Dynamics in Dipole- and Higher-Moment Conserving Systems

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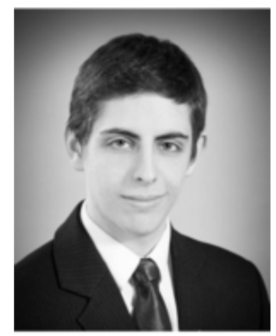
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P. Sala


R. Verresen


M.Knap


Scherg
Kohlert
Madhusudhana
Bloch
Aidelsburger

Non-Equilibrium Universality in Many-Body Physics, Sep 27, 2021

Thermalizing , Many-Body Localized, or….

Fully ergodic

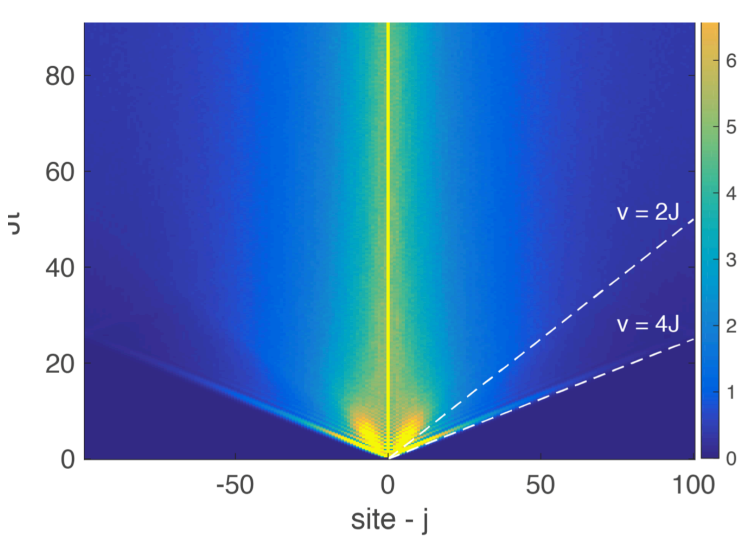
- Few conserved quantities
- Observables relax to thermal values
- All eigenstates look thermal

MBL (or integrable)

- Extensively many local integrals of motion (LIOM)
- Memory of initial state
- Eigenstates are non-thermal

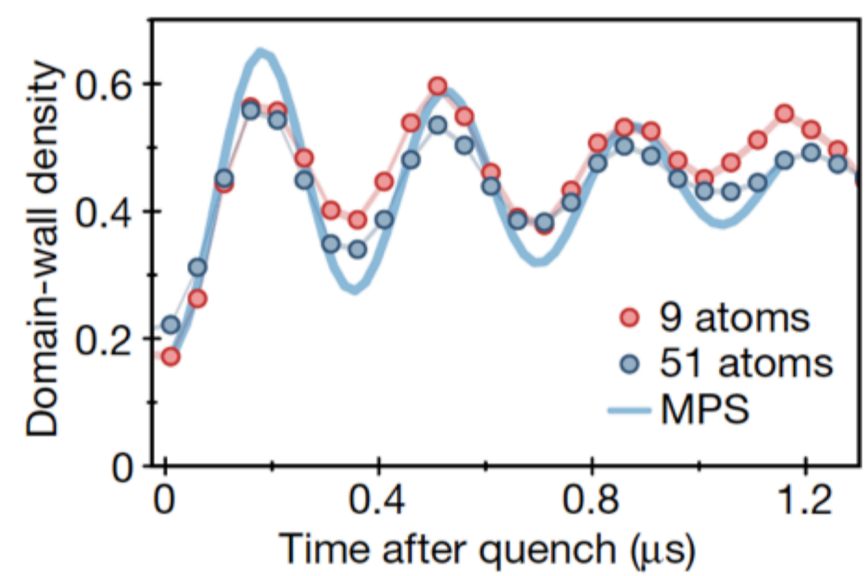
Intermediate behavior: Systems with constrained dynamics

- Localization in gauge theories



[Smith el al '17]

- Quantum many-body scars



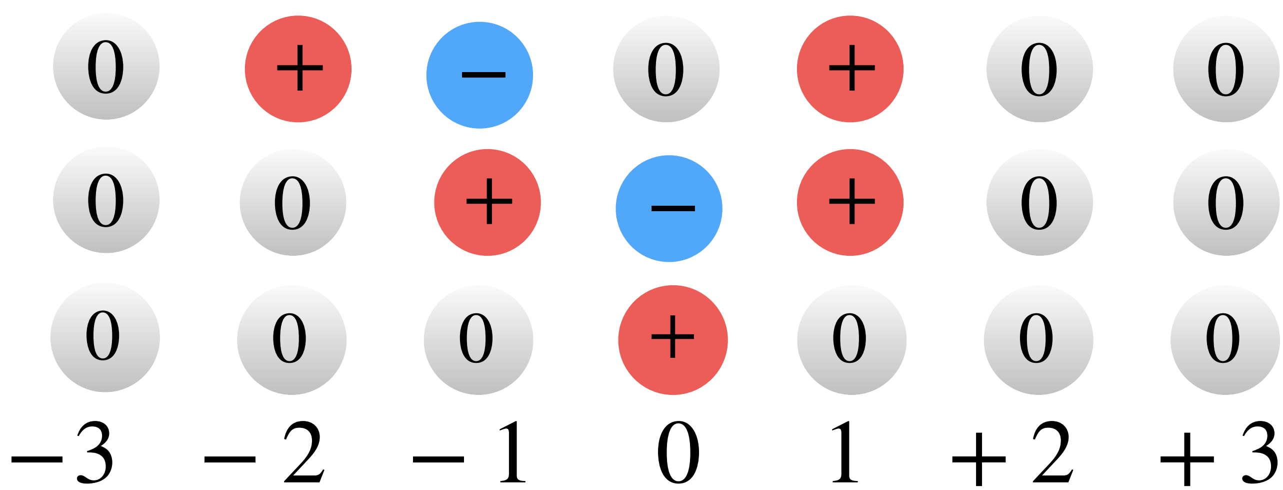
[Bernien et al '17, Turner et al ' 18, Moudgalya et al. '18]

Charge and dipole conservation

Conservation of a U(1) charge Q and its associated dipole moment P in a spin-1 chain:

$$Q = \sum_n S_n^z \qquad\qquad P = \sum_n (n - n_0) S_n^z$$

- Combination of Q and P symmetry puts constraints on the mobility of excitations: **“fractons”:** $|\pm\rangle = S^{\pm}|0\rangle$ [Pretko '18]



$Q = 1 \quad P = 0$

- Certain random local unitary dynamics with such symmetries fail to thermalize: **How robust is this phenomenon?** [Pai et al '18]

Overview

Dynamics in Dipole- and Higher-Moment Conserving Systems

- Fragmentation and fracton hydrodynamics in D=1

  [Sala, Rakovszky, Verresen, Knap, FP, Phys. Rev. X **10**, 011047 (2020)]
  [Feldmeier, Sala, De Tomasi, FP, Knap, PRL 125, 245303, 2020]

- Kinetic constraints in tilted Fermi-Hubbard chains

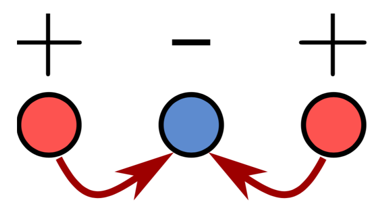
  [Scherg et al., Nat Commun **12**, 4490 (2021)]
  [Kohlert et al., arXiv:2106.15586]

- Fragmentation in D>1

  [Sala, Lehmann, Rakovszky, FP (in progress)]

Charge and dipole conservation in spin-1 chains

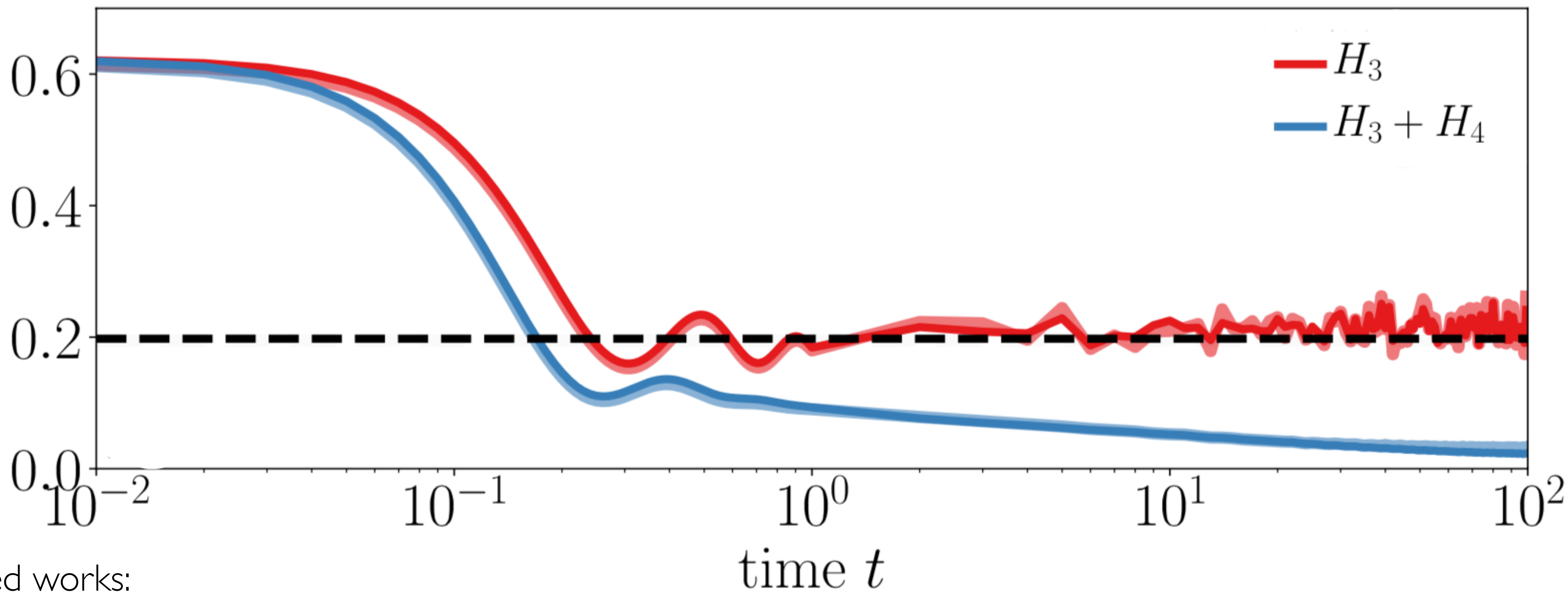
Q and P conserving spin-1 Hamiltonians



$$H_3 = -\sum_n \left[S_n^+ \left(S_{n+1}^-\right)^2 S_{n+2}^+ + \text{H.c.} \right]$$

$$H_4 = -\sum_n \left[S_n^+ S_{n+1}^- S_{n+2}^- S_{n+3}^+ + \text{H.c.} \right]$$

Autocorrelation function at $T = \infty$: $C(t) = \langle S_0^z(t) S_0^z(0) \rangle - 2/3N$



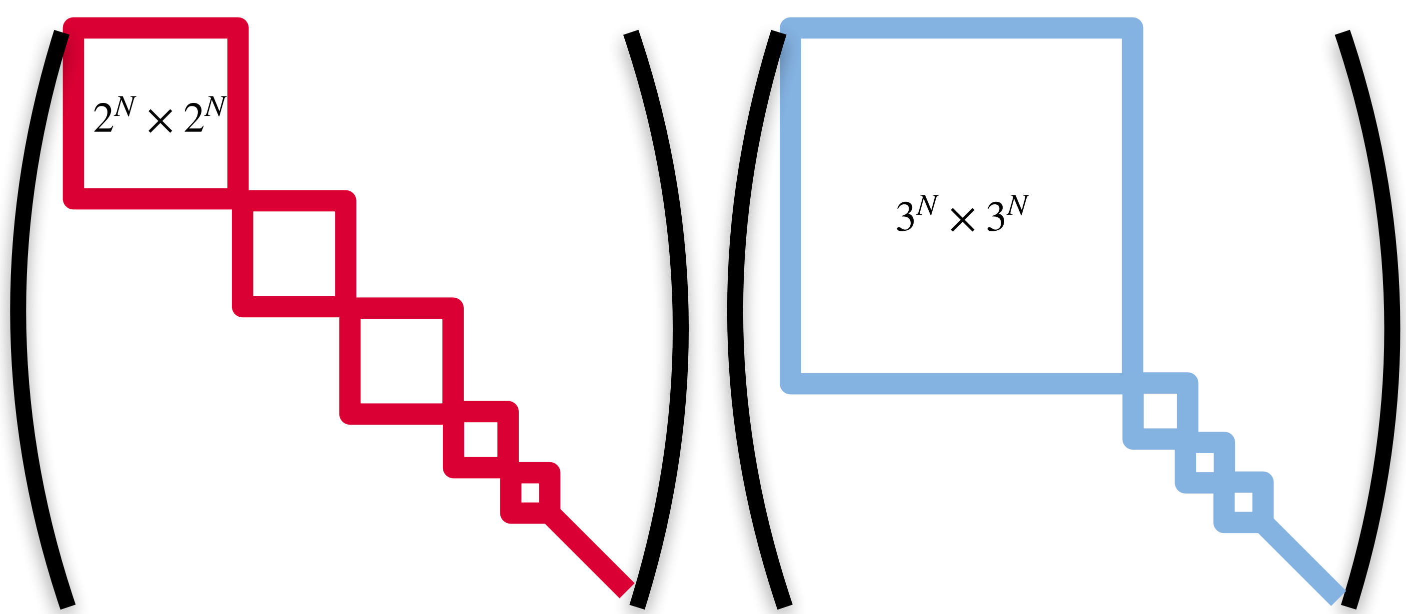
Related works:
Khemani, Nandkishore '20
Moudgalya et al. '20

[Sala, Rakovszky, Verresen, Knap, FP, PRX **10**, 011047 (2020)]

Hilbert space fragmentation

Fragmentation for an S=1 system

Fragmentation	Strong	Weak
# of sectors	~exp[N]	~exp[N]
Size of largest sector	$\sim 2^N \times 2^N$	$\sim 3^N \times 3^N$



[Sala, Rakovszky, Verresen, Knap, FP, PRX **10**, 011047 (2020)]

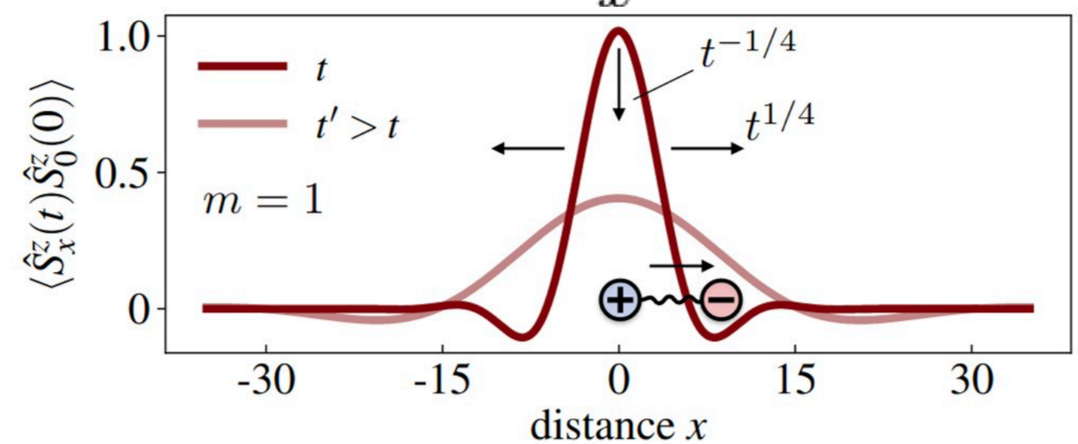
Hydrodynamics with higher-moment conservation

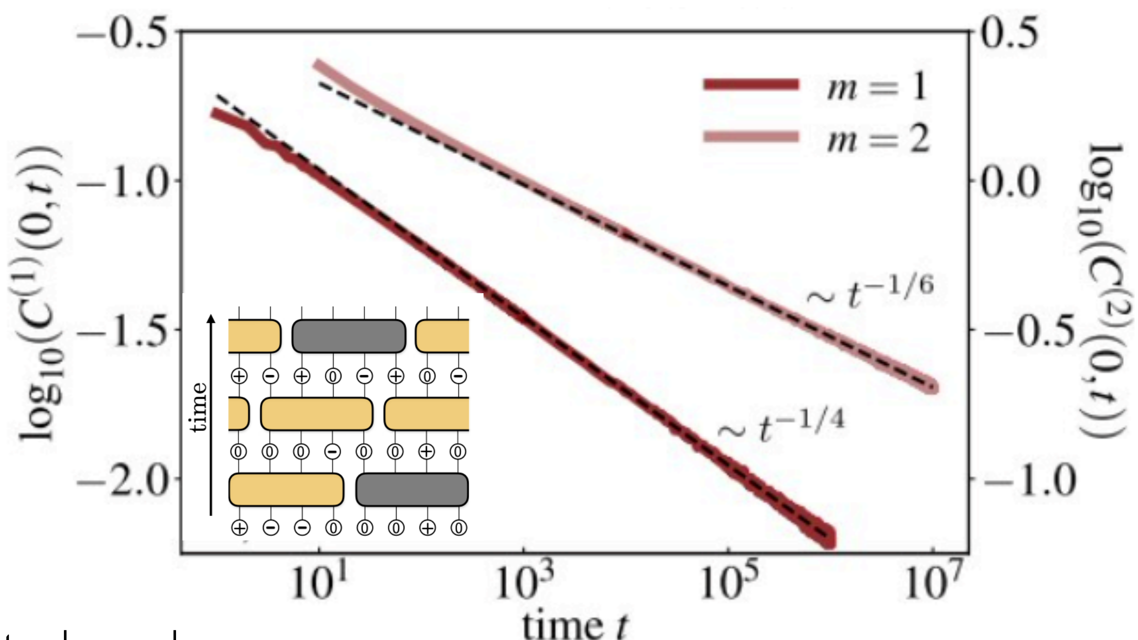
Universal late-time features in the weakly fragmented regime at infinite temperature:

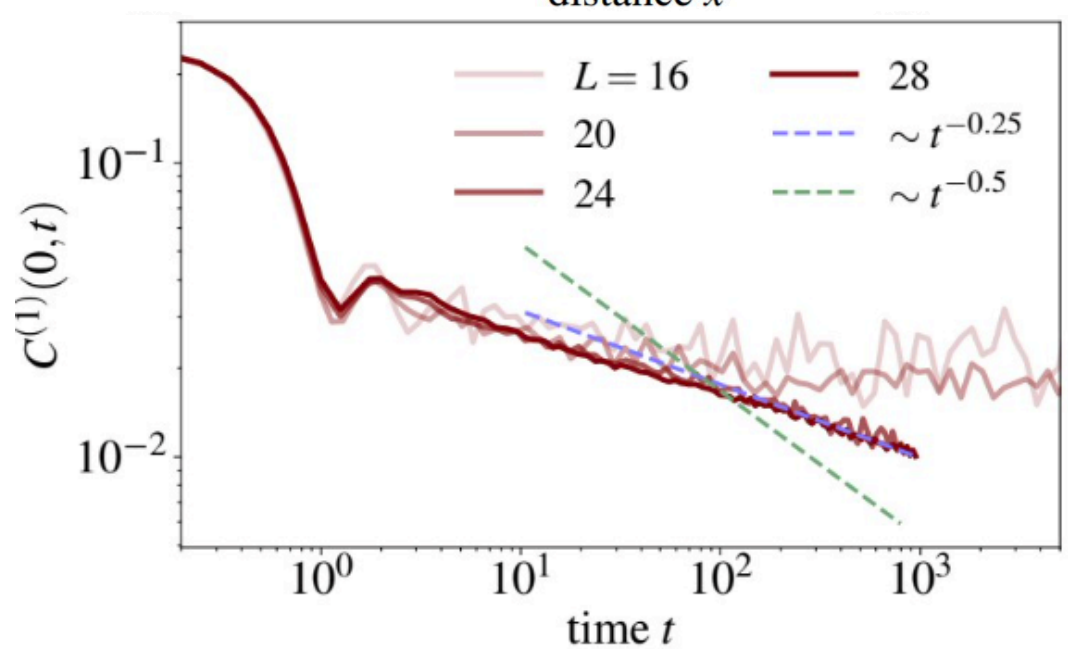
$$\mathcal{Q}^{(m)} = \sum_x x^m S^z_x$$

$$\partial_t \langle \hat{S}^z_x \rangle = -D(-1)^{m+1} \partial_x^{2(m+1)} \langle \hat{S}^z_x \rangle$$

$$C_0(t) \sim t^{-1/2(m+1)}$$





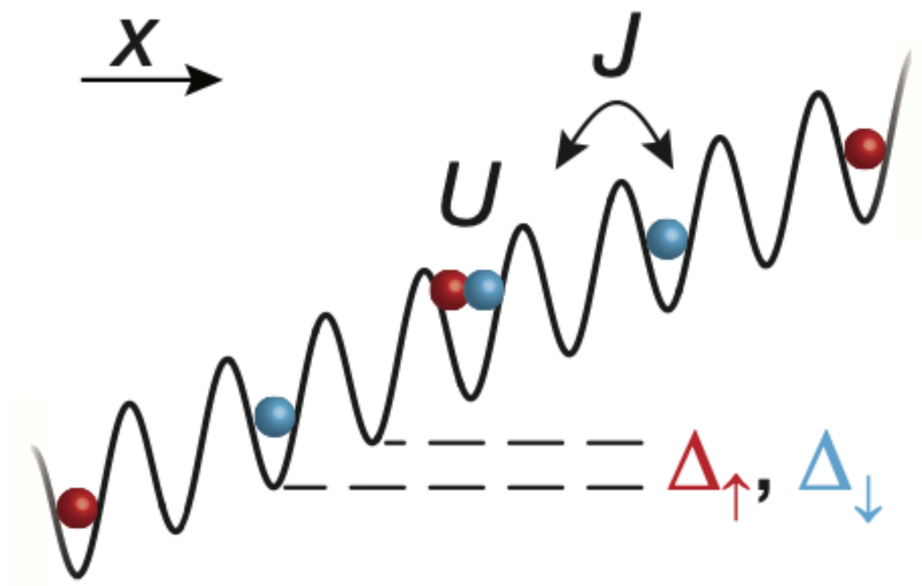


Related works:
Gromov et al. '20; Zhang '20;
Khemani, et al '20; Medenjak et al. '17
Morningstar et al. '20

[Feldmeier, Sala, De Tomasi, FP, Knap, PRL **125**, 245303, 2020]

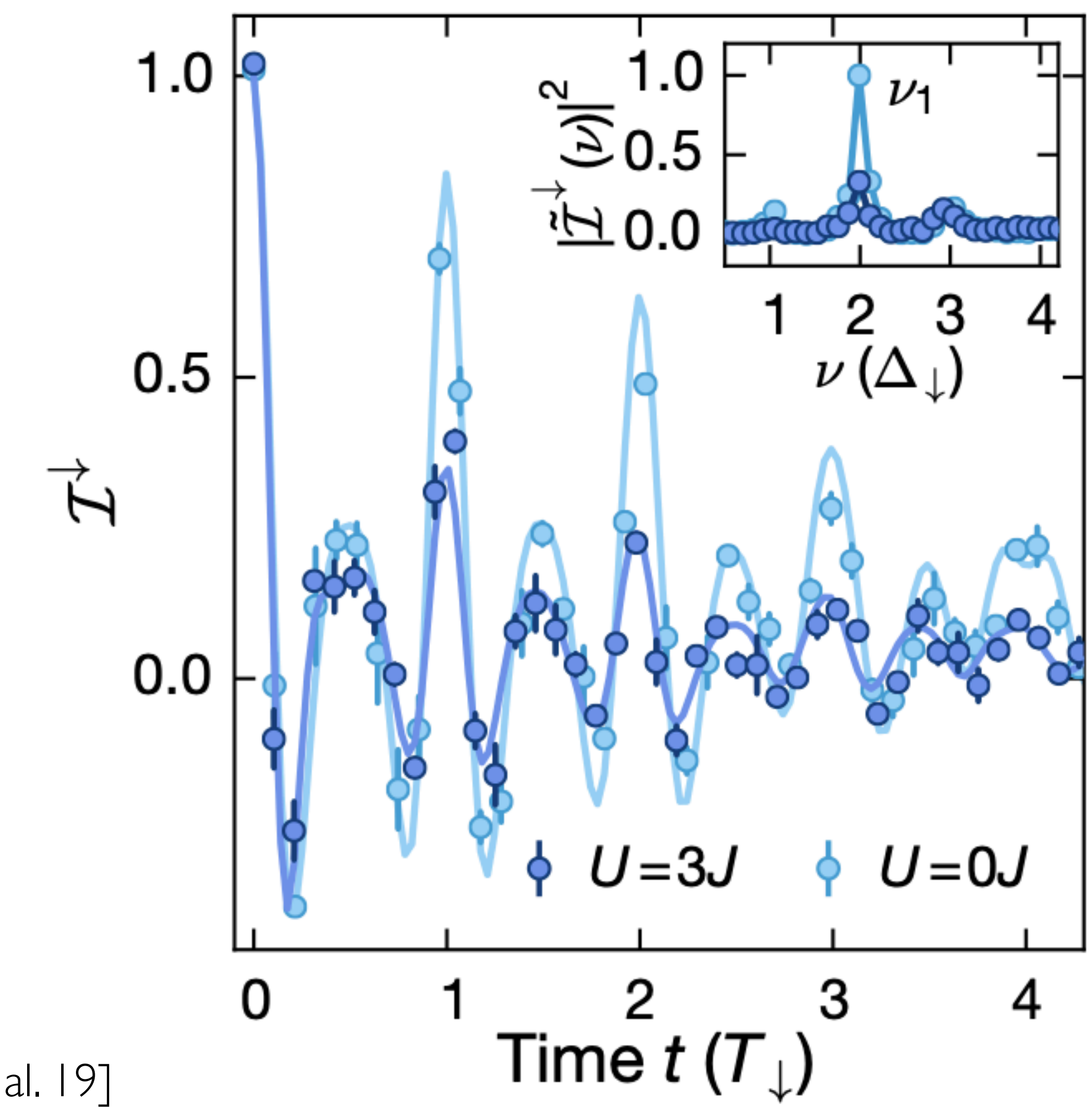
Experimental realization of Hilbert space fragmentation

Experimental setup and short timed dynamics



$$\hat{H} = \sum_{i,\sigma=\uparrow,\downarrow} \left(-J\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + \text{h.c.} + \Delta_\sigma i \hat{n}_{i,\sigma}\right) + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}\,,$$

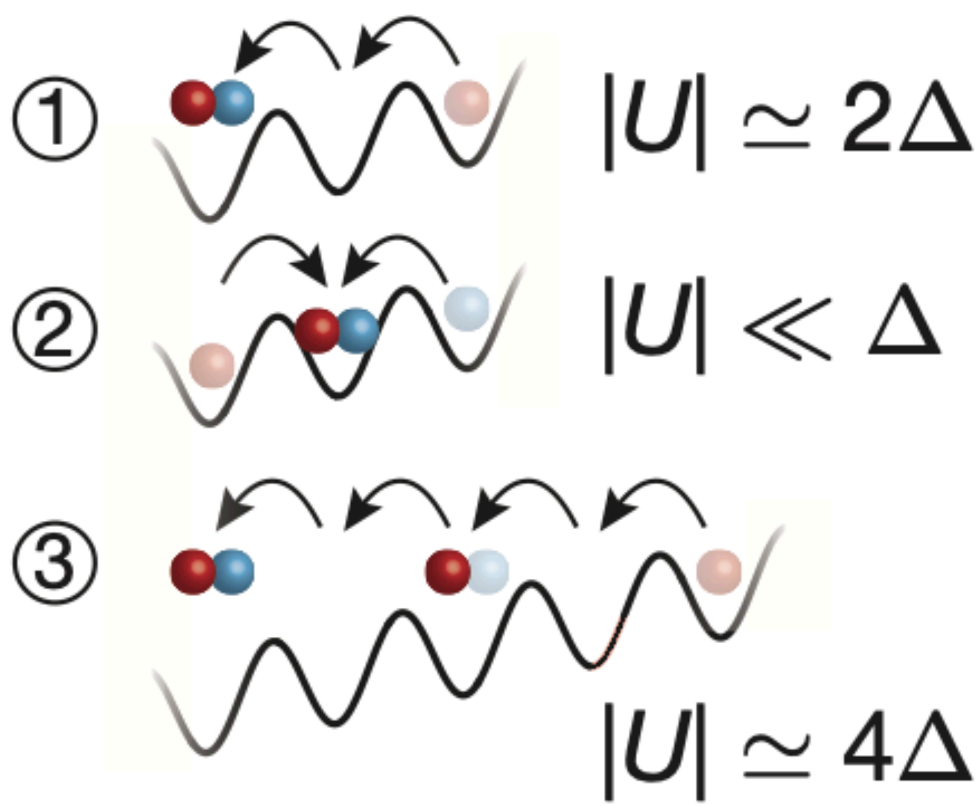
(Stark MBL in presence of perturbations) [Schulz et al. 19, van Nieuwenburg et al. 19]

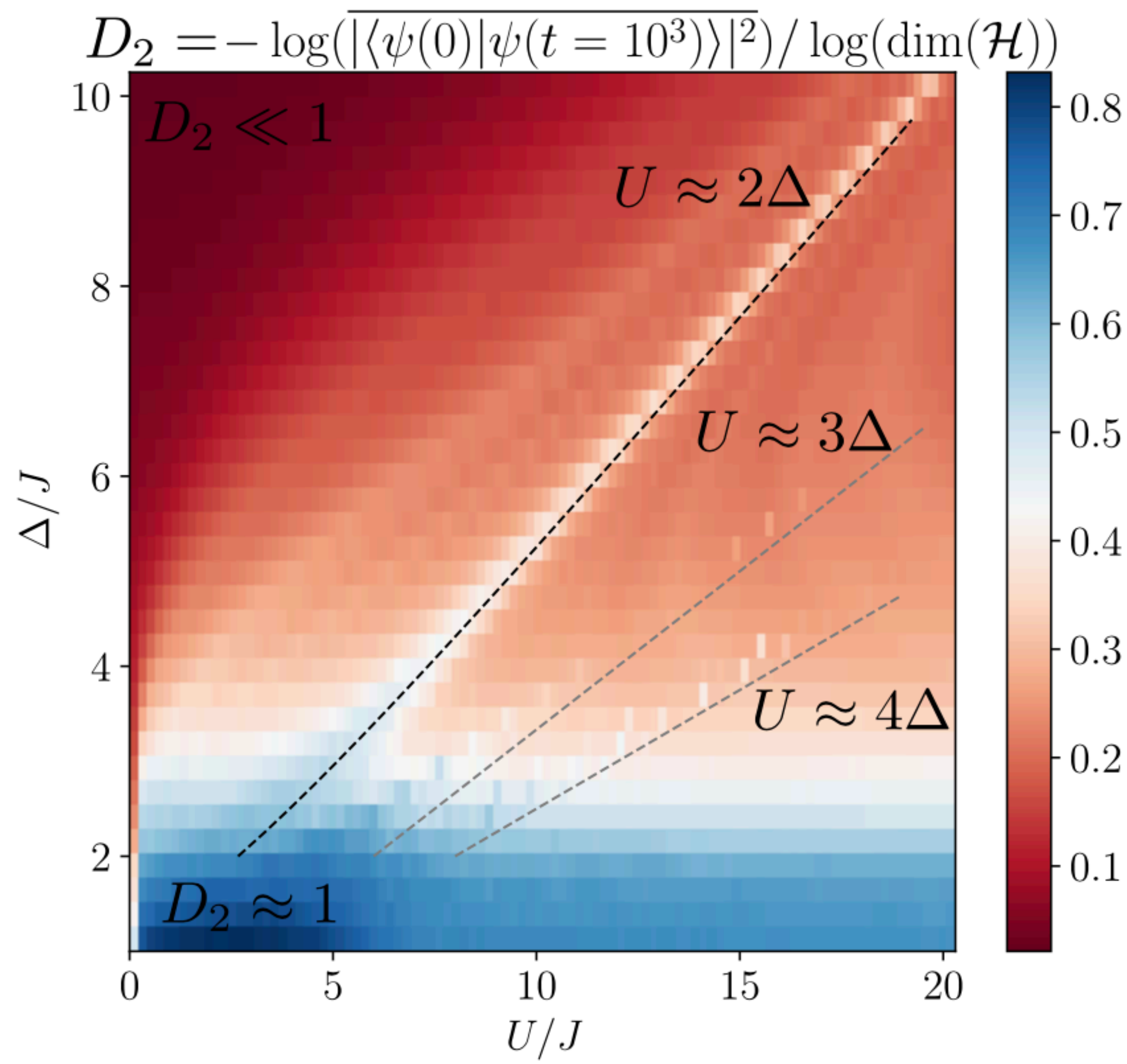


[Scherg et al., Nat Commun **12**, 4490 (2021)]

Constrained dynamics in tilted Fermi-Hubbard chains

Effective Hamiltonians in the large tilt/interaction limit: Constrained dynamics



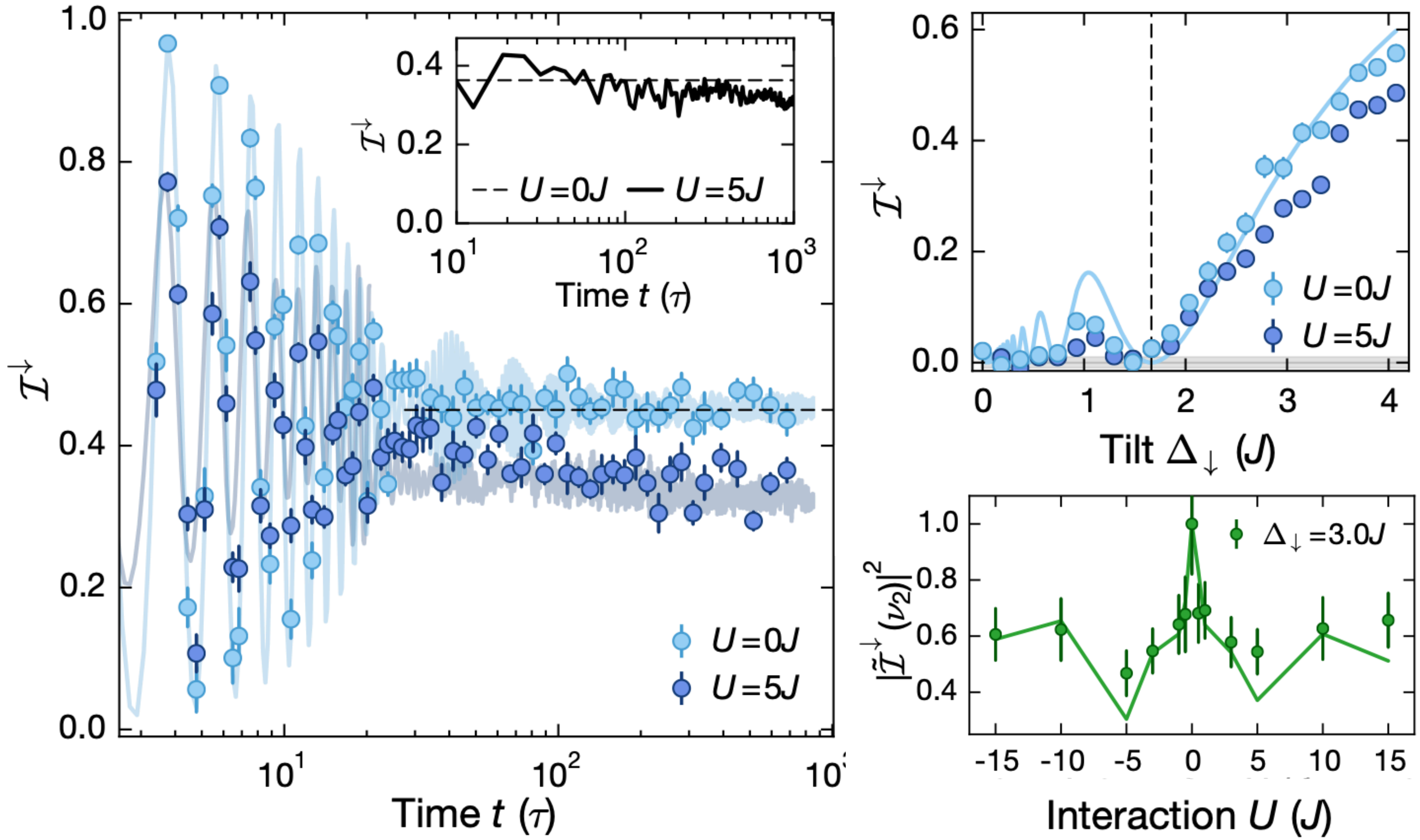


CDW Initial state of singlons

[Scherg et al., Nat Commun **12**, 4490 (2021)]

Constrained dynamics in tilted Fermi-Hubbard chains

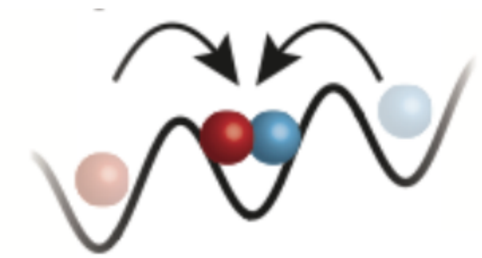
Long-time dynamics of the imbalance



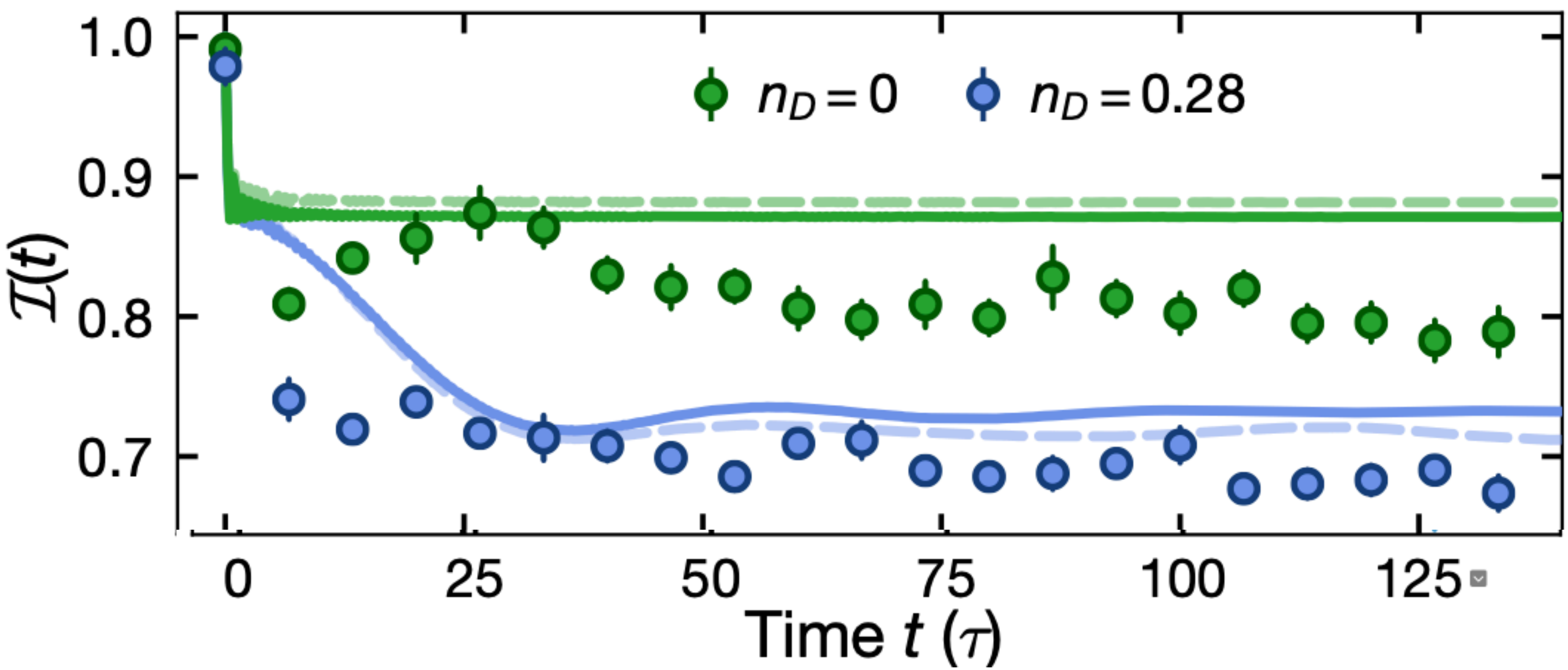
[Scherg et al., Nat Commun **12**, 4490 (2021)]

Constrained dynamics in tilted Fermi-Hubbard chains

Effective Hamiltonian dynamics
for $|U|, J \ll \Delta$



$\Delta/J = 8.0(2),\ U/J = 2.7(2)$



[Kohlert et al., arXiv:2106.15586]

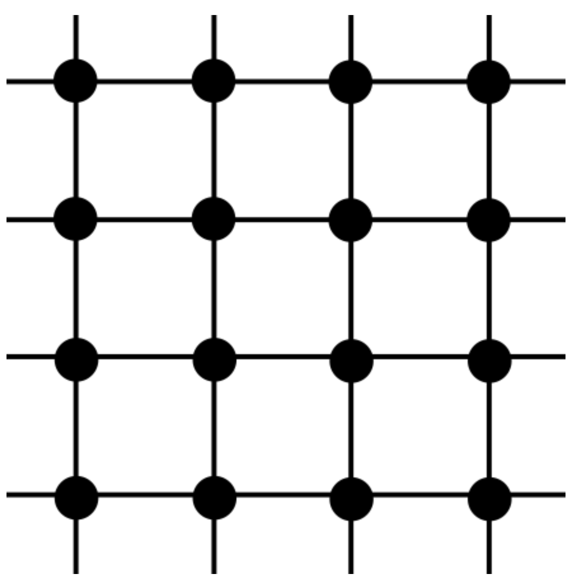
Hilbert space fragmentation in D > 1

Consider square lattice with higher moment conservation

Charge $\mathcal{Q}^{(0)} = \sum_{\boldsymbol{r}} s_{\boldsymbol{r}};$

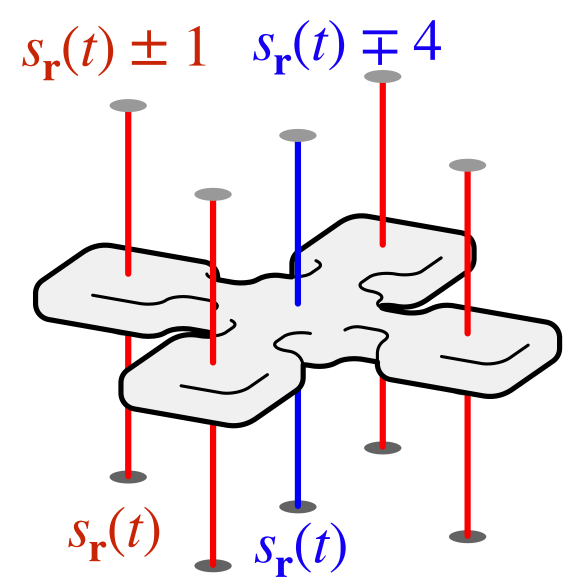
Dipole $\mathcal{Q}_x^{(1)} = \sum_{\boldsymbol{r}} x s_{\boldsymbol{r}},$ $\mathcal{Q}_y^{(1)} = \sum_{\boldsymbol{r}} y s_{\boldsymbol{r}};$

Quadrupole $\mathcal{Q}_{xy}^{(2)} = \sum_{\boldsymbol{r}} xy s_{\boldsymbol{r}},$ $\mathcal{Q}_{x^2-y^2}^{(2)} = \sum_{\boldsymbol{r}} (x^2 - y^2) s_{\boldsymbol{r}}$



$s_{\boldsymbol{r}} \in \{-S, -S+1, \ldots, S-1, +S\}$

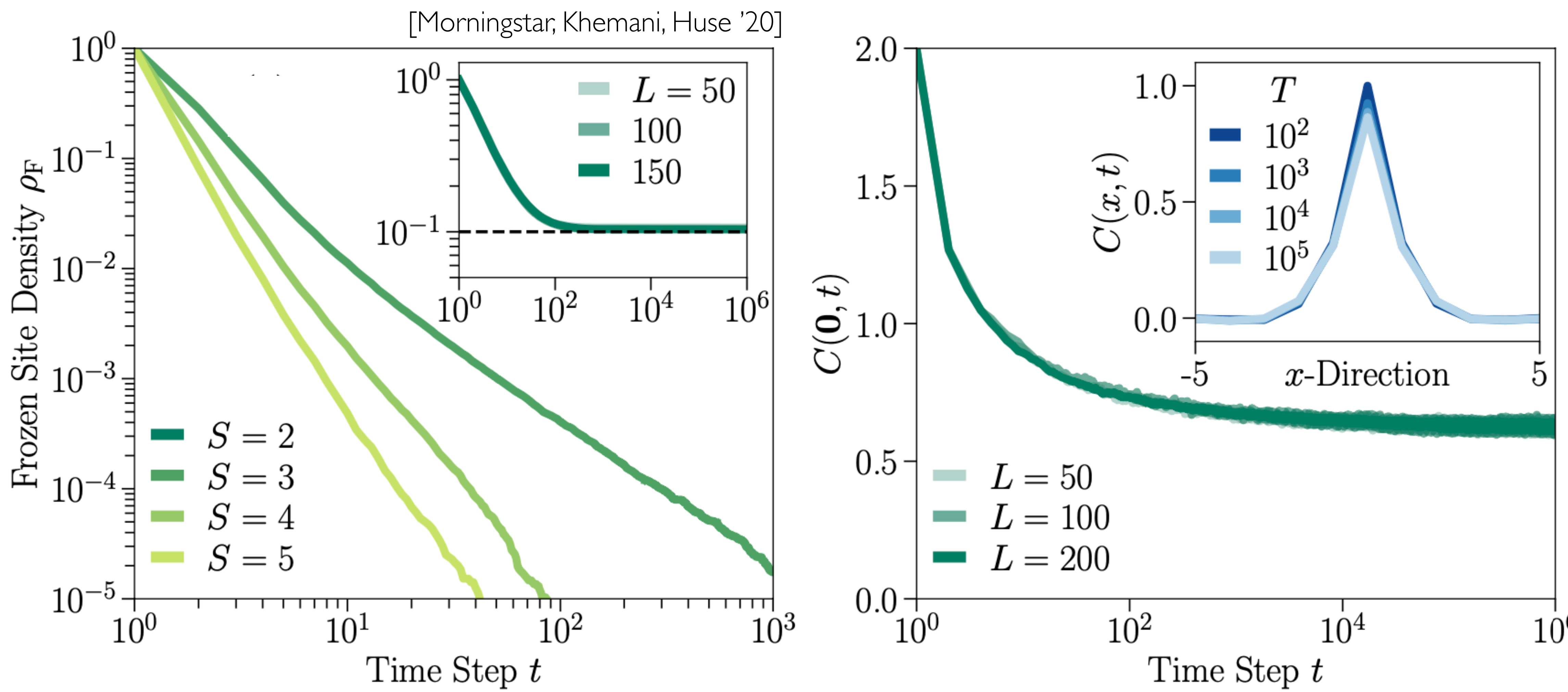
[cf. Khemani, Nandkishore '20; Khudorozhkov et al., 21]



[Sala, Lehmann, Rakovszky, FP (in progress)]

Hilbert space fragmentation in D > 1

Consider square lattice with higher moment conservation

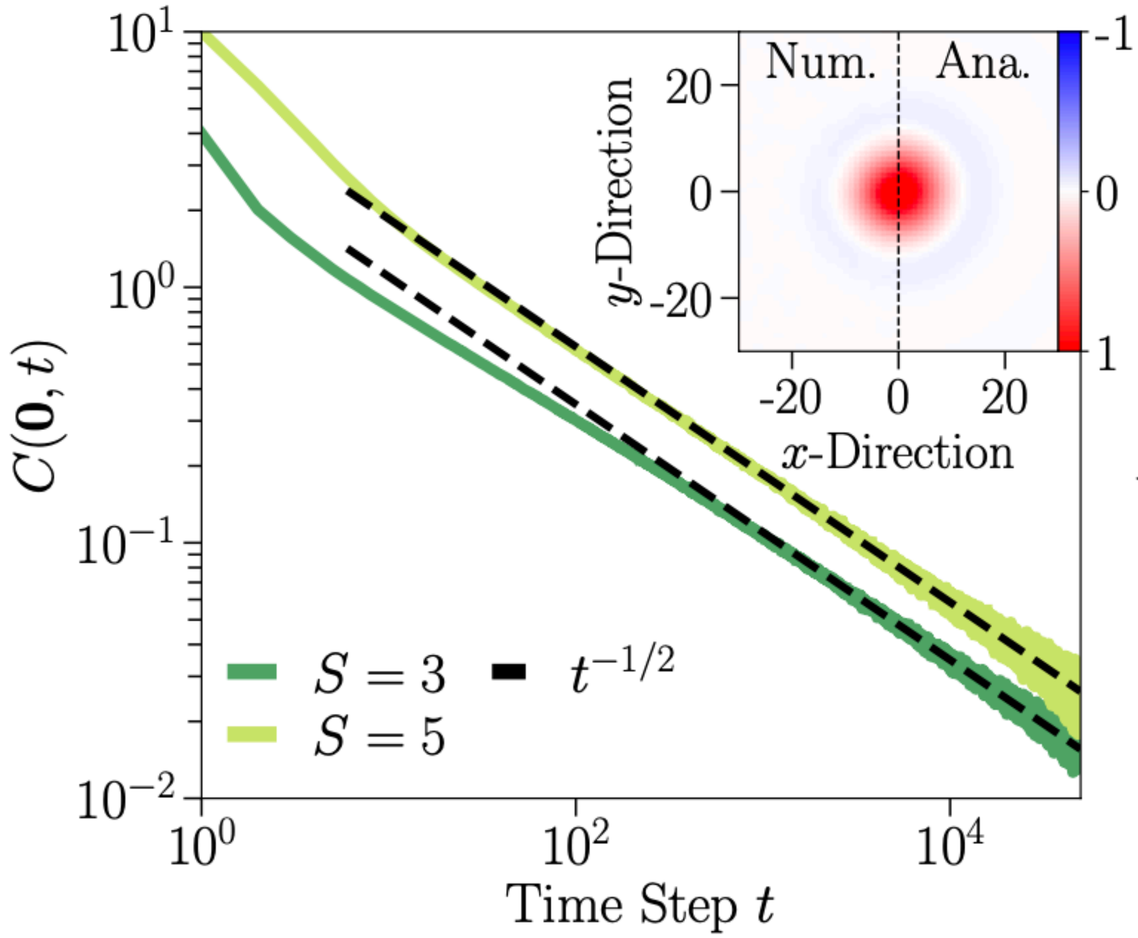


- Strong fragmentation for $S \leq 2$

[Sala, Lehmann, Rakovszky, FP (in progress)]

Hydrodynamic description

When diffusion conserves quadrupole moments



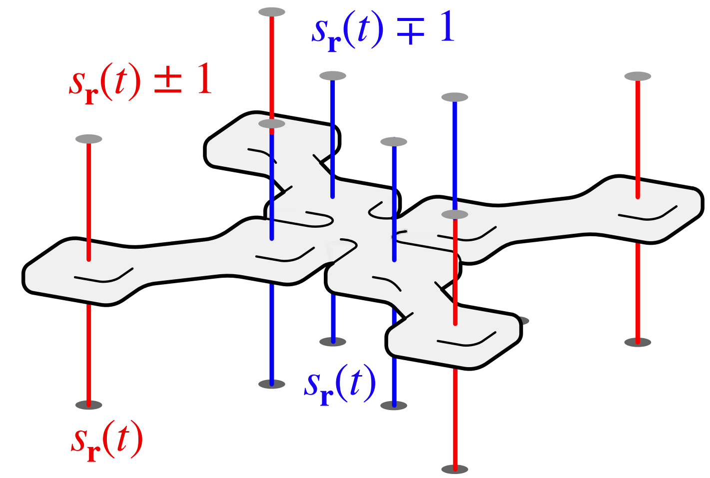
- Same description as for just dipole conservation!

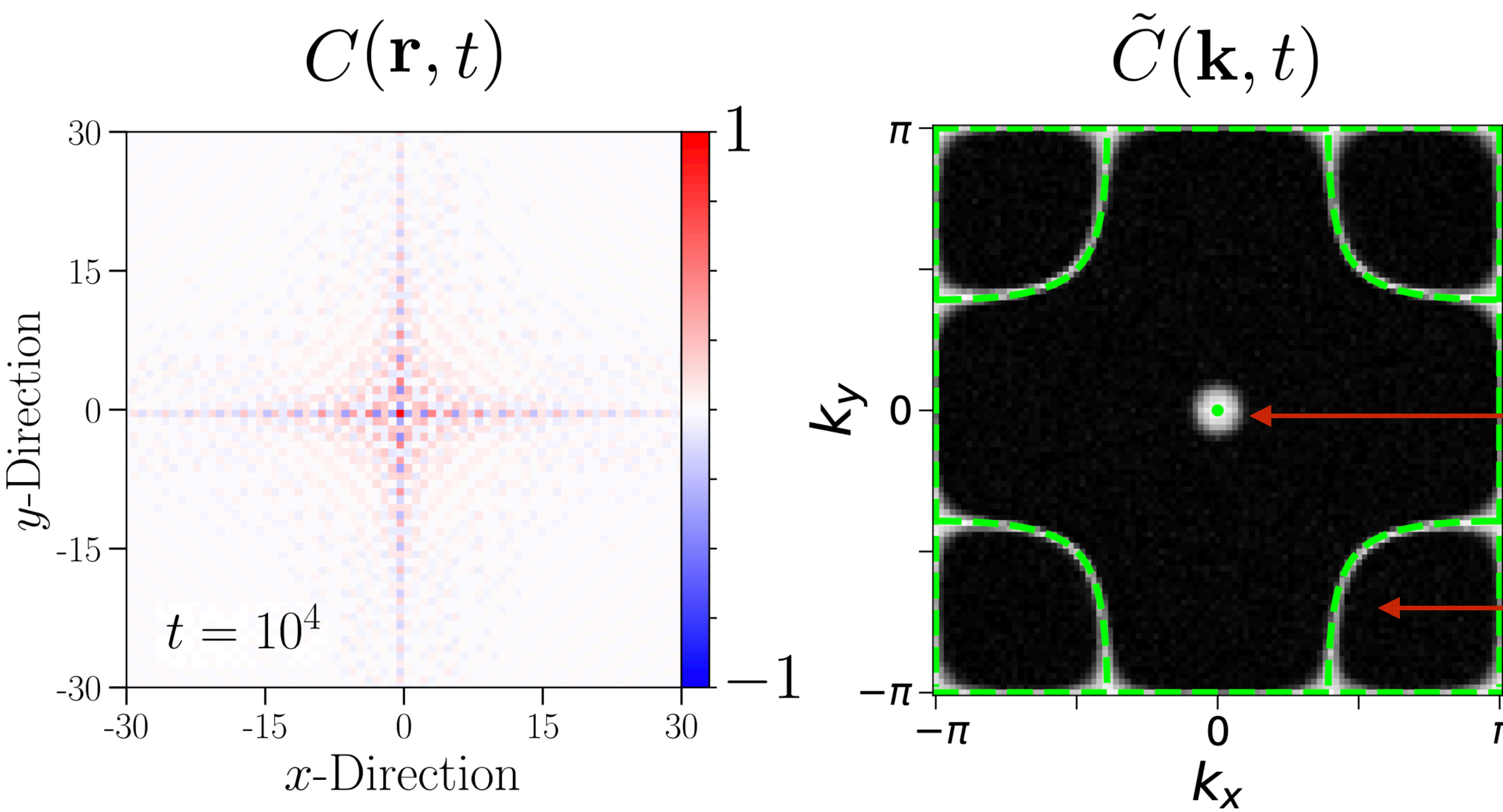
$$\partial_t\langle s_{\boldsymbol{r}}(t)\rangle + D\Delta^2\langle s_{\boldsymbol{r}}(t)\rangle = 0 \Rightarrow \quad C(0,t) \propto \frac{1}{\sqrt{Dt}}$$

[Sala, Lehmann, Rakovszky, FP (in progress)]

Exotic symmetries

Additional "modulated symmetries"





Subsystem Sym.

$$\sum_x e^{i\pi x} s_{x,y_0}$$

Moments

"Modulated Sym."

$$\sum_{\mathbf{r}} e^{i\tilde{\mathbf{k}}\cdot\mathbf{r}} s_{\mathbf{r}}$$

- Certain s_{k} are conserved! Hydrodynamic description?

[Sala, Lehmann, Rakovszky, FP (in progress)]

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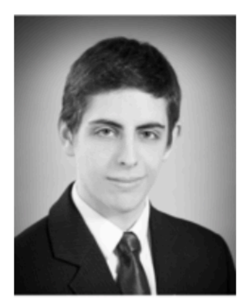
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Thank You!