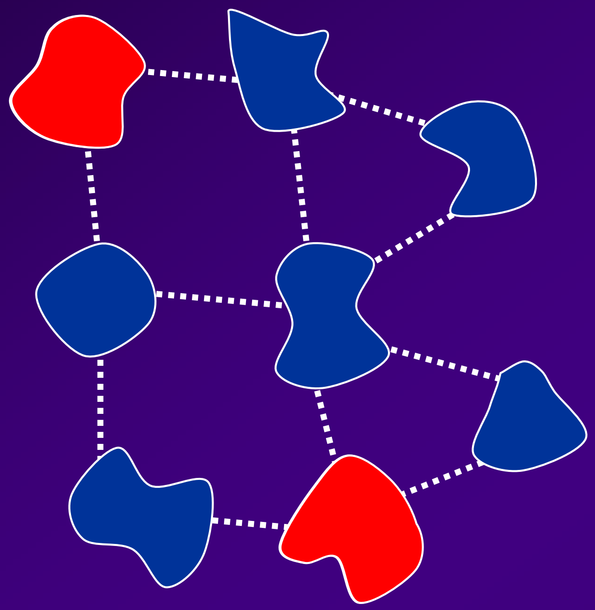
Local quantum dynamics and information flow

Benjamin Schumacher

Department of Physics
Kenyon College

Mike Westmoreland	Denison University
Lee Kennard	Kenyon College ('07)
Michael Nathanson	Kenyon / St. Mary's College
Katharina Christandl	Kenyon / Cal Poly
Reinhard Werner	U. of Braunschweig

Universe as information network



Universe is divided into subsystems.

Subsystems interact and exchange information.

Locality: Not all subsystems exchange information directly.

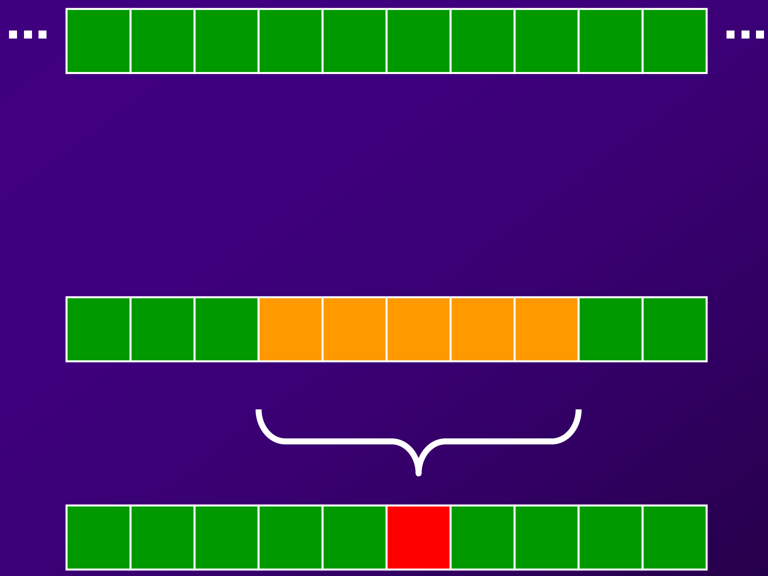
What does quantum mechanics say about the rules of this web?

What does quantum mechanics say about locality?

Classical cellular automata

Classical cellular automata

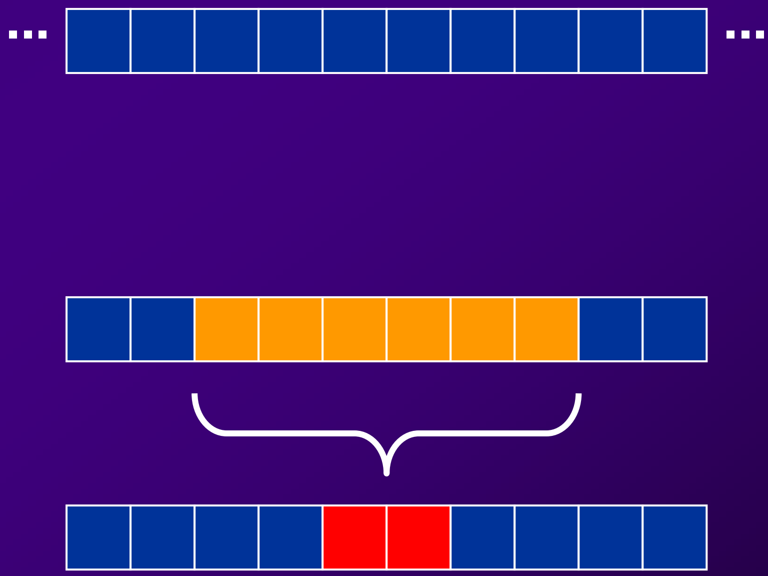
- Uniform grid of cells (1-D, 2-D, ...)
- Each cell has a finite number of states.
- At each discrete time step, cell states update according to a local rule – we need only know the previous states of a finite “neighborhood” of cells.
- Any local rule is okay.
- Global update rule can be reversible or irreversible.



Quantum cellular automata

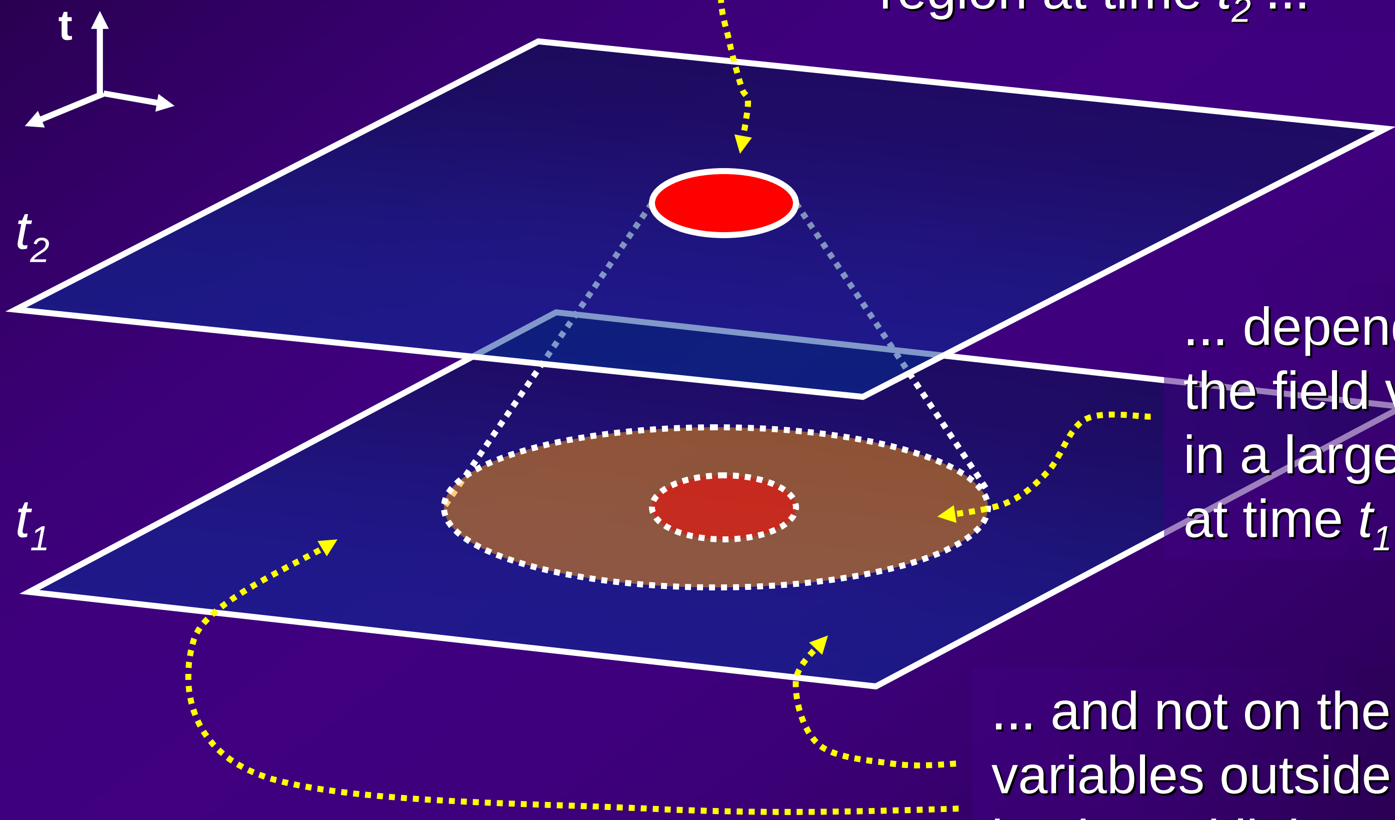
Quantum cellular automata

- Uniform grid of cells (1-D, 2-D, ...)
- Each cell is a quantum system.
- To find the next state of any bounded region, we only need to know the previous state of a “neighborhood” of that region
- Not all local rules can be woven together into a global update rule.
- Global evolution can be unitary or non-unitary



Causal structure

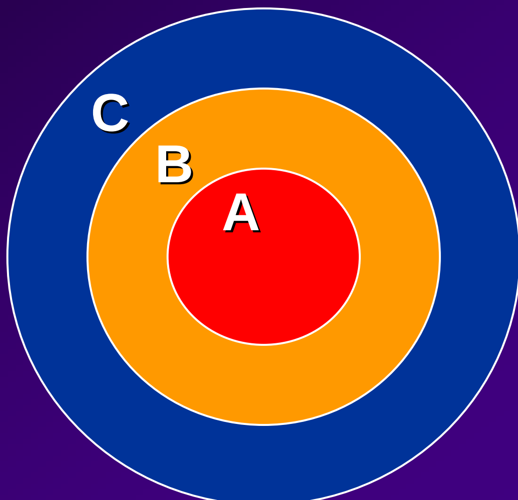
Field variables in a bounded region at time t_2 ...



... depend only on the field variables in a larger region at time t_1 ...

... and not on the field variables outside the backward light cone.

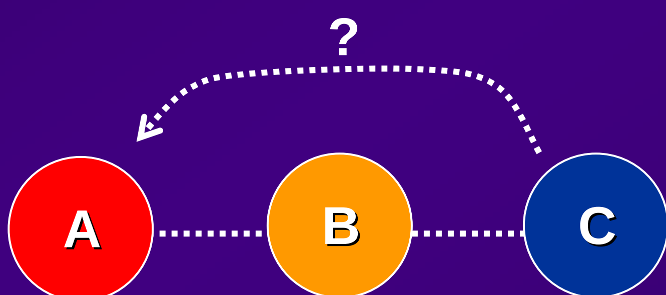
Bulls-eye and chain



A is the system of interest.

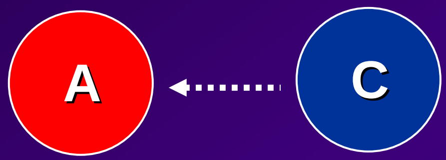
C is the distant “rest of the world”

B is the rest of A’s “neighborhood”



Locality: In one time step, there is no information transfer from C to A.

Information flow



When does information “flow” from C to A?

- Information flows from C to A if the final state of A depends on the initial state of C.
- Information does not flow from C to A if the final state of A does not depend on the initial state of C.

Note: We must consider all possible initial states of A and C.

Quantum difficulties!

Initial state of AC is not determined by the initial states of A and C separately – quantum entanglement.

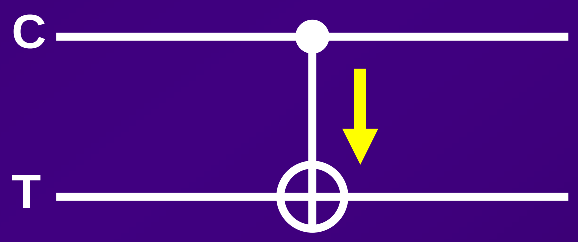
Two bits (classical)

Two classical bits.

Interaction: Controlled-NOT

C = control bit

T = target bit



Note: CNOT operation is reversible

CT		CT
0 0	→	0 0
0 1	→	0 1
1 0	→	1 1
1 1	→	1 0

Final T state does depend on initial C state. There is information flow from C to T.

Final C state does not depend on initial T state. There is no information flow from T to C.

Classical CNOT has one-way information flow from C to T.

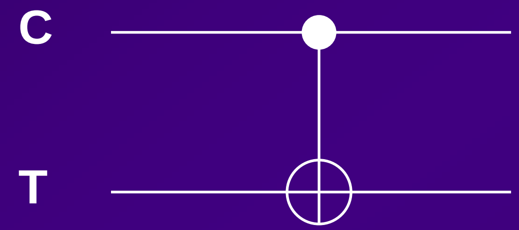
Two qubits

Two qubits.

Interaction: quantum CNOT

C = control bit

T = target bit



$\lvert CT\rangle$	$\rightarrow$	$\lvert CT\rangle$
$\lvert 0\,0\rangle$	$\rightarrow$	$\lvert 0\,0\rangle$
$\lvert 0\,1\rangle$	$\rightarrow$	$\lvert 0\,1\rangle$
$\lvert 1\,0\rangle$	$\rightarrow$	$\lvert 1\,1\rangle$
$\lvert 1\,1\rangle$	$\rightarrow$	$\lvert 1\,0\rangle$

One-way information flow? **No!**

CNOT is unitary

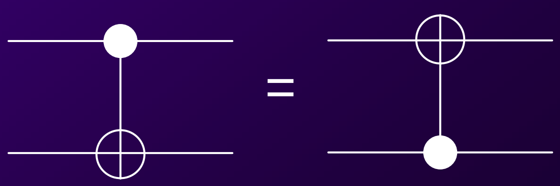
Look at CNOT in a conjugate basis:

$$\lvert + \rangle = \frac{1}{\sqrt{2}}\left(\lvert 0\rangle + \lvert 1\rangle\right)$$

$$\lvert - \rangle = \frac{1}{\sqrt{2}}\left(\lvert 0\rangle - \lvert 1\rangle\right)$$

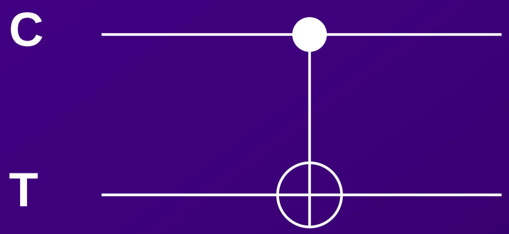
$\lvert CT\rangle$	$\rightarrow$	$\lvert CT\rangle$
$\lvert +\,+\rangle$	$\rightarrow$	$\lvert +\,+\rangle$
$\lvert +\,-\rangle$	$\rightarrow$	$\lvert -\,-\rangle$
$\lvert -\,+\rangle$	$\rightarrow$	$\lvert -\,+\rangle$
$\lvert -\,-\rangle$	$\rightarrow$	$\lvert +\,-\rangle$

In the conjugate basis, control and target qubits switch roles!



No one-way information flow

- No unitary interaction can yield one-way information flow between quantum systems.
- Quantum measurement



  C = system of interest

  T = measuring apparatus

  We'd like to have information flow C $\rightarrow$ T only, so that we do not disturb the system. But any unitary interaction can make information flow either way.

- Non-unitary quantum operations can have one-way information flow.

Quantum dynamics

Dynamics of an isolated quantum system is unitary.

$$|\psi\rangle \rightarrow U|\psi\rangle$$

pure states $\rightarrow$ pure states

mixed states $\rightarrow$ mixed states

$$\rho \rightarrow U \rho U^\dagger$$

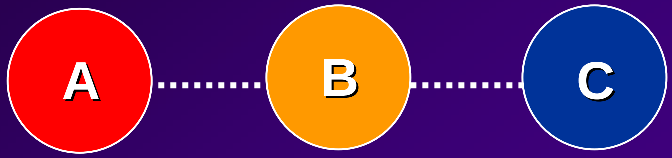
density operator

Open systems: General quantum evolution is described by a map on density operators. Pure states may evolve to mixed states and vice versa.

$$\rho \rightarrow \mathcal{E}(\rho)$$

$\mathcal{E}$ must be linear (in ρ), trace-preserving, and completely positive (CP).

Locality



Global evolution map E^{ABC}

<u>Locality</u>

The evolution map E^{ABC} is local – that is, there is no information flow from C to A – provided the final state of A is determined by the initial state of AB alone.

output

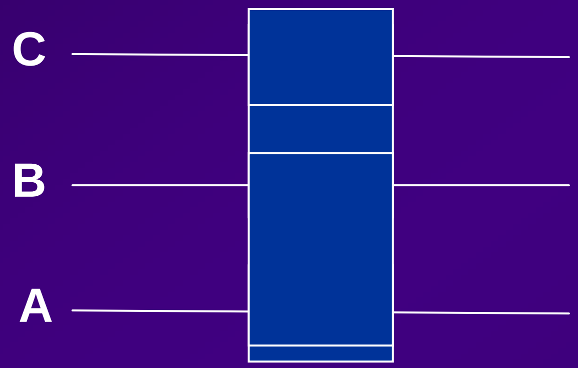
E^{A}_{AB} exists

input

Global unitarity

Can we have

- Global evolution of ABC unitary; and
- No information transfer from C to A ?



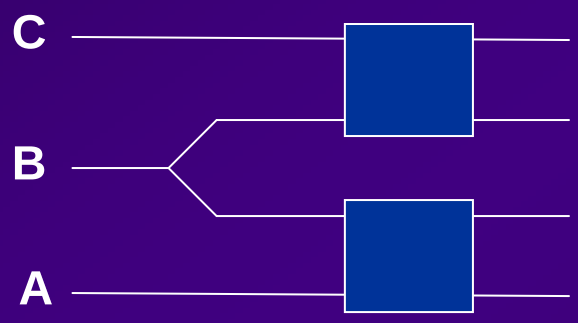
Yes, of course. Trivial cases:

A or C are isolated.

Global unitarity

Can we have

- Global evolution of ABC unitary; and
- No information transfer from C to A ?

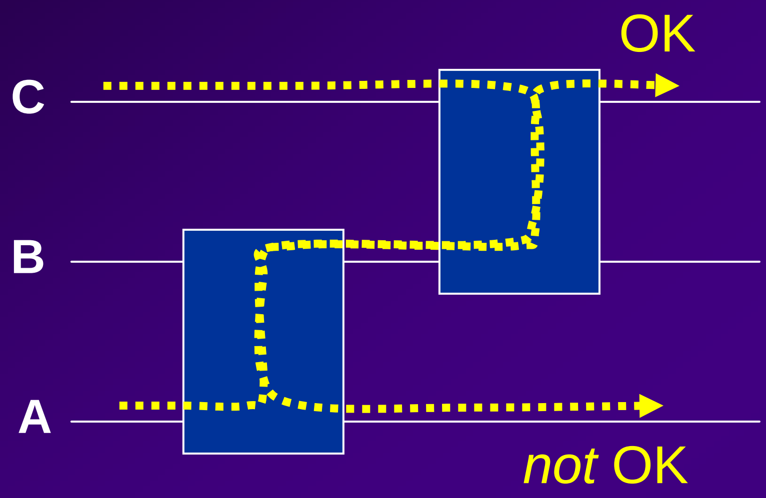


Yes, of course. Trivial cases:

A or C are isolated.

A and C interact separately with parts of composite system B.

A more interesting example



Points to note

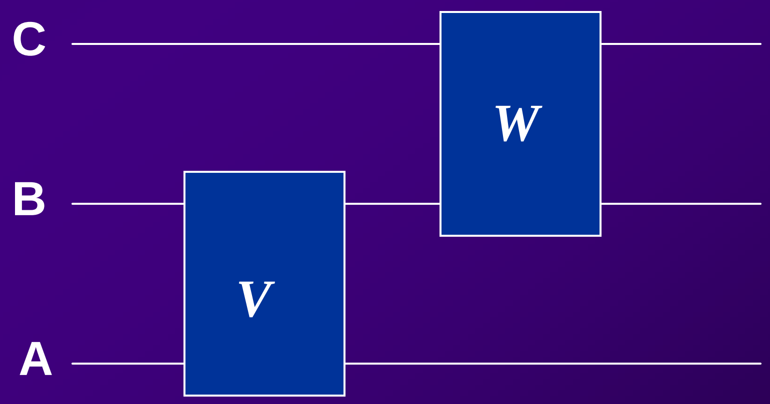
- AB interaction followed by BC interaction.
- One-way information transfer: A $\rightarrow$ C but **not** C $\rightarrow$ A
- Previous examples can be converted to this general form

Remarkable fact: This is the *only* possibility!

A decomposition theorem

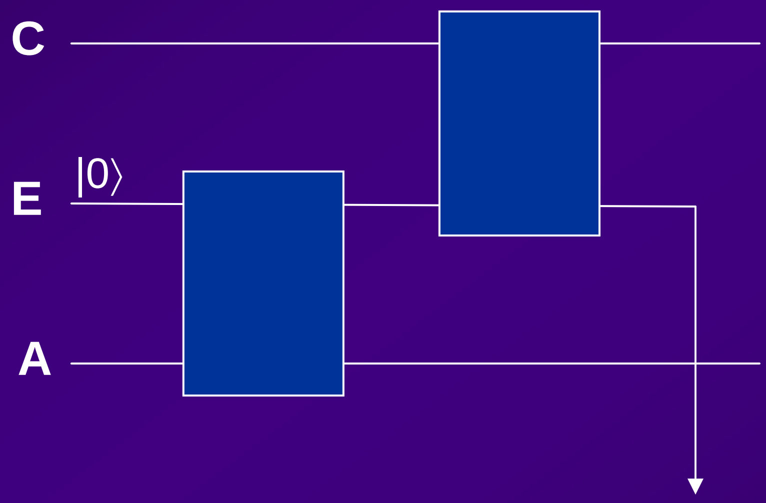
Suppose system ABC evolves via unitary U^{ABC}, such that no information transfer is possible from C to A ("locality"). Then

$$U^{ABC} = \left(1^{A} \otimes W^{BC}\right)\left(V^{AB} \otimes 1^{C}\right)$$



A two-system result

Suppose E^{AC} is a CP map such that no information is transferred from C to A. Then there is a unitary representation for E^{AC} of the form



A and C interact with a common environment, but A's interaction is finished before C's interaction starts.

Semicausal operations are semilocalizable

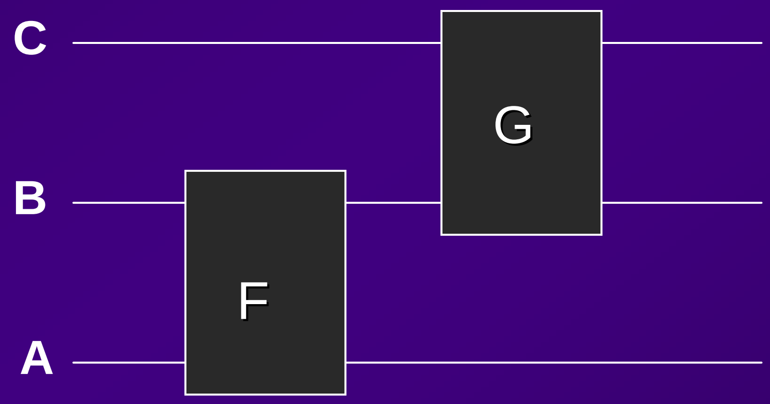
Beckman et. al. (2001)

Eggeling et al. (2002)

General decomposition?

Suppose ABC evolves according to a general CP map E, and no information is transferred from C to A.

Can we always decompose such a map into F and G as follows?

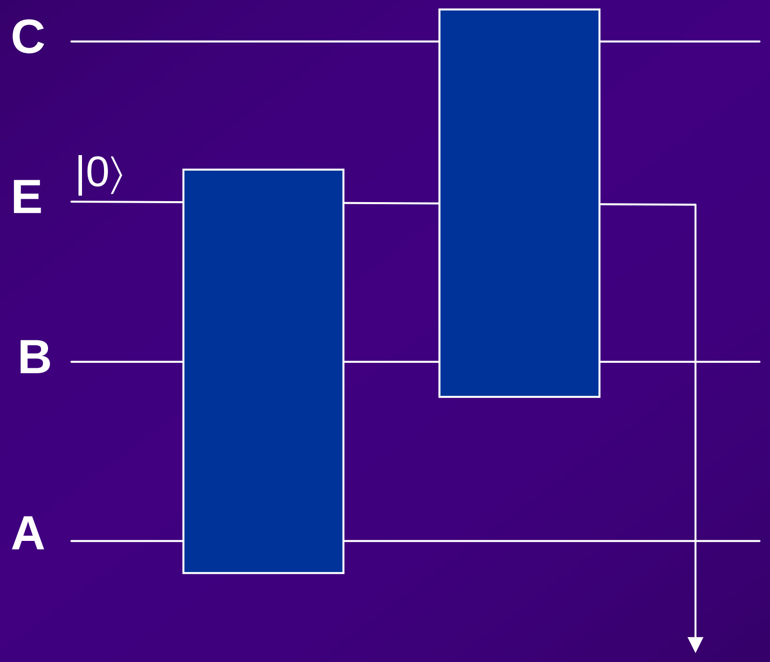


No. There are local maps that are not of this form.

However . . . .

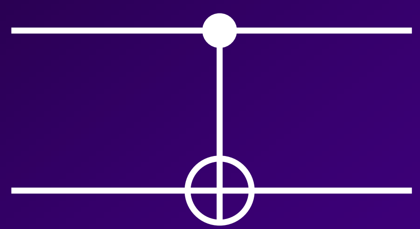
Locality in general

Suppose E^{ABC} is a general CP map that is local – that is, no information can flow from C to A. Then the map has a unitary representation of the form:



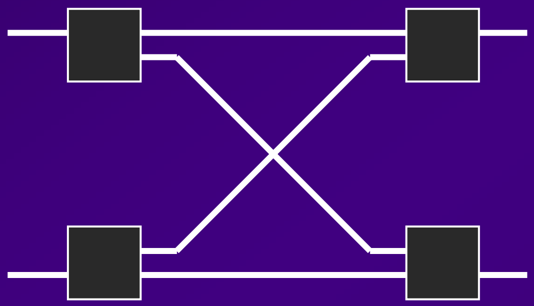
C can interact with B and E, but only after A has finished interacting with them.

Dissecting CNOT



We know that the quantum CNOT gate involves information flow in both directions.

Can we model this in an explicit way? What is the structure of information flow inside CNOT?



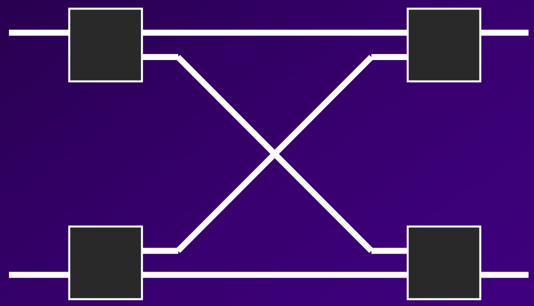
Model: Simple information exchange

Note that every classical gate can be modeled in this way. (Exchange copies!)

Can CNOT be modeled by local CP maps and simple information exchange?

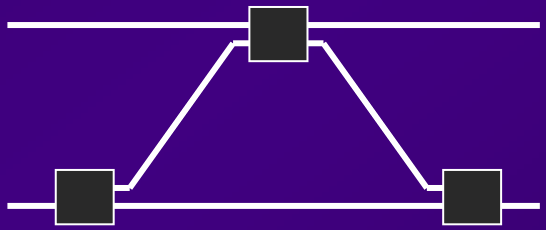
No!

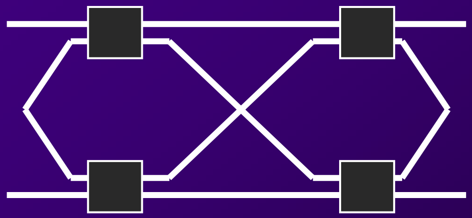
Dissecting CNOT



M. Nathanson: No entangling unitary two-qubit gate can be modeled by local CP maps and simple information exchange

Here are two ways that you **can** model CNOT:

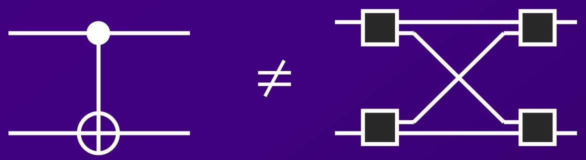




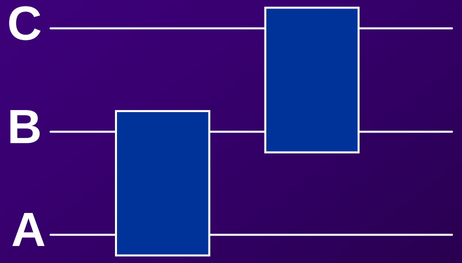
What is the essential difference between these information flow patterns and simple information exchange?

Big ideas

Unitary interactions always allow information to flow both ways. But this is not just “simple information exchange”!



In order to prevent information transfer from C to A, we must somehow “hide” C from A in the interaction. The only place to hide C is *in the causal future*.



References

- D. Beckman, D. Gottesman, M. A. Nielsen and John Preskill, “Causal and localizable quantum operations”, *Phys. Rev. A* **64**, 052309 (2001).
- T. Eggeling, D. Schlingemann and R. F. Werner, “Semicausal operations are semilocalizable”, *Europhys. Lett*. **57 (6)**, 782 (2002).
- E. Hawkins, F. Markopoulou, and H. Sahlmann, “Evolution in Quantum Causal Histories”, *Class. Quant. Grav*. **20**, 3839 (2003).
- B. Schumacher and R. F. Werner, “Reversible Quantum Cellular Automata”, quant-ph/0405174
- B. Schumacher and M. D. Westmoreland, “Locality and information transfer in quantum operations”, *Quant. Info. Proc.* **4**, 13 (2005).