# Breakdown of hydrodynamics below four dimensions in a fracton fluid

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Non-equilibrium universality in many-body physics
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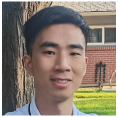

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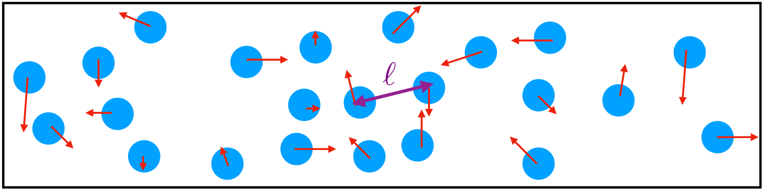
**1** Hydrodynamics is the effective field theory of thermalization.



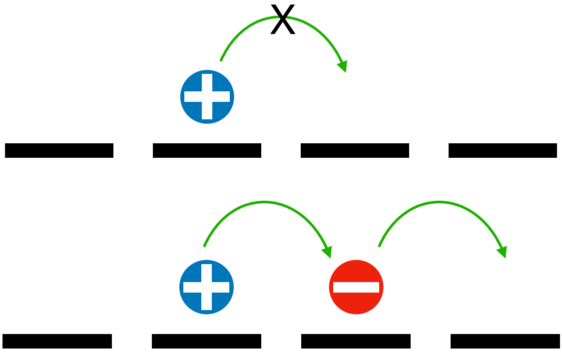

It is a perturbative expansion in the small parameter $\ell\partial_i$, and the equations depend only on symmetries and conserved quantities.

$$\partial_t\rho + \partial_i(\rho v_i) = 0,$$
$$\partial_t(\rho v_i) + \partial_j(P(\rho)\delta_{ij} + \rho v_i v_j) - \eta\partial_j\partial_j v_i = 0.$$

Sometimes, hydrodynamics is *unstable*. The fluid above does not exist in $d = 1$ spatial dimension; instead, we have KPZ.

2

What happens if the microscopic dynamics is subject to a non-trivial constraint, such as dipole conservation?

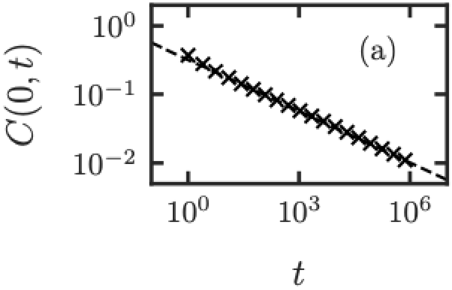



We say that such theories have **fractons** – elementary excitations are immobile, and only move with other fractons.

3 There is a novel universality class of hydrodynamics with (just) charge and dipole conservation:

$$\partial_t n = -B\partial_x^4 n$$

[Gromov, Lucas, Nandkishore; 2003.09429]




$$C(0,t) = \langle n(0,0)n(0,t)\rangle$$

$$C(0,t) \sim t^{-1/4}$$

[Morningstar *et al*; 2004.00096], [Feldmeier *et al*; 2004.00635]

4

We derived this result by constructing a Schwinger-Keldysh action for hydrodynamics demanding suitable constraints. Due to dipole conservation

$$\partial_t \int \mathrm{d}^d x \; x_i n = 0,$$

the Ward identity for charge conservation must be

$$\partial_t n + \partial_i \partial_j J_{ij} = 0.$$

For $J_{ij}$ to obey a local fluctuation-dissipation theorem, we require

$$J_{ij} = B_1 \partial_i \partial_j n + B_2 \delta_{ij} \partial_k \partial_k n + \xi_{ij}.$$

This formalism further reveals an infinity of new hydrodynamic universality classes with exotic multipole/subsystem ("fractonic") conservation laws!

[Gromov, Lucas, Nandkishore; 2003.09429]

5 In common speak, we usually associate hydrodynamic theories with **momentum conservation**:

$$\partial_t \rho + \partial_i(\rho v_i) = 0,$$
$$\partial_t(\rho v_i) + \partial_j(P(\rho)\delta_{ij} + \rho v_i v_j) - \eta \partial_j \partial_j v_i = 0.$$

So, what happens if we consider a dipole conserving theory with momentum conservation?

The conserved quantities are

$$P_i = \int \mathrm{d}^d x \ \pi_i,$$

$$Q = \int \mathrm{d}^d x \ n,$$

$$D_i = \int \mathrm{d}^d x \ x_i n.$$

The multipole algebra relating them is:

$$\{P_i, Q\} = \{D_i, Q\} = 0, \quad \{D_i, P_j\} = Q\delta_{ij}.$$

This last Poisson bracket/commutator will make hydrodynamics subtle!

7

We expect that hydrodynamic equations will take the form

$$\partial_t n + \partial_i \partial_j J_{ij} = 0,$$
$$\partial_t \pi_i + \partial_j T_{ij} = 0.$$

But, what are the constitutive relations for $J_{ij}$ and $T_{ij}$? The dipole algebra demands the theory be invariant under

$$\pi_i \to \pi_i + c_i n$$

Or, if we write

$$v_i = \frac{\pi_i}{n},$$

the theory is invariant under $v_i \to v_i + c_i$. This includes thermodynamics! So we conclude

$$\text{entropy density} = s(n, \partial_i v_j).$$

[Grosvenor *et al*; 2105.01084], [Glorioso *et al*; 2105.13365]

8

After some algebra, we find the ideal constitutive relations:

$$
\begin{aligned}
J_{ij} &= \psi_{ij} = T\frac{\partial s}{\partial(\partial_i v_j)}, \\
T_{ij} &= P\delta_{ij} + V_i\pi_j - \psi_{ik}\partial_j v_k, \\
P &= Ts - nT\frac{\partial s}{\partial n}, \\
V^i &= \frac{1}{n}\partial_j\psi_{ji},
\end{aligned}
$$

with first order dissipative corrections and noise:

$$
\begin{pmatrix} J_{ij}^{(1)} \\ T_{ij}^{(1)} \end{pmatrix} = \begin{pmatrix} -C_{ijkl} & -\kappa_{ijkl} \\ \alpha_{ijkl} & \eta_{ijkl} \end{pmatrix} \begin{pmatrix} \partial_k\partial_l\mu \\ \partial_k V_l \end{pmatrix} + \begin{pmatrix} \xi_{ij} \\ \tau_{ij} \end{pmatrix}
$$

A standard fluctuation dissipation theorem holds.

[Glorioso *et al*; 2105.13365]

9

Is hydrodynamics stable in $d$ space dimensions? Within linear response:

$$z = \frac{[t]}{[x]} = 4, \quad [n] = \frac{d}{2}, \quad [\pi] = \frac{d}{2} - 1, \quad [\tau] = [\xi] = \frac{d}{2} + 2,$$

Let's consider the pressure nonlinearity

$$P(n) = P_0 + P_1 \delta n + P_2 (\delta n)^2 + \cdots .$$

$P_1$ is "relevant" but non-dissipative, so we do not worry about it. (Linear hydrodynamics makes sense!). But

$$[P_2] = 2 - \frac{d}{2},$$

meaning hydrodynamics is unstable when $d < 4$. This will lead to a fractonic "dipole-KPZ" non-equilibrium universality class!

10

So, is it possible to actually look for this? A one dimensional model with dipole, momentum (and energy) conservation is the Hamiltonian system

$$H = \sum_{i=1}^{N-1} \frac{(p_i - p_{i+1})^2}{2} + V(x_i - x_{i+1}),$$

$$V(x) = \frac{1}{2}x^2 + \frac{g}{3}x^3 + \frac{g'}{4}x^4 + \cdots .$$

Just like momentum conservation corresponds to $x_i \to x_i + c$, dipole conservation corresponds to $p_i \to p_i + c$.

We can also add dissipation and noise to this model to break energy conservation. We will call this **Model A**. The linearized Model A is equivalent to the Heisenberg ferromagnet:

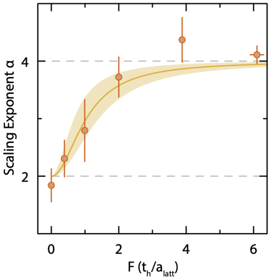
$$H = -\sum_{i=1}^{N-1} S_i^\alpha S_{i+1}^\alpha, \qquad \{S_i^\alpha, S_j^\beta\} = \epsilon^{\alpha\beta\gamma} S_i^\gamma \delta_{ij}.$$

$$S_x \sim D, \quad S_y \sim P, \quad S_z \sim Q.$$

11 **Model B** is a model with *emergent momentum and dipole conservation*:

$$H = \sum_{i=1}^{N} [-\cos p_i - F x_i] + \sum_{i=1}^{N-1} V(x_i - x_{i+1}).$$

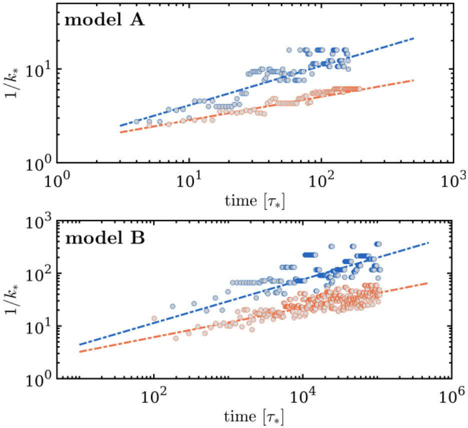
We take $F$ to be very large, so that the short time dynamics consists of fast Bloch oscillations. The non-momentum conserving version has been realized in a tilted Fermi-Hubbard model in an experimental optical lattice.




[Guardado-Sanchez *et al*; 1909.05848]

12 We simulated Models A and B. With random initial conditions, the typical wave number of correlations should obey

$$k_{\mathrm{typ}}(t) \sim t^{-1/z}.$$




When $P_2 = 0$, $z \approx 4$; but when $P_2 \neq 0$, $z \approx 2.5$!

We have found a new non-equilibrium universality class in a "fracton fluid". However, we know that infinitely many more new universality classes of this kind exist.

Interplays between spacetime symmetries (and their breaking) and fracton hydrodynamics is quite subtle, and may not always be captured by naive Landau prescription.

Work on field theoretic derivation of our results is (slowly) being written up. May help understand how to couple fractons to gravity.