

# Extracting complexity of quantum dynamics using machine learning

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Uni Cal., Berkeley



Jožef Stefan Institute

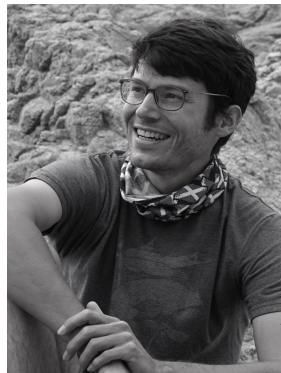


University of Cologne



Moore Foundation  
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Markus Schmitt

# Detect local complexity of quantum states

- A new measure for the local complexity of quantum many-body states
- Local complexity:
  - **number of parameters** needed to reproduce all **local observables**
- Applications:
  - detect stages of noneq dynamics
  - detect (lack of) thermalization
  - learn  $H$  from local observables
  - analyse and characterize quantum simulators
  - detect noise-type

# Complexity dynamics: information bottleneck

## Short times:

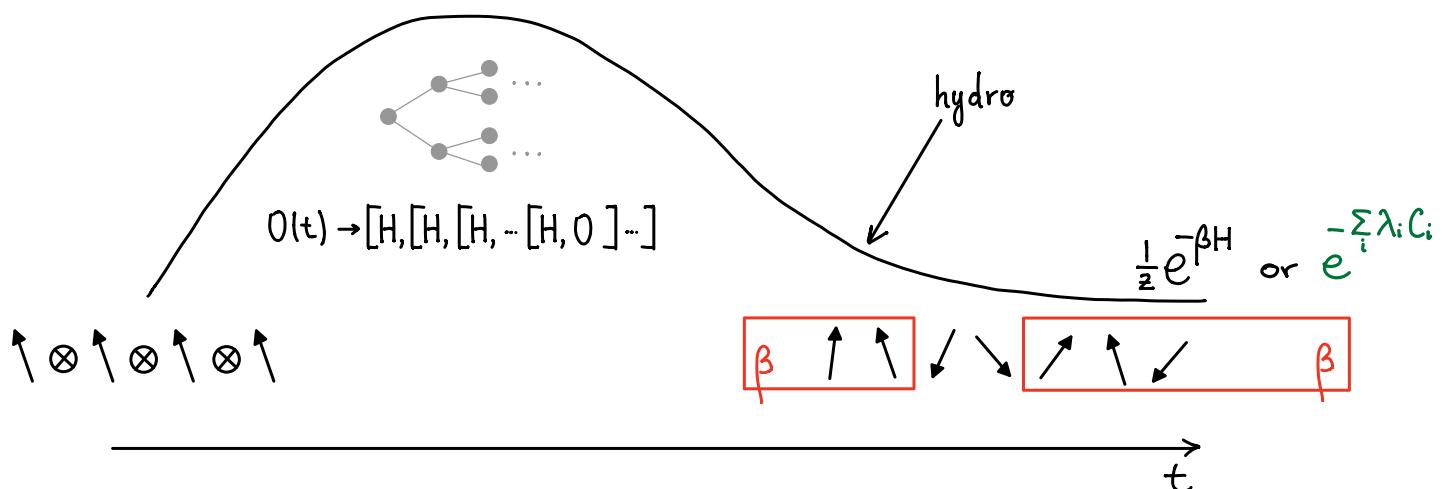
- Initial product state
- Operator growth,

Parker et al, PRX 2019

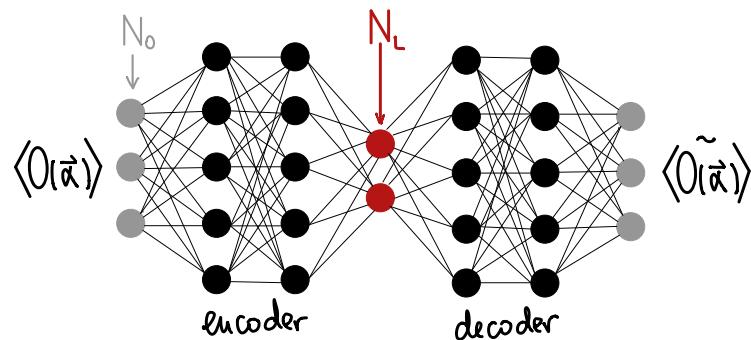
$$O(t) = \exp(\hat{\mathcal{L}}^\dagger t) O(0)$$

## Long times:

- Emergent hydro description,  
Lux et al PRA 2014, Bohrdt et al NJP 2017
- Only conservation laws matter
- Statistic description



# Autoencoders → efficient data encoding (compression)



**Input**  $x$ : expectation values  $\text{tr}[O(\alpha)\rho]$  of local operators

$$O(\alpha) = \sigma_1^{\alpha_1} \dots \sigma_{|\mathcal{S}|}^{\alpha_{|\mathcal{S}|}}, \quad \alpha = (\alpha_1, \dots, \alpha_{|\mathcal{S}|}) \in \{0, x, y, z\}^{|\mathcal{S}|}$$

Data sets:

- one data set:  $N_O$  operators
- need  $N_{\text{real}}$  sets for learning

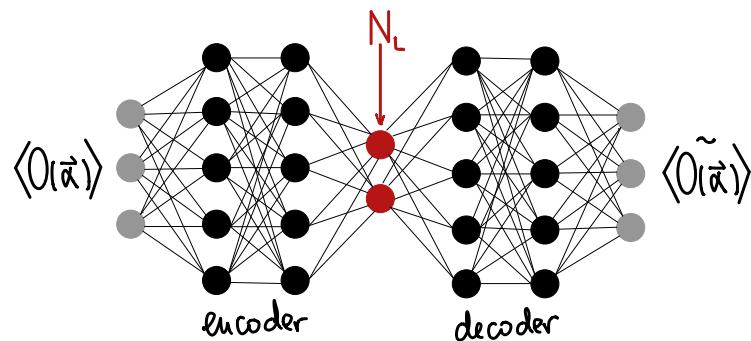
Dimensional reduction by encoder:

- $N_O$ -dim to  $N_L$ -dim

**Bootleneck:**  $N_L$  neurons

**Output**  $f_\theta(x)$ : network reproduction of local observables

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**Bootleneck:**  $N_L$  neurons

**Output**  $f_\theta(x)$ : network reproduction of local observables

1. Initialize the network by training on a subset of realizations

$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} (f_\theta(x) - x)^2$$

2. How well unseen  $\langle O(\alpha) \rangle$  can be reproduced by the network: Test error

# Complexity dynamics: information bottleneck

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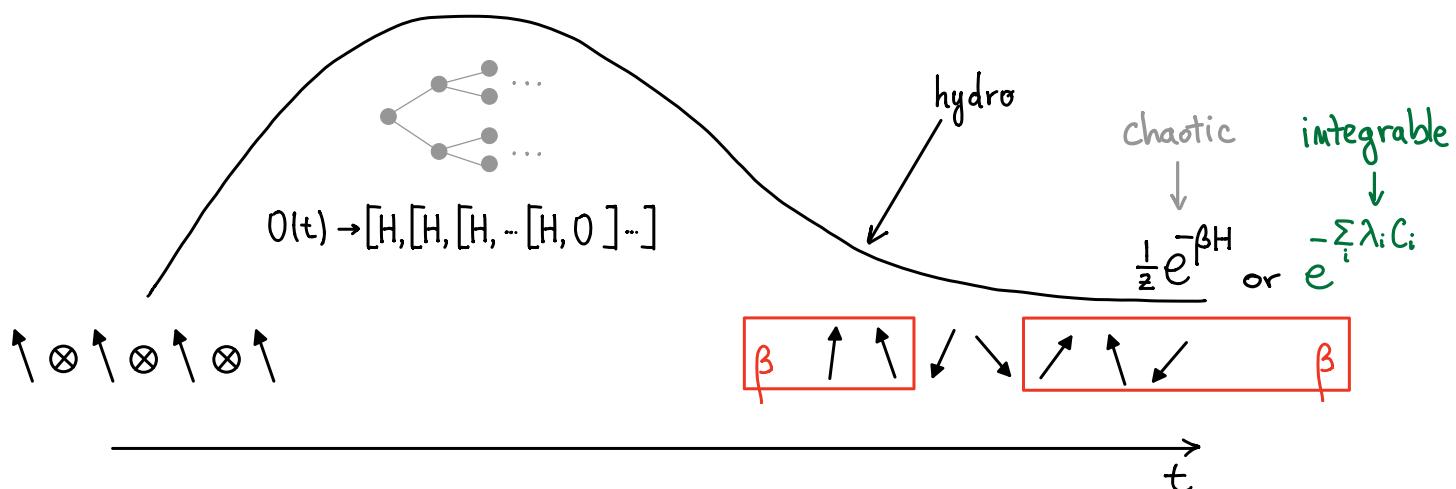
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# Detecting (generalized) Gibbs ensemble

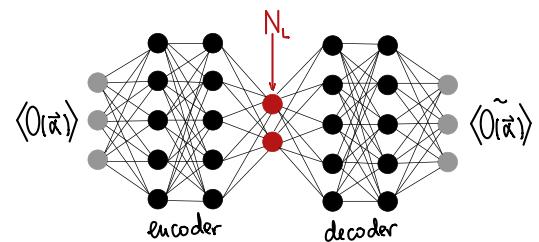
Hamiltonian: integrable t-Ising

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x \quad \rightarrow \quad [C_i, H] = 0$$

Data set: 'synthetic' GGE

$$\langle O(\alpha) \rangle = \text{tr}[O(\alpha)\rho_{GGE}], \quad \rho_{GGE} = \frac{1}{Z} e^{\sum_{i=1}^{N_c} \lambda_i C_i}, \quad C_0 = H$$

with random  $\lambda_i \in [-2, 2]$



# Detecting (generalized) Gibbs ensemble

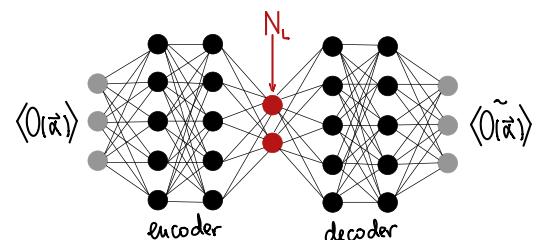
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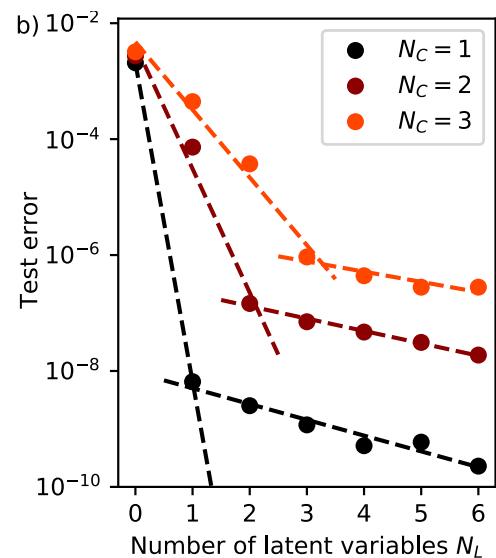
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Test error:

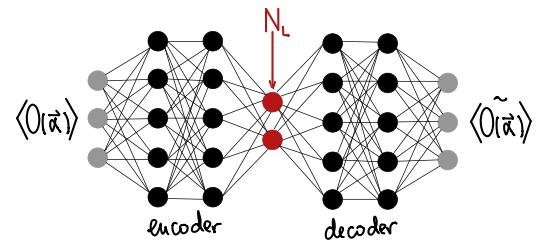
$$\mathcal{L}_{\mathcal{D}_T}(\theta) = \frac{1}{|\mathcal{D}_T|} \sum_{x \in \mathcal{D}_T} (f_\theta(x) - x)^2$$



# Detecting Gibbs ensemble

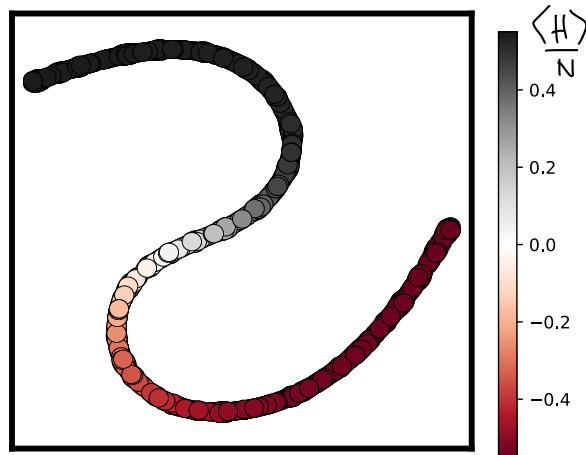
Thermal states

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \rho_{GE} = \frac{1}{Z}e^{\lambda_0 C_0},$$



Towards interpretable learning: What does it learn?

- mapping of  $N_L$  dimensional latent space to 2D (t-SNE)
- color by  $\langle H \rangle / N \rightarrow$  monotonic function of energy

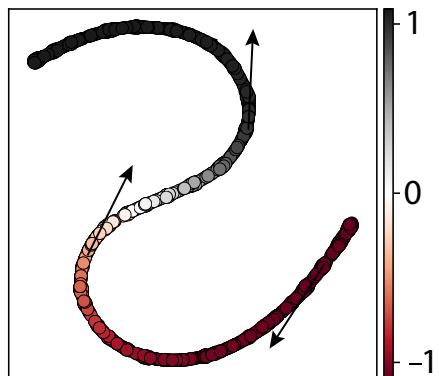


# Reconstructing $H$ from measurements in Gibbs ensemble

Consider  $\rho_{GE} = \frac{1}{Z} e^{\lambda_0 C_0}$  of

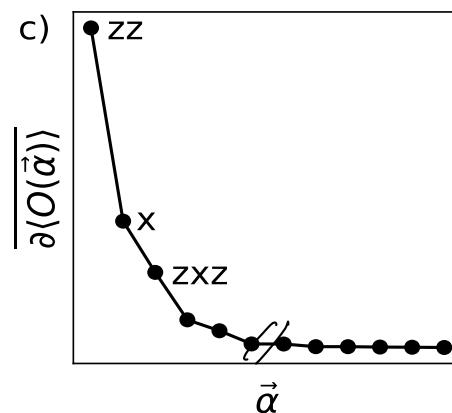
$$H = \sum_j J \sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x$$

t-SNE projection of latent space



**Reconstruction of  $H$  from  $\langle O(\alpha) \rangle$ :**

0. t-sne, test error  $\rightarrow$  expect  $\rho \sim e^{-\beta H}$
1. Find operators  $\langle O(\alpha) \rangle$  with largest gradient along the line:



2. Newtons method  $\rightarrow$  get  $\tilde{h}_x = h_x/J$

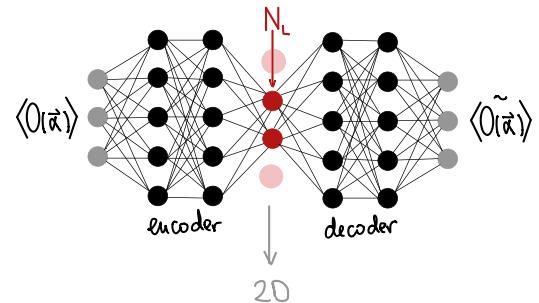
$$\text{tr}[\sigma_j^z \sigma_{j+1}^z e^{-\beta(\sigma_j^z \sigma_{j+1}^z + \tilde{h}_x \sigma_j^x)}] - \langle \sigma_j^z \sigma_{j+1}^z \rangle_{mes} = 0$$

$$\text{tr}[\sigma_j^x e^{-\beta(\sigma_j^z \sigma_{j+1}^z + \tilde{h}_x \sigma_j^x)}] - \langle \sigma_j^x \rangle_{mes} = 0$$

# Detecting (generalized) Gibbs ensemble

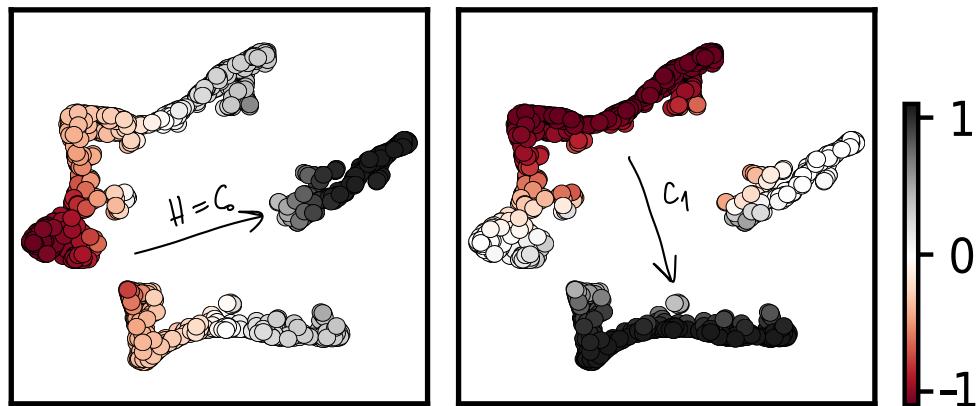
Consider integrable t-Ising Hamiltonian

$$H = \sum_j J\sigma_j^z\sigma_j^z + h_x\sigma_j^x, \quad \rho_{GGE} = \frac{1}{Z} e^{\lambda_0 C_0 + \lambda_1 C_1},$$



Towards interpretable learning: What does it learn?

- mapping of  $N_L$  dimensional latent space to 2D (t-SNE)
- color by  $\langle H \rangle / N, \langle C_1 \rangle / N$



# Complexity dynamics: information bottleneck

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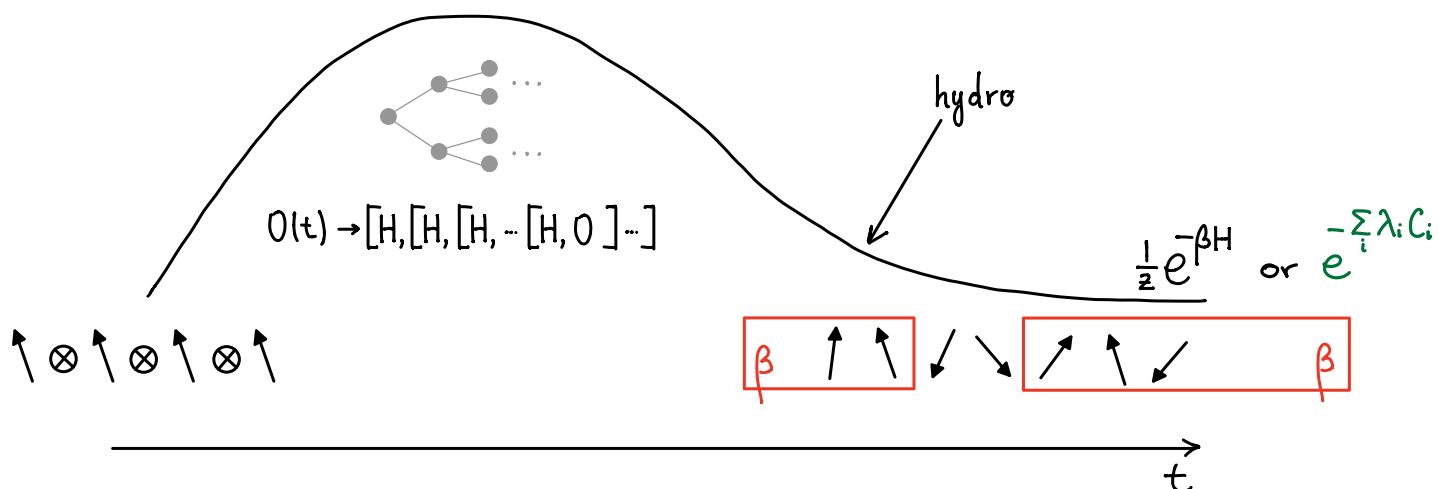
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$$O(t) = \exp(\hat{\mathcal{L}}^\dagger t) O(0)$$

## Long times:

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# Open setup: parametrization of steady states

$$\hat{\mathcal{L}}\rho = -i[H, \rho] + \epsilon \hat{\mathcal{D}}\rho = 0, \quad \hat{\mathcal{D}}\rho = \sum_k L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\}$$

- Known simpler limit  $\epsilon \rightarrow 0$

$$\rho = \begin{cases} \frac{1}{Z} e^{-\beta H}, & H \text{ chaotic } (\text{ZL, [Alberton] Altman, Rosch, PRL '18, [PRL '20])} \\ \frac{1}{Z} e^{-\sum_i \lambda_i C_i}, & H \text{ integrable } (\text{Lange, ZL, Rosch, Nat Comm '17, PRB '18}) \end{cases}$$

- Finite small  $\epsilon$  (ZL, Lange, Rosch, PRB 97, 024302 (2018))

$$\rho = \rho_{(G)GE} + \delta\rho(\epsilon)$$

- Setup studied here: randomly rotated 1-site and 2-site Lindblads

$$L_i^{(1)} = \left( R_z(\theta) R_y(\phi) S_i^- R_y^{-1}(\phi) R_z^{-1}(\theta) \right) P_{i+1}^\downarrow, \quad L_i^{(2)} = R_z(\theta') R_y(\phi') P_{i+1}^\uparrow R_y^{-1}(\phi') R_z^{-1}(\theta')$$

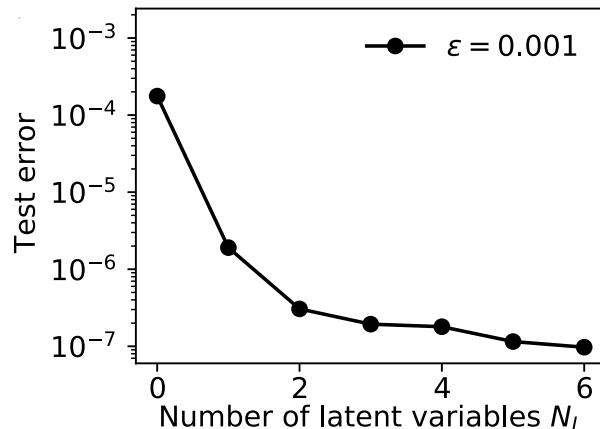
# Open setup: weak coupling to baths $\epsilon \ll 1$

**Chaotic**  $H \rightarrow \rho \approx \frac{1}{Z} e^{-\beta H}$ ,

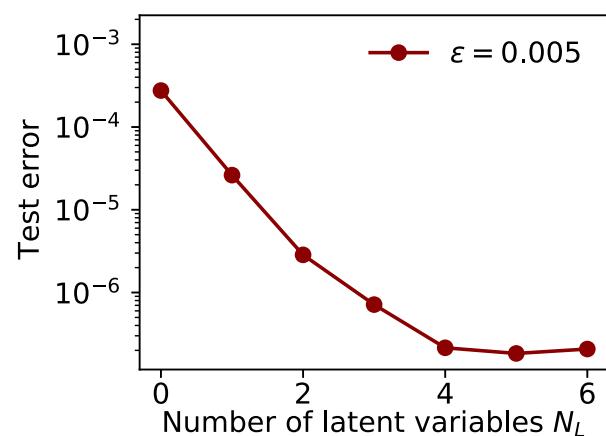
$$H = \sum_j J \sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x + h_z \sigma_j^z$$

**Integrable**  $H \rightarrow \rho \approx \frac{1}{Z} e^{-\sum_i \lambda_i C_i}$ ,

$$H = \sum_j J \sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x$$



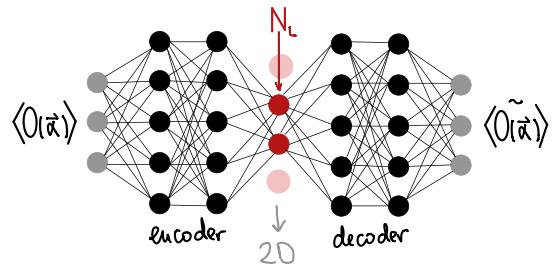
1st latent var.  $\rightarrow$  energy  
2nd latent var.  $\rightarrow$  noise



more latent vars.  $\rightarrow$  cons. laws  $C_i$   
additional latent var.  $\rightarrow$  noise

# What is learned?

Do 2D projection of the latent space

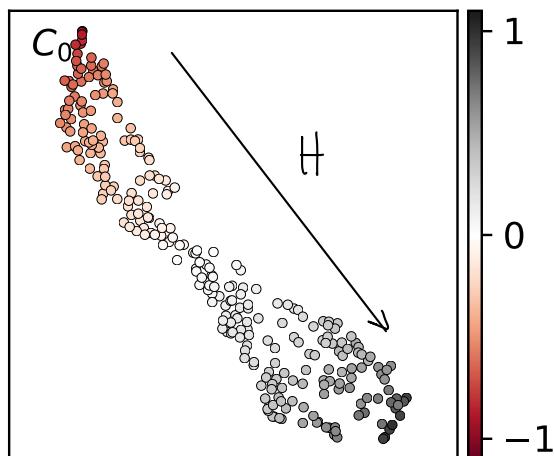


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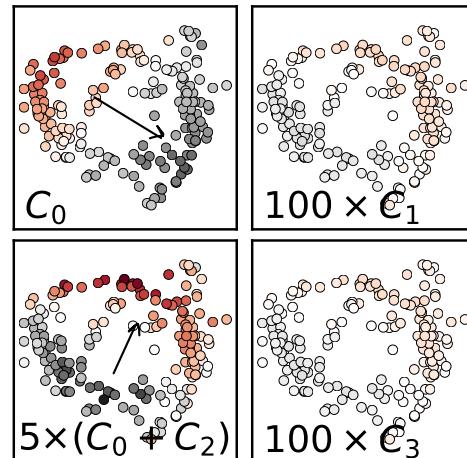
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→ energy



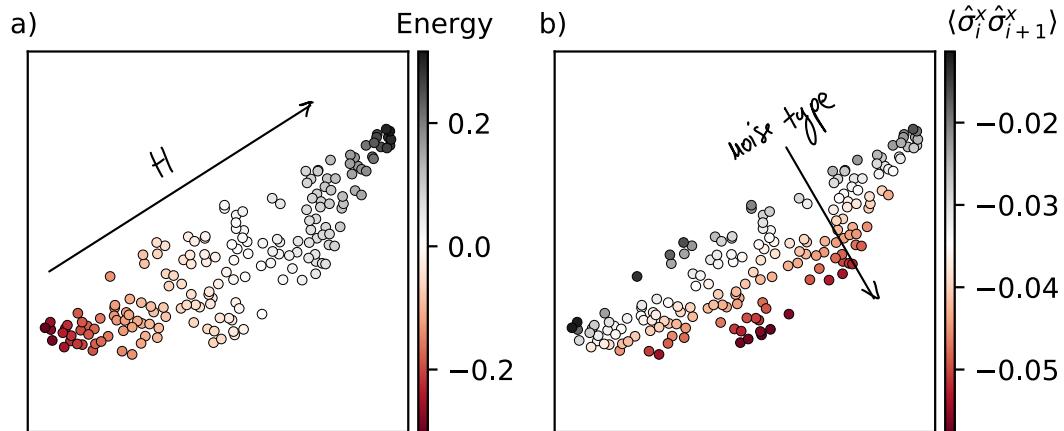
→ most local parity-even cons. laws  
→ learn which truncated GGE

# Open setup: noise-type reconstruction

$$H = \sum_i J_z S_i^z S_{i+1}^z + h_x S_i^x + h_z S_i^z \quad (1)$$

with Lindblad operators that favour AFM  $\sigma_i^x \sigma_{i+1}^x$  correlations

$$\begin{aligned} L_i^{(1a)} &= S_i^{+,x} P_{i+1}^{\downarrow,x}, & L_i^{(1b)} &= P_i^{\downarrow,x} S_{i+1}^{+,x}, \\ L_i^{(2a)} &= S_i^{-,x} P_{i+1}^{\uparrow,x}, & L_i^{(2b)} &= P_i^{\uparrow,x} S_{i+1}^{-,x}, \\ L_i^{(3)} &= S_i^z \end{aligned} \quad (2)$$



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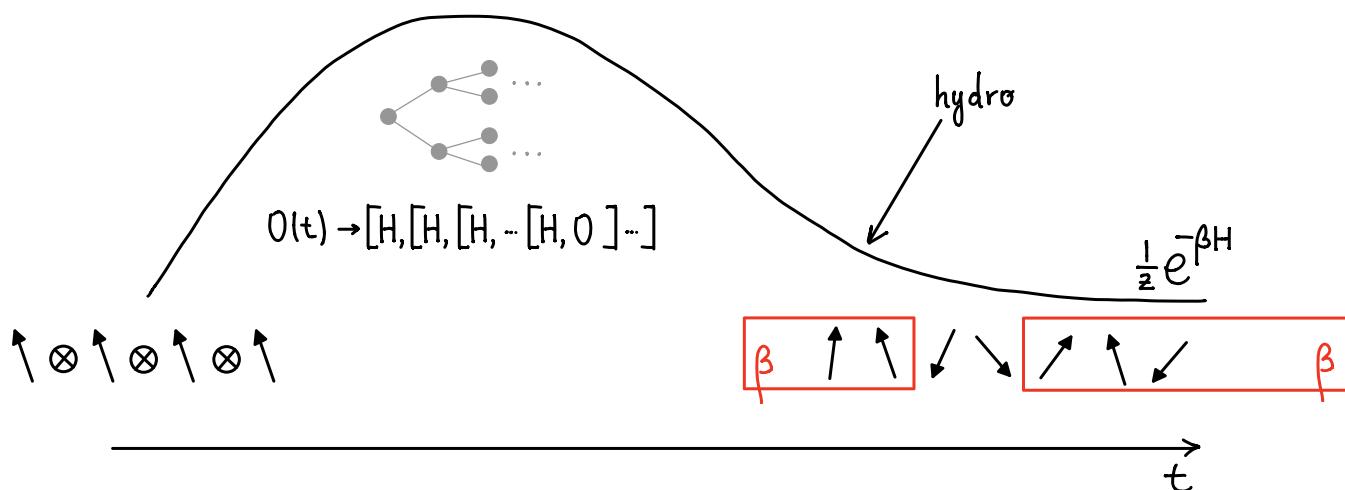
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Parker et al, PRX 2019

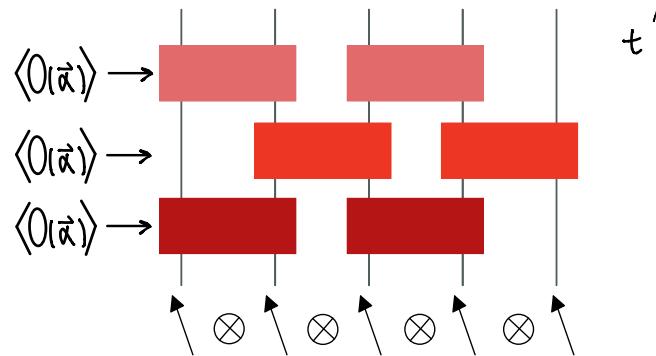
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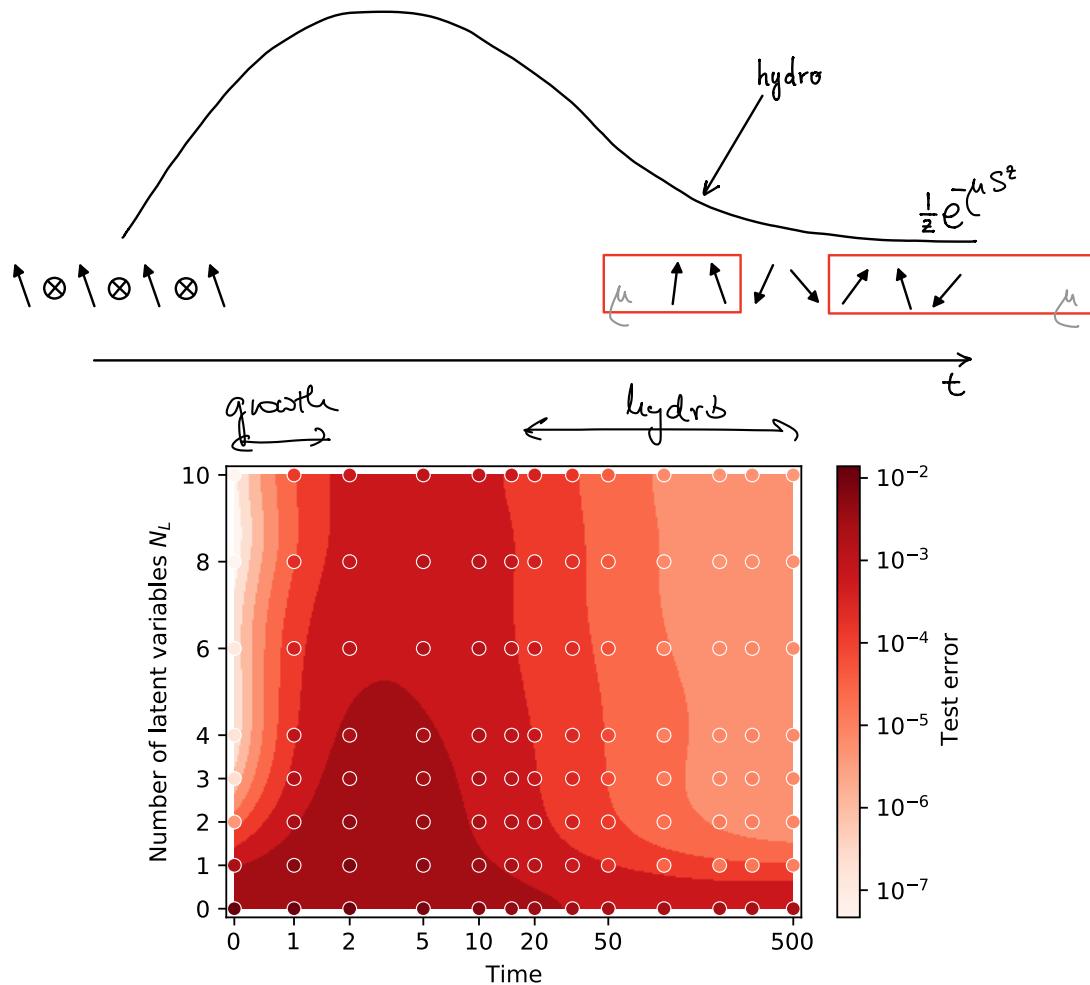


# Complexity growth in random circuits with $S_z$ conservation



- Start with a product state  $\rightarrow$  need 2 variables
- Propagate with **random unitaries with magnetization conservation**  $\rightarrow \mu$
- Measure local observables at different times

# Complexity growth in random circuits with $S_z$ conservation



# Conclusions

## Detect low-dimensional physical representation

- Closed systems
  - Distinguish chaotic from integrable system
  - Reconstruction of  $H$  from measurements
- Open systems
  - Detect the complexity of density matrix
  - Reconstruction of noise-type
- Advantages
  - Suitable for experimental data
  - Ansatz-free measure of local complexity

