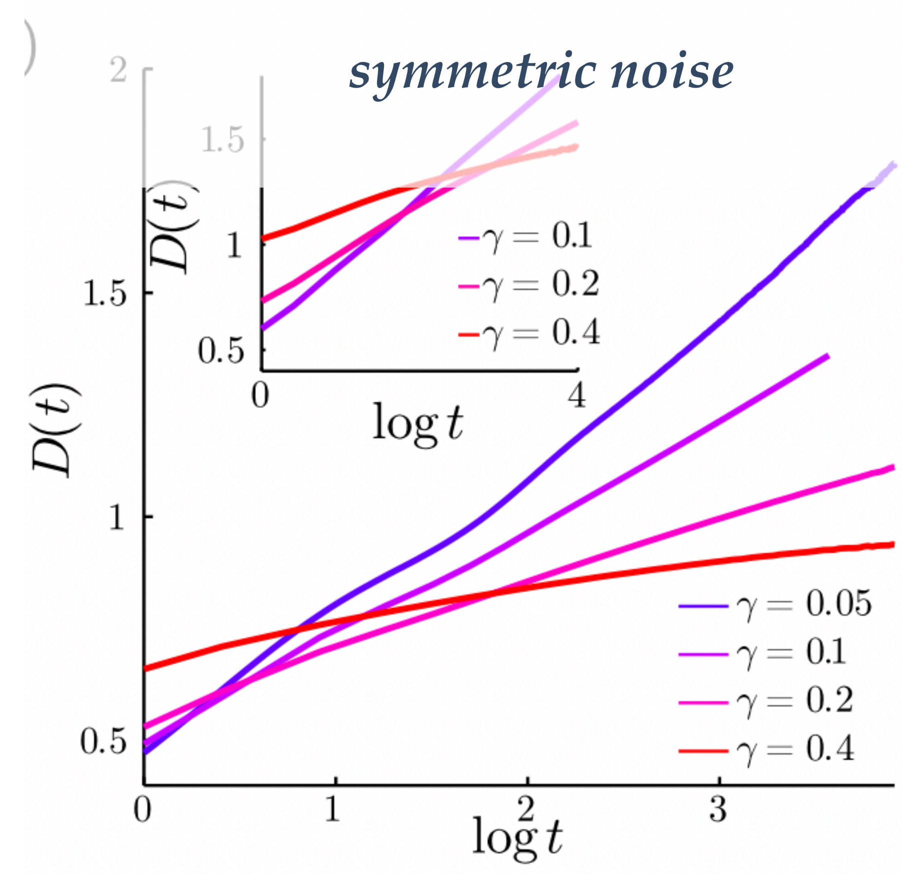
- Very similar numerical results for SU(3), SU(4), Sp(6)
- Full analytical solution of GHD equations for SU(N)
- What's the ultimate origin? [cf. Bulchandani, PRB (2020)]
    - Start with the ferromagnet; z = 2 critical theory
    - Spin transport at field h is dominated by spin solitons of size 1/h
    - These have velocity v(h) ~ h (as needed)



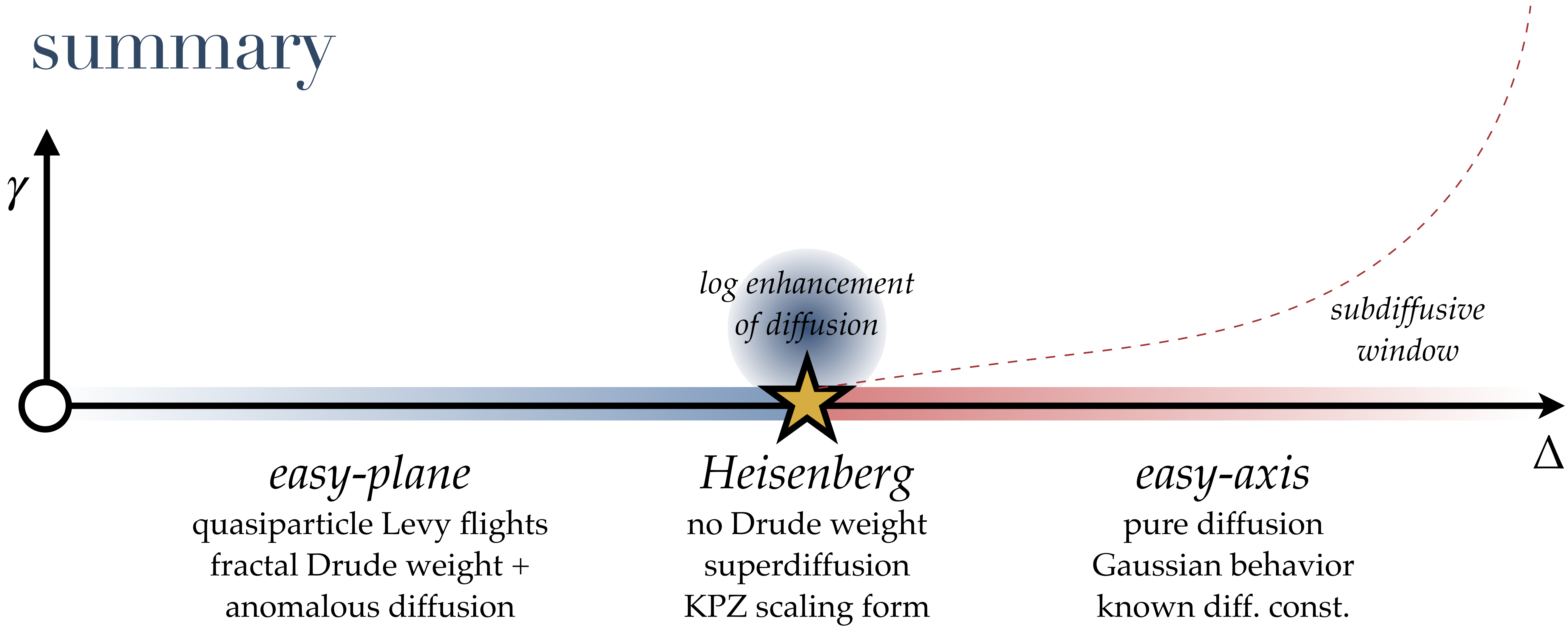
away from integrability

- Two classes of perturbations:
  - SU(2)-violating perturbations immediately destroy / scatter Goldstones
  - SU(2)-conserving local perturbations cannot couple directly to Goldstones, must couple through higher derivatives
  - Simple estimate of Goldstone lifetime: $\tau_s \geq s^2 \Rightarrow \sigma(\omega) \sim \log \omega$ (or slower!)
  - But this ignores higher-order / nonperturbative phenomena (cf. Lamacraft talk)





summary



- Integrable limit: dense gases of stable elementary excitations
  - Diffusion from depolarization of magnons passing through large domains
  - Superdiffusion with $z = 3/2$ from Goldstone modes at finite density
- Surprises away from integrability: anomalous suppression *and* enhancement of transport
- Open question/task: framework for asymptotics away from integrability

outlook: experiments!

Article

Spin transport in a tunable Heisenberg model realized with ultracold atoms

https://doi.org/10.1038/s41586-020-3033-y
Received: 18 May 2020

Paul Niklas Jepsen[1,2,3] ✉, Jesse Amato-Grill[1,2,3], Ivana Dimitrova[1,2,3], Wen Wei Ho[3,4], Eugene Demler[3,4] & Wolfgang Ketterle[1,2,3]

PHYSICAL REVIEW X **8**, 021030 (2018)
Featured in Physics

Thermalization near Integrability in a Dipolar Quantum Newton's Cradle

Yijun Tang,[1,2] Wil Kao,[2,3] Kuan-Yu Li,[2,3] Sangwon Seo,[1,2,3] Krishnanand Mallayya,[4] Marcos Rigol,[4] Sarang Gopalakrishnan,[5,6] and Benjamin L. Lev[1,2,3]

PHYSICAL REVIEW LETTERS **122**, 090601 (2019)
Editors' Suggestion
Featured in Physics

Generalized Hydrodynamics on an Atom Chip

M. Schemmer,[1] I. Bouchoule,[1] B. Doyon,[2] and J. Dubail[3]

LETTERS
https://doi.org/10.1038/s41567-021-01191-6
nature physics
Check for updates

Detection of Kardar–Parisi–Zhang hydrodynamics in a quantum Heisenberg spin-1/2 chain

A. Scheie[1,7], N. E. Sherman[2,3,7], M. Dupont[2,3], S. E. Nagler[1], M. B. Stone[1], G. E. Granroth[1], J. E. Moore[2,3] ✉ and D. A. Tennant[4,5,6] ✉

QUANTUM GASES

Generalized hydrodynamics in strongly interacting 1D Bose gases

Neel Malvania[1]†, Yicheng Zhang[1]†, Yuan Le[1], Jerome Dubail[2], Marcos Rigol[1], David S. Weiss[1]*

Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion

David Wei,[1,2] Antonio Rubio-Abadal,[1,2,*] Bingtian Ye,[3] Francisco Machado,[3,4] Jack Kemp,[3] Kritsana Srakaew,[1,2] Simon Hollerith,[1,2] Jun Rui,[1,2,†] Sarang Gopalakrishnan,[5,6] Norman Y. Yao,[3,4] Immanuel Bloch,[1,2,7] and Johannes Zeiher[1,2]

outlook

- How does weak integrability-breaking affect dynamics?
  - Relaxation rates without reference to form factors? [arXiv:1912.08826]
  - Can the relaxation rates depend "interestingly" on the perturbation?

PHYSICAL REVIEW X **8**, 021030 (2018)

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Thermalization near Integrability in a Dipolar Quantum Newton's Cradle

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  - How to go beyond the GHD limit? ("Orthogonality catastrophes")

- Realistic experimental consequences (NMR, cold atoms, etc.)

Spin transport in a Mott insulator of ultracold fermions

Matthew A. Nichols[1,2,3], Lawrence W. Cheuk[2,4], Melih Okan[1,2,3], Thomas R. Hartke[1,2,3], Enrique Mendez[1,2,3], T. Senthil[1], Ehs...

+ See all authors and affiliations

Science 25 Jan 2019: