Thermalization in quantum chromodynamics: Ab initio approaches and interdisciplinary connections

Raju Venugopalan

Brookhaven National Laboratory

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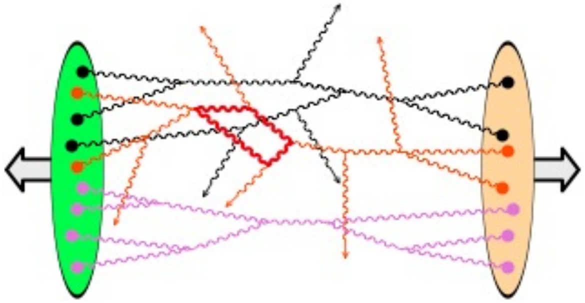
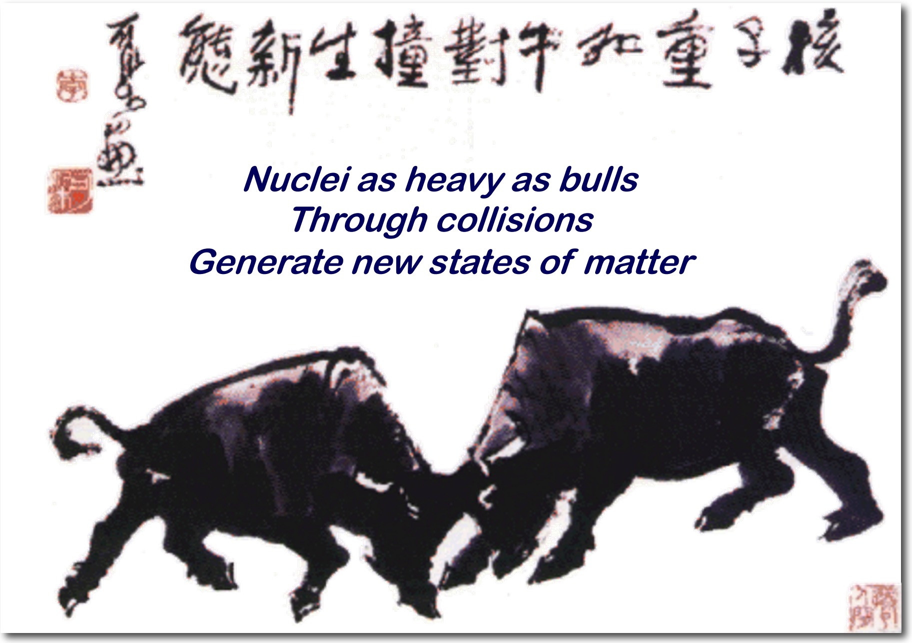
Boiling the QCD vacuum in ultrarelativistic heavy-ion collisions

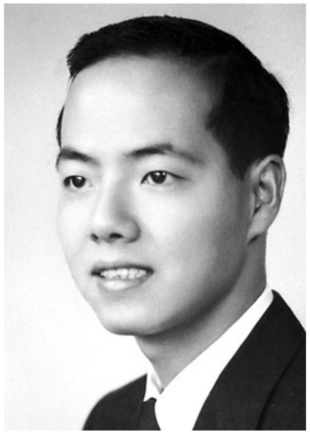
Collisions of heavy-ions at 200 GeV/nucleon
at BNL's Relativistic Heavy Ion Collider (RHIC)
and 5.5 TeV/nucleon
at CERN's Large Hadron Collider (LHC)
create a deconfined non-Abelian fluid,
the Quark-Gluon Plasma (QGP)



Hottest matter on earth:
$T \sim 4 * 10^{12}$ Kelvin



Nuclei as heavy as bulls
Through collisions
Generate new states of matter



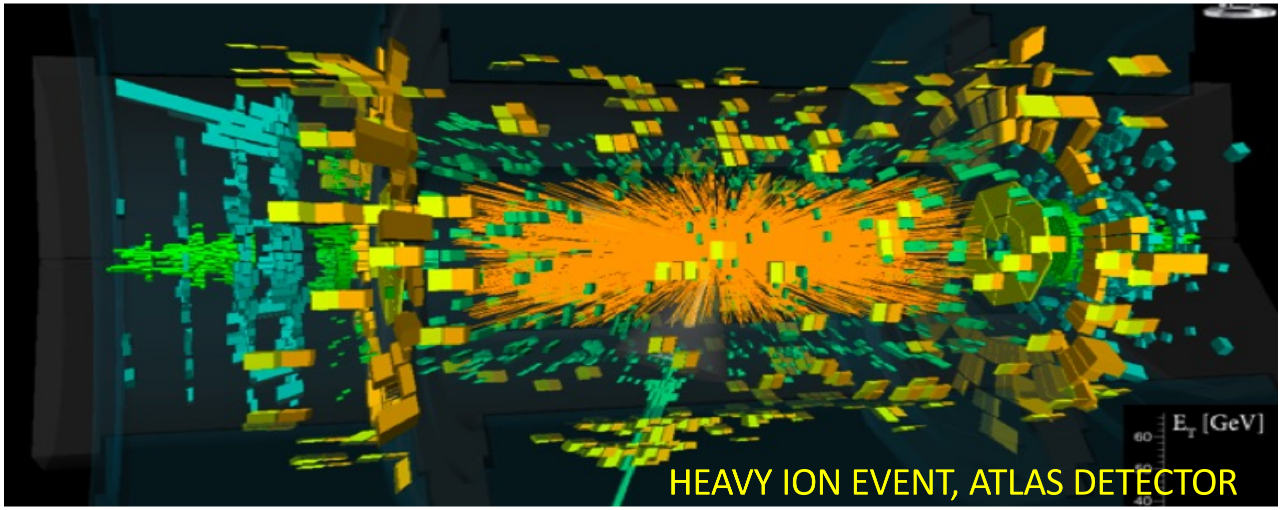
TD Lee
Nobel Laureate (1957)

At lower energy, physics sensitive to critical phenomena in the T-μ_B phase diagram

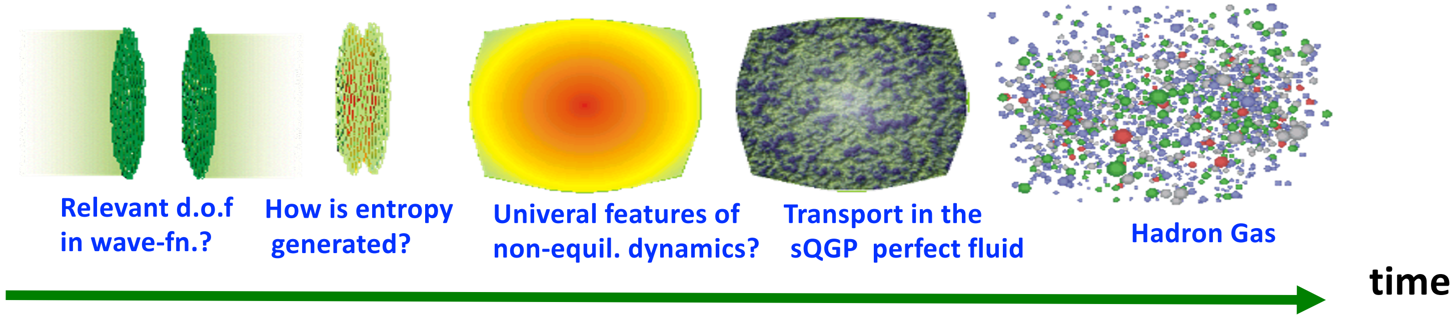
Beam energy scan at RHIC

“Standard model” of a heavy-ion collision

Initial energy
density > 10^3 that
at cross-over
to hadron gas



HEAVY ION EVENT, ATLAS DETECTOR



Relevant d.o.f
in wave-fn.?

How is entropy
generated?

Univeral features of
non-equil. dynamics?

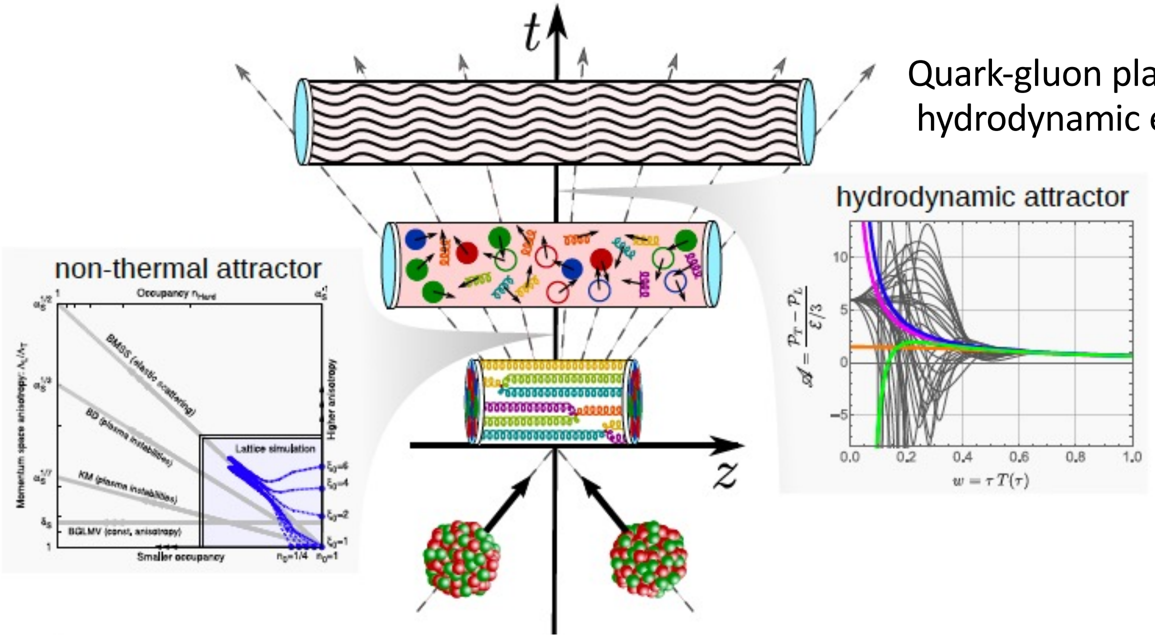
Transport in the
sQGP perfect fluid

Hadron Gas

time

Thermalization a very difficult problem in quantum field theory even for simple systems:
Ab initio theory, phenomenology and insight from other sub-fields, all essential for progress

Spacetime evolution of a heavy-ion collision



Quark-gluon plasma undergoing
hydrodynamic expansion

QCD thermalization: Ab initio approaches and interdisciplinary connections

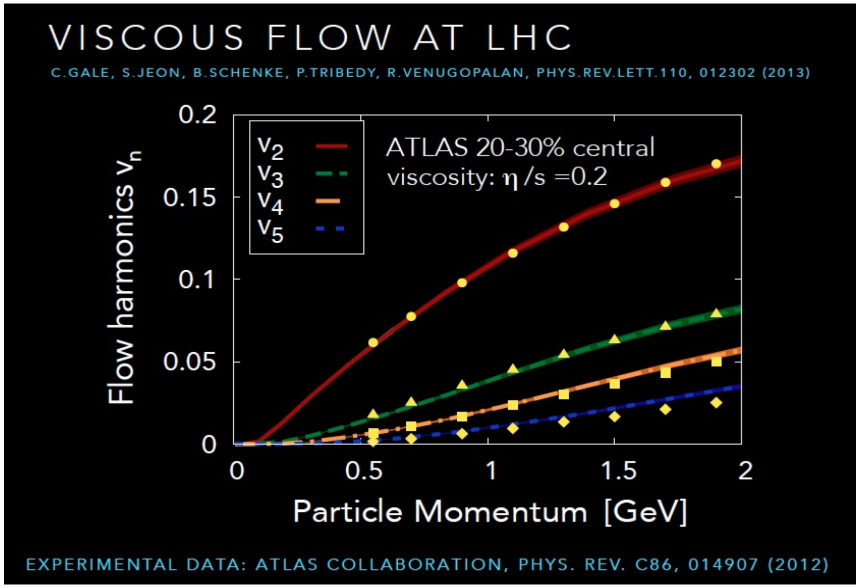
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, and Raju Venugopalan

Rev. Mod. Phys. **93**, 035003 (2021)

The unreasonable effectiveness of hydrodynamics

Initial anisotropic
spatial geometry

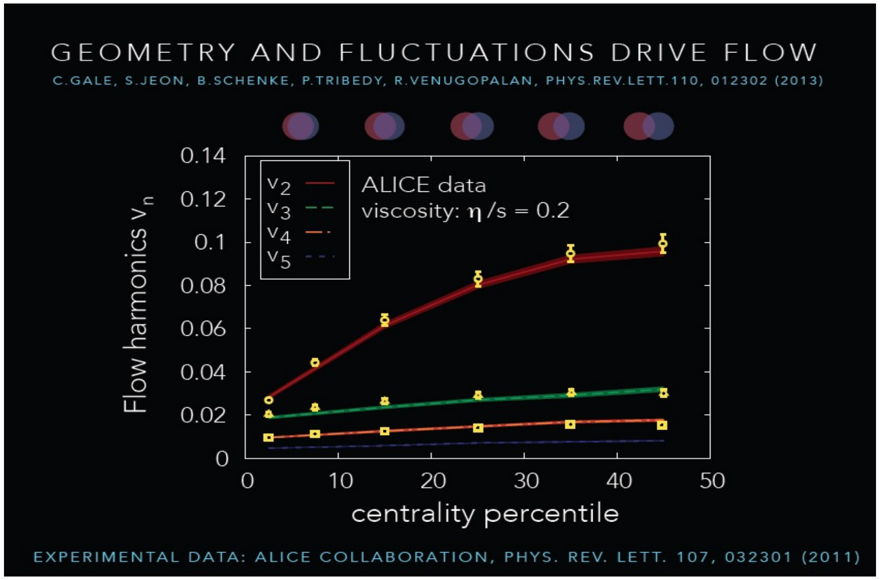
Final momentum anisotropy
from hadron correlations



VISCOUS FLOW AT LHC

C.GALE, S.JEON, B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PHYS.REV.LETT.110, 012302 (2013)

EXPERIMENTAL DATA: ATLAS COLLABORATION, PHYS. REV. C86, 014907 (2012)



GEOMETRY AND FLUCTUATIONS DRIVE FLOW

C.GALE, S.JEON, B.SCHENKE, P.TRIBEDY, R.VENUGOPALAN, PHYS.REV.LETT.110, 012302 (2013)

EXPERIMENTAL DATA: ALICE COLLABORATION, PHYS. REV. LETT. 107, 032301 (2011)

Approaches to thermalization in heavy-ion collisions

Two “clean” *ab initio* theoretical approaches:

- Holographic thermalization (paradigmatic example: AdS/CFT duality of strongly coupled ($g^2 N_c \to \infty$, $N_c \to \infty$) N=4 SUSY YM to classical gravity in $AdS_5 \times S_5$)
  Not the right theory but valuable insight into universal features of strongly correlated non-equilibrium dynamics. Example: transport coefficients, hydrodynamics far-from-equilibrium

- Highly occupied (occupancy f >> 1) QCD at weak coupling ($g^2 \to 0$, g^2 f ~ 1)

Much of this talk will be describe the second of these two approaches

Applications to heavy-ion collisions require *phenomenological extrapolations* of ab initio theory

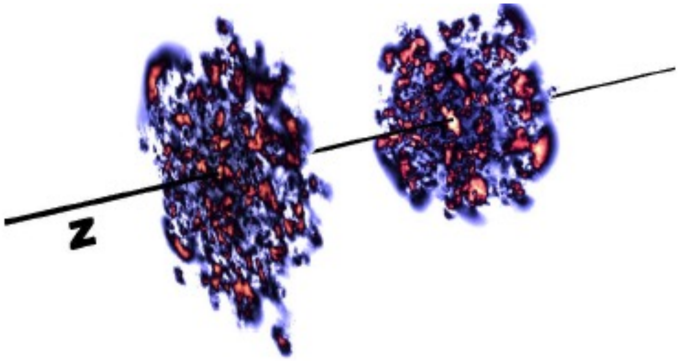
Universal features of the dynamics (non-thermal fixed points, hydrodynamization), and interdisciplinary connections thereof, provide powerful guidance

Highly occupied glue far off-equilibrium: the Glasma

In QCD's high energy (Regge) limit,
nuclei can be described as highly occupied $\sim\frac{1}{\alpha_S}$
gluon shock waves with emergent hard scale Q_S

EFT description: Color Glass Condensate

Review: Gelis,Jalilian-Marian,Iancu,RV, arXiv:1002.0333



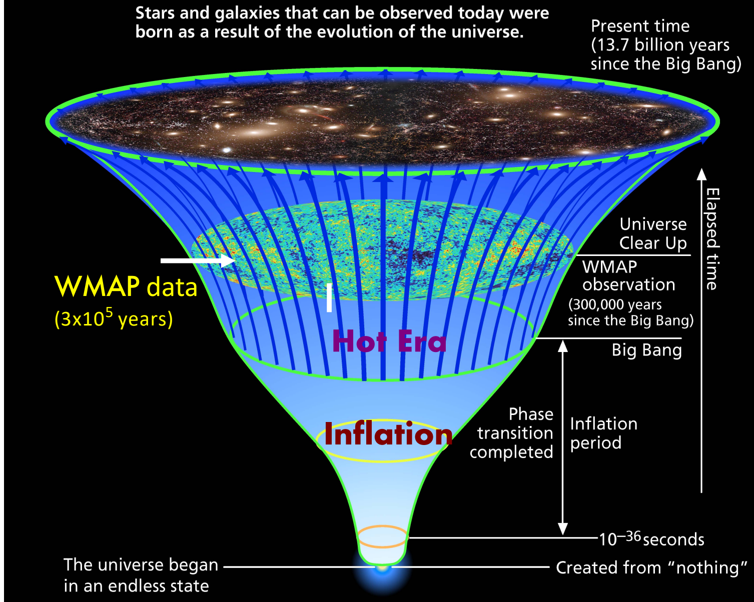
Collisions of "lumpy" gluon shock waves

Leading order solution shock wave solution: QCD Yang-Mills eqns in presence of light-cone sources

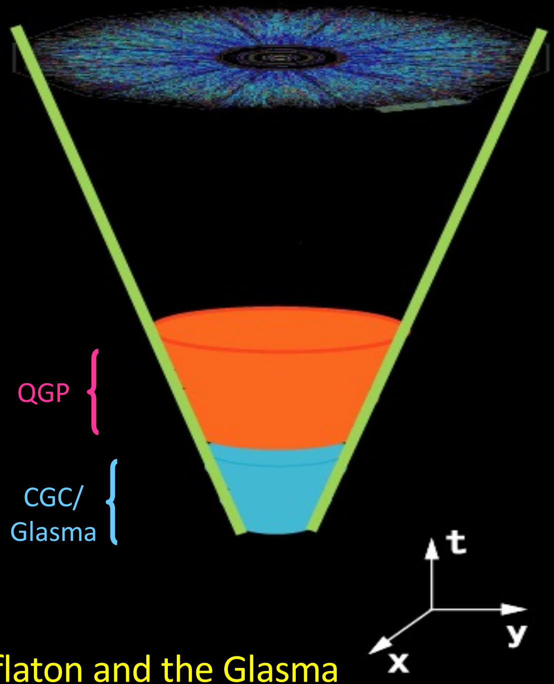
$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_A^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_B^a(x_\perp)\delta(x^+)$$

$$\langle\rho_{A(B)}^a(x_\perp)\rho_{A(B)}^a(y_\perp)\rangle = Q_{S,A(B)}^2\delta^{(2)}(x_\perp - y_\perp)$$

Big Bang



Little Bang



Non-equilibrium dynamics: the inflaton and the Glasma

Plot by T. Hatsuda

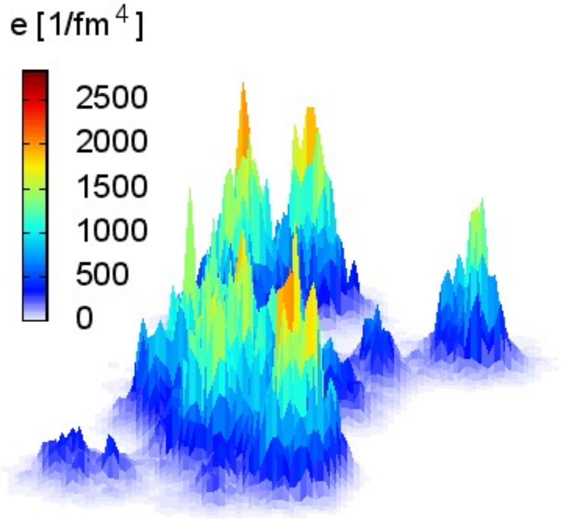
Big Bang vs. Little Bang

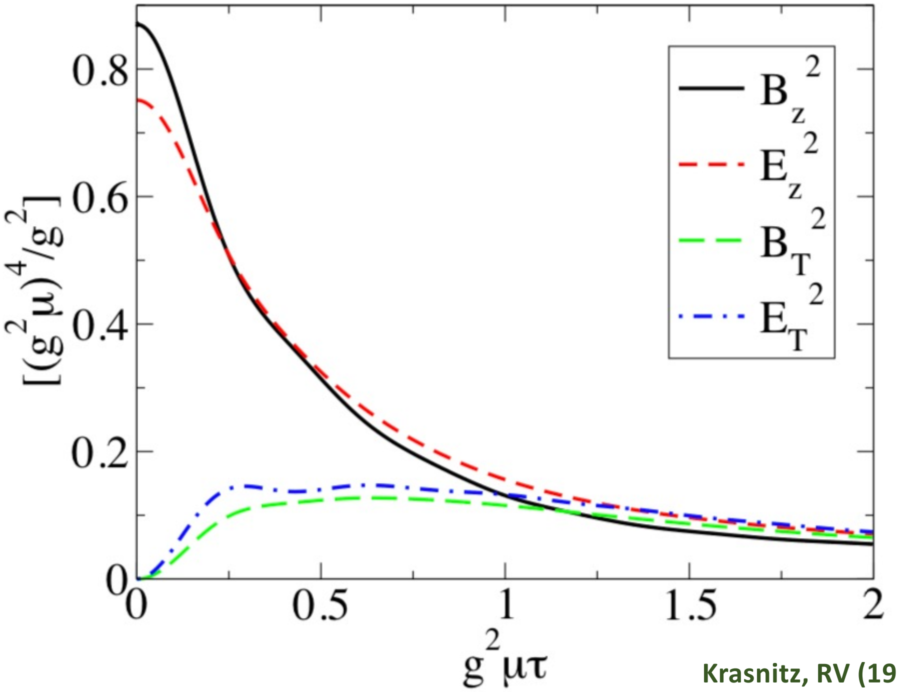
Decaying Inflaton with occupation # $1/g^2$	↔	Decaying Glasma with occupation # $1/g^2$
Explosive amplification of low momentum small fluctuations (parametric resonance in preheating)	↔	Explosive amplification of low momentum small fluctuations (Weibel instabilities)
Interaction of fluctuations/inflaton -> thermalization?	↔	Interaction of fluctuations/Glasma -> thermalization?

Other common features: turbulence, topological defects,…

***Turbulent thermalization*,**
Micha and Tkachev, arXiv:hep-ph/0403101

Boost invariant Yang-Mills: numerical solutions





Glasma energy density and pressure

Krasnitz, RV (1998)
Lappi,hep-ph/0606207

$$T_{\mu\nu}(\tau=0) = \frac{1}{2}(B_z^2 + E_z^2) \times \mathrm{diag}(1,1,1,-1)$$

Initial longitudinal pressure is negative: Goes to $P_L = 0$ from below with time evolution

Decoherence and prethermalization from quantum fluctuations

“Toy” example: scalar Φ^4 theory

Gaussian random variable

$$\langle c_{\nu k} c_{\mu l} \rangle = 0$$

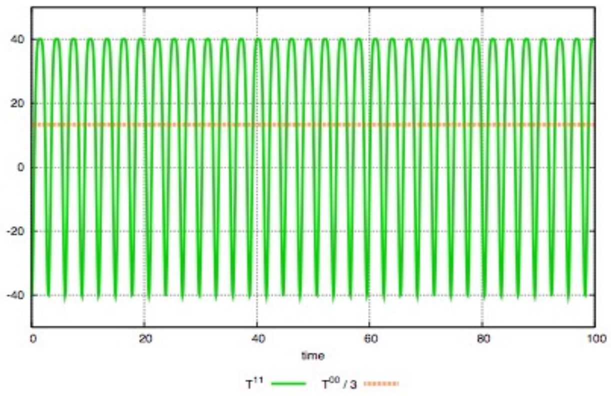
$$\langle c_{\nu k} c^*_{\mu l} \rangle = 2\pi \delta(\nu - \mu)\delta_{kl}$$

$$\phi(\tau, \eta, x_\perp) = \phi_{\rm cl.}(\tau, x_\perp) + \frac{1}{2}\int \frac{d\nu}{2\pi}\, d\mu_k\; c_{\nu k}\; e^{i\nu\eta} \chi_k(x_\perp) H_{i\nu}(\lambda_k \tau) + c.c$$

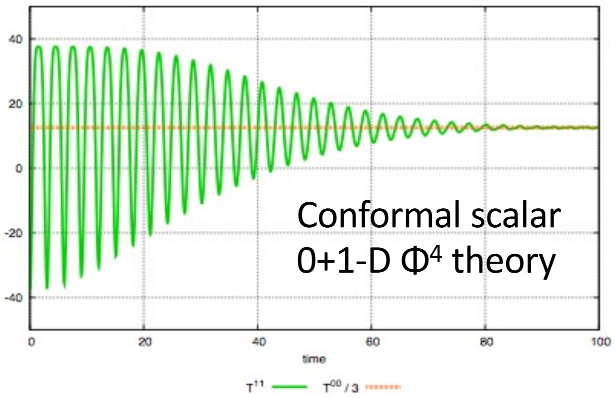
satisfies “small fluctuation” equation

$$[-\partial_\perp^2 + V''(\phi_0)]\chi_k = \lambda_k^2\, \chi_k$$

Energy density and pressure without averaging over fluctuations

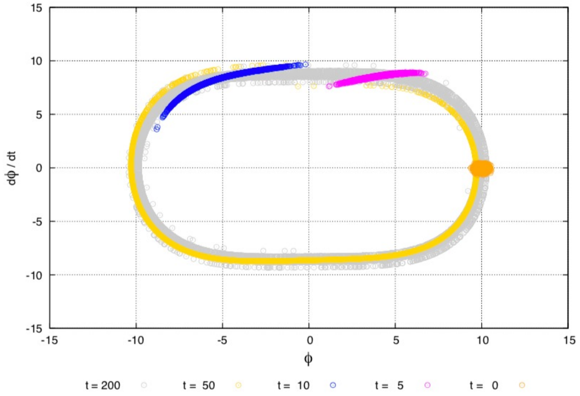


Energy density and pressure after averaging over fluctuations



Conformal scalar
0+1-D Φ^4 theory

Phase space density in Poincaré plane



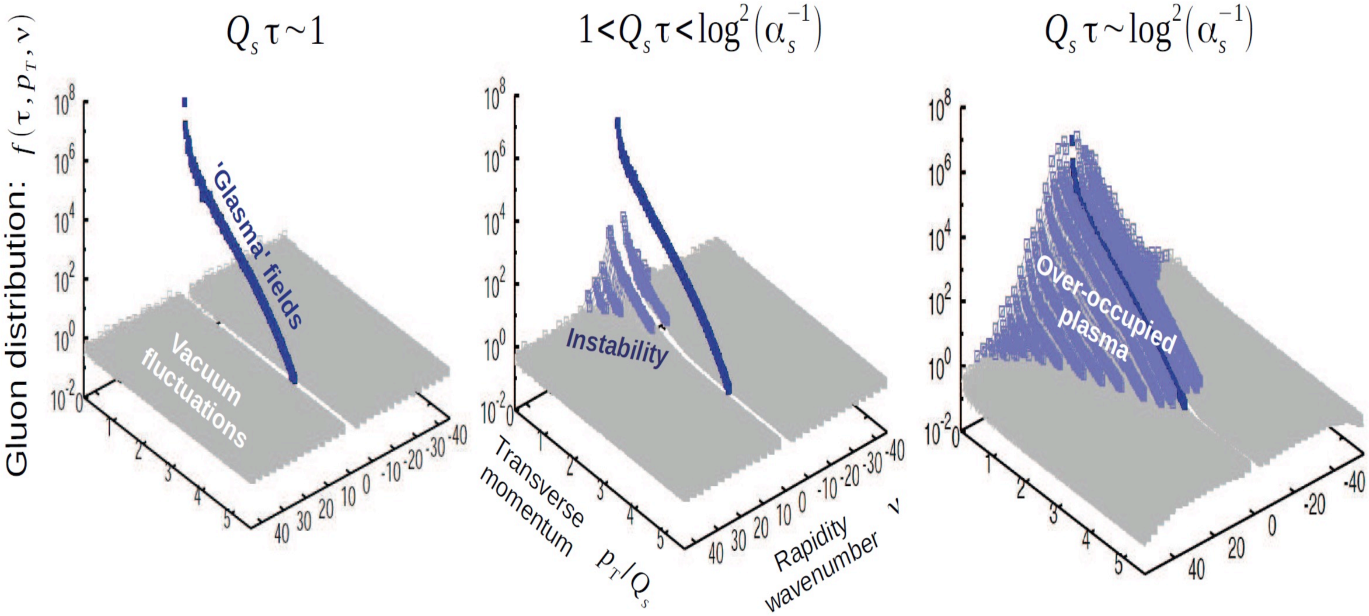
These quantum modes satisfy an “eigenstate thermalization” criteria conjectured by Berry (and significantly developed by Srednicki and others) as essential for thermalization of a quantum fluid

Berges, Borsanyi, Wetterich (2004)
Dusling,Epelbaum,Gelis,RV (2011)

Explosive amplification of quantum fluctuations

Longitudinally expanding Glasma fields are unstable to quantum fluctuations… leading to an explosive "Weibel"-like instability.

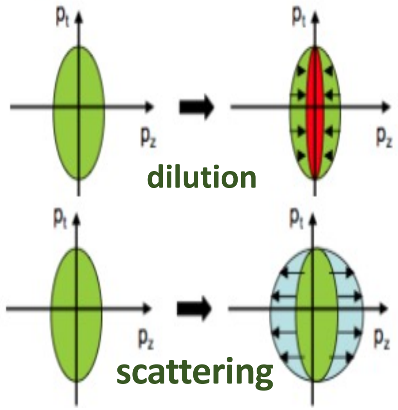
Rapid decoherence and overpopulation of all momentum modes



Classical-statistical lattice simulations of 3+1-D gluon fields exploding into the vacuum

Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

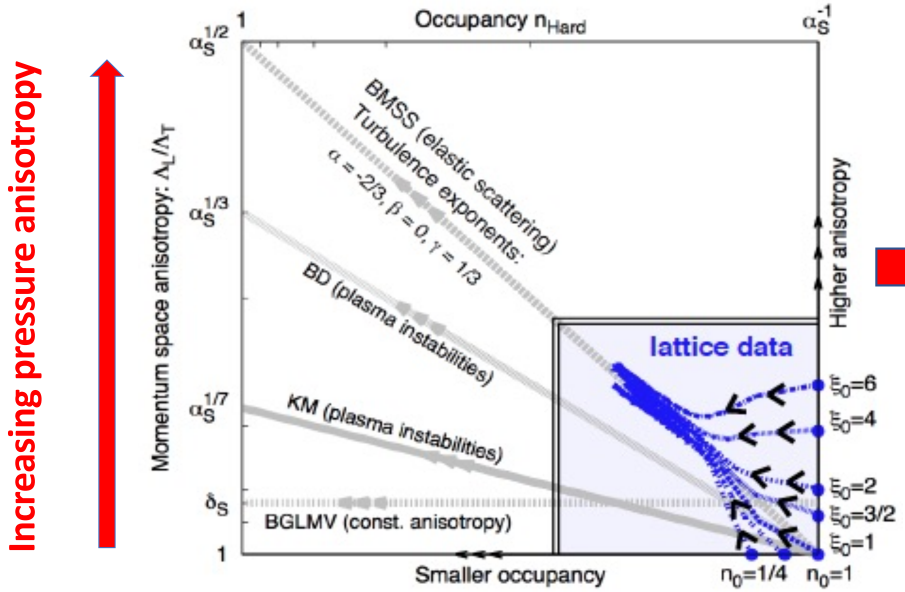
Classical-statistical simulations of 3+1-D Yang-Mills

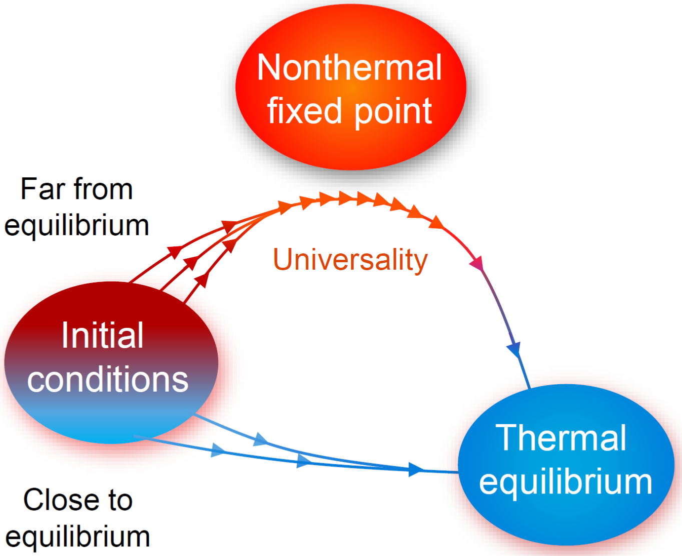


After rapid scrambling of information by quantum fluctuations, competition between dilution due to expansion and isotropization due to scattering

Single particle distributions become self-similar in time characterized by universal exponents – helps identify “right” kinetic theory

$$f(p_\perp, p_z, t) = t^\alpha\ f_S(t^\beta p_T, t^\gamma p_z)$$





Nonthermal fixed point

Far from equilibrium

Universality

Initial conditions

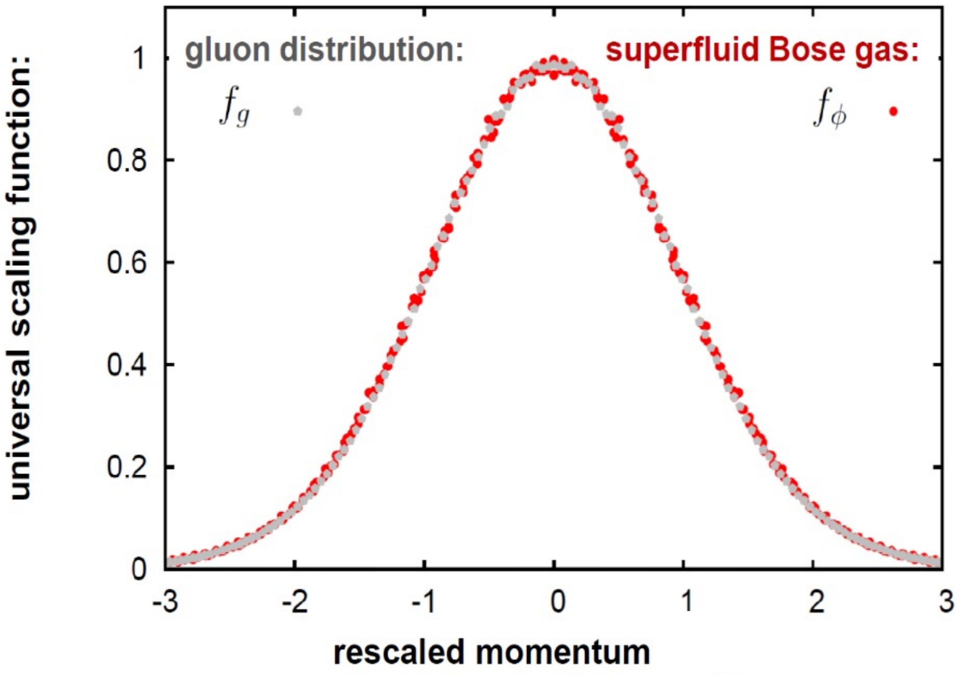
Thermal equilibrium

Close to equilibrium

Berges,Boguslavski,Schlichting,RV (2014)

The Glasma and overoccupied ultracold quantum gases

Simulations of self-interacting scalar fields with identical initial conditions demonstrates remarkable *universality of longitudinally expanding world's hottest and coolest fluids*



In a wide inertial range, scalar & gauge fields have identical scaling exponents & functions

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

$$\tau = \sqrt{t^2 - z^2}$$

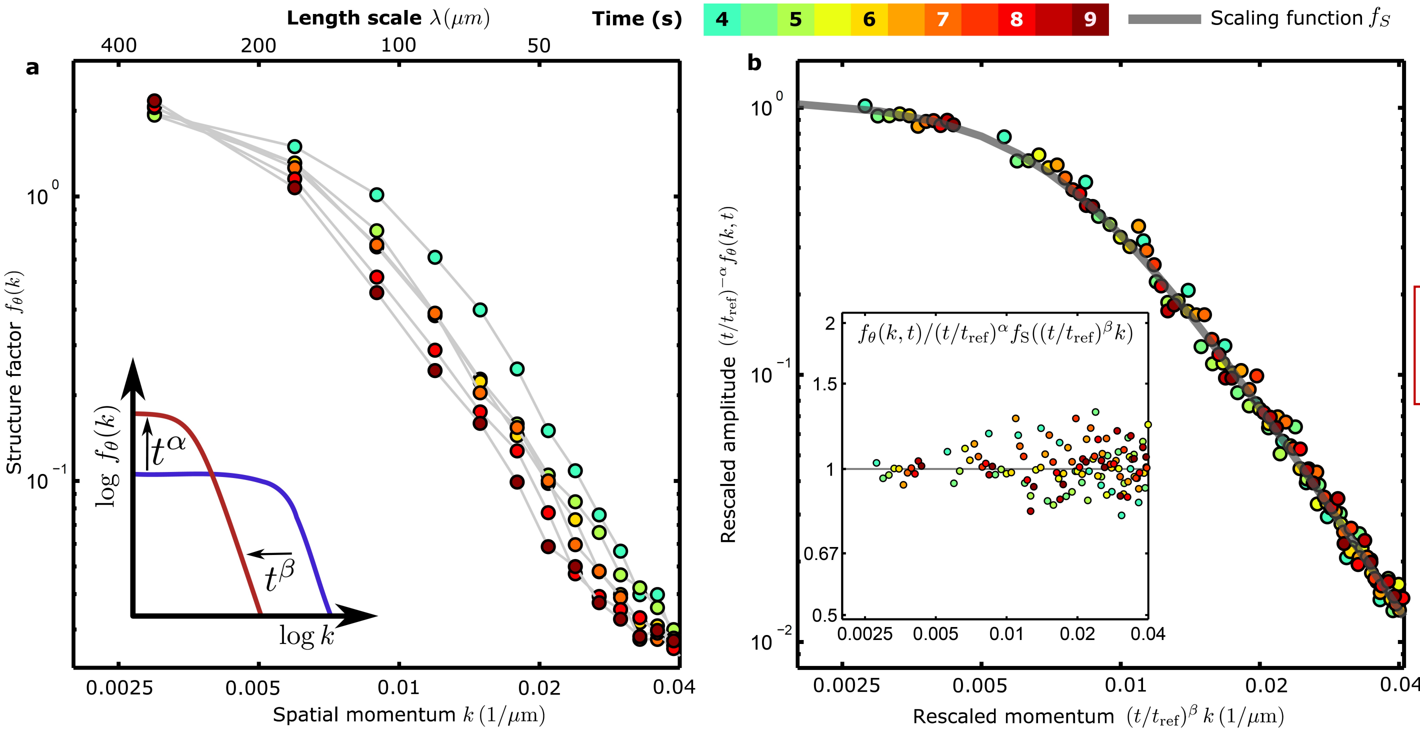
$$\alpha = -\frac{2}{3}, \beta = 0, \gamma = 1/3$$

Berges,Boguslavski,Schlichting,RV, PRL (2015)
Editor's suggestion

The Glasma and over-occupied quantum gases

Similar non-thermal fixed points discovered in cold atom experiments
- albeit only for static geometry so far

^{87}Rb BEC in a quasi 1D optical trap

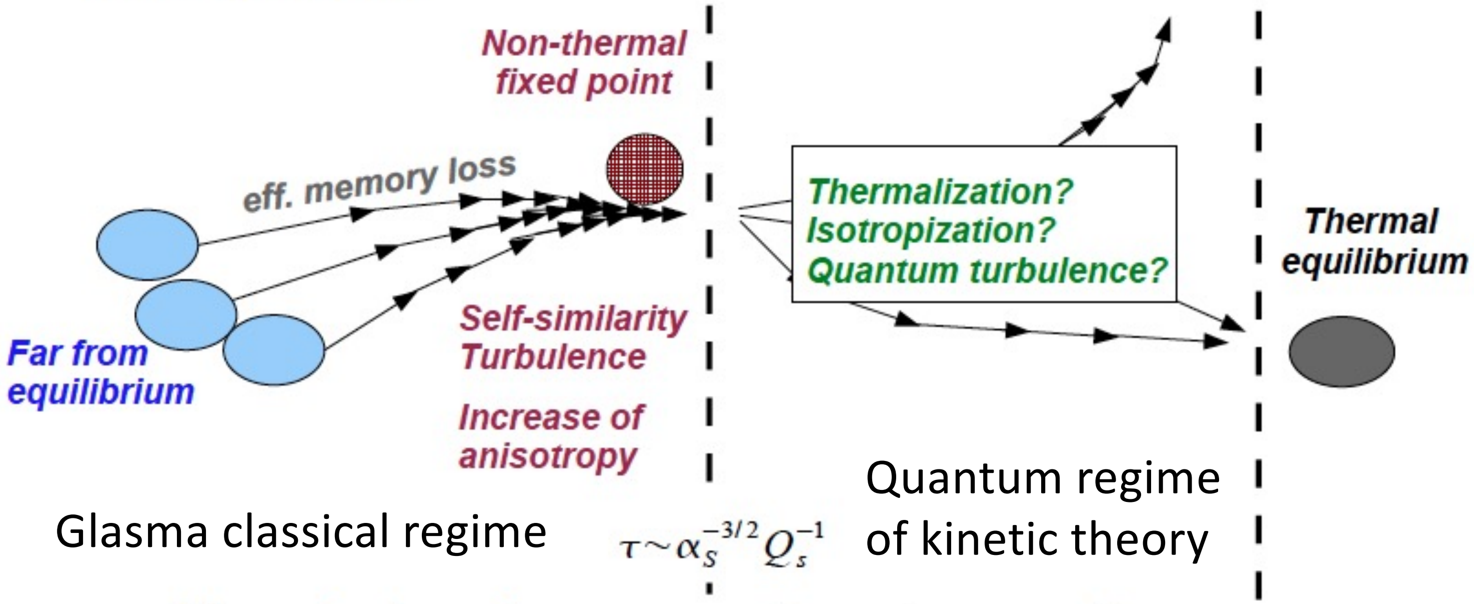


$$f_\theta(k,t) = t^\alpha f_S(t^\beta k)$$

$\alpha = 0.33 \pm 0.08$ $\beta = 0.54 \pm 0.06$

Oberthaler BEC Labs
Prüfer et al, arXiv:1805.11881, *Nature* (2018)

From nuts to soup: bottom-up thermalization



Non-thermal fixed point

eff. memory loss

Far from equilibrium

Self-similarity Turbulence

Increase of anisotropy

Thermalization?
Isotropization?
Quantum turbulence?

Thermal equilibrium

Glasma classical regime

$\tau \sim \alpha_S^{-3/2} Q_s^{-1}$

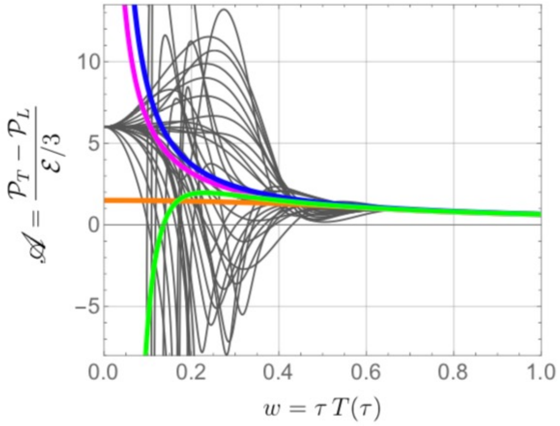
Quantum regime of kinetic theory

Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Baier,Mueller,Schiff,Son, hep-ph/0009237

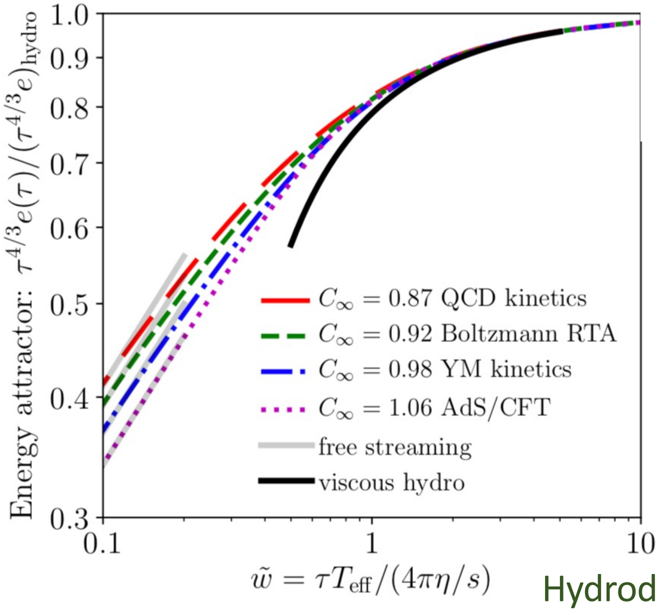
Hydrodynamics far-from-equilibrium



Magenta, Blue, Green, 1st, 2nd, 3rd, hydro constitutive relations. Orange curve is hydro attractor; different init. conditions (grey)

Heller, Janik, Witaszczyk, arXiv:1203.0755
Romatschke, arXiv:1704.08699

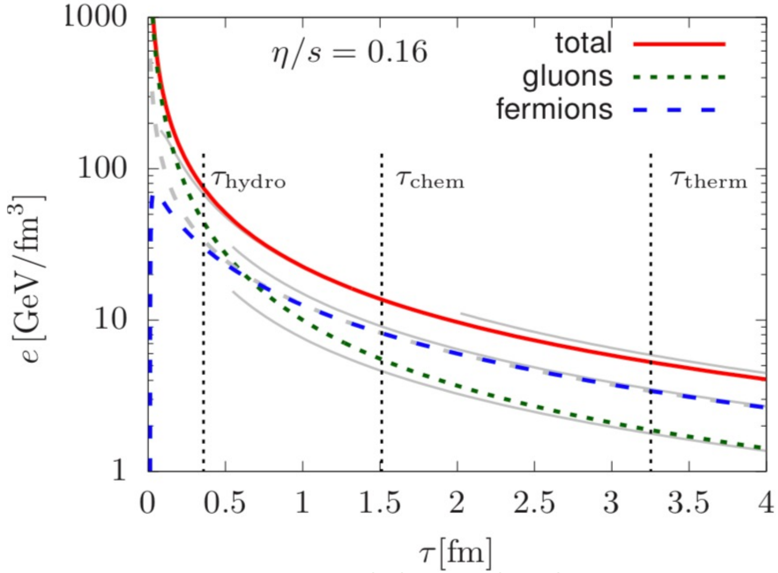
Rapid convergence to viscous hydrodynamics



Hydrodynamization variable
-inverse Knudsen #

Giacalone,Schlichting,Mazeliauskas,arXIv:1908.02866

Hydrodynamization < chemical < thermal equilibration



Kurkela,Mazeliauskas,arXiv:1811.03040

From nuts to soup: bottom-up thermalization

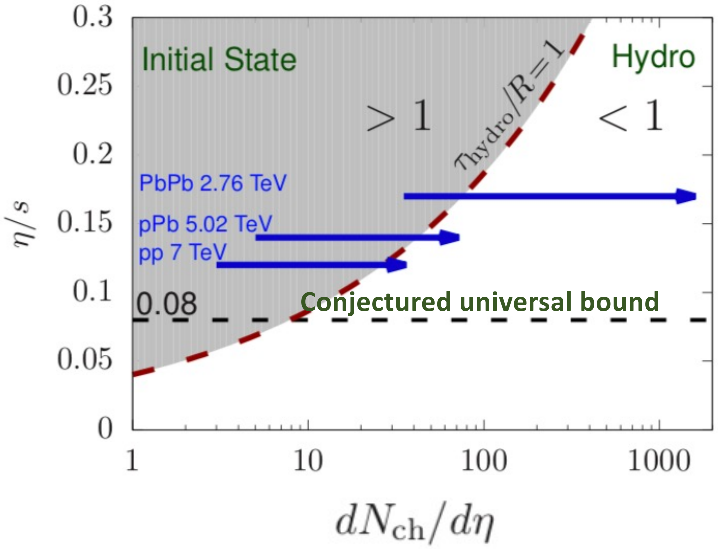
Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Since $\alpha_S \propto \frac{1}{\log Q_S}$

then $\tau_{\text{therm}} \propto \frac{(\log Q_S)^{5/2}}{Q_S} \rightarrow 0$ as $Q_S \rightarrow \infty$

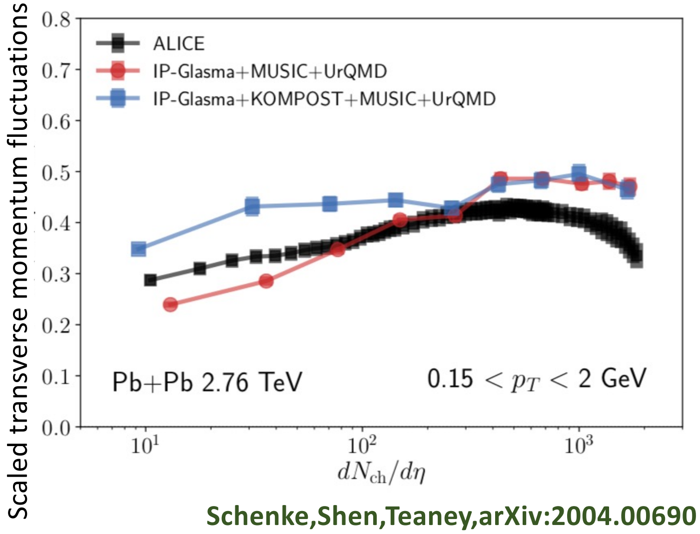
Glasma/bottom-up prediction: *In the Regge limit, matter thermalizes almost instantaneously*



Event engineering:

Promising tool
to quantify thermalization
times in small systems

Mazeliauskas, arXiv:1807.05586
See also Kurkela,Wiedemann,Wu,
arXiv:1905.05139

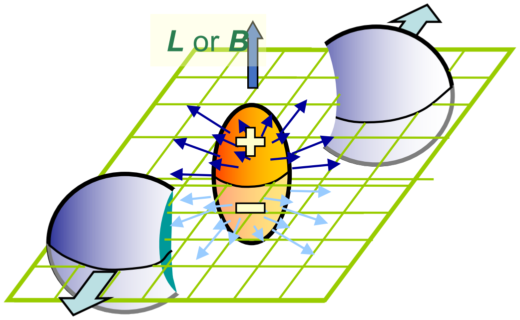


Schenke,Shen,Teaney,arXiv:2004.00690

KOMPOST implementation of bottom-up:

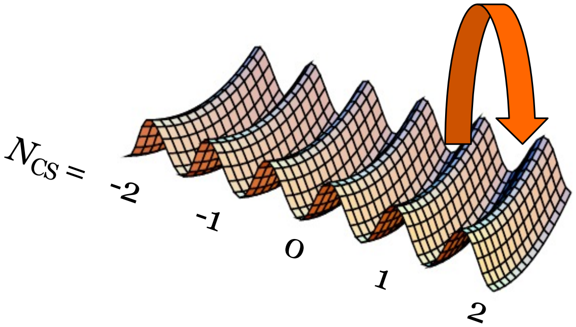
Kurkela et al., arXiv:1805.00961

Early-time dynamics and topology: The Chiral Magnetic Effect



External (QED) magnetic fields
10^{18} Gauss, of Magnetar strength!
-dies very rapidly (~1 fm) after collision

+

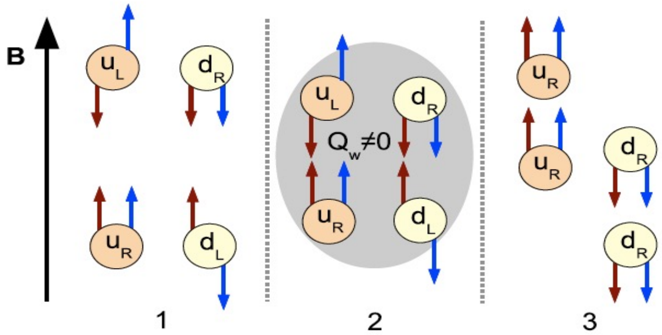


Kharzeev,McLerran,Warringa (2007)
Kharzeev, Fukushima, Warringa (2008)

Considerable experimental activity in HI collisions and condensed matter systems

Over barrier topological (sphaleron) transitions … analogous to proposed mechanism for electroweak baryogenesis

$$\vec{J}_{\mathrm{CME}} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$$



Chiral magnetic effect

Ruled out definitively in isobar RHIC collisions at the highest energy.

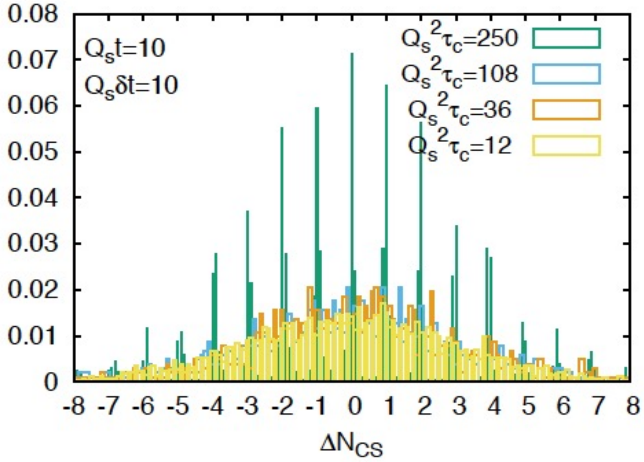
STAR collaboration, arXiv:2109.00131

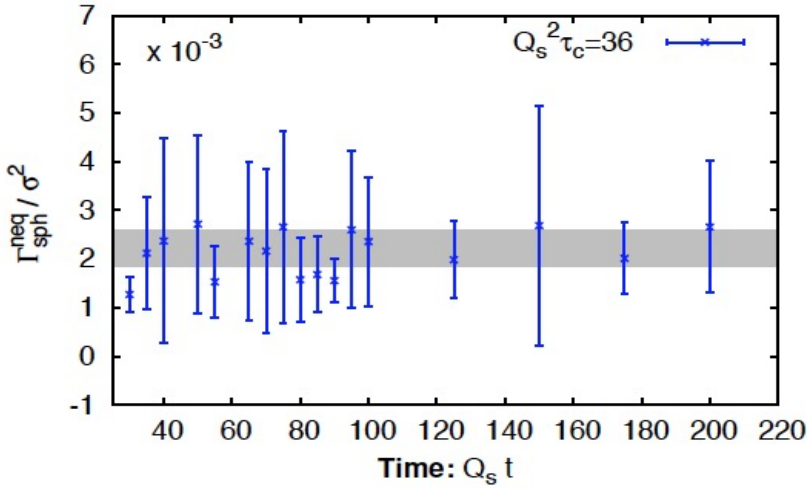
Prospects may still exist at lower energies

Early-time dynamics and topology: The Chiral Magnetic Effect

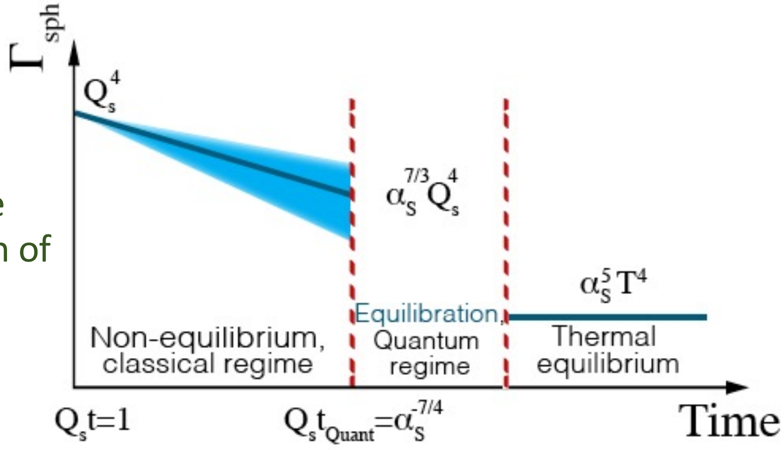
Mace,Schlichting, RV: PRD (2016)

Chern-Simons # dist. in the Glasma





Sphaleron transition rate scales with string tension of spatial Wilson loops



My perspective on some outstanding open questions

What is the role of entanglement in this complex many-body system ?
Example: Entanglement “induced” thermalization of a 1+1-D QCD string

Berges, Floerchinger, RV,
arXiv: 1705.05338, 1712.09362

Are there other systems in nature which have dynamical features universal to the CGC/Glasma ?
Example: a CGC-Black Hole correspondence

Dvali, RV, arXiv:2106.11989

Can some of its real-time dynamical features be simulated by quantum computers ?
Example: Since particle digitization strategy motivated by Feynman’s parton model

Barata,Mueller,Tarasov,RV
arXiv:2012.00020