Fully Quantum Scalable Approaches to Driven-Dissipative Lattice Models

Marzena Szymańska

KITP, Sep 2021

Acknowledgements

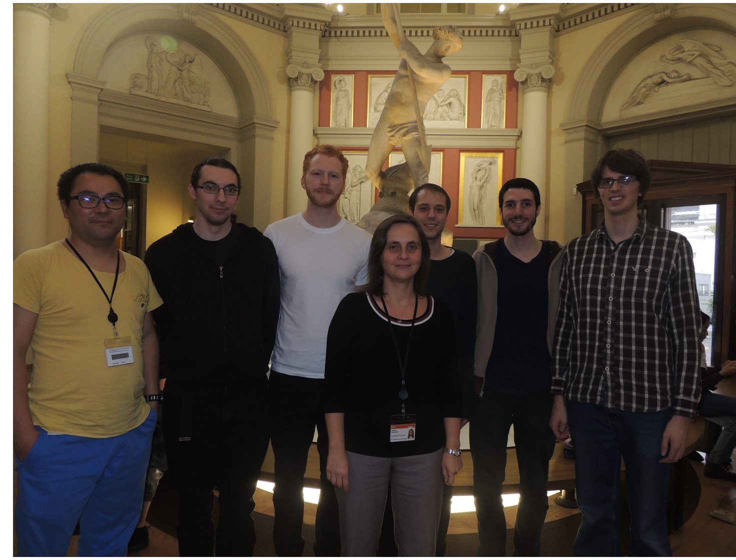
UCL

Group:

A. Ferrier

C. McKeever

C. Lledo Veloso



In collaboration with:

Piotr Deuar

Michal Matuszewski

Giuliano Orso

Interpol consortium

Funding:

EPSRC

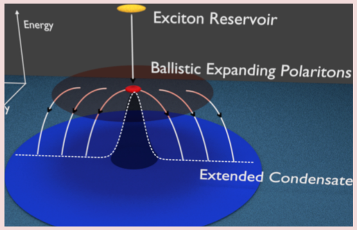
Engineering and Physical Sciences Research Council

QUANTERA

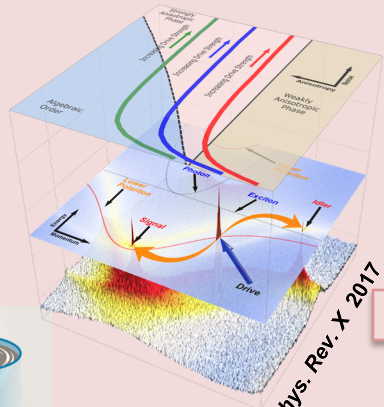
Interacting Photons

Interacting Photons

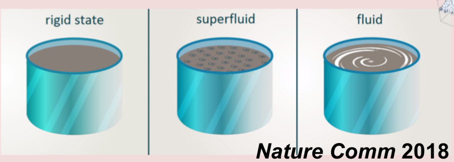
New Area: **Quantum Fluids of Light**



Nature Materials **2018**

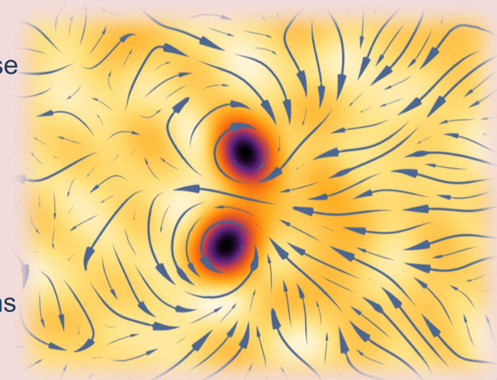


Phys. Rev. X **2017**



Nature Comm **2018**

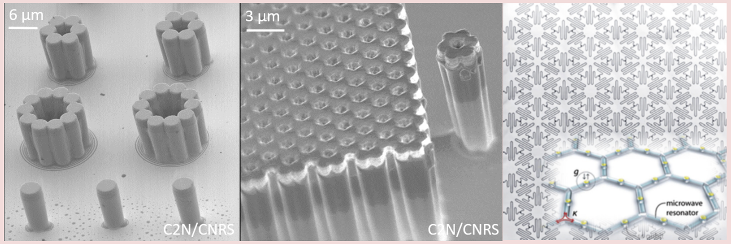
- Non-equilibrium phase transitions
- Critical phenomena
- Non-equilibrium superfluidity
- Topological transitions and defects

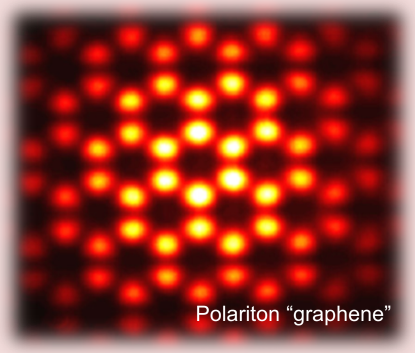


Vortices in polariton quantum fluid

Lattice Potentials for Interacting Photons

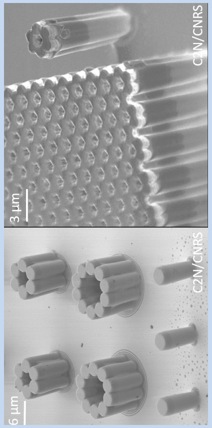
Emerging Area: **Quantum Solids of Light**

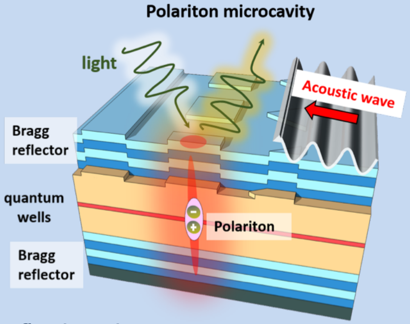




Quantum Solids of Light

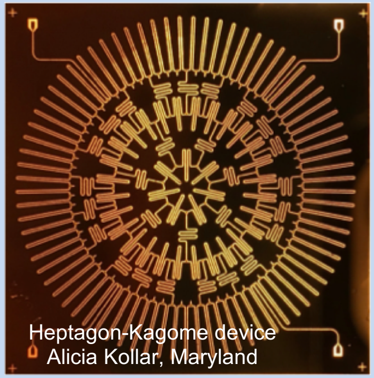
Polariton Lattices





- flat bands
- photonic "graphene",
- topologically protected states

Circuit QED Lattices

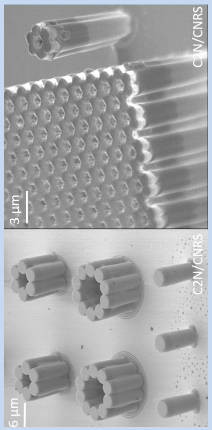


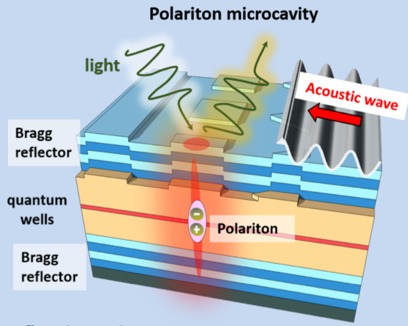
- Dissipative phase transitions

synthetic materials for exploring brand new physics, e.g. interacting quantum mechanics in curved space

Quantum Solids of Light

Polariton Lattices





- flat bands
- photonic “graphene”,
- topologically protected states

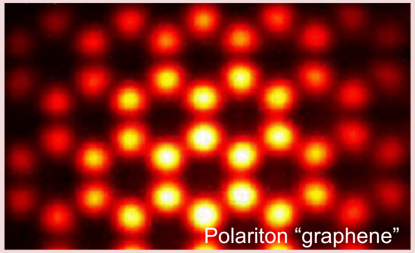
Circuit QED Lattices


Heptagon-Kagome device
Alicia Kollar, Maryland

- Dissipative phase transitions
  synthetic materials for exploring brand new physics, e.g. interacting quantum mechanics in curved space

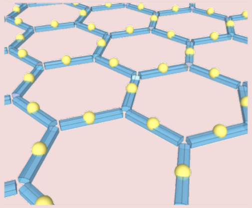
Strongly Interacting Photons

correlated or non-trivial topological states, but …
in driven-dissipative environment


Polariton “graphene”

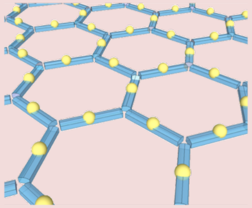
Theory Challenge

We need effective methods to treat **strong correlations** and **entanglement** in **driven-dissipative lattice systems**



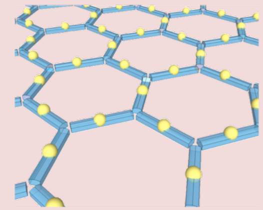
Theory Challenge

Light-matter platforms
strong competition between coherent and dissipative dynamics



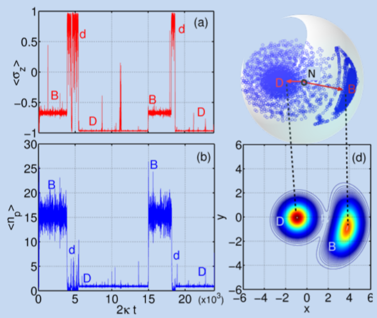
Theory Challenge

Light-matter platforms
strong competition between coherent and dissipative dynamics



Phase Space Methods

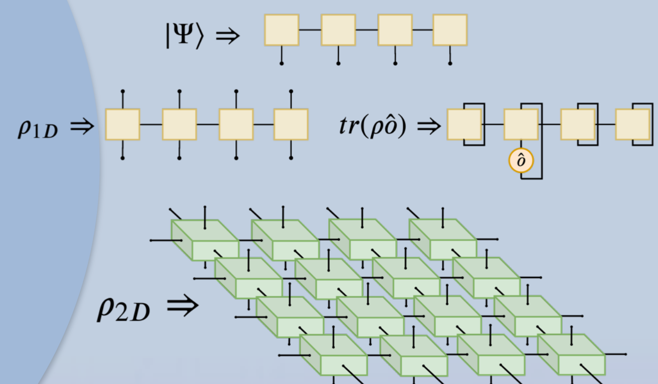
Good for driven-dissipative **non-equilibrium** but **weakly correlated** systems



Extend to stronger correlations and entanglement

Tensor Network Methods

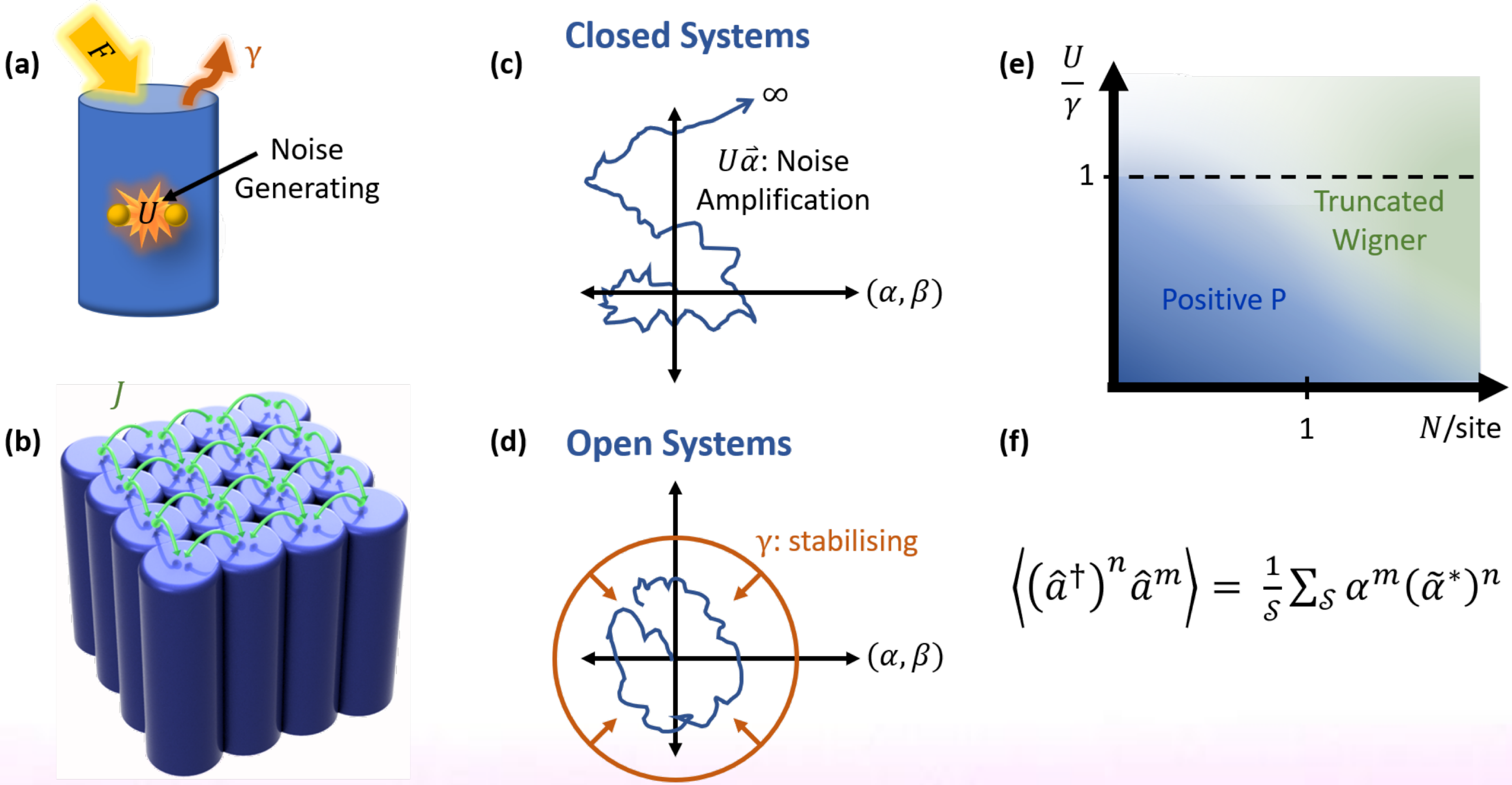
Good for **strongly correlated** (especially 1D) **equilibrium** systems



Extend to driven-dissipative non-equilibrium systems

Positive P for Driven-Dissipative Lattices of Bosons

P. Deuar *at. al*. *PRX Quantum* (2021)



$$\left\langle \left(\hat{a}^{\dagger}\right)^{n} \hat{a}^{m} \right\rangle = \frac{1}{S} \sum_{S} \alpha^{m} (\tilde{\alpha}^{*})^{n}$$

Positive P Method

- Lindblad Master equation: $\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}, \hat{\rho}] + \frac{\gamma}{2}\sum_j 2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\hat{\rho} - \hat{\rho}\hat{a}_j^\dagger\hat{a}_j$

- Fokker-Planck equation for $P(\alpha, \beta)$: $\frac{\partial P(\vec{\alpha},\vec{\beta})}{\partial t} = \mathcal{L}[P(\vec{\alpha},\vec{\beta})]$

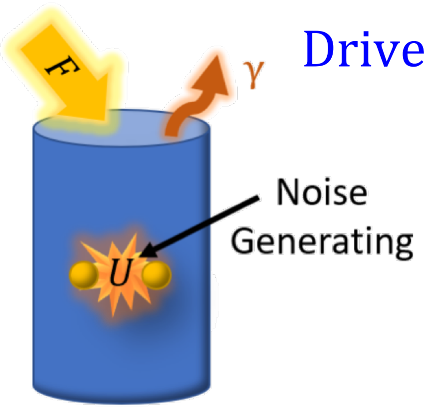
(where $\mathcal{L}$ is some differential operator in phase space α_j, β_j)

- Stochastic equations for phase space variables α_j, β_j: $\frac{\partial \alpha_j}{\partial t} = \ldots \quad \frac{\partial \beta_j}{\partial t} = \ldots$

- Quantum observables correspond to stochastic averages $\langle \ldots \rangle_{PP}$ of products of phase space variables: $\left\langle \left(\hat{a}_j^\dagger\right)^m \hat{a}_j{}^n \right\rangle = \langle \alpha_j{}^n \beta_j{}^m \rangle_{PP}$

  – Exact in the limit of infinite realisations.

Bose-Hubbard Model

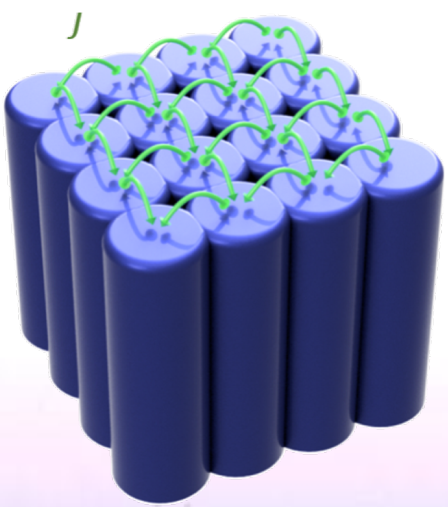


Driven Dissipative Bose-Hubbard model:

$$H = \sum_i\left[-\Delta\hat{a}_i^\dagger\hat{a}_i + \frac{U}{2}\hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i\hat{a}_i + F_i\left(\hat{a}_i^\dagger + \hat{a}_i\right)\right] - \sum_{i,j}J_{i,j}\left[\hat{a}_j^\dagger\hat{a}_i + \hat{a}_i^\dagger\hat{a}_j\right]$$

Master Equation: $\frac{\partial\hat{\rho}}{\partial t} = -i[\hat{H},\hat{\rho}] + \frac{\gamma}{2}\sum_j 2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\hat{\rho} - \hat{\rho}\hat{a}_j^\dagger\hat{a}_j$

Ito Stochastic equations:



$$\frac{\partial\alpha_j}{\partial t} = \left(i\Delta - \frac{\gamma}{2} - iU\alpha_j\beta_j + \frac{iU}{2}\right)\alpha_j - iF_j + i\sqrt{iU}\alpha_j\xi_j^{(1)} + i\sum_i J_{i,j}\alpha_i,$$

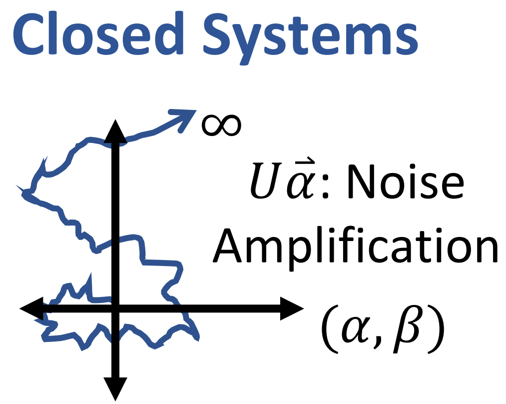
$$\frac{\partial\beta_j}{\partial t} = \left(-i\Delta - \frac{\gamma}{2} + iU\alpha_j\beta_j - \frac{iU}{2}\right)\beta_j + iF_j^* + \sqrt{iU}\beta_j\xi_j^{(2)} - i\sum_i J_{i,j}\beta_i.$$

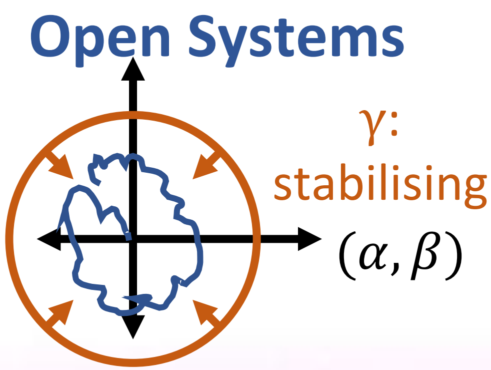
with real Gaussian noises

$$\left\langle\xi_j^{(\lambda)}(t)\right\rangle = 0\ ,\ \left\langle\xi_j^{(\lambda)}(t)\xi_{j'}^{(\lambda')}(t')\right\rangle = \delta_{\lambda,\lambda'}\delta_{j,j'}\delta(t-t')$$

Applicability of Positive P

- Closed systems: noise self amplification results in instabilities in positive P trajectories





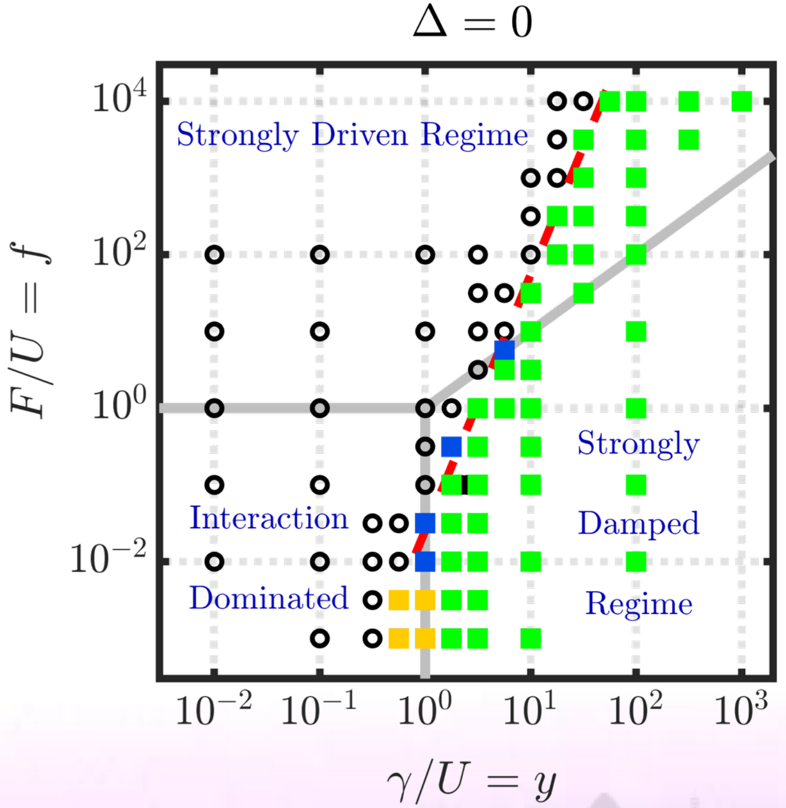
Applicability of Positive P

P. Deuar *at. al*. *PRX Quantum* (2021)

- Closed systems: noise self amplification results in instabilities in positive P trajectories

- Open systems: sufficient dissipation can stabilise trajectories fully!

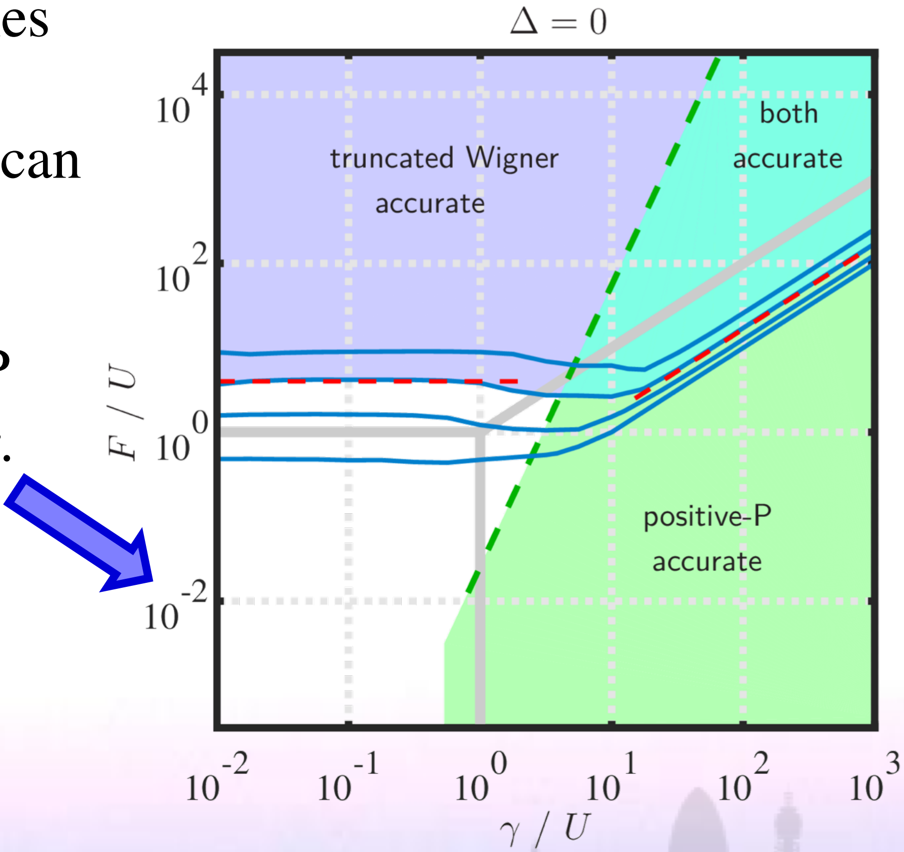
(green = fully stable, blue/yellow = marginal cases)



Applicability of Positive P

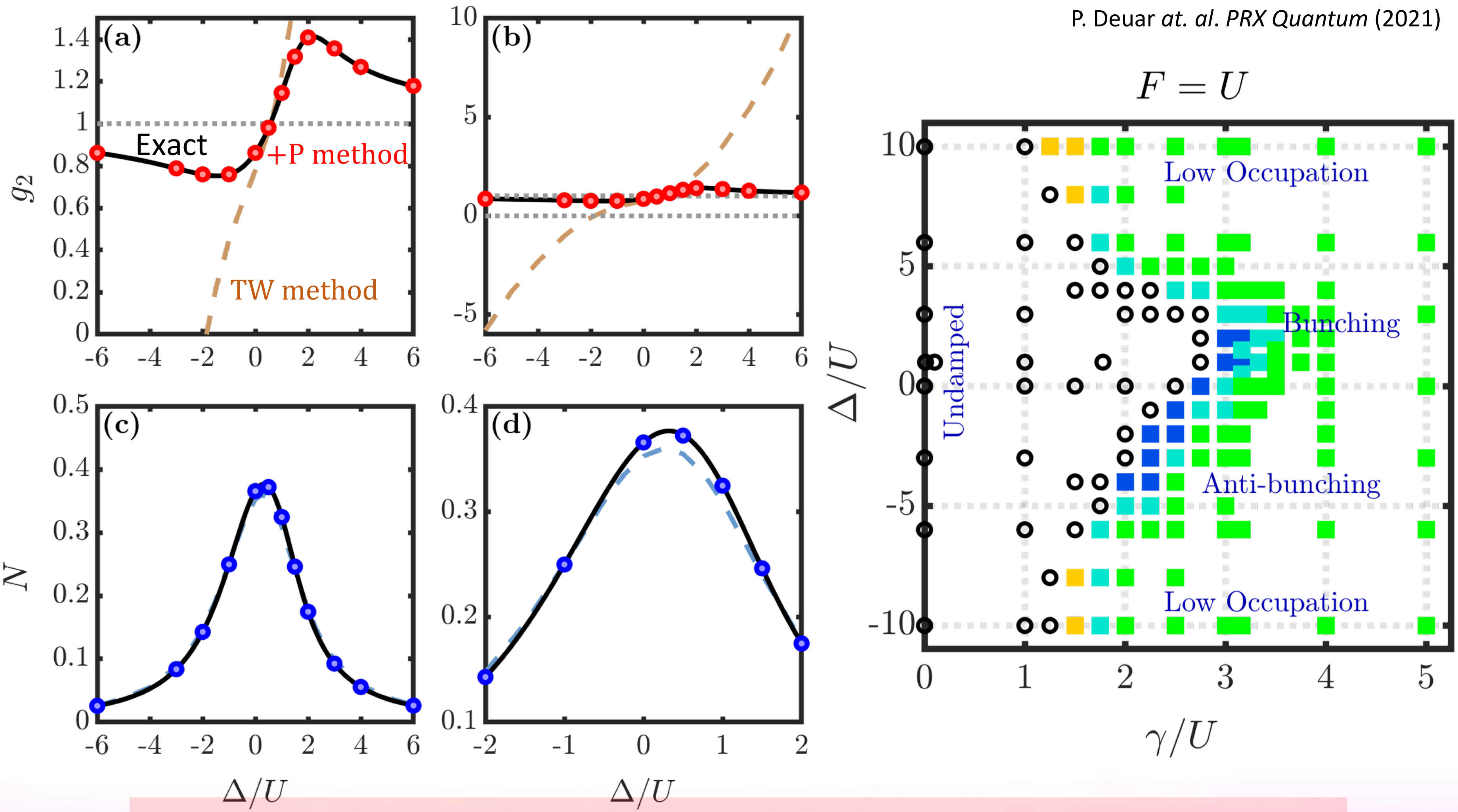
P. Deuar *at. al. PRX Quantum* (2021)

- Closed systems: noise self amplification results in instabilities in positive P trajectories
- Open systems: sufficient dissipation can stabilise trajectories fully!
- Region of applicability for positive P complementary to Truncated Wigner.



Bunching and Antibunching

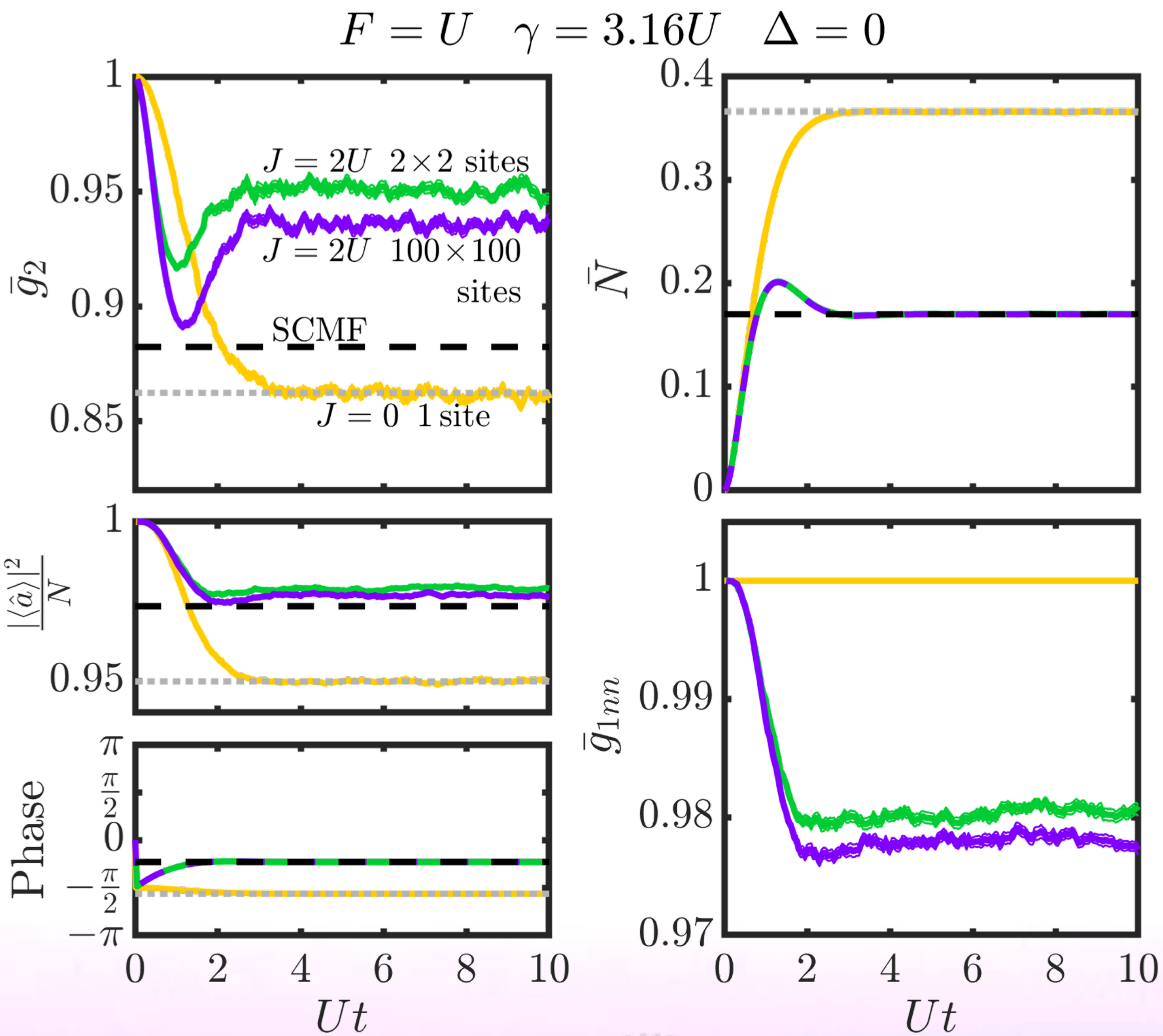
P. Deuar *at. al. PRX Quantum* (2021)



Quantum Effects accessible by Positive P

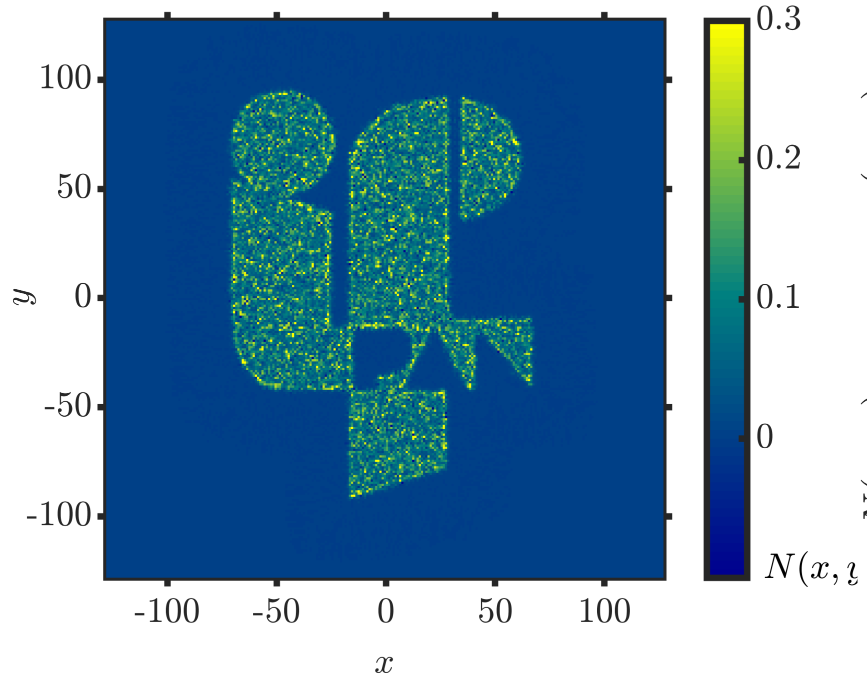
Square Lattices

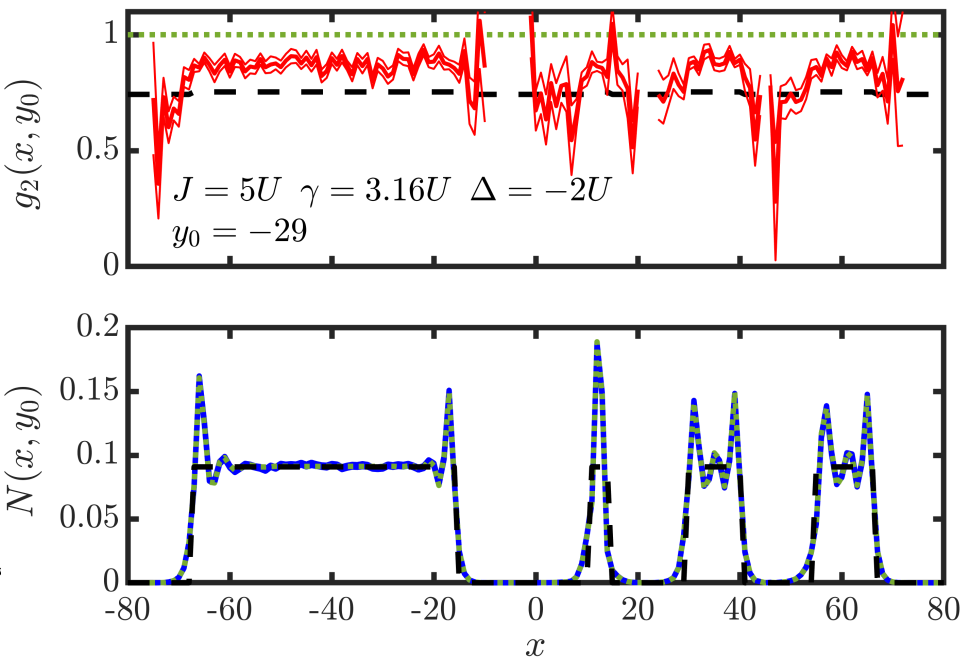
P. Deuar *at. al. PRX Quantum* (2021)



- Quantum correlations not well described by approximate methods
- Large systems needed for convergence!
- Positive P scales linearly with system size

Non-unform Lattices





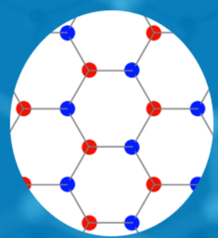
Space and time-dependence of parameters easily incorporated with no extra numerical effort

Nonlocal interactions can be efficiently treated

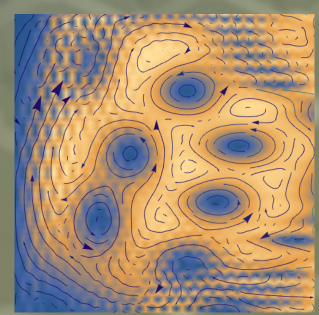
Perfect to look at correlations, interference, tunneling and nonlocal effects

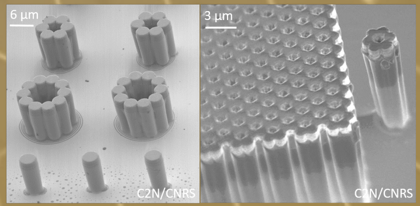
Synthetic magnetic field of strained honeycomb lattice

MODEL



BIG
LATTICES

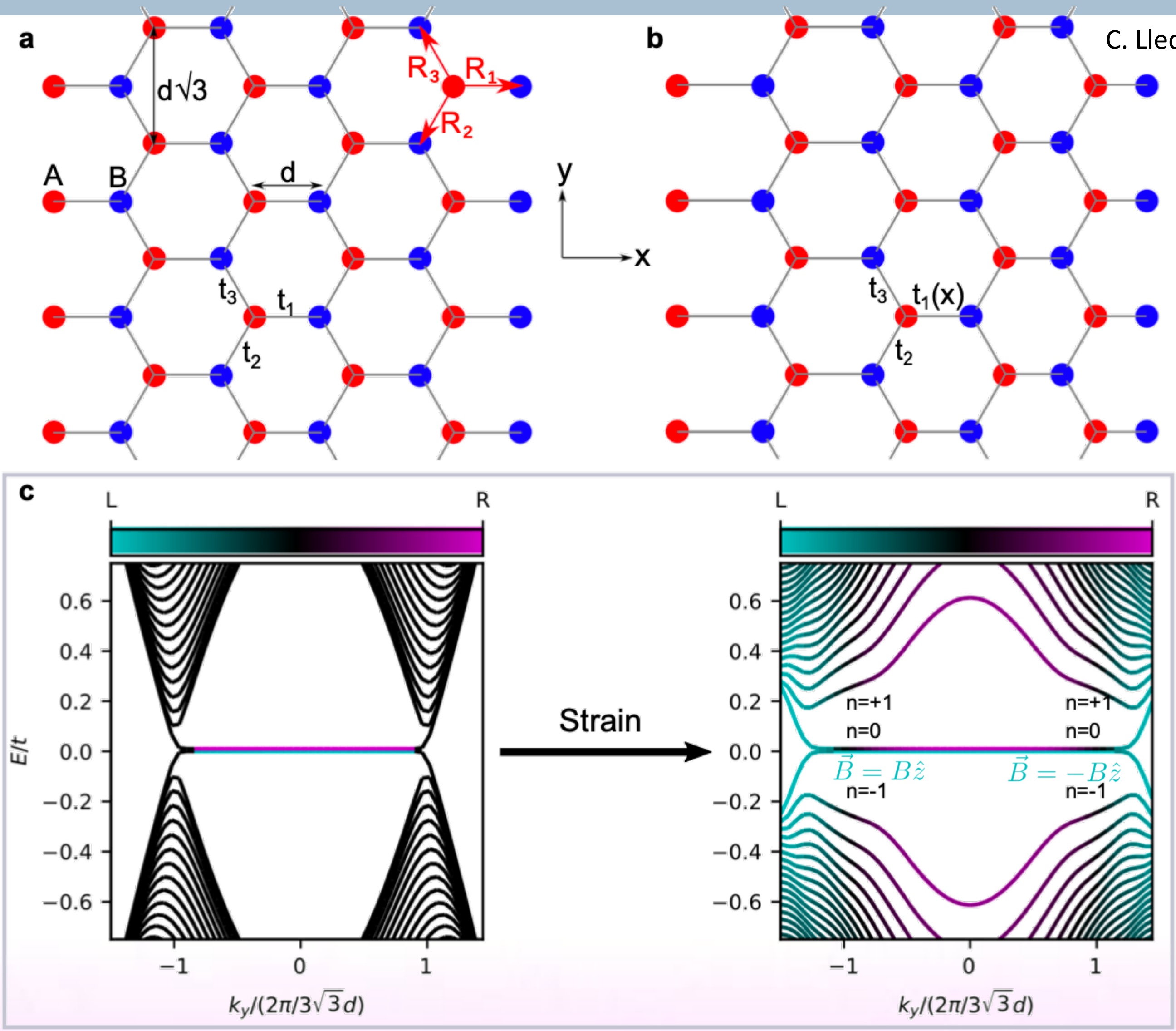




EXPERIMENTAL
LATTICES

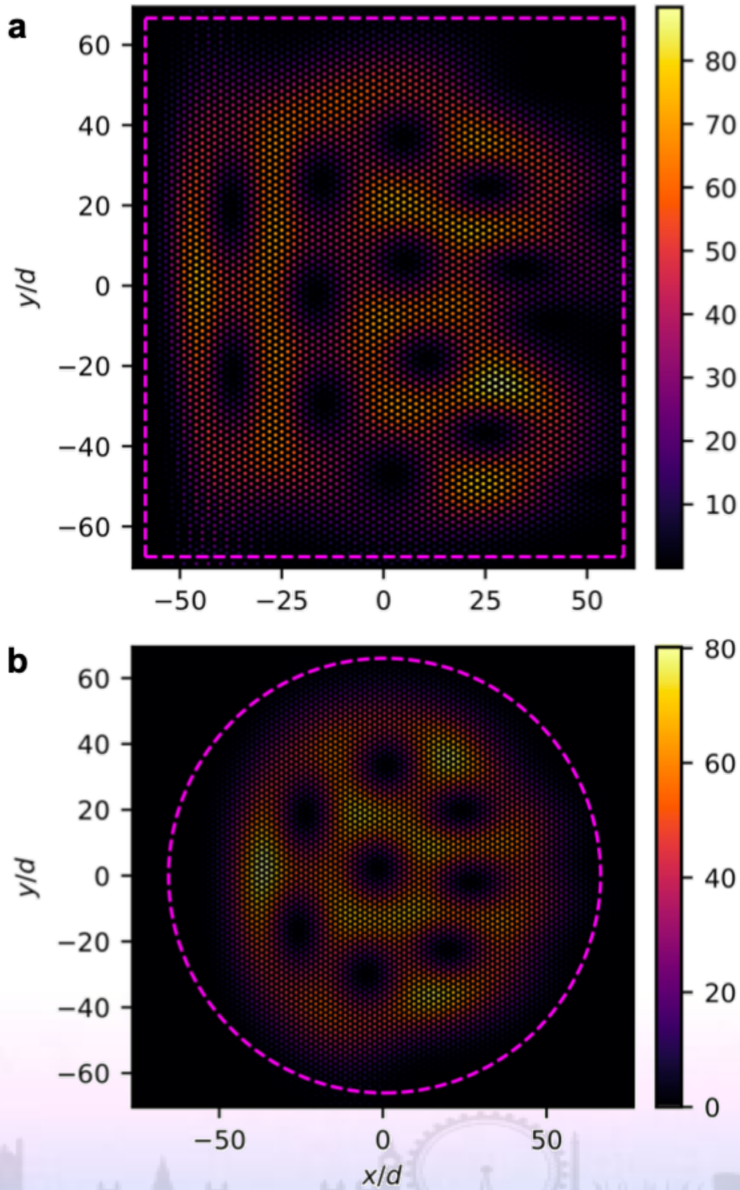
Polariton Strained Graphene and Landau Levels

C. Lledo *at. al*. *arxiv* 2103.07509 (2021)



Homogenous Pump

C. Lledo *at. al. arxiv* 2103.07509 (2021)



Magnetic length:

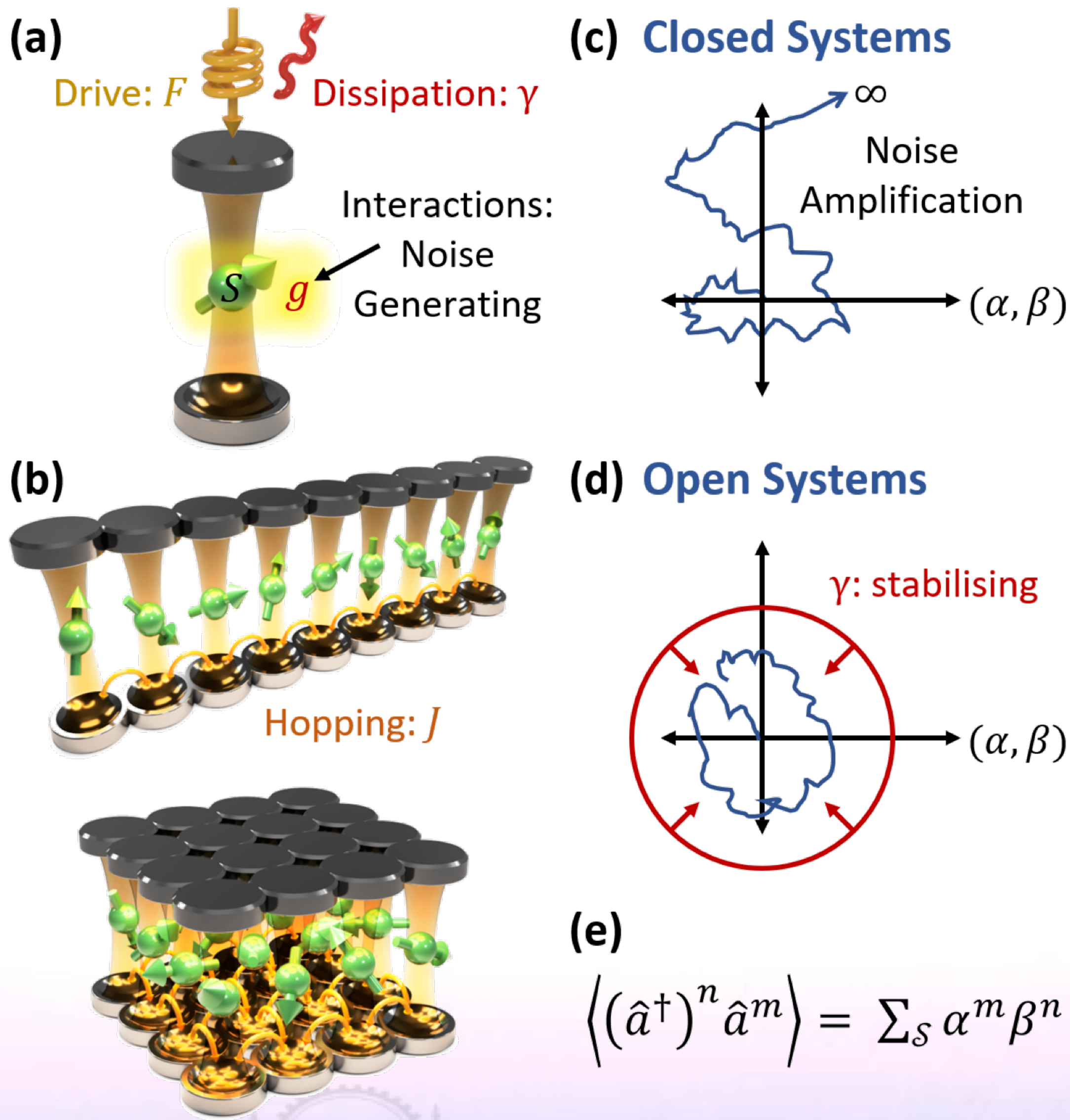
$$l_B = \sqrt{\frac{1}{|eB|}} = \frac{3d}{\sqrt{2\tau}} \approx 8.7d$$

n=0 Landau-level states

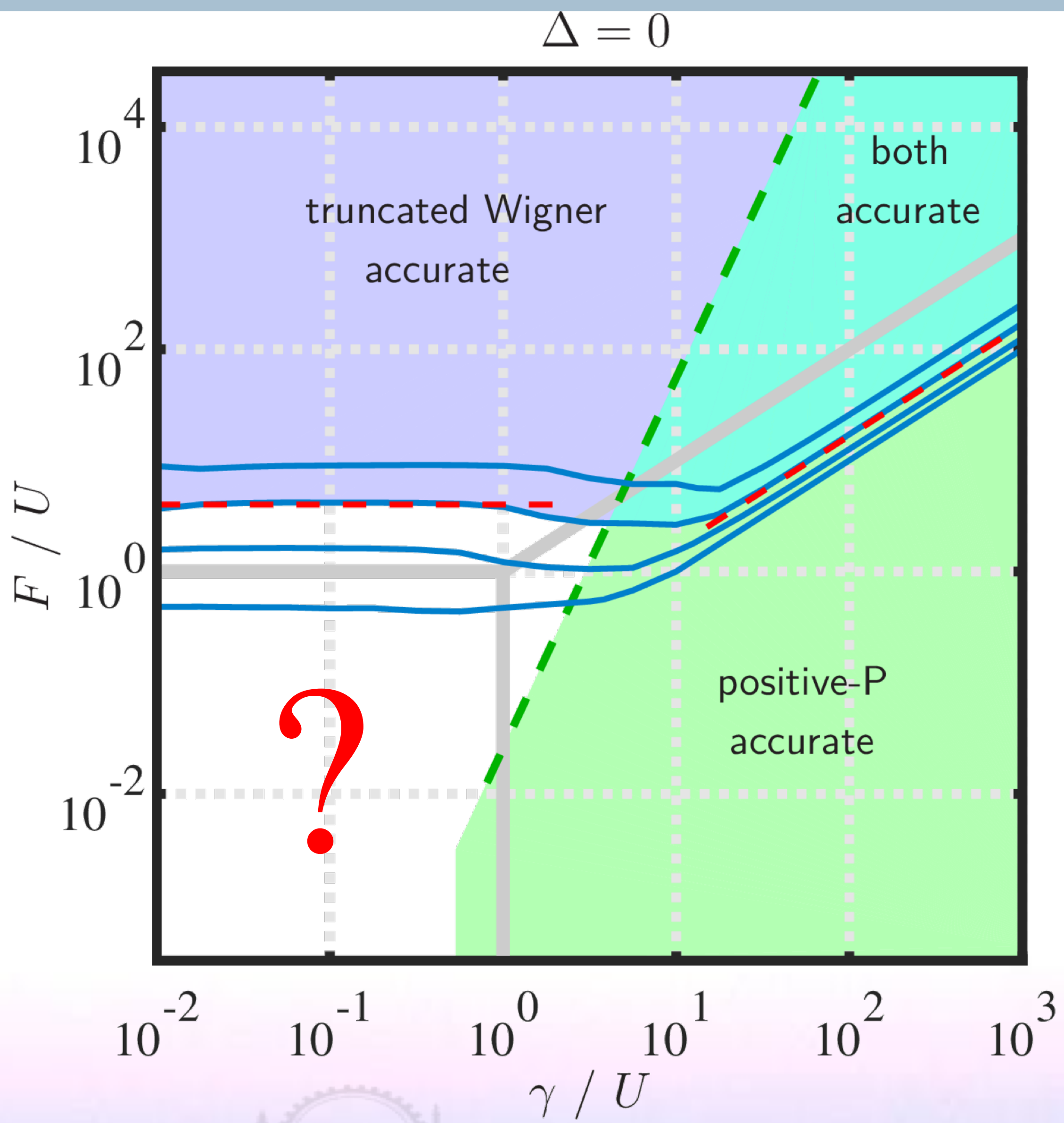
$$\psi_A(x_i, y_j) = 0$$

$$\psi_B(x_i, y_j) \propto e^{iq_y y_j} e^{-(x_i - l_B^2 q_y)^2/(2l_B^2)}$$

New Direction: Positive P for Spin-Boson Systems



Strong Interactions and Low Dissipation Limit



Numerical Tensor Network Methods

✧ Restrict the problem to the “physical corner” of Hilbert space

✧ Represent the quantum state as a network of tensors

✧ Tensor network with loops (cyclic) pose a challenge

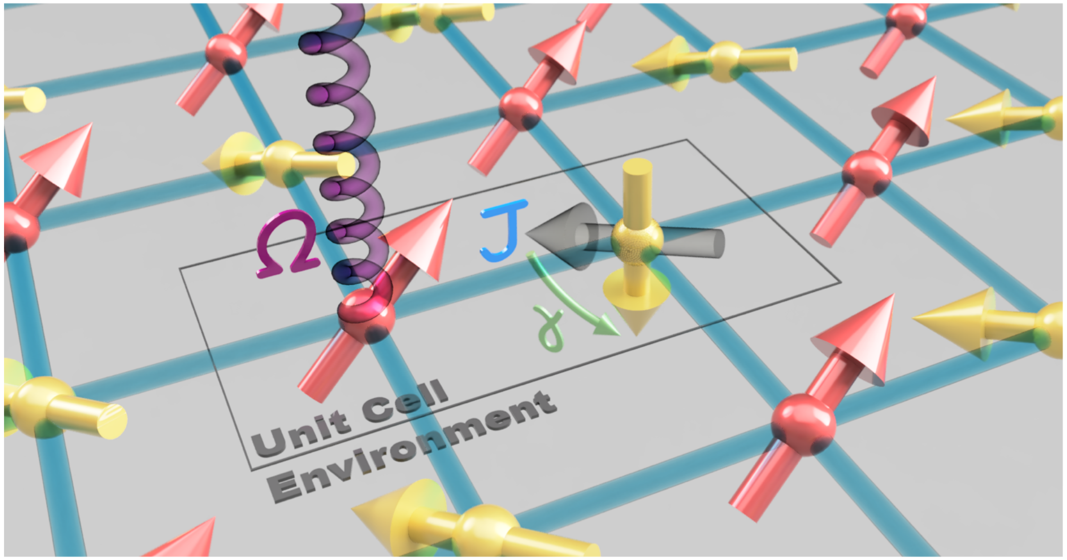
✧ First attempt at designing an algorithm for 2D open dissipative systems not fully successful

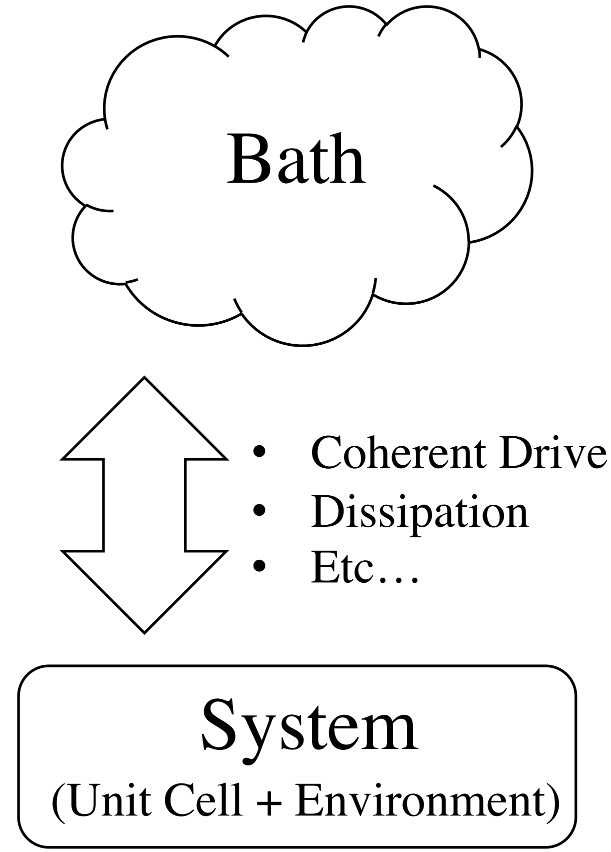
A. Kshetrimayum *at. al*. Nature Comm 8, 1 (2017)
D. Kilda *at. al*. SciPost (2021)

✧ **Our Goal:** accurate **dynamics** and **steady states** of 2D open quantum lattice models ideally in the thermodynamic limit using a tensor network method

Open Quantum Lattice Models

C. Mc Keever, MH Szymanska *PRX* (2021)



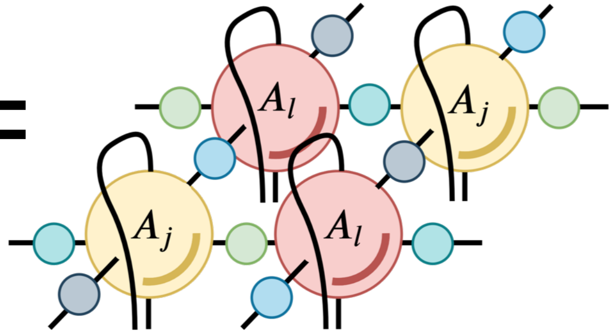


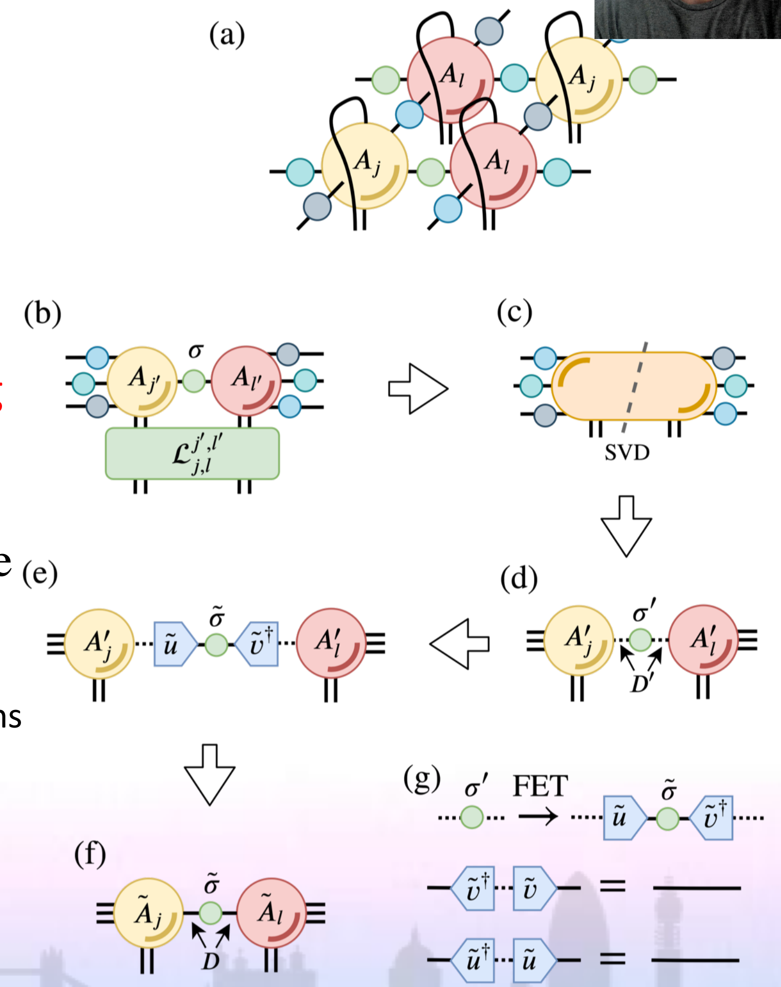
$$\frac{d\rho}{dt} = \mathcal{L}\left(\rho\right) = -i\left[\mathcal{H}, \rho\right] + \mathcal{D}\left(\rho\right),$$

$$\hat{\mathcal{D}}(\hat{\rho}) = \sum_{\alpha}\left(\hat{L}_{\alpha}\rho\hat{L}_{\alpha}^{\dagger} - \frac{1}{2}\{\hat{L}_{\alpha}^{\dagger}\hat{L}_{\alpha}, \hat{\rho}\}\right),$$

Numerical Tensor Network Methods

C. Mc Keever, MH Szymanska *PRX* (2021)

$\rho =$ 



- ✧ Represent the state as an infinite Projected Entangled Pair Operator (iPEPO)
- ✧ Integrate equation of motion using Time Evolving Block Decimation (TEBD)
- ✧ Full Environment Truncation (FET) and fixing the network to Weighted Trace Gauge (WTG)
  - optimal truncations of enlarged network bonds by optimising an objective function appropriate for open systems
- ✧ For details of the algorithm see [C. Mc Keever, MH Szymanska *PRX* (2021)]

A Benchmark

Transverse field dissipative Ising model

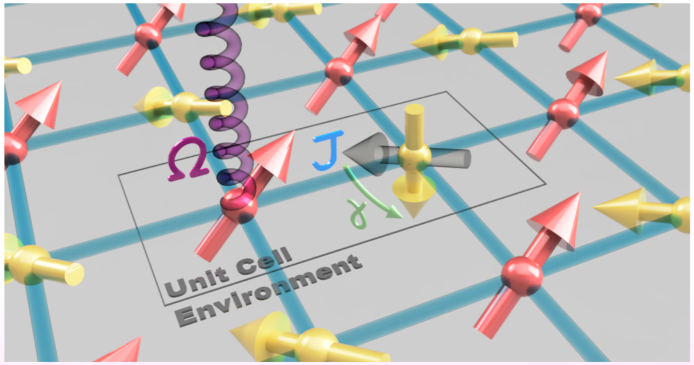
$$\hat{H} = \frac{V}{z}\sum_{\langle j,l\rangle}\hat{\sigma}_j^z\hat{\sigma}_l^z + \sum_j \frac{h_x}{2}\hat{\sigma}_j^x \qquad \hat{L}_j = \sqrt{\gamma}\tfrac{1}{2}\left(\hat{\sigma}_j^y - i\hat{\sigma}_j^z\right)$$

✧ Steady state of the dissipator does not commute with the Hamiltonian

✧ In the special case $hx/\gamma = 0$ exactly solvable for local observables in time [M. Foss-Feig et. al. PhysRevLett.119.190402]

✧ Consider four sets of parameters:

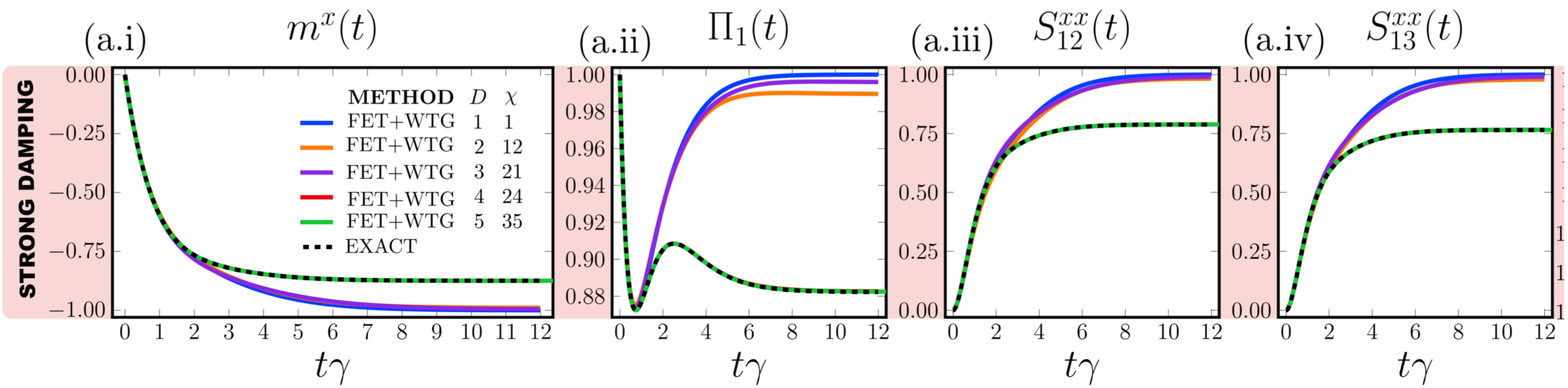
- ✧ Strong Damping V/$\gamma = 0.2$, hx/γ =0
- ✧ Moderate Damping V/$\gamma = 1.2$, hx/γ =0
- ✧ Weak Damping V/$\gamma = 4.0$, hx/γ =0
- ✧ No Exact Solution V/$\gamma = 0.5$, hx/γ =1



Strong Damping

C. Mc Keever, MH Szymanska *PRX* (2021)

$V/\gamma = 0.2$, $hx/\gamma = 0$

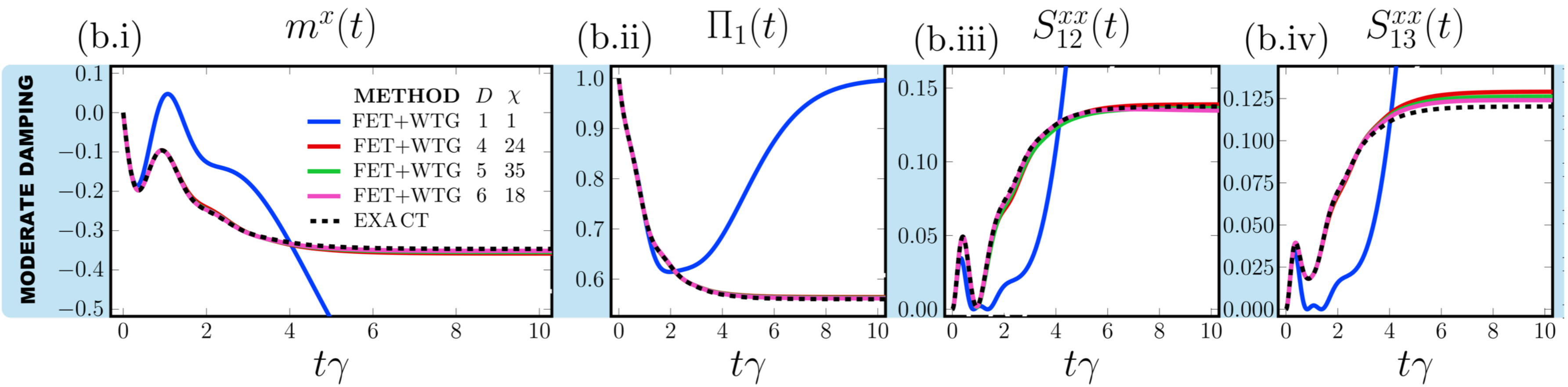


- ✧ Correction to mean field (D=1)
- ✧ Excellent agreement for D>3

Moderate Damping

C. Mc Keever, MH Szymanska *PRX* (2021)

V/γ = 1.2 , hx/γ =0

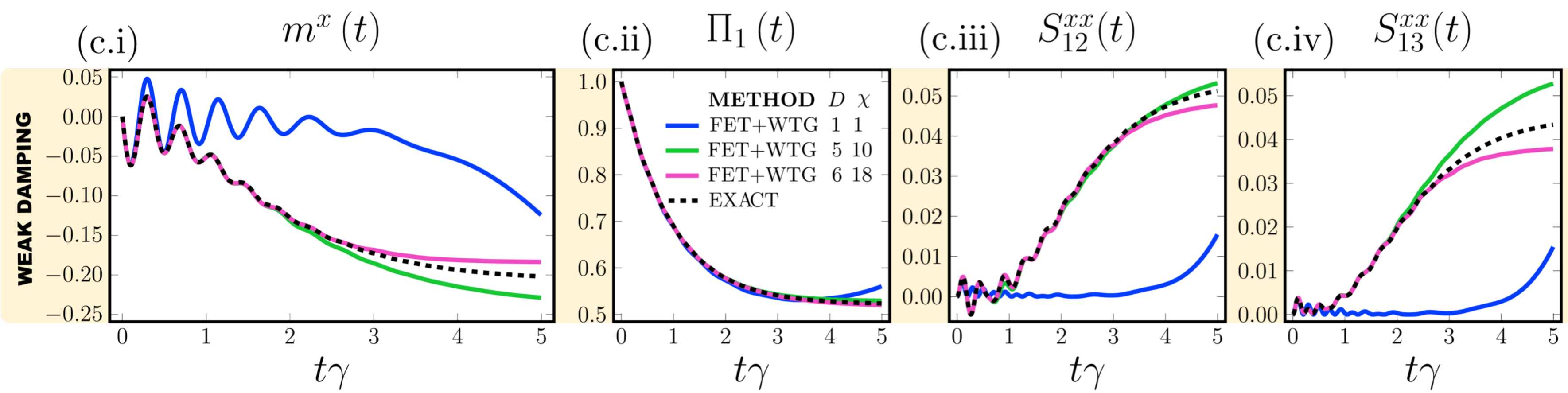


- Significant correction to mean field (D=1) result
- Good agreement for D>3

Weak Damping

C. Mc Keever, MH Szymanska *PRX* (2021)

V/$\gamma = 4.0$, hx/γ =0



✧ Significant correction to mean field (D=1) result

✧ Accurate at early times, begins to deviate from exact result after a few tγ

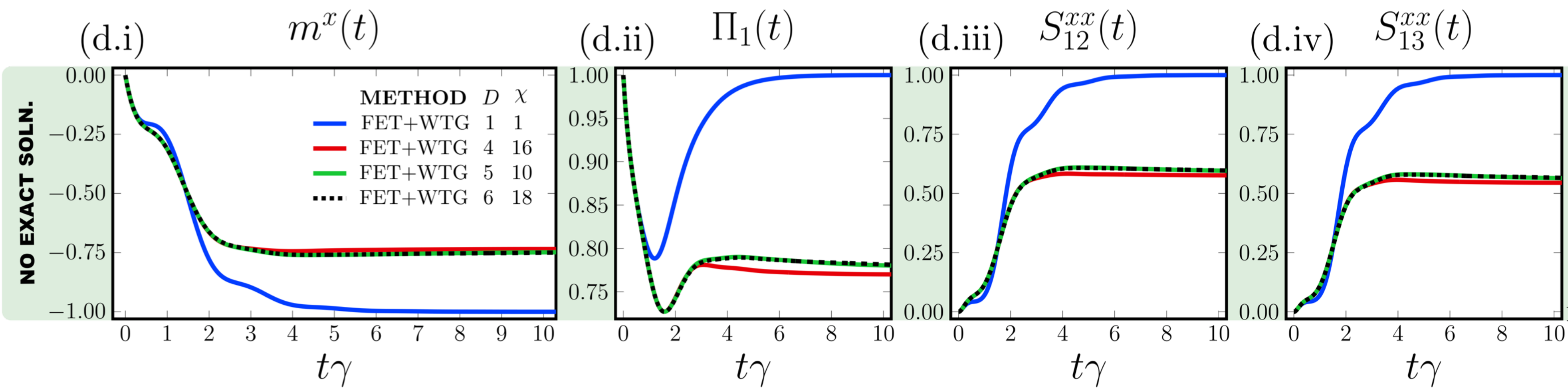
Accurate dynamics for strong and moderate damping

Significant correction to the mean field theory

Finite Transverse Field

C. Mc Keever, MH Szymanska *PRX* (2021)

$V/\gamma = 0.5$, $hx/\gamma = 1.0$



✧ Significant correction to mean field (D=1) result

✧ Convergence achieved for D>4

No exact solution to compare with
Quantum Simulator needed

Anisotropic Dissipative XY model

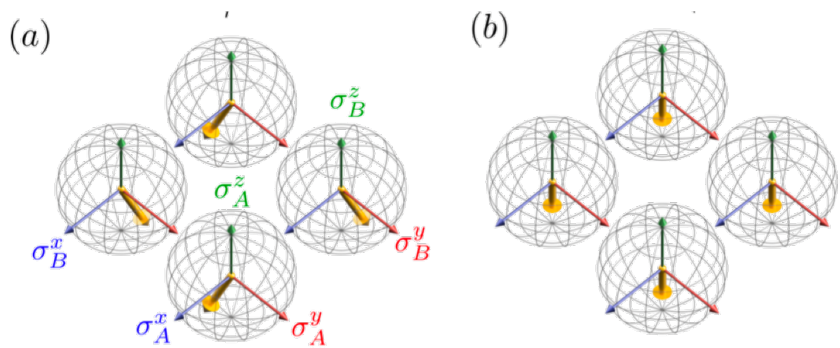
C. Mc Keever, MH Szymanska *PRX* (2021)

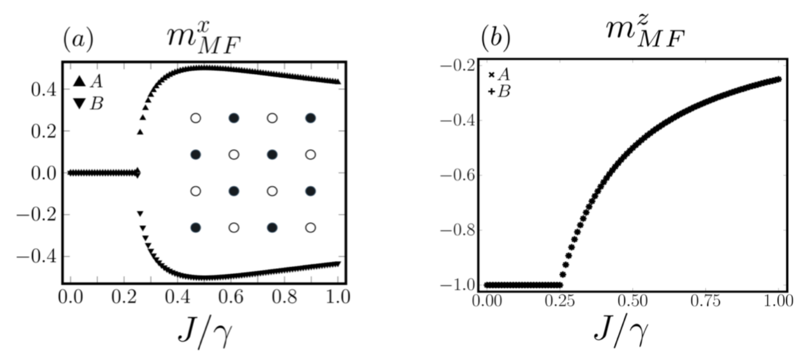
$$\hat{H} = \frac{J}{z}\sum_{\langle j,k\rangle} \hat{\sigma}^x_j\hat{\sigma}^x_k - \hat{\sigma}^y_j\hat{\sigma}^y_k \qquad \hat{L}_j = \sqrt{\Gamma}\hat{\sigma}^-_j$$

Anisotropic coupling: effective magnetic field perpendicular to the spin
Dissipation: causes decaying towards $|\downarrow^z\rangle$
PT-symmetric model

✧ Mean-field theory: spontaneously symmetry broken staggered-XY (sXY) steady-state phase stable in 2D

E. Lee et. al., PRL. 110, 257204 (2013)

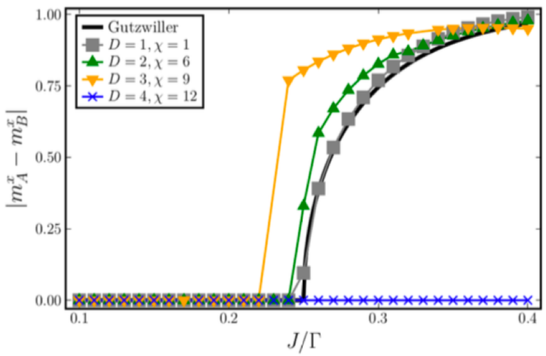


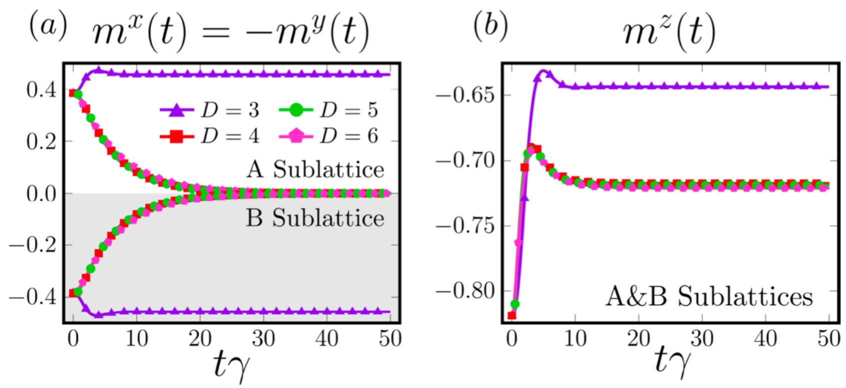


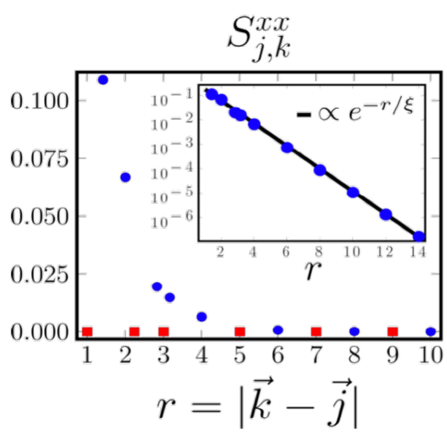
✧ Keldysh Field theory: long wavelength limit, classical XY always above BKT transition i.e sXY destroyed by fluctuations

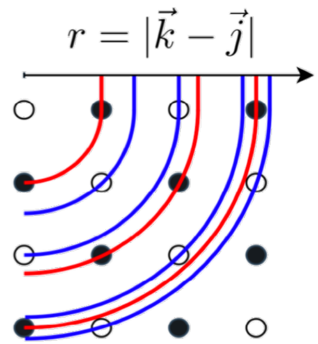
M. F. Maghrebi et. al., PRB 93, 014307 (2016)

Anisotropic Dissipative XY model





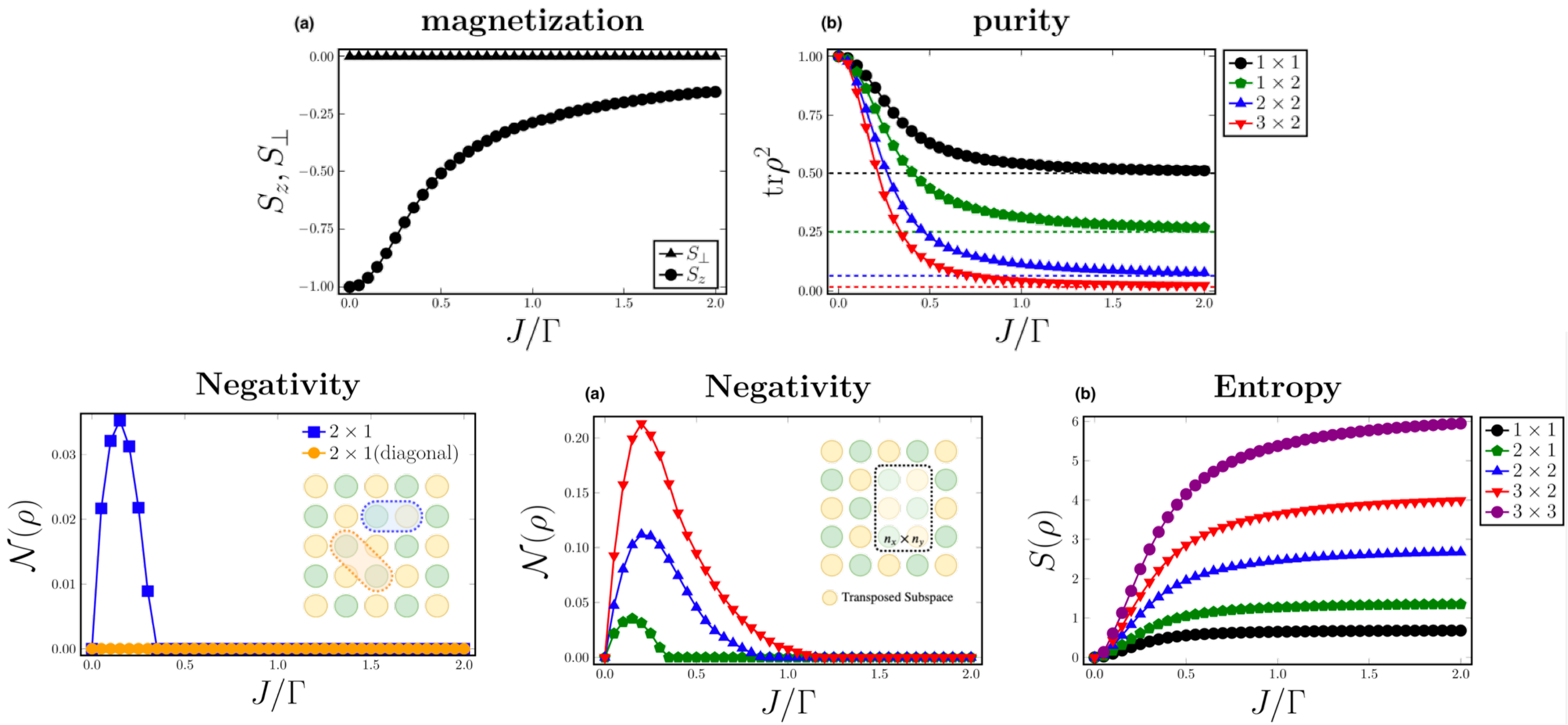




Any long range algebraic order associated to the symmetry broken phase is not present in the iPEPO solution - disordered phase

C. Mc Keever, MH Szymanska *PRX* (2021)

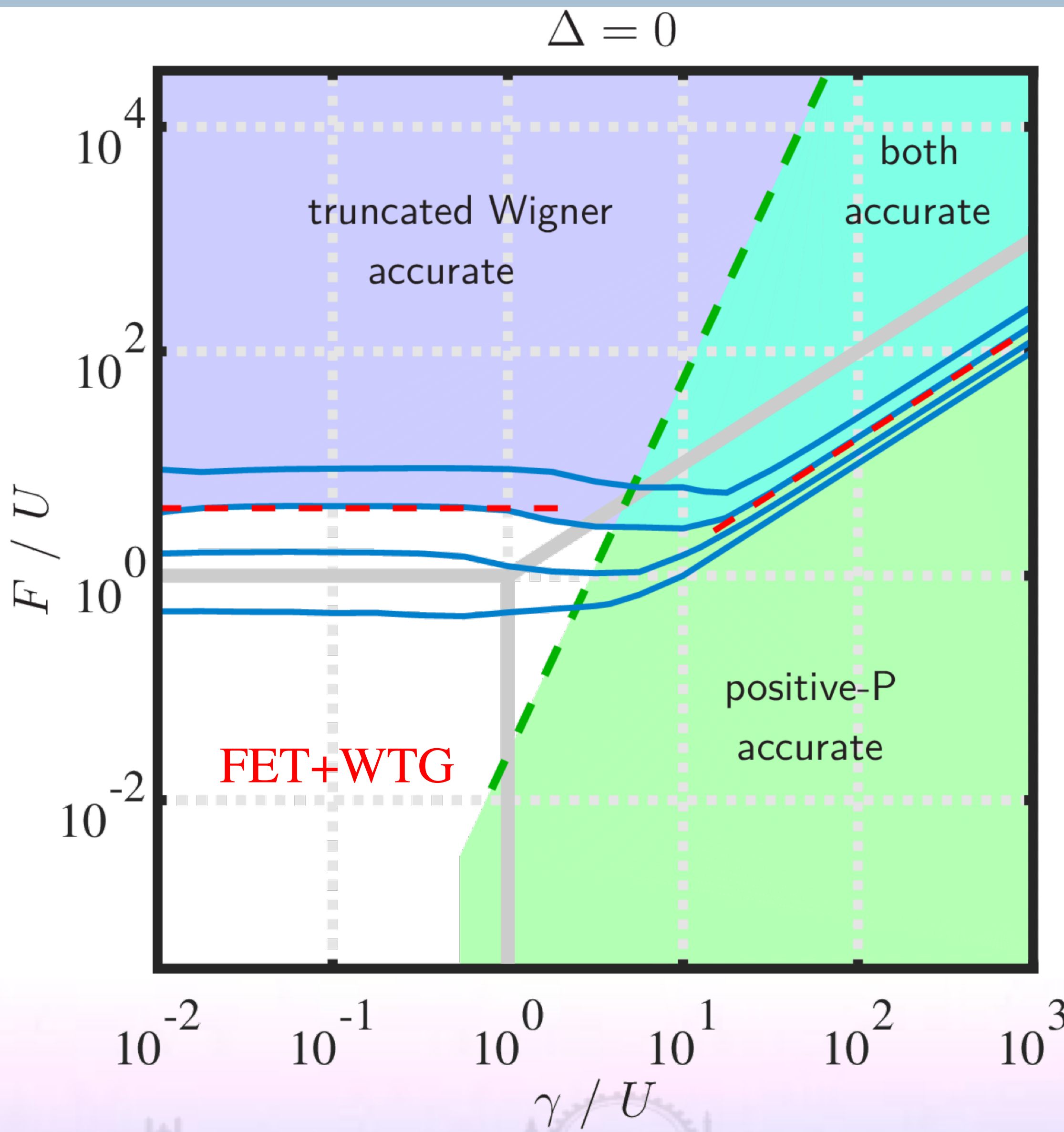
Anisotropic Dissipative XY model



Smooth crossover between maximally mixed and pure state –
no phase transition despite PT symmetry

C. Mc Keever, MH Szymanska *PRX* (2021)

Conclusions



- Dissipation in e.g. photonic platforms helps in performance of both stochastic and TN methods
- Quantum correlation and entanglement possible to describe with these methods
- Physical Quantum Simulators needed for verification and to cross the limits