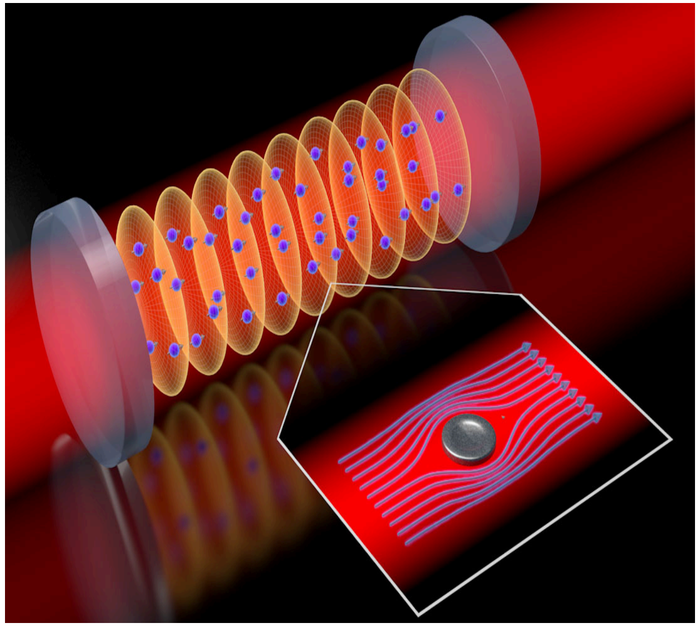
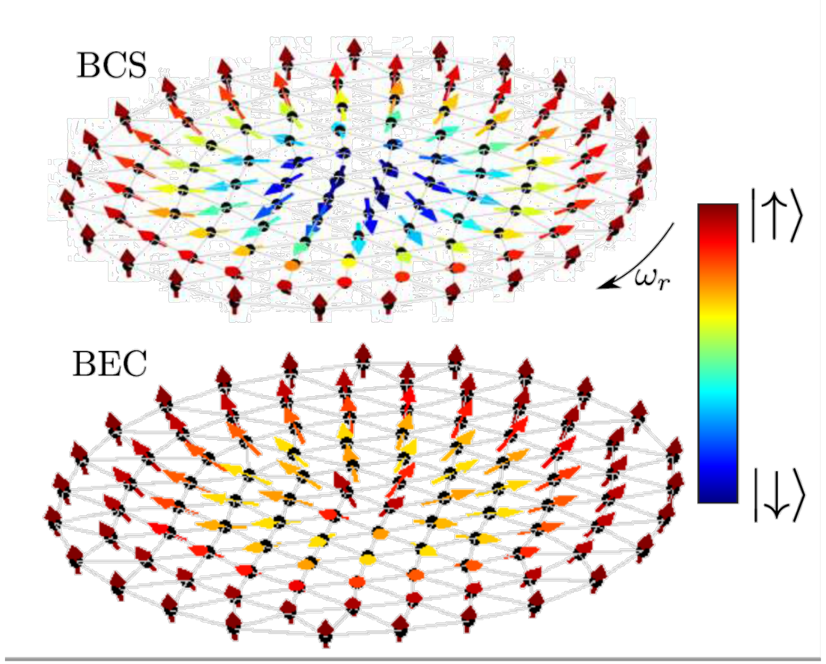
Simulating dynamical phases of BCS Superconductors with cavity QED and trapped ion systems

Ana Maria Rey

JILA NIST CU





Non-Equilibrium Universality, September 28th (2021)

Equilibrium Phase Transitions

Phase Transition: Abrupt (non-analytic) change in the free energy of a many-body system as a result of a slow changes in system's parameters, such as temperature (classical), or others control parameter (quantum @ T=0).

Typically describe by an **order parameter**: Quantity that changes non-analytically at the transition point.

Non-Equilibrium Phase Transitions

Here: Dynamical Phase Transitions (Closed systems, Unitary Dynamics)

A long-time (averaged) order parameter distinguishes two dynamical phases and features a non-analytic behavior.

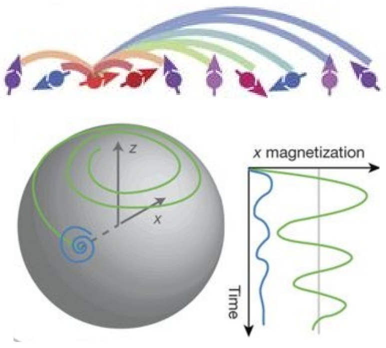
- ❑ Dynamical: Not found by minimization of free energy
- ❑ New symmetries
- ❑ Robust generation of useful entangled states

Other Definitions: Innsbruck, PRL(2017) & Hamburg, Nat. Phys (2018), M. Heyl, EPLA (2019)

Great deal of experimental interest

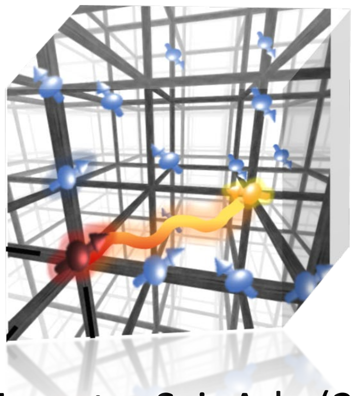
53 ions

Several Thousands to Million



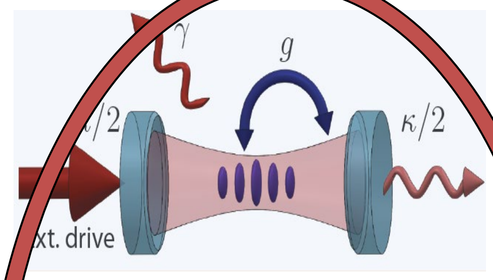
JQI, Nature(2018)

Using power law interactions in a 1D ion chain



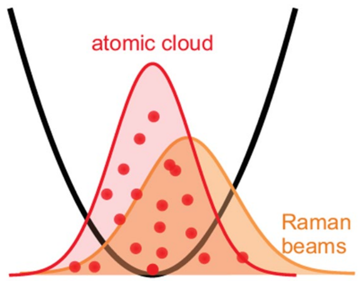
Toronto, Sci. Adv (2019)
10.1126/sciadv.aax1568

Using a Feshbach resonance in a quantum degenerate gas of fermionic K atoms in a dipole trap



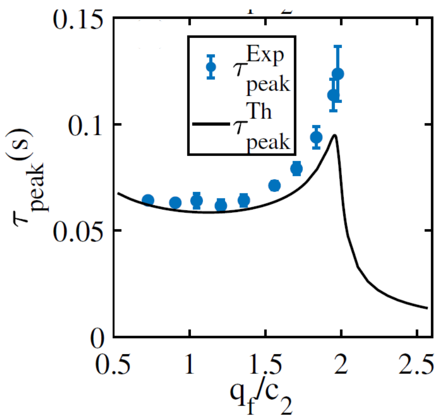
JILA, Nature 580,602 (2020)

Using cavity mediated interactions between Sr atoms in an optical cavity



Hannover, PRL 125, 240504(2020)

Using a sideband transition in a thermal gas of Rb atoms



Tsinghua, PRL 124, 043001 (2020)

Using spin changing collisions in a Na BEC

Even richer physics in the presence of dissipation: Esslinger, Lev, Hemmerich

Simulating a BCS Superconductor

S-wave BCS Hamiltonian in momentum space k

$$\hat{H} \sim -\chi \sum_{\boldsymbol{kq}} \hat{c}^{\dagger}_{-\boldsymbol{q}\uparrow} \hat{c}^{\dagger}_{\boldsymbol{q}\downarrow} \hat{c}_{-\boldsymbol{k}\uparrow} \hat{c}_{\boldsymbol{k}\downarrow} + \sum_{\boldsymbol{k}\sigma} \varepsilon_k \hat{c}^{\dagger}_{\boldsymbol{k}\sigma} \hat{c}_{\boldsymbol{k}\sigma}$$

$\varepsilon_k = \frac{\hbar^2 k^2}{2m} - \mu$: Single-Particle Dispersion

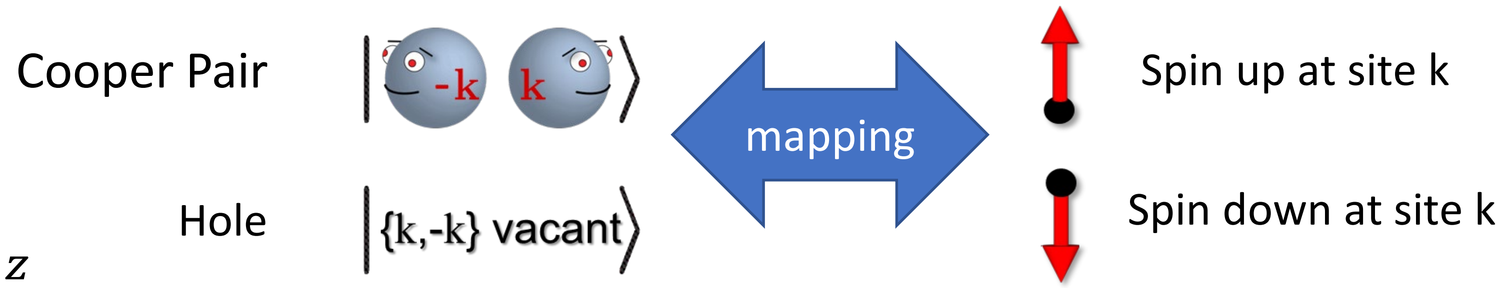
Attractive s − wave Interactions

Anderson Pseudo-spins

Relevant Hilbert space

$$\hat{c}_{-\boldsymbol{k}\downarrow} \hat{c}_{\boldsymbol{k}\uparrow} \Leftrightarrow \hat{s}^{-}_{\boldsymbol{k}}$$

$$\hat{c}^{\dagger}_{\boldsymbol{k}\uparrow} \hat{c}_{\boldsymbol{k}\uparrow} + \hat{c}^{\dagger}_{-\boldsymbol{k}\uparrow} \hat{c}_{-\boldsymbol{k}\uparrow} - 1 \Leftrightarrow 2\hat{s}^{z}_{\boldsymbol{k}}$$

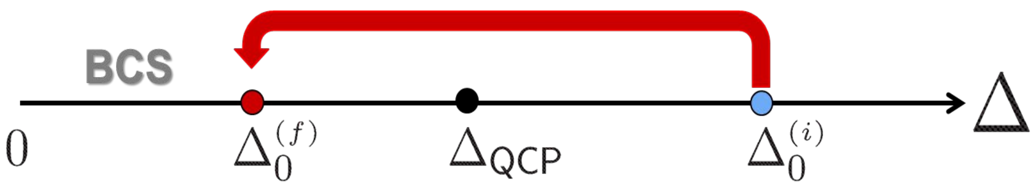


$$H \sim \sum_{\boldsymbol{k}} 2\varepsilon_{\boldsymbol{k}} \hat{s}^{z}_{\boldsymbol{k}} + \chi \sum_{\boldsymbol{k}q} \hat{s}^{+}_{\boldsymbol{k}} \hat{s}^{-}_{\boldsymbol{q}} = \sum_{\boldsymbol{k}} 2\varepsilon_{\boldsymbol{k}} \hat{s}^{z}_{\boldsymbol{k}} + \chi \hat{S}^{+} \hat{S}^{-}$$

- **Integrable (Richardson-Gaudin model)**

Dynamical Phases of BCS Model

$\Delta \equiv \chi \langle \hat{S}^- \rangle$ Order parameter or gap function



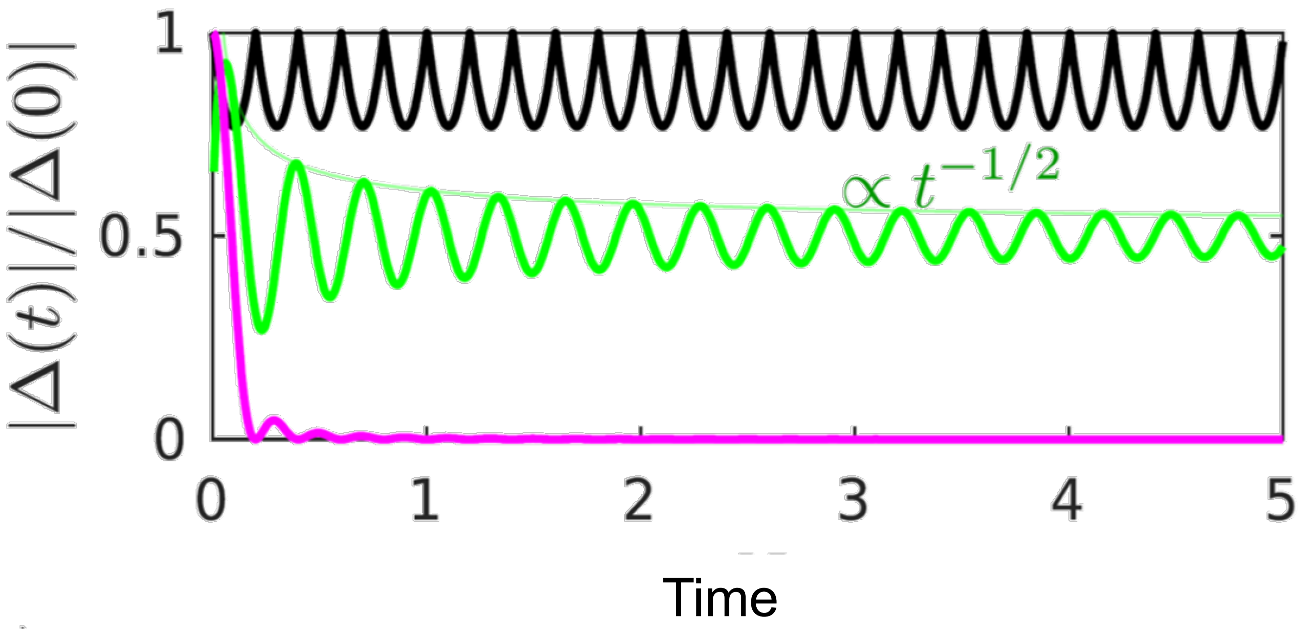
Gap dynamics

Method: Self-consistent non-equilibrium mean field theory

(Foster, Gurarie, Yuzbashyan,Levitov…)

- Exact solution to nonlinear classical spin dynamics via integrability,
- Lax construction: Frequency spectrum ruling dynamics

Three Dynamical Phases



III Persistent Oscillationa $|\Delta(\infty)| = \Delta_f(t)$

II Nonzero constant $|\Delta(\infty)| = \Delta_f$

I Dephasing $|\Delta(\infty)| = 0$

Dynamical Phases of BCS Model

REPORTS

week ending
15 OCTOBER 2004

- Hard in real materials.
- Need of ultra-fast pulses
- Small quenches

ondensate

54, USA

ado 80309, USA

, Texas 7700, USA

15)

Ryusuke Matsunaga,[1*] Naoto Tsuji,[1] Hiroyuki Fujita,[1] Arata Sugioka,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3†] Hirotaka Terai,[2] Zhen Wang,[2‡] Hideo Aoki,[1,4] Ryo Shimano[1,5*]

Can we see them in Cold atoms?

- Require too cold temperatures for observation in ultracold paired fermionic gases

Using internal levels in trapped quantum degenerate gas: Phase I and Phase II

Smale et al., Sci. Adv. 2019;5: eaax1568 J. Thywissen group at U. Toronto

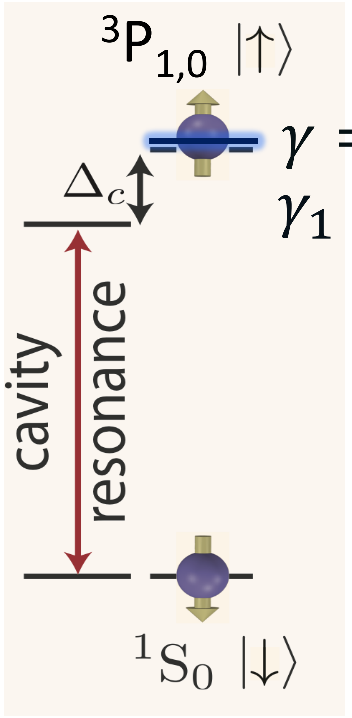
Using Cavity QED systems: Phase I-III

- *Science* **361**, 6399, 259 (2018) ; *Nature* **580** 602-607 (2020) ; PRL **126**, 173601(2021) J. Thompson group, JILA

Strontium: New Regime of Cavity QED

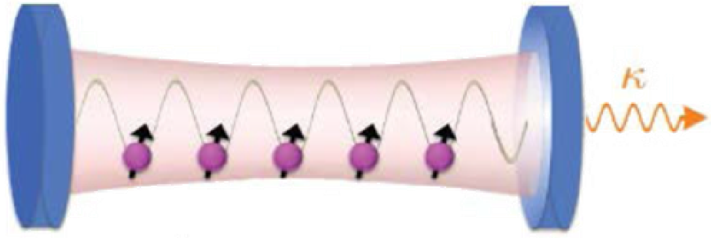
J. Thompson

Ultra-long lived optical excited states



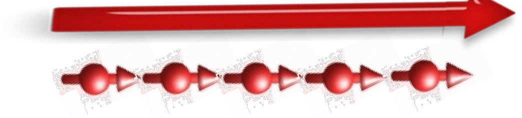
$\gamma = 2\pi \times 1 \text{ mHz}$

$\gamma_1 = 2\pi \times 7.5 \text{ kHz}$



$\kappa = 2\pi \times 160\ kHz$

Collective spin



$$\hat{H} = -\Delta_c \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger \hat{S}^- + \hat{a}\hat{S}^+)$$

$$\Delta_c \gg \kappa, \sqrt{N} g \gg \gamma$$

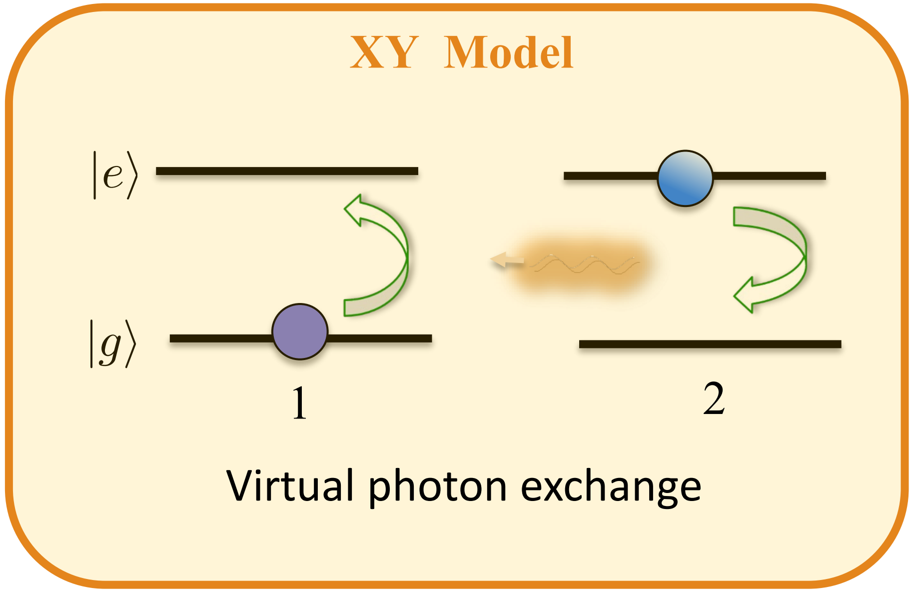
"Far detuned " limit

Field follows atomic dipole

$$\hat{a}^\dagger \approx \left(\frac{g}{\Delta_c - i\kappa/2}\right)\hat{S}^+$$

Unitary Spin Exchange Interactions

$$\hat{H} = \chi \hat{S}^+ \hat{S}^- \propto \sum_{i,j}^{N} \hat{\sigma}_i^+ \hat{\sigma}_j^- \qquad \chi = \frac{g^2 \Delta_c}{(\kappa/2)^2 + {\Delta_c}^2}$$



Similar Exchange Hamiltonian:

- Polar Molecules
- Magnetic Atoms
- Rydberg Atoms
- Trapped ions
- Cavities (Raman transitions)

Here: Collectively Enhanced and Infinite Range

Superradiant Emission

$$\mathcal{L}_{spin} = \sqrt{\Gamma}\hat{S}^-$$

$$\Gamma = \frac{\ldots\kappa}{(\kappa\ldots + {\Delta_c}^2} \qquad \frac{\chi}{\Gamma} = \frac{\Delta_c}{\kappa} \gg 1$$

Superradiant emission negligible if $\Delta_c \gg \kappa$

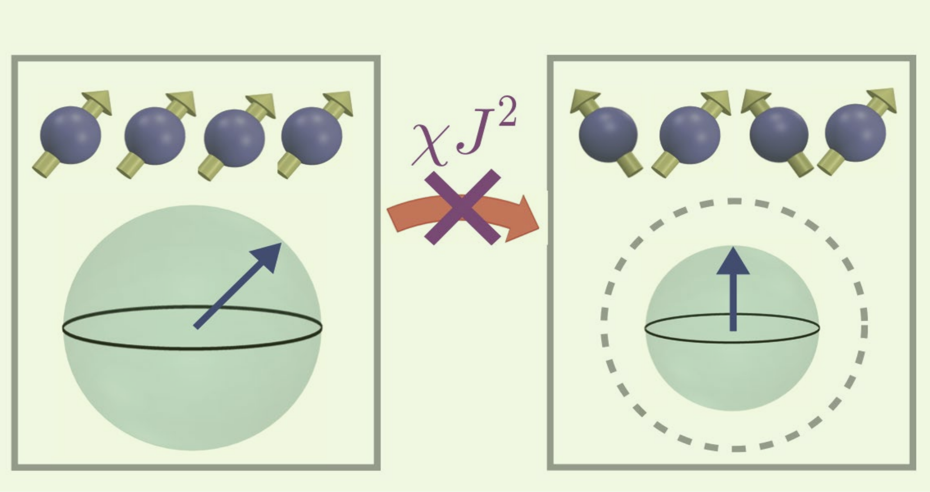
Focus on unitary dynamics

Dynamical Consequences of Spin-Exchange

Norcia, Lewis-Swan, Cline, Zhu, Rey, Thompson, *Science* **361**, 6399, 259 (2018)

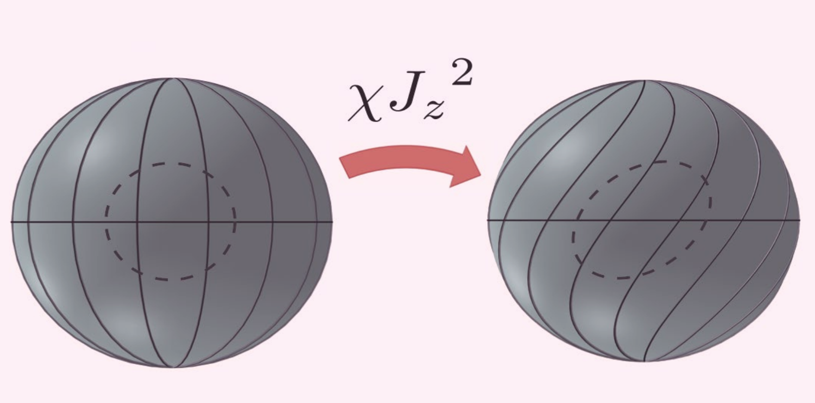
$$\hat{H} = \chi \hat{S}^{+}\hat{S}^{-} = \chi\left(\hat{S}^{2} - \hat{S}_{z}^{2}\right)$$

Gap protection:
Energy cost for changing *S*



Enhanced quantum coherence

One axis twisting:
Precession rate depends on S_z



Entanglement generation

See also E. J. Davis et al Monika H. Schleier-Smith Phys. Rev. Lett. 125, 060402 (2020)

Gap Protection

$$\hat{H} = \chi \hat{S}^+\hat{S}^- = \chi\left(\hat{\mathbf{S}}^{2} - \hat{S}_z^{2}\right)$$

In the collective Dicke manifold the dynamics of $\chi\hat{S}^+\hat{S}^-$ and $-\chi\left(\hat{S}_z^{2}\right)$ are identical:

$\hat{S}^{2}$ acts as a constant of motion

But they are different in the presence of dephasing

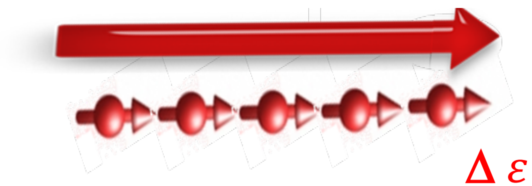
$$\hat{H}_{env} = \frac{1}{2}\sum_{i=1}^{N}\varepsilon_i\sigma_i^z$$

Dephasing can be detrimental in OAT

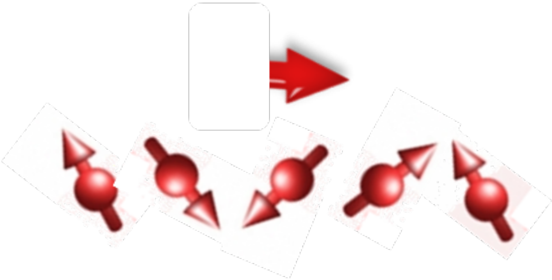
ε_i: random variables: noise

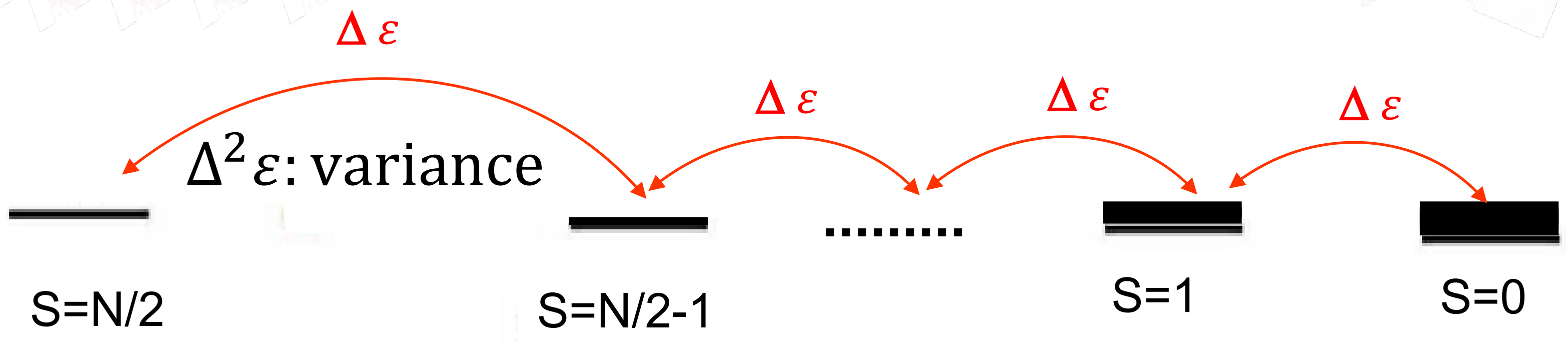
$$\hat{H} = -\chi\hat{S}_z^{2}$$

$$\left(\vec{S}\cdot\vec{S}\right)|S,M\rangle = \mathbf{S}(\mathbf{S}+\mathbf{1})\,|S,M\rangle$$



H_{env} does not conserve S



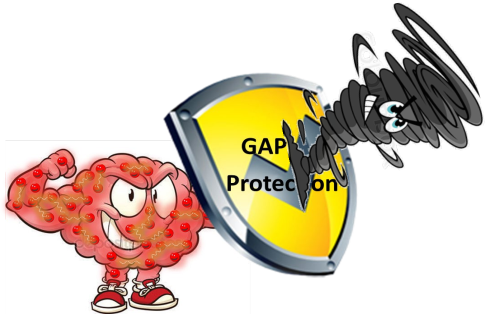


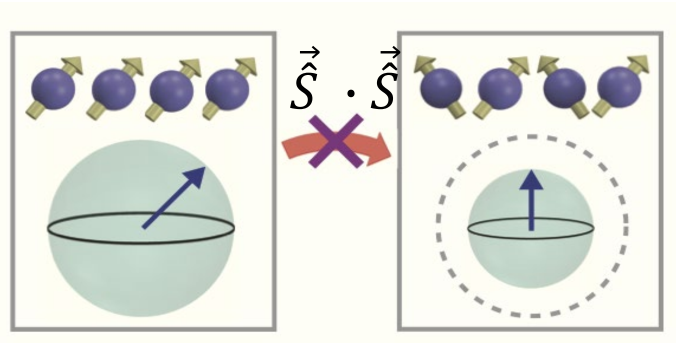
Gap Protection: Control of Decoherence

Idea: Gapped Hamiltonians create states that are only sensitive to global perturbations. NOT sensitive to local perturbations: **Many body protection**

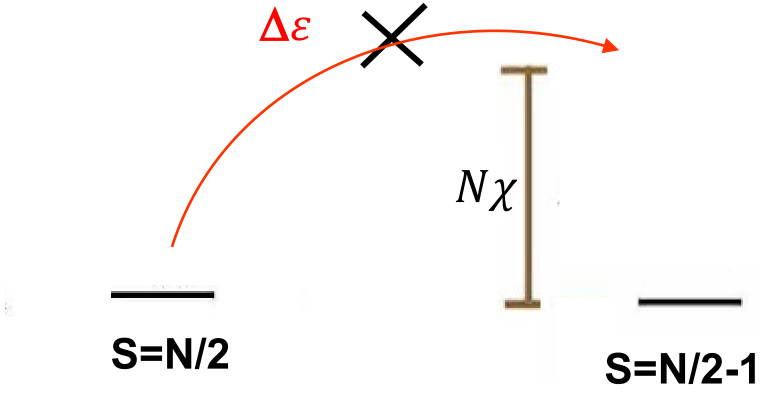
$$H = \hat{H}_z + \hat{H}_{env} + \hat{H}_{prot} = -\chi \hat{S}_z^{\,2} + \frac{1}{2}\sum_{i=1}^{N} \varepsilon_i \sigma_i^z + \chi \hat{S}^2$$

Gap protection





Stabilizes alignment

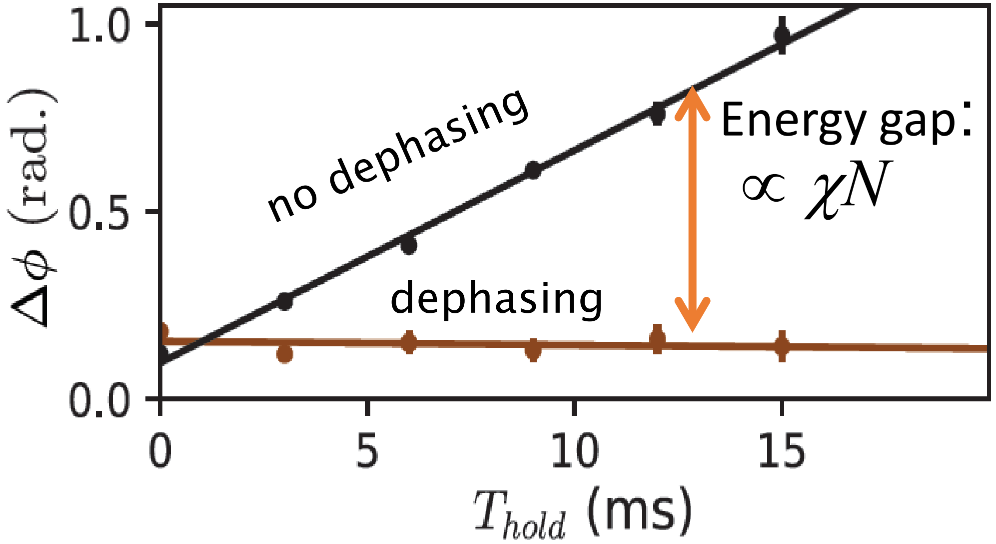


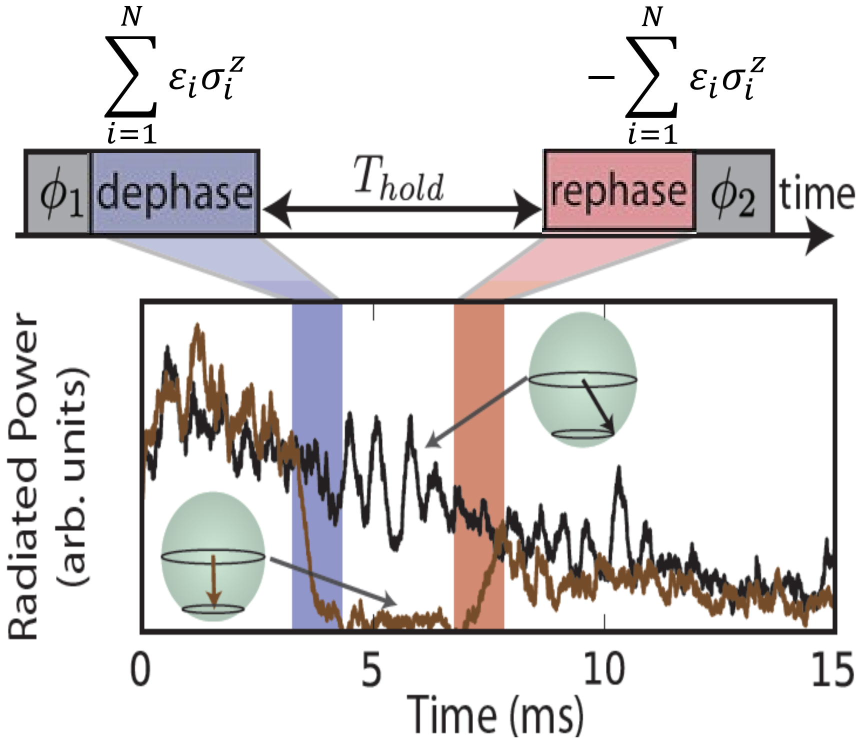
Gap: Exchange energy change from

$S = N/2 \rightarrow S = N/2 - 1$

Rey *et al* PRA 77, 052305(2008)

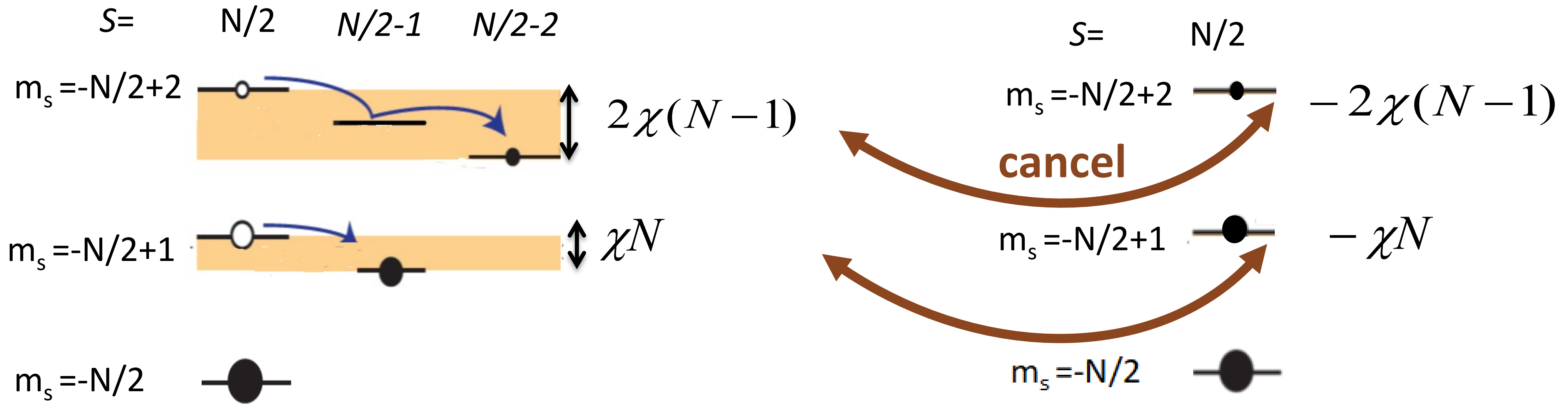
Can we measure the gap?





$$\hat{H} = \chi \hat{S}^2$$

$$\hat{H} = -\chi \hat{S}_z^2$$



Simulating Phases I,II & III in our Cavity

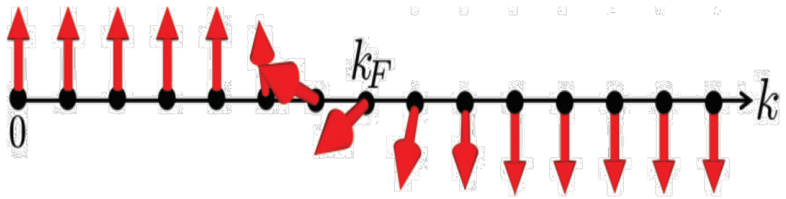
momentum k→ position x

$$H = \sum_j 2\varepsilon_j \hat{s}_j^z - \chi \hat{S}^+ \hat{S}^-$$

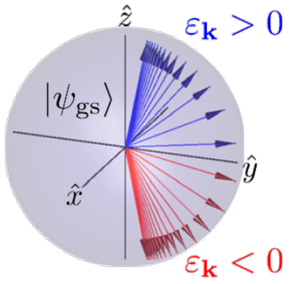
Photon-mediated interactions

BCS Ground State

$\varepsilon_k > 0$ $\varepsilon_k < 0$



$$\langle \hat{\sigma}_{\mathbf{k}}^z \rangle_{\mathrm{gs}} = \frac{\varepsilon_{\mathbf{k}}}{\sqrt{\Delta_{\mathrm{gs}}^2 + \varepsilon_{\mathbf{k}}^2}},$$

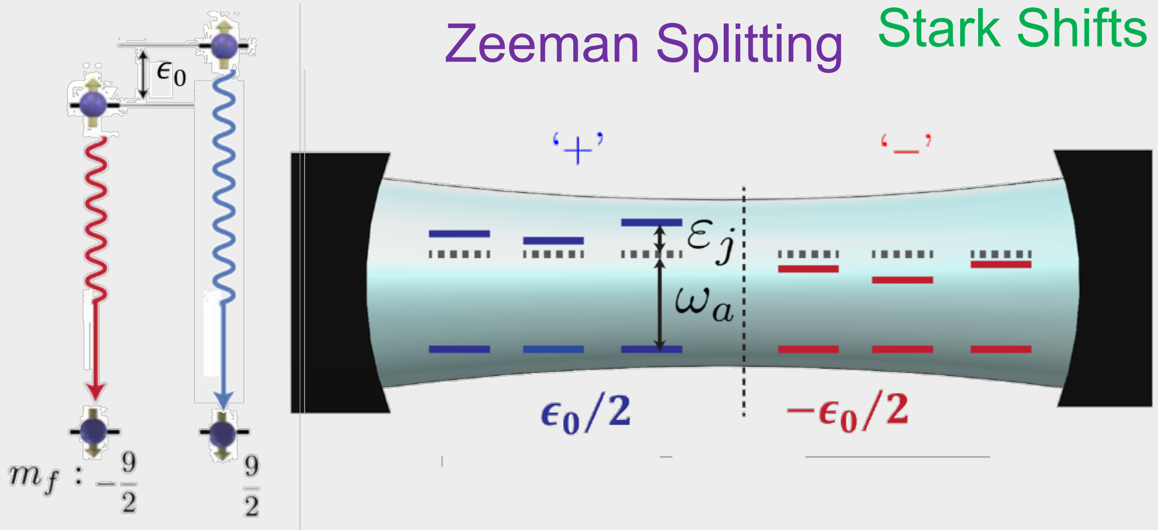


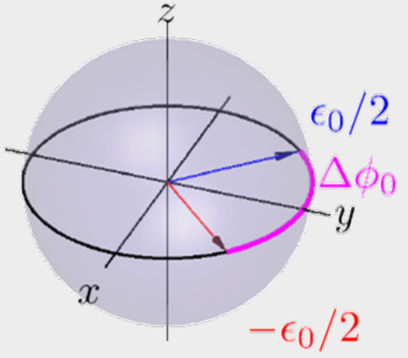
Axial magnetization correlated with sign of ε_k

Engineered Dispersion to emulate Fermi Sea

Two ensembles $\varepsilon_j = \pm\frac{\epsilon_0}{2} + \eta_j$ $\eta_j \in [-W, W]$

Zeeman Splitting Stark Shifts



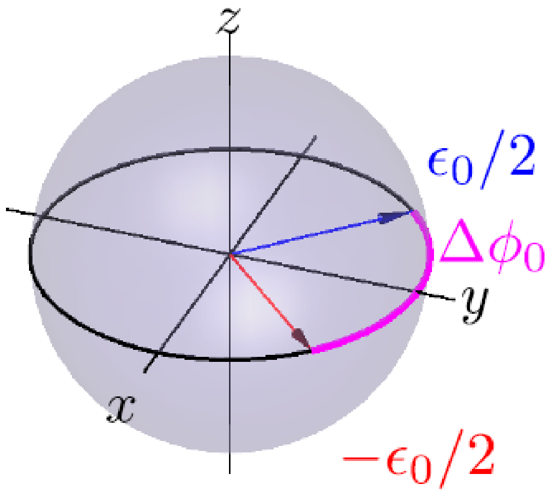


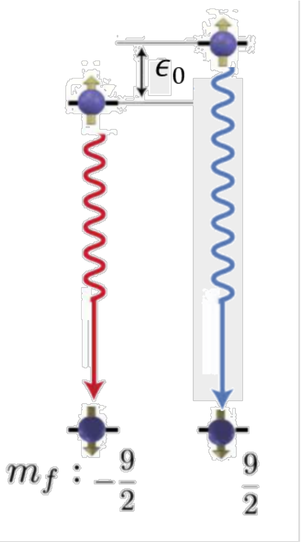
Transverse magnetization correlated with sign of ε_k

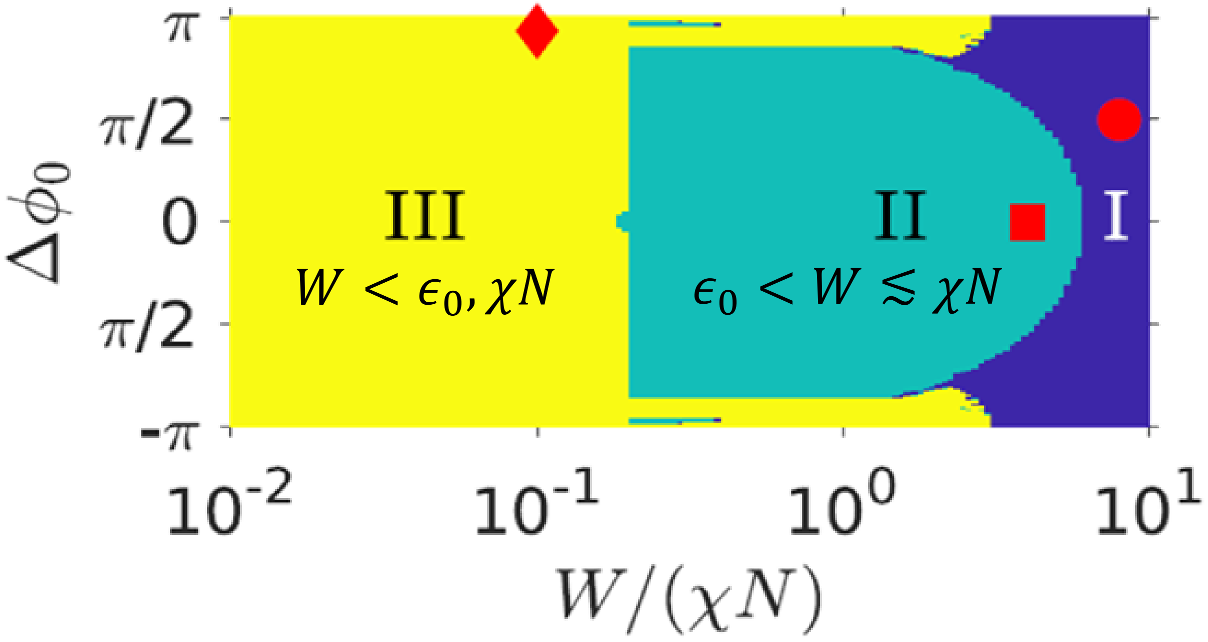
R. Lewis-Swan *et al* PRL 126, 173601(2021)

Simulating Phases I,II & III in our Cavity

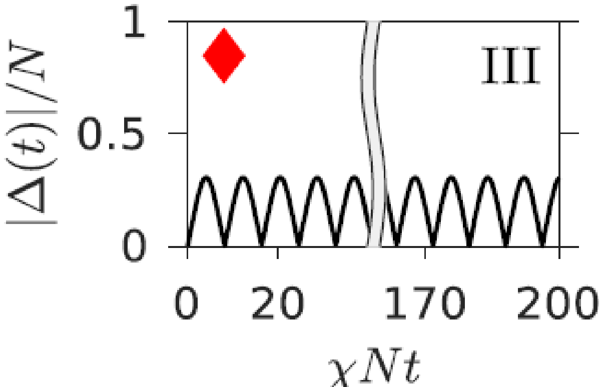
R. Lewis-Swan *et al* **PRL 126, 173601(2021)**

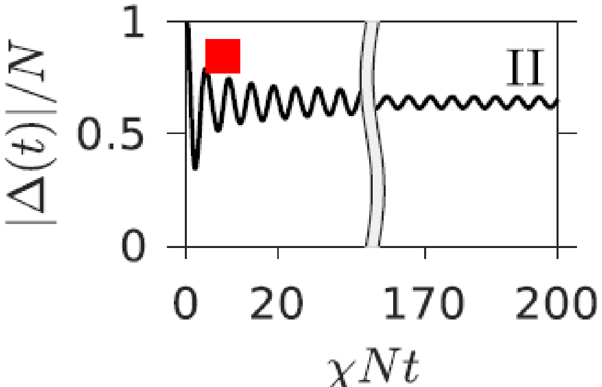


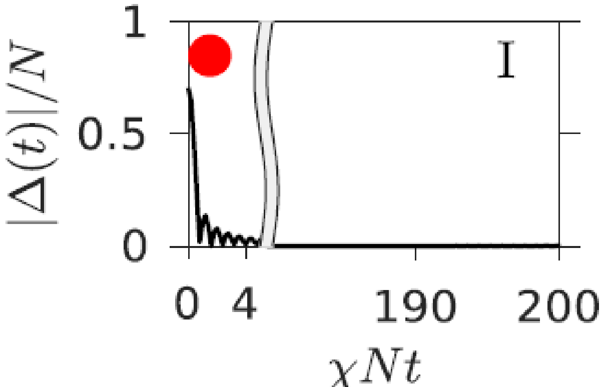




$\epsilon_0/(\chi N) = 0.1$

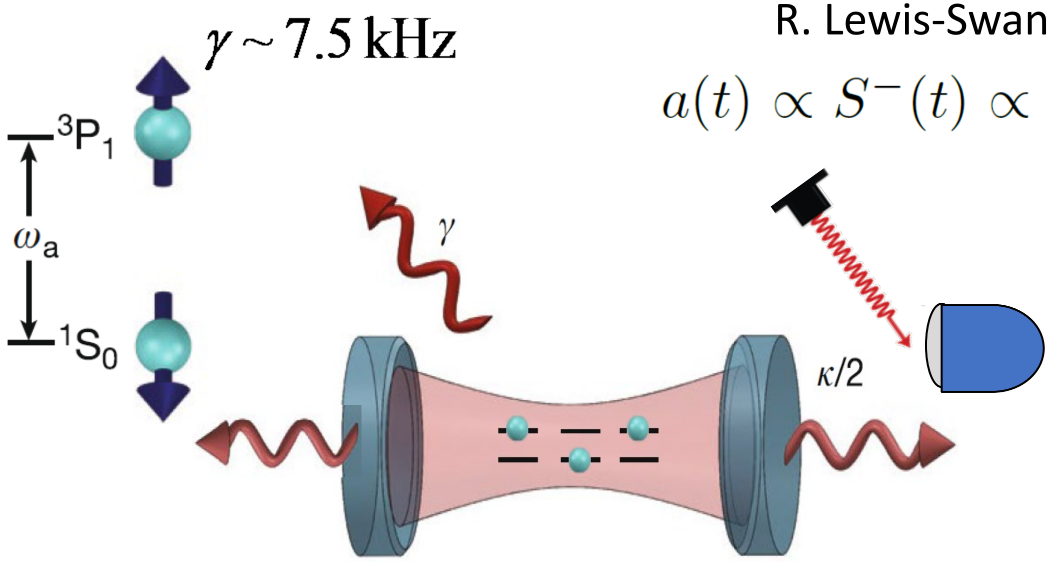






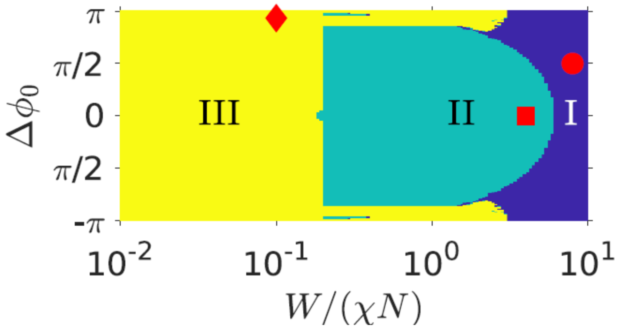
Experimental Observation

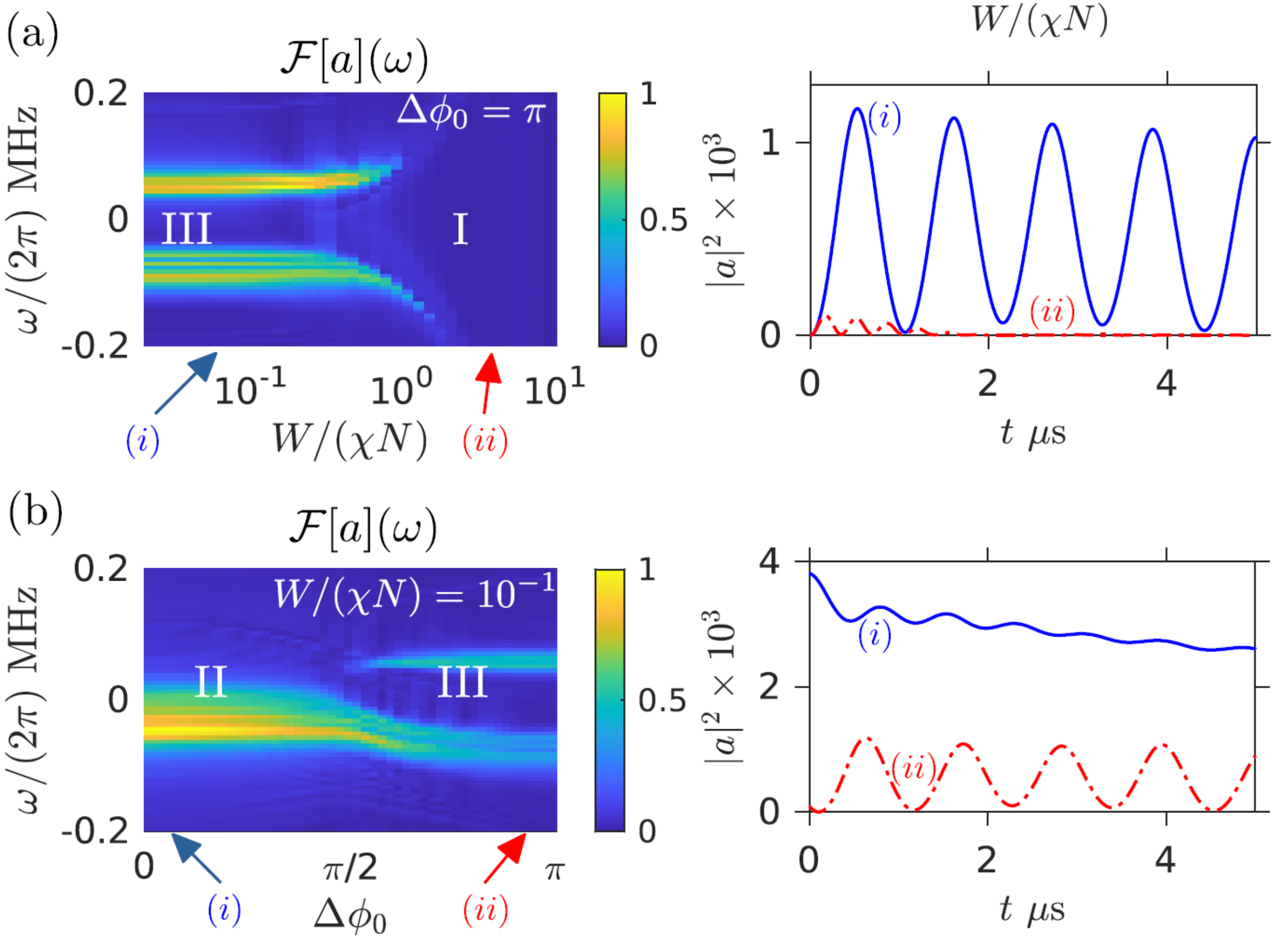
R. Lewis-Swan *et al* PRL 126, 173601(2021)



$$a(t) \propto S^-(t) \propto \Delta(t)$$

$$|a(t)|^2 \propto |\Delta(t)|^2$$





Simulations include:

- Single particle emission
- Cavity inhomogeneities:

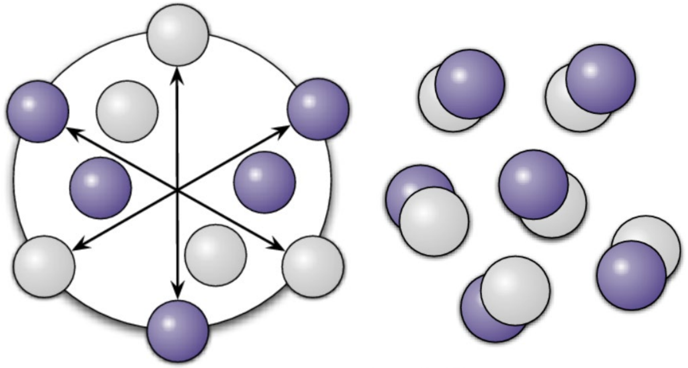
Incommensurate wavelengths of the standing wave optical lattice confining the atoms and the relevant cavity mode

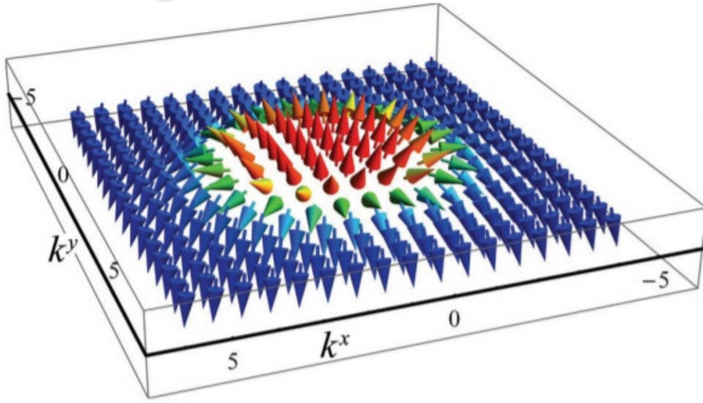
$$\chi \rightarrow \chi_{i,j}$$

Engineering 2D Topological p+ i p Superfluids

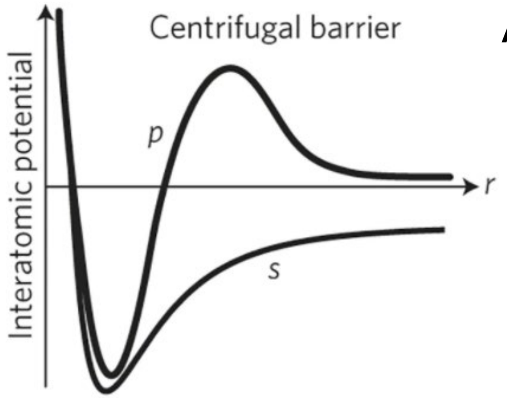
P-wave interactions enable interesting phases of matter: Superfluidity & Topology

- Cooper pairing of electrons or neutral fermionic atoms which carry angular momentum
- Gapless, chiral edge states that circulate around the boundary: analog of the quantum Hall effect (energy instead of charge).
- Host so-called “Majorana zero modes” : non-abelian anyons that could be exploited for topological quantum computation



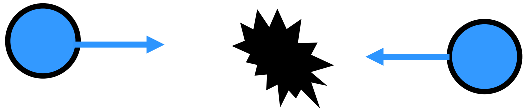


- Unfortunately, they’re weaker than s-wave
- Are other routes to engineer them? Idea use trapped ions



Amplify with Feshbach resonance?

Yes, but not too much due to losses!



2D Topological p+i p Superfluids

BCS p-wave Hamiltonian in momentum modes space k

$$H \sim -\frac{G}{2m}\sum_{\boldsymbol{kq}} \boldsymbol{q}\cdot\boldsymbol{k}\,\hat{c}^{\dagger}_{-\boldsymbol{q}}\hat{c}^{\dagger}_{\boldsymbol{q}}\hat{c}_{-\boldsymbol{k}}\hat{c}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \varepsilon_k \hat{c}^{\dagger}_{\boldsymbol{k}}\hat{c}_{\boldsymbol{k}}$$

Attractive p − wave Interactions

$\varepsilon_k = \frac{\hbar^2 k^2}{2m} - \mu$: Single-Particle Dispersion

Relevant Hilbert space

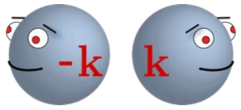
$$H \sim \sum_{\boldsymbol{k}} 2\varepsilon_k \hat{s}^z_{\boldsymbol{k}} - \frac{G}{2m}\sum_{\boldsymbol{kq}} \boldsymbol{q}\cdot\boldsymbol{k}\,\hat{s}^+_{\boldsymbol{q}}\hat{s}^-_{\boldsymbol{k}}$$

Anderson Pseudo-spins

$$\mathbf{k}\cdot\mathbf{q} = \tfrac{1}{2}\left[(k^x - ik^y)(q^x + iq^y) + (k^x + ik^y)(q^x - iq^y)\right],$$

Cooper pairs

$\langle s^z_{\mathbf{k}}\rangle = +\frac{1}{2}$ $\quad | \ldots \rangle$



$\langle s^z_{\mathbf{k}}\rangle = -\frac{1}{2}$ $\quad |\{k,-k\}\ \text{vacant}\rangle$

$$H \sim \sum_{\boldsymbol{k}} 2\varepsilon_k \hat{s}^z_{\boldsymbol{k}} - \frac{G}{2m}\sum_{\boldsymbol{kq}} qk\, e^{-i(\phi_{\boldsymbol{k}}-\phi_{\boldsymbol{q}})}\hat{s}^+_{\boldsymbol{q}}\hat{s}^-_{\boldsymbol{k}}$$

Time-reversal ymmetry broken model

- **Integrable (Richardson-Gaudin model)**

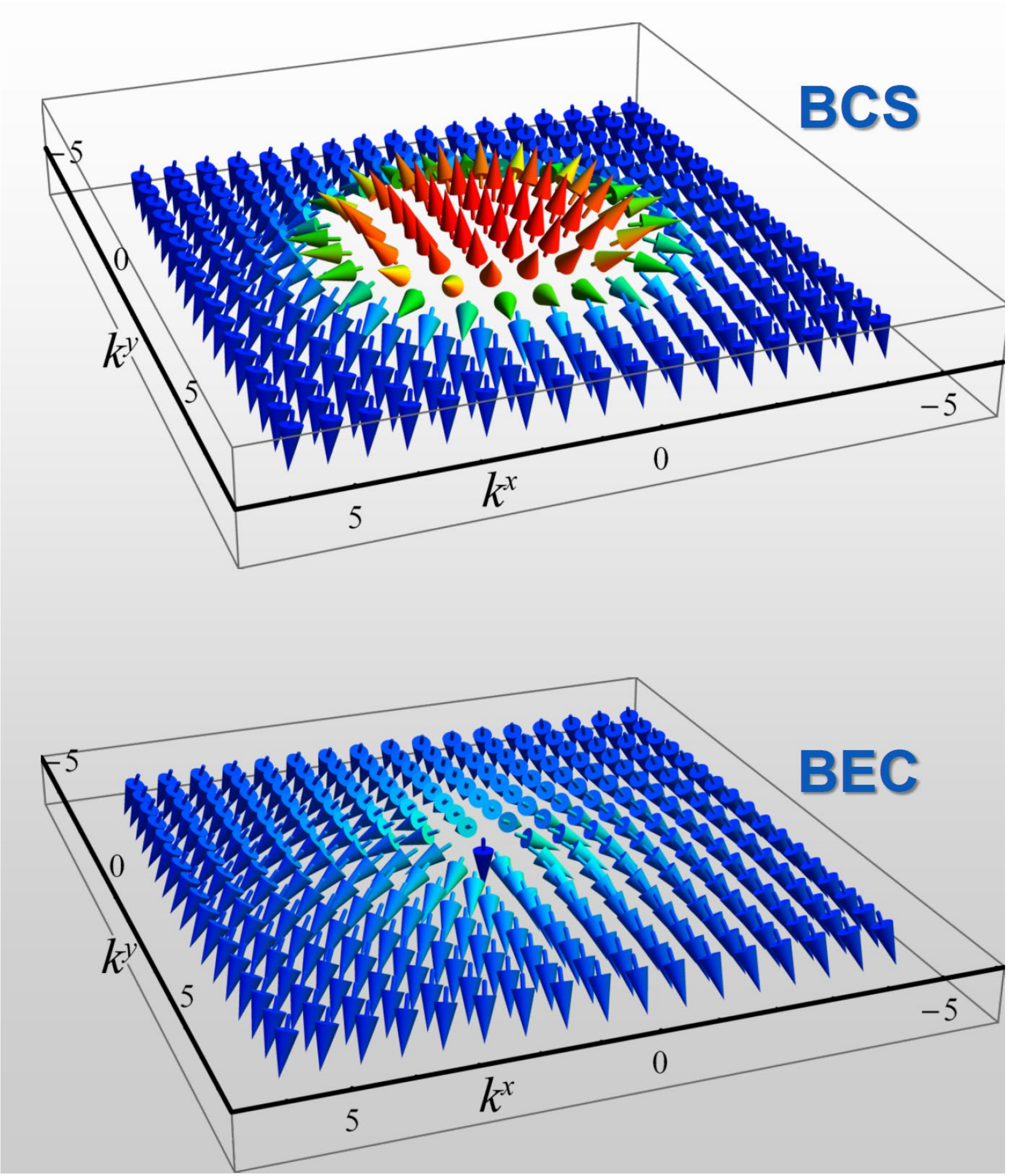
2D Topological p+i p Superfluids

Two distinct topological behaviors

- **Fully gapped when μ ≠ 0**
- ***Weak-pairing* BCS state topologically non-trivial**
- ***Strong-pairing* BEC state topologically trivial**
- **Winding number in the ground state directly connected to the orientation of the central spin**

$$Q \equiv \int_{\mathbf{k}} \rho(\mathbf{k}) = [\langle s^z_{\mathbf{k}=0}\rangle - \langle s^z_{\mathbf{k}=\infty}\rangle]$$
$$= \begin{cases} 1, & \mu > 0 \text{ (BCS)} \\ 0, & \mu < 0 \text{ (BEC)} \end{cases}$$

Volovik 88; Read and Green 00

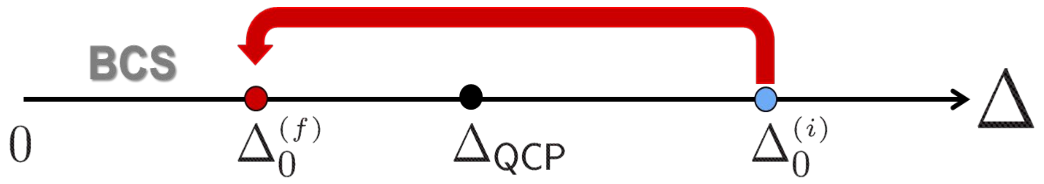


M. Foster et al PHYSICAL REVIEW B 88, 104511 (2013)

Dynamical Phases of BCS Model

$$\Delta \equiv \frac{G}{2m} \sum_{\boldsymbol{k}} k\, e^{-i\phi_{\boldsymbol{k}}} \langle \hat{s}_{\boldsymbol{k}}^{-} \rangle$$

Order parameter or gap function



Gap dynamics

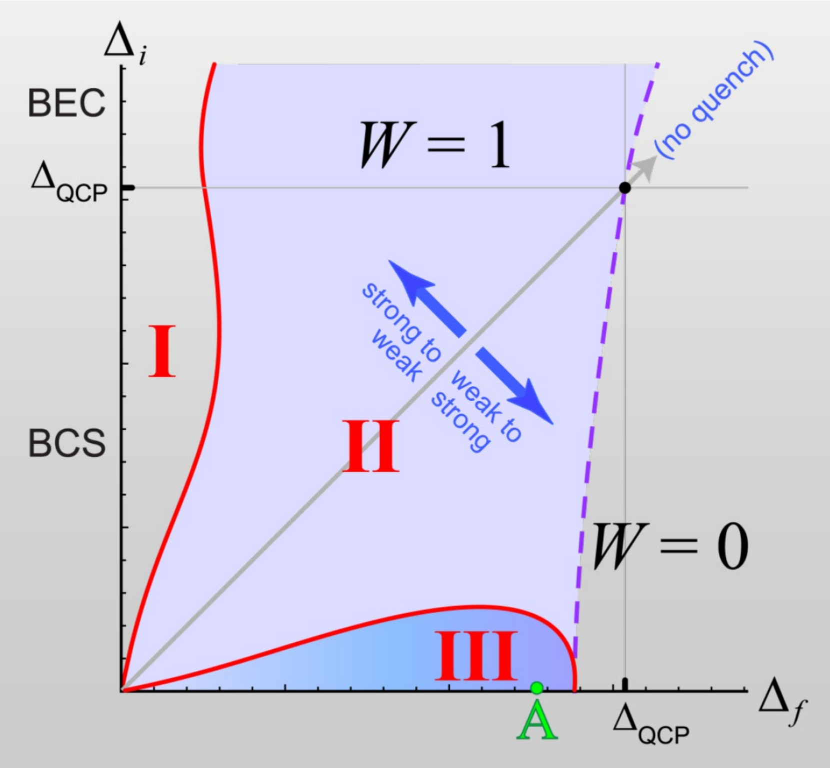
Dynamical phases: $\Delta(t \to \infty)$

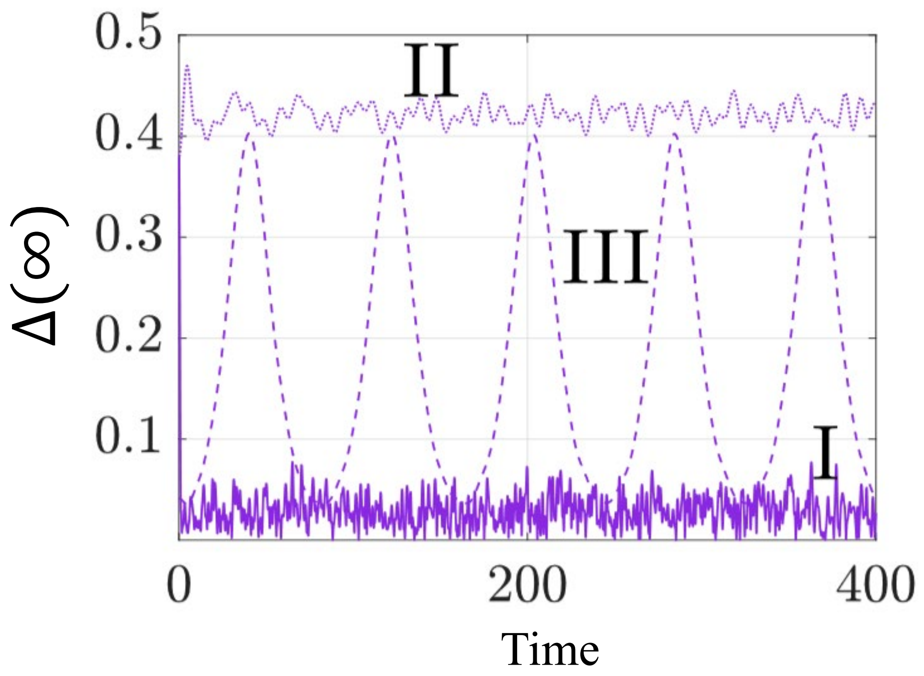
Phase I:
Gap decays to zero.

Phase II:
Gap goes to a constant.

Phase III:
Gap oscillates.

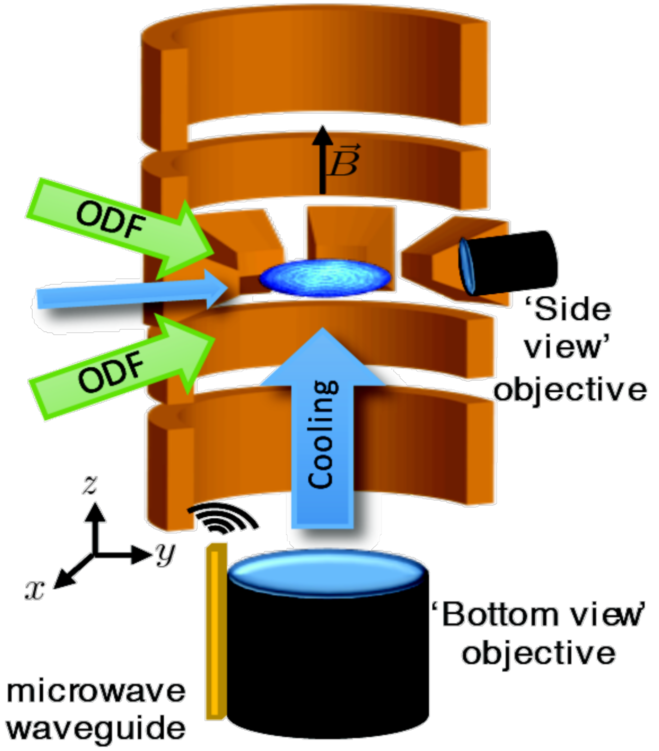
Three Dynamical Phases



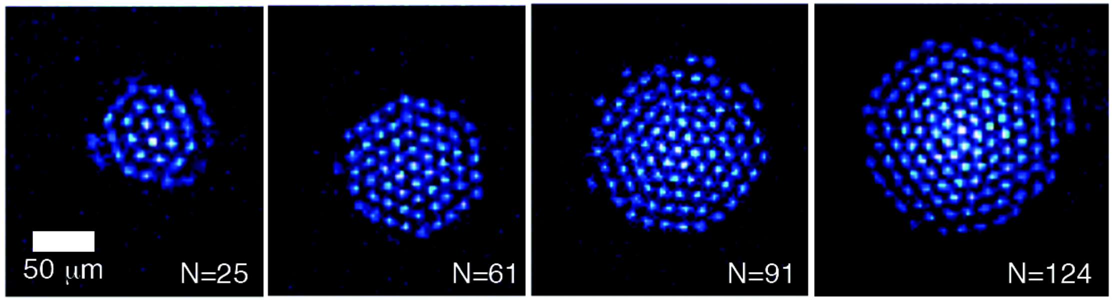


M. Foster et al PHYSICAL REVIEW B 88, 104511 (2013)

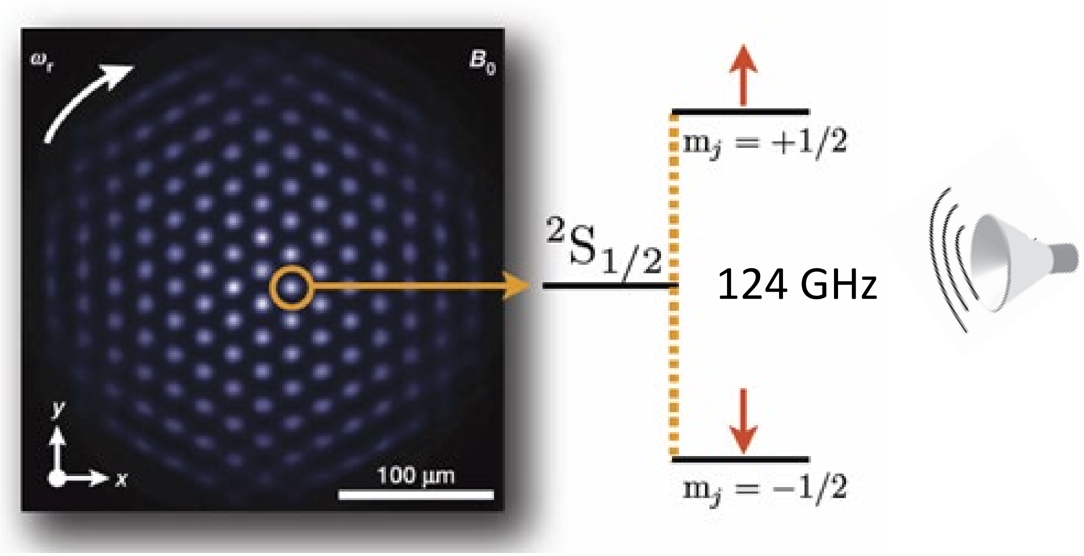
Penning Trap Experiments: $^9Be^+$



- **Penning trap: 2D triangular crystals of 20-300 ions**



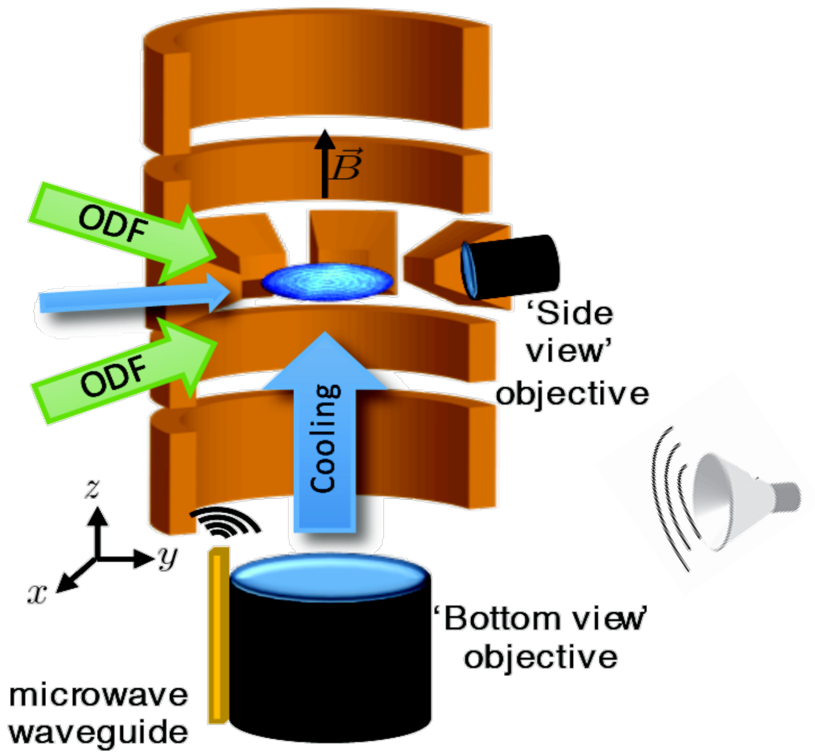
- **Two hyperfine states used as spin ½ system**



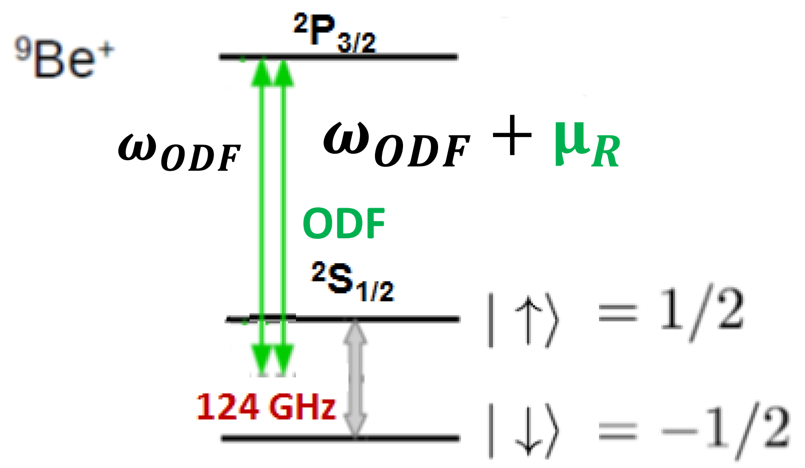
Magnetic field $B = 4.46\ T$
Axial freq. $\omega_z = 2\pi \times 1.57$ MHz
Rotation freq. $\omega_r = 2\pi \times 180$ kHz

Single qubit gates with 99.9 fidelity

Penning Trap Experiments: $^9Be^+$



- Spin-phonon interactions generated by lasers



- Spin dependent force

$$H(t) = f_0 \sum_i \sin(\Delta k_x \hat{x}_i + \Delta k_z \hat{z}_i - \mu_R t)\ \hat{s}_i^z - \Omega_0 \sum_i \hat{s}_i^x$$

$\hat{s}_i^{x,z}$ Spin operators on spin i

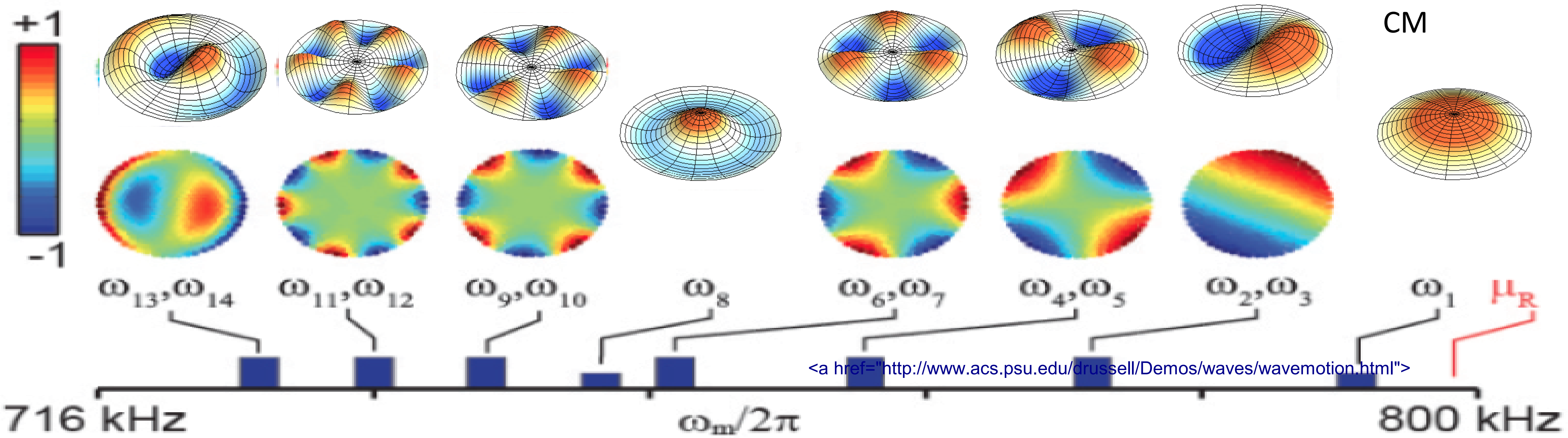
Phonons mediate Spin-spin interactions

- Ions are not independent: form a crystal due to Coulomb interactions

$$H(t) = -\Omega_0 \sum_i \hat{s}_i^x + f_0 \sum_i \sin(\Delta k_x \hat{x}_i + \underbrace{\Delta k_z \hat{z}_i} - \mu_R t)\ \hat{s}_i^z$$

Written in terms of drumhead eigenmodes

J. Britton et al Nature **484**, 489(2012)
C.C. J. Wang *et al* PRA **87**, 013422 (2013)



Phonons mediated interactions

- Ions are not independent: form a crystal due to Coulomb interactions

$$H(t) = -\Omega_0 \sum_i \hat{s}_i^z + f_0 \sum_i \sin(\Delta k_x \hat{x}_i + \Delta k_z \hat{z}_i - \mu_R t)\ \hat{s}_i^x$$

- Written in terms of drumhead eigenmodes

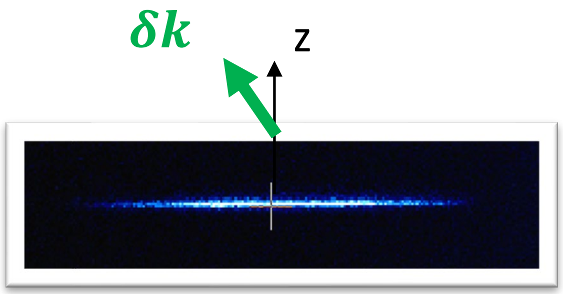
$$\hat{z}_i = \sum_\nu b_i^\nu (\hat{a}_\nu e^{-i\omega_\nu t} + \hat{a}_\nu^\dagger e^{i\omega_\nu t})$$

- Ions are also rotating

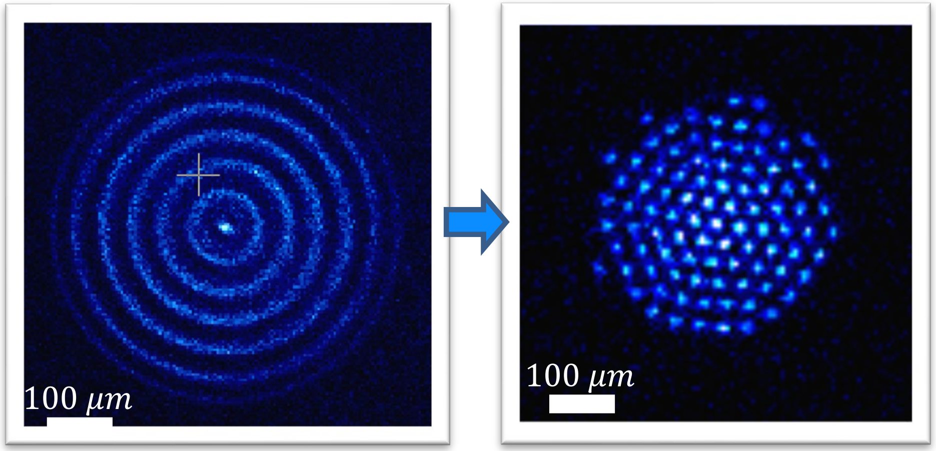
$$\hat{x}_i = r_i \cos(\omega_r t - \phi_i)$$

- Strong Ω_0: go to rotating frame

$$\hat{s}_i^\pm \Rightarrow \hat{s}_i^\pm e^{\pm i\Omega_0 t}$$



Side view



Lab frame

Rotating frame

Bottom view

Only Excite Center of Mass Mode

$$\mu_R = -\delta + \Omega_0 + \omega_r + \omega_1$$

Near resonance condition $\delta \ll \Omega_0$

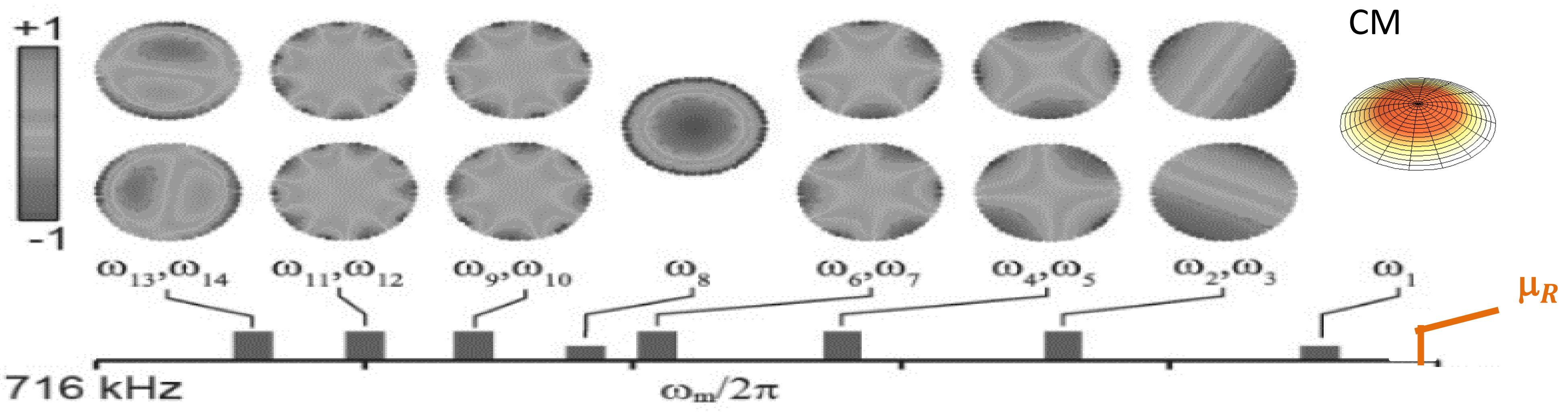
Only this term is slowly oscillating

$$H_{\rm rot}(t) \approx -\frac{g}{\sqrt{N}} \sum_j \tilde{r}_j \left(\hat{s}_j^+ \hat{a}\, e^{-i(\delta t + \phi_j)} + \hat{s}_j^- \hat{a}^\dagger e^{i(\delta t + \phi_j)} \right)$$

$$g = f_0(\Delta k_x R)(\Delta k_z a_{ho}) \quad \tilde{r}_j = \frac{r_j}{R} \qquad a_{ho} = \sqrt{\frac{\hbar}{(m\omega_1)}}$$

$\hat{a}^\dagger$: CM phonons creation operator

R: Radius of crystal

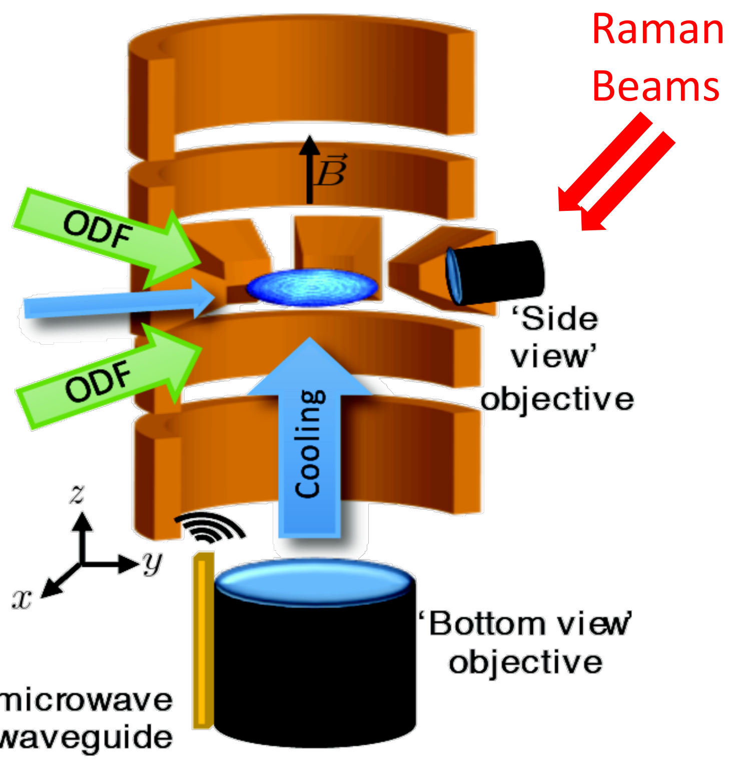


<a href="http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html">

Spin-spin interactions

- If $g \ll \delta$ CM phonons can be adiabatically eliminated. Back to the non-rotating frame

$$H_{\rm eff}(t) \approx \sum_j \left(-\Omega_0 - \frac{g^2}{\delta}\tilde{r}_j^2\right)\hat{s}_j^z - \frac{g^2}{\delta N}\sum_{j\neq l}\tilde{r}_j\tilde{r}_l e^{-i(\phi_l-\phi_j)}\hat{s}_l^+\hat{s}_j^- \qquad J = \frac{g^2}{\delta}$$



$$\Omega_0 \Rightarrow \Omega_0\left(1 - \frac{\tilde{r}_j^2}{w^2}\right)$$ Tunability of single particle dispersion

$$H_{\rm eff}(t) \approx \sum_j \left(\Omega\,\tilde{r}_j^2 - \Omega_0\right)\hat{s}_j^z - \frac{J}{N}\sum_{j\neq l}\tilde{r}_j\tilde{r}_l e^{-i(\phi_l-\phi_j)}\hat{s}_l^+\hat{s}_j^-$$

Engineered the desired p+ip BCS Hamiltonian

$$k \Leftrightarrow \tilde{r}_j \qquad \Omega\,\tilde{r}_j^2 - \Omega_0 \Leftrightarrow \frac{k^2}{2m} - \mu \qquad \frac{J}{N} \Leftrightarrow \frac{G}{2m}$$

Preparing Initial State

$$\Delta \equiv \frac{G}{2m} \sum_{\boldsymbol{k}} k\, e^{-i\phi_k} \langle \hat{s}_{\boldsymbol{k}}^- \rangle$$

$$H(t) = f_0 \sum_i \sin(\Delta k_x \hat{x}_i + \Delta k_z \hat{z}_i - \mu t)\ \hat{s}_i^x$$

$\mu_R = \omega_r$ Detuning

Then only the following term survives

$$\hat{H}_{\mathrm{rot}} = \frac{B_0}{2} \sum_j \tilde{r}_j \left(\hat{s}_j^+ e^{-i\phi_j} + \hat{s}_j^- e^{i\phi_j} \right)$$

$$B_0 = f_0(\Delta k_x R)$$

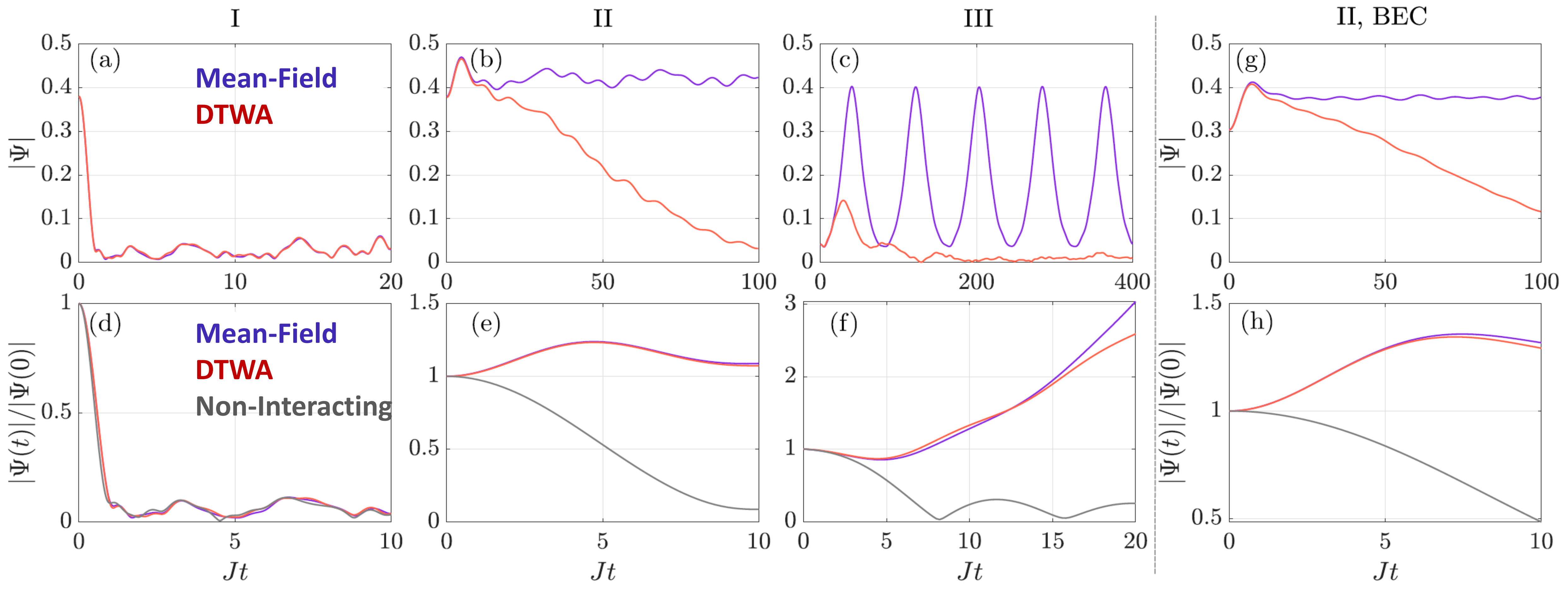
We can use $\hat{H}_{\mathrm{rot}}$ to:

- ❑ Initialization into a state with non-trivial order parameter
- ❑ Read out of the order parameter without single site resolution

$$\Delta \propto \sum_j \tilde{r}_j e^{-i\phi_j} \left\langle \hat{s}_j^- \right\rangle$$

Numerical Calculations for N=200

$$\Psi \equiv \frac{2}{N}\sum_j \tilde{r}_j e^{-i\phi_j}\left\langle \hat{s}_j^- \right\rangle$$

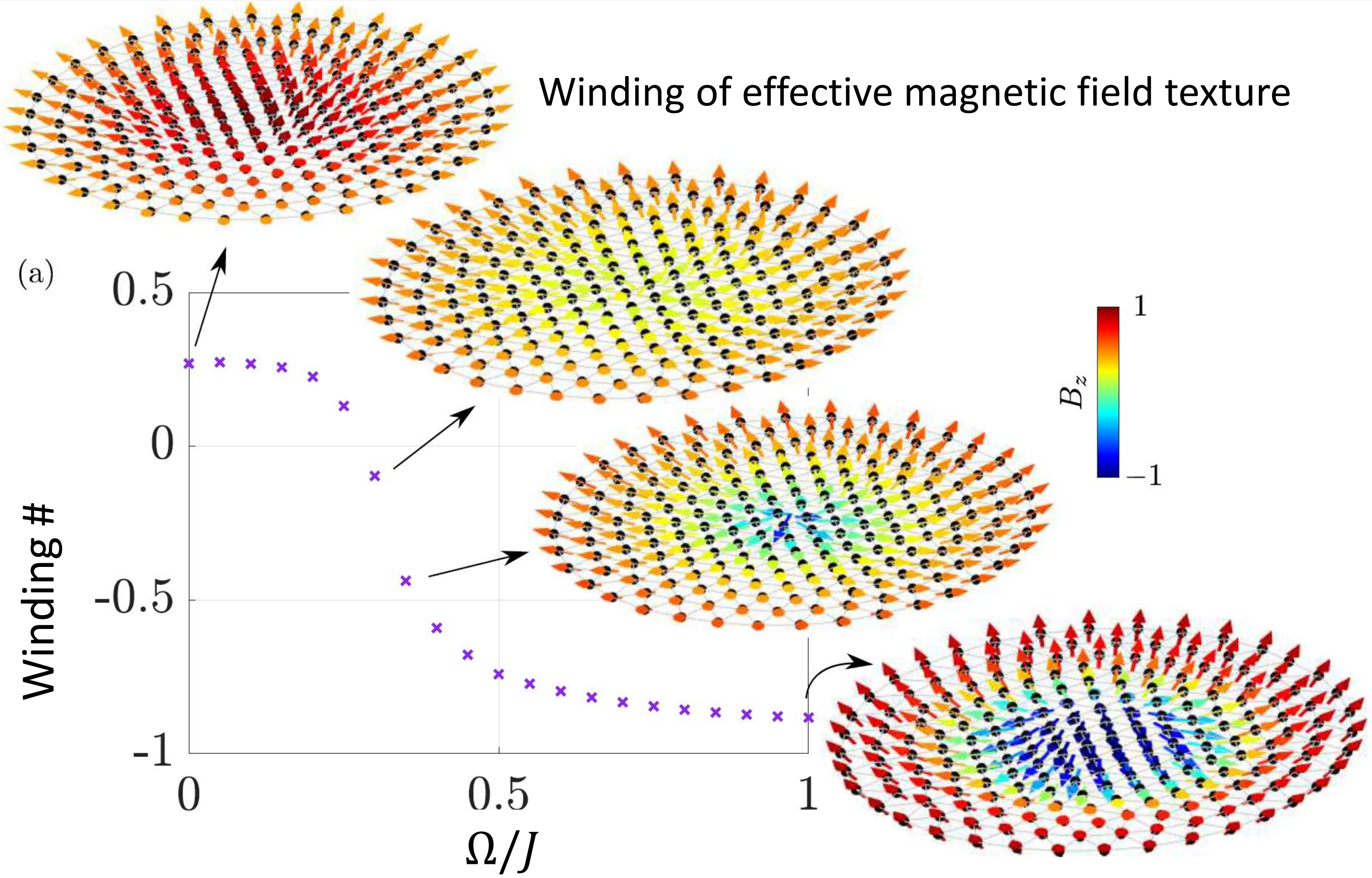


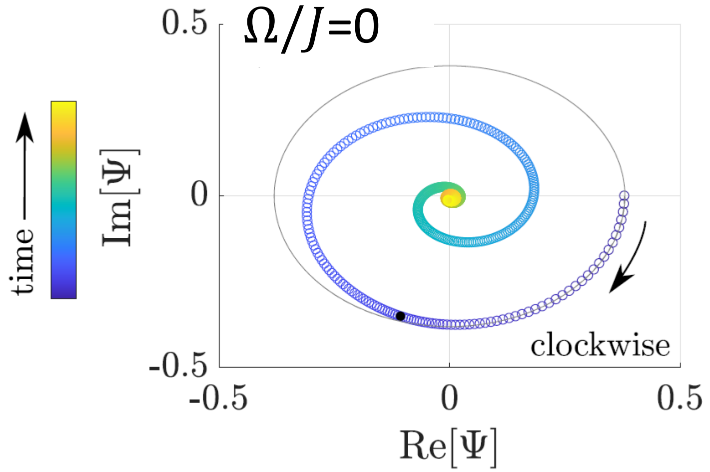
- ❑ DTWA accounts for quantum fluctuations during dynamics
- ❑ For the # of particles in consideration, they can not be neglected
- ❑ Nevertheless, all the phases can be distinguished at short times

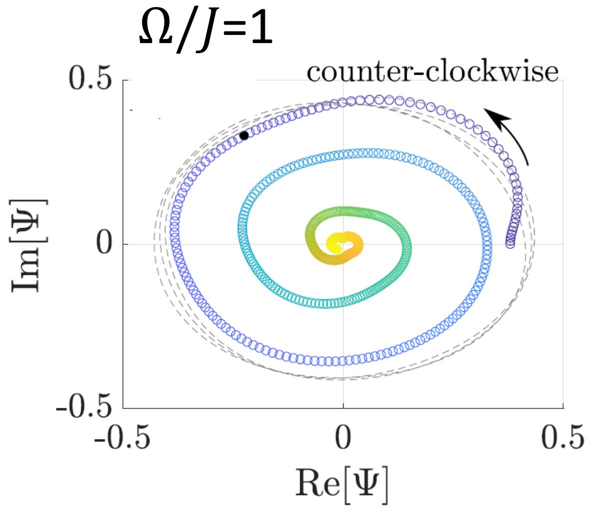
Probing Chirality

$$\Psi \to \Psi_\infty e^{-i\,t\,\mu_\infty} \qquad \begin{cases} \mu_\infty > 0 & \text{BCS: Topologically Non-trivial} \\ \mu_\infty < 0 & \text{BEC: Topologically trivial} \end{cases}$$

Winding of effective magnetic field texture







New directions in quantum simulations

- **Observation of exchange interactions: Gap protection and OAT:** *Science* **361**, 6399, 259 (2018)
- **Observation dynamical Phase Transition:** *Nature* **580** 602-607 (2020)
- **Proposal for simulation of BCS phases:** S-wave: PRL **126,** 173601(2021)**,** P-wave: In preparation

New opportunities

- Beyond Mean Field corrections: Metrological utility
- Include the role of active bosons
- Generalization to multi-level systems

Theory

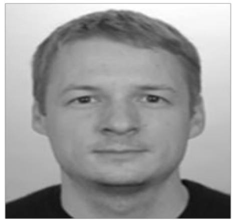
The Toronto team:

Joseph Thywissen

S. Smale

B. Olsen

K. Jackson



S Trotzky

The JILA Sr team:

James Thompson

M. Norcia

J. Cline

J. Silva

D. Young

The Trapped Ions team:

J. Bollinger

A. Shankar

P. Zoller

Rb team:

Jan Arlt

Carsten Klempt

Thank You!!