Synthetic Quantum Matter with Atoms and Photons

Ryan Wilson

US Naval Academy

TUI-3

KITP

Preliminaries

Ryan

- CU/JILA (grad) to NIST/UMD/JQI (NRC postdoc) to USNA (current)
- Asst. Professor @USNA, Aug. 2014-present
- Broadly curious about quantum many-body physics (in practice, ultracold atoms)
- 2015-2017 KITP Scholar

Preliminaries

Ryan

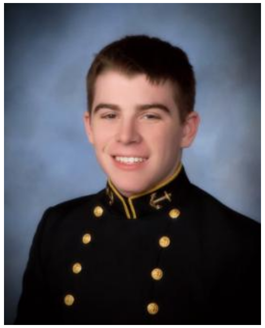
- CU/JILA (grad) to NIST/UMD/JQI (NRC postdoc) to USNA (current)
- Asst. Professor @USNA, Aug. 2014-present
- Broadly curious about quantum many-body physics (in practice, ultracold atoms)
- 2015-2017 KITP Scholar

USNA

- 4400 “Midshipmen” representing all congressional districts
- 1100 intro physics students, ~20-30 physics majors graduated/year
- “Trident Scholar” program provides 18-24 research credits during senior year

Research “Group”

Q. Info



Chaos

BEC

Preliminaries

Quantum Many-Body Physics

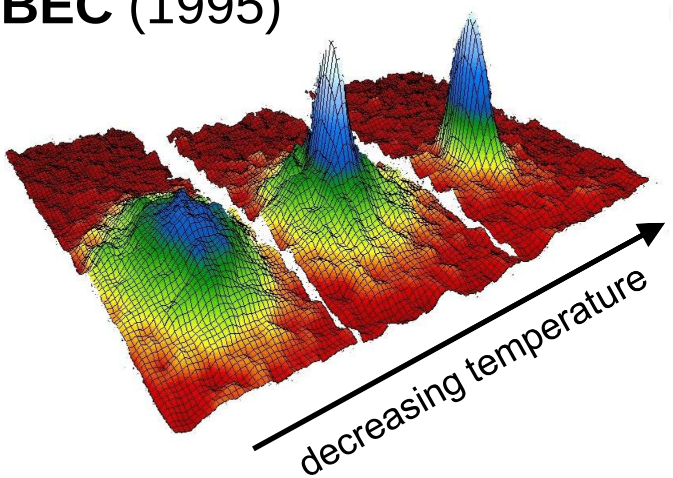
- Condensed matter / solid state / ultracold atoms (laser cooling to $T<10$ nK)
  (non-linear photonics, exciton/polariton gases, too)
- Quantum statistics are important at low temperatures
- Ground states, non-equilibrium phases (ordering, topology)
- Challenges: strong correlations, entanglement, large Hilbert spaces…

Preliminaries

Quantum Many-Body Physics

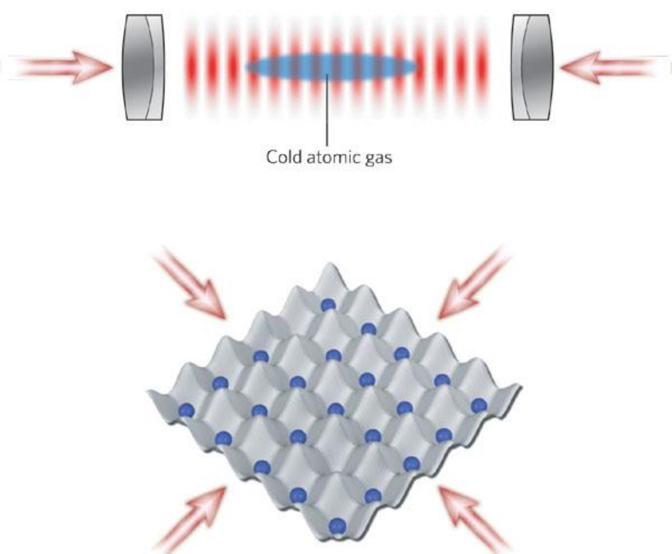
- Condensed matter / solid state / **ultracold atoms** (laser cooling to T<10 nK)
  (non-linear photonics, exciton/polariton gases, too)
- Quantum statistics are important at low temperatures
- Ground states, non-equilibrium phases (ordering, topology)
- Challenges: strong correlations, entanglement, large Hilbert spaces…

BEC (1995)

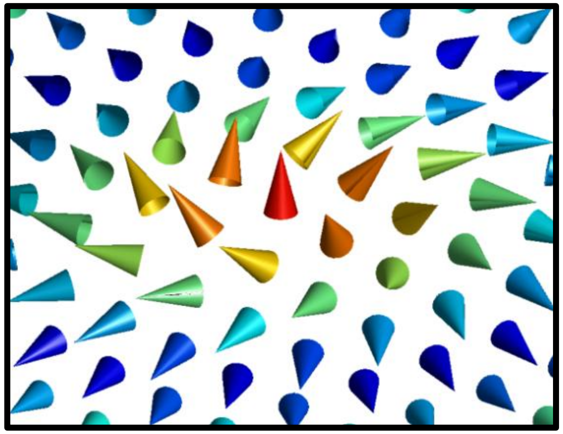


(Wiemann/Cornell Rb87 BEC, JILA)

Optical Lattices



Spinors



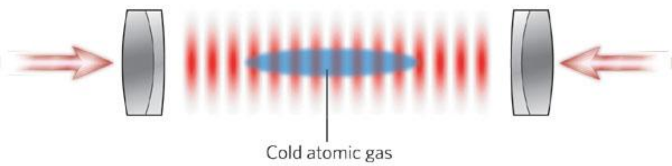
S=1/2 – magnetism
S=1 – nematic order
S=N – SU(N) magnetism

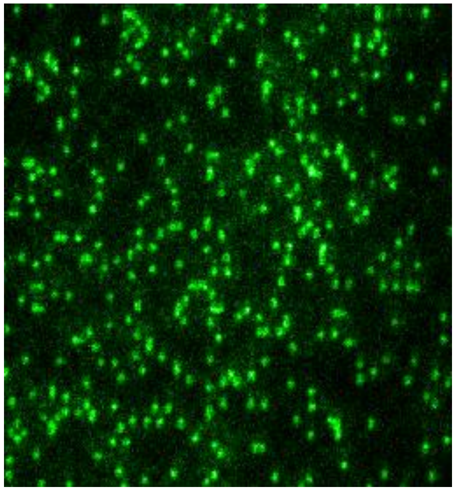
Preliminaries

Quantum Many-Body Physics

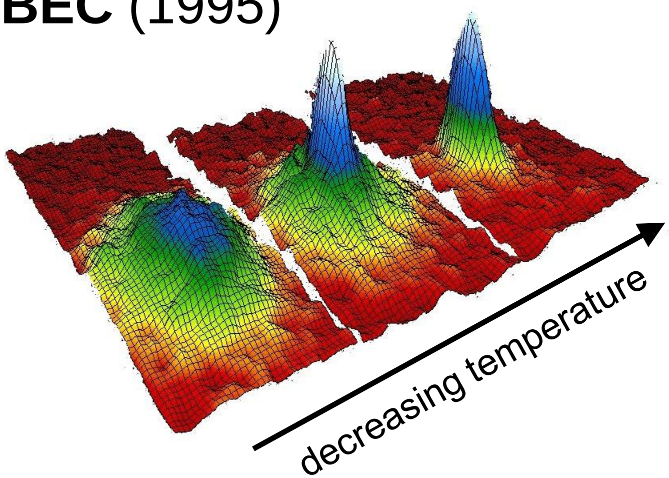
- Condensed matter / solid state / **ultracold atoms** (laser cooling to T<10 nK)
  (non-linear photonics, exciton/polariton gases, too)
- Quantum statistics are important at low temperatures
- Ground states, non-equilibrium phases (ordering, topology)
- Challenges: strong correlations, entanglement, large Hilbert spaces…

Optical Lattices



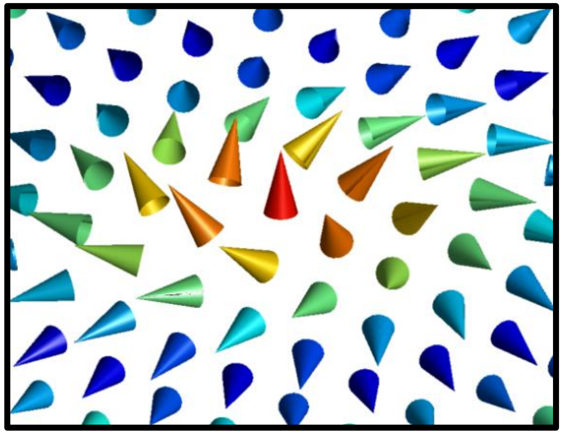


BEC (1995)



(Wiemann/Cornell Rb87 BEC, JILA)

Spinors



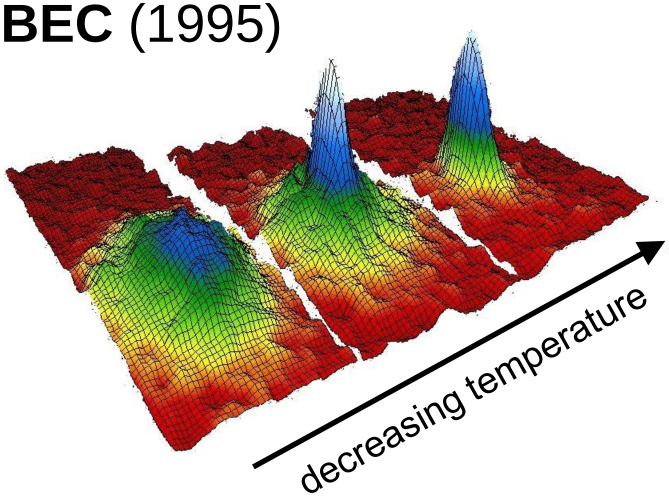
S=1/2 – magnetism
S=1 – nematic order
S=N – SU(N) magnetism

Preliminaries

Quantum Many-Body Physics

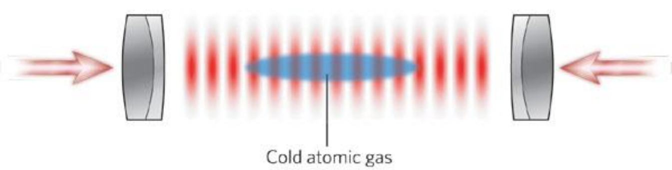
- Condensed matter / solid state / **ultracold atoms** (laser cooling to T<10 nK)
  (non-linear photonics, exciton/polariton gases, too)
- Quantum statistics are important at low temperatures
- Ground states, non-equilibrium phases (ordering, topology)
- Challenges: strong correlations, entanglement, large Hilbert spaces…

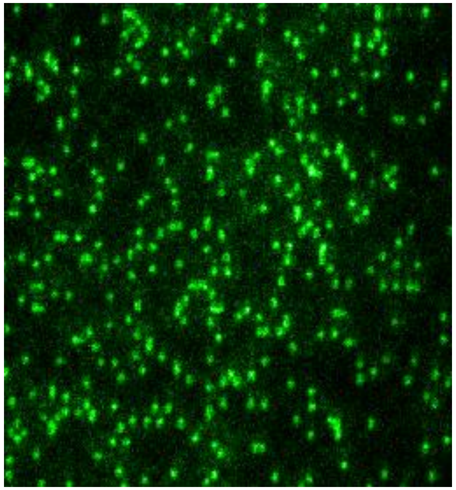
BEC (1995)



(Wiemann/Cornell Rb87 BEC, JILA)

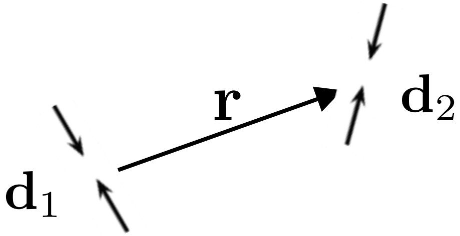
Optical Lattices





Dipoles

$$V(\mathbf{r}) = \frac{\mathbf{d}_1 \cdot \mathbf{d}_2 - 3(\mathbf{d}_1 \cdot \hat{r})(\mathbf{d}_2 \cdot \hat{r})}{r^3}$$

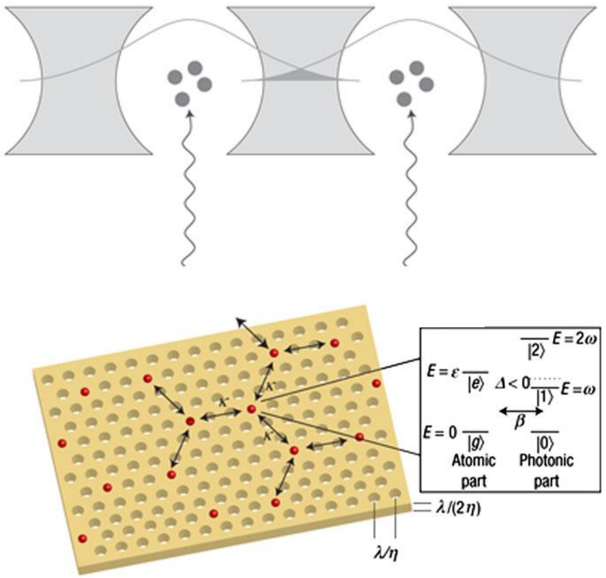


Preliminaries

Quantum Many-Body Physics

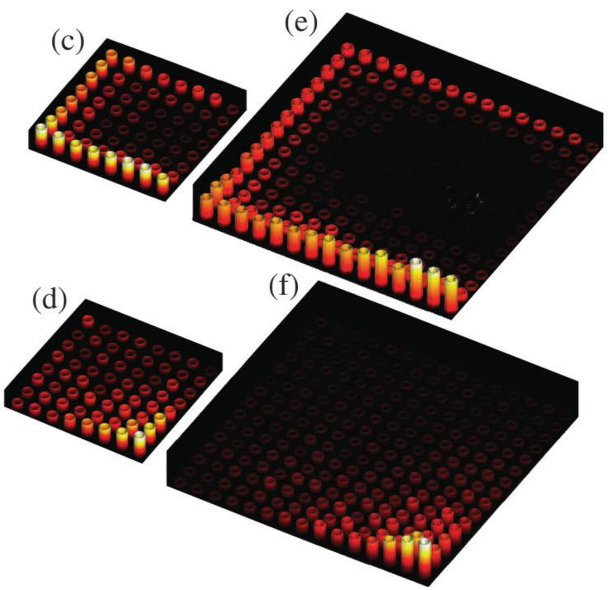
- Condensed matter / solid state / ultracold atoms (laser cooling to T<10 nK) (**non-linear photonics**, exciton/polariton gases, too)
- Quantum statistics are important at low temperatures
- Ground states, non-equilibrium phases (ordering, topology)
- Challenges: strong correlations, entanglement, large Hilbert spaces…

Optical cavities (nonlinear)



(silicon, typically)

Topological light



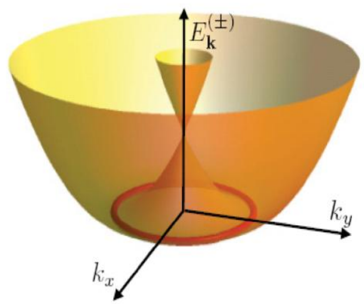
Inherently **dissipative**, must be driven/pumped

Carusotto & Ciuti, RMP **85** 2013

Synthetic Quantum Matter with Atoms and Photons

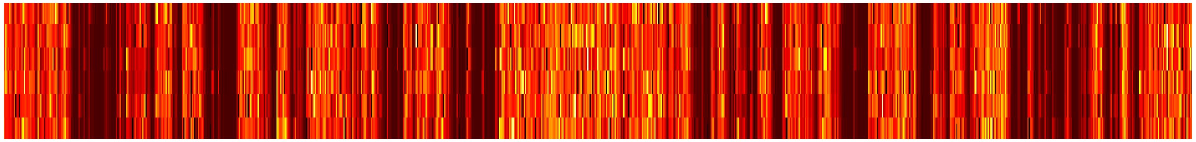
Dipolar BEC in synthetic gauge field (spin-orbit coupling)

Ground state ordering



Driven-dissipative array of nonlinear optical cavities

Emergence in open quantum systems



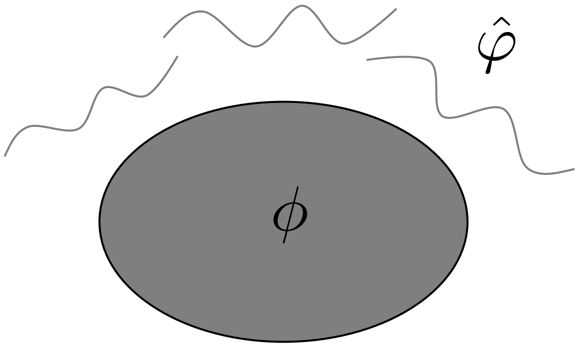
BEC

$$\hat{\mathcal{H}} = \int d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r})\hat{H}_0(\mathbf{r})\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r} \int d\mathbf{r}'\, \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})$$

Order parameter (mean field): $\hat{\psi}(\mathbf{r}) \simeq \phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$

U(1) symmetry, broken by emergence of BEC

Superfluidity, quantized rotation, etc.



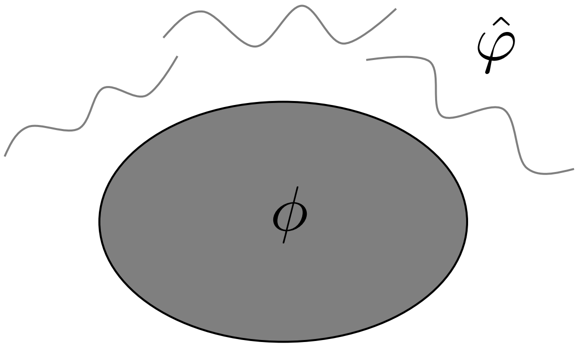
BEC

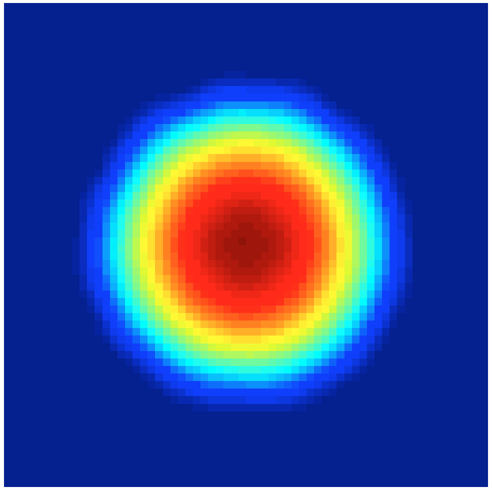
$$\hat{\mathcal{H}} = \int d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r})\hat{H}_0(\mathbf{r})\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r} \int d\mathbf{r}'\, \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})$$

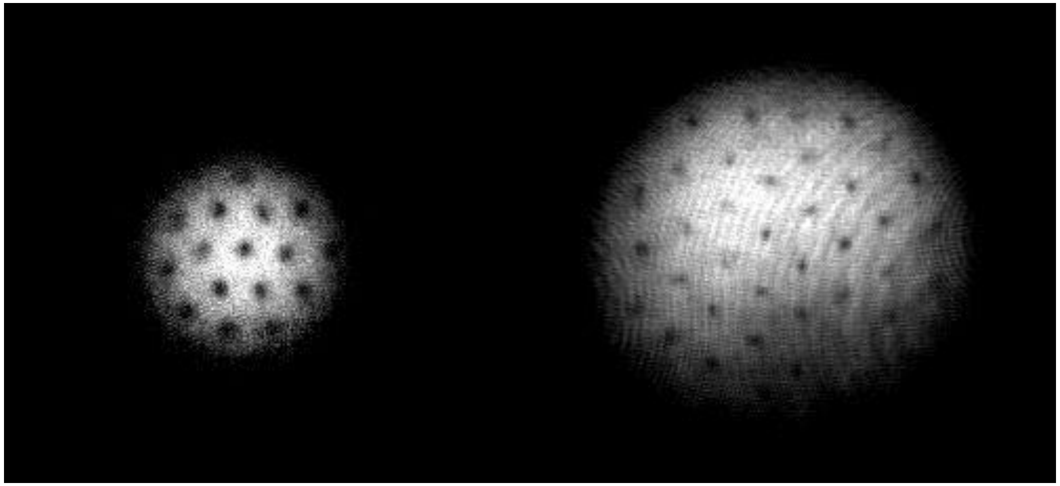
Order parameter (mean field): $\hat{\psi}(\mathbf{r}) \simeq \phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$

U(1) symmetry, broken by emergence of BEC

Superfluidity, quantized rotation, etc.







(Ketterle Na23 Lab, MIT)

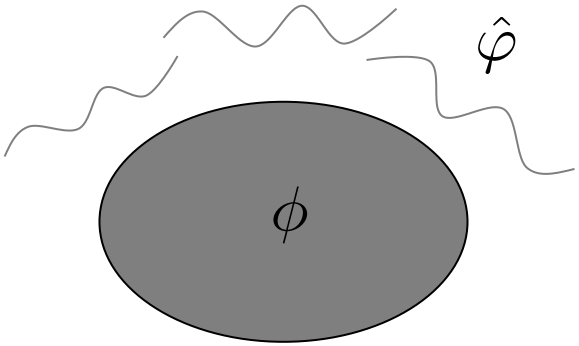
Non-linear Schrodinger equation

Great for undergraduates

BEC

$$\hat{\mathcal{H}} = \int d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r})\hat{H}_0(\mathbf{r})\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r} \int d\mathbf{r}'\, \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})$$

Order parameter (mean field): $\hat{\psi}(\mathbf{r}) \simeq \phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$



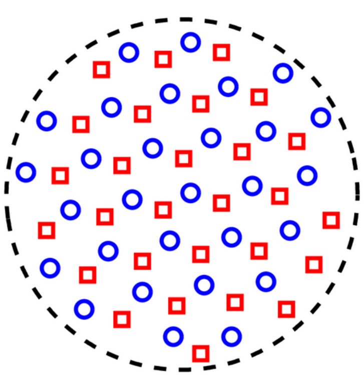
U(1) symmetry, broken by emergence of BEC

Superfluidity, quantized rotation, etc.

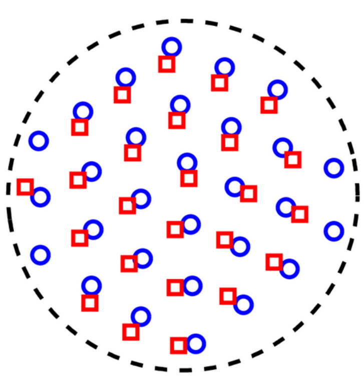
PRL **113,** 165301 (2014) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2014

Half-Quantum Vortex Molecules in a Binary Dipolar Bose Gas

Wilbur E. Shirley,[1,2] Brandon M. Anderson,[2] Charles W. Clark,[2] and Ryan M. Wilson[2]
[1]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[2]*Joint Quantum Institute, National Institute of Standards and Technology and the University of Maryland, College Park, Maryland 20742, USA*



hexagons



molecules

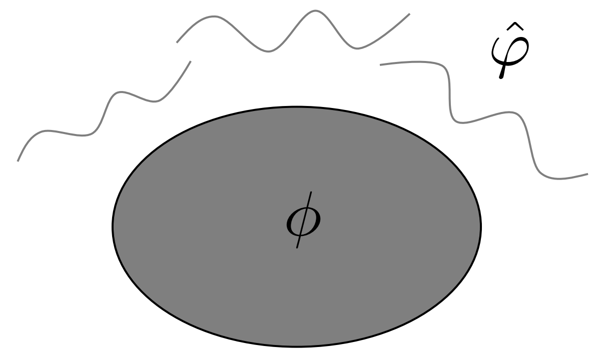
BEC

$$\hat{\mathcal{H}} = \int d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r}) \hat{H}_0(\mathbf{r}) \hat{\psi}(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}'\, \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') V(\mathbf{r}-\mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r})$$

Order parameter (mean field): $\hat{\psi}(\mathbf{r}) \simeq \phi(\mathbf{r}) + \hat{\varphi}(\mathbf{r})$

U(1) symmetry, broken by emergence of BEC

Superfluidity, quantized rotation, etc.



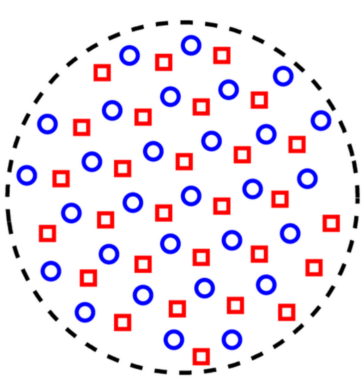
PRL **113,** 165301 (2014) PHYSICAL REVIEW LETTERS week ending 17 OCTOBER 2014

Half-Quantum Vortex Molecules in a Binary Dipolar Bose Gas

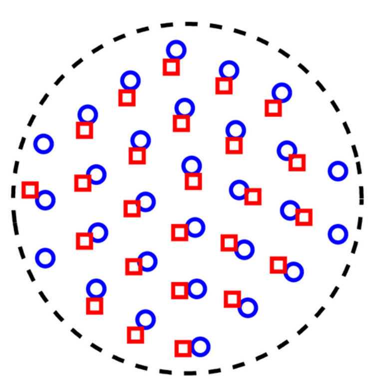
Wilbur E. Shirley,[1,2] Brandon M. Anderson,[2] Charles W. Clark,[2] and Ryan M. Wilson[2]

[1]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

[2]*Joint Quantum Institute, National Institute of Standards and Technology and the University of Maryland, College Park, Maryland 20742, USA*



hexagons



molecules

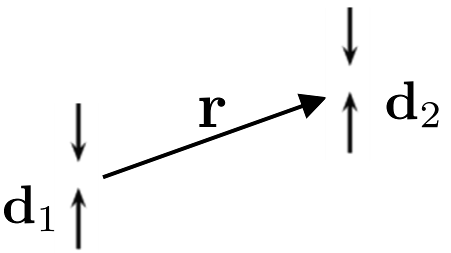
Vortices crystalize

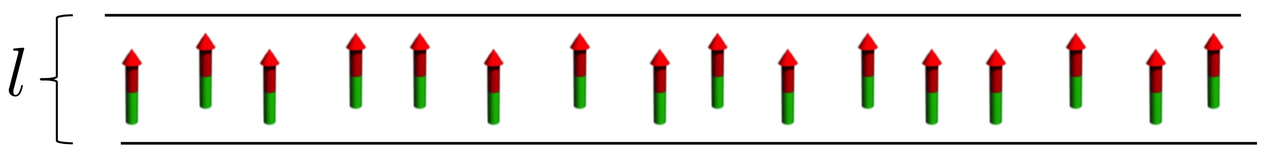
Other types of ordering?

(density crystals, supersolids, etc.)

Magnetic dipolar atoms (high-spin)

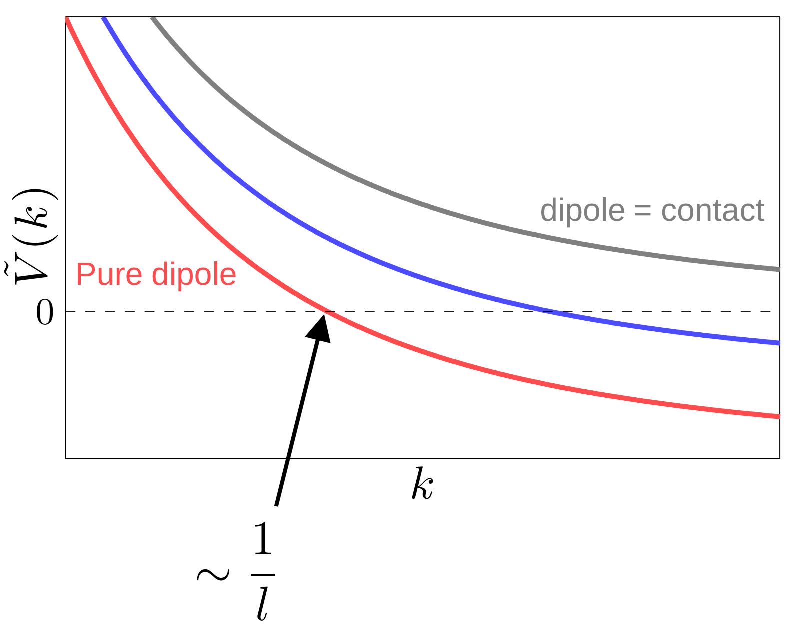
- ^{52}Cr (2005, Pfau, Stuttgart) PRL **94** 160401 (2005)
- ^{164}Dy (2011, Lev Lab, Illinois/Stanford) PRL **107** 190401 (2011)
- ^{168}Er (2012, Ferlaino, Innsbruck) PRL **108** 210401 (2012)
- ^{160}Dy & ^{162}Dy (2014, Lev Lab, Stanford) arXiv:1311.3069 (2014)





$$V(\mathbf{r}) = \frac{1 - 3\cos^2\theta}{r^3}$$

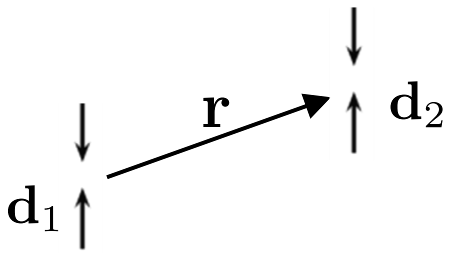
k-space interaction potential

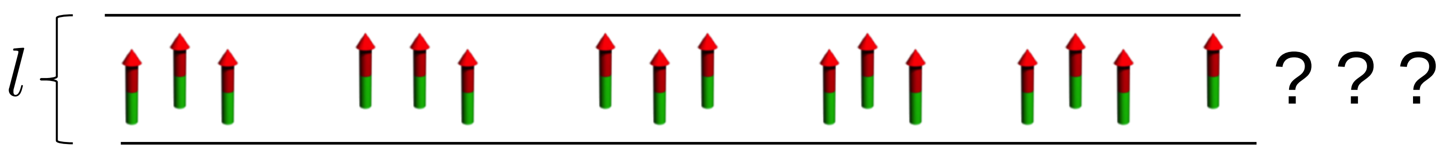


Dipolar BEC

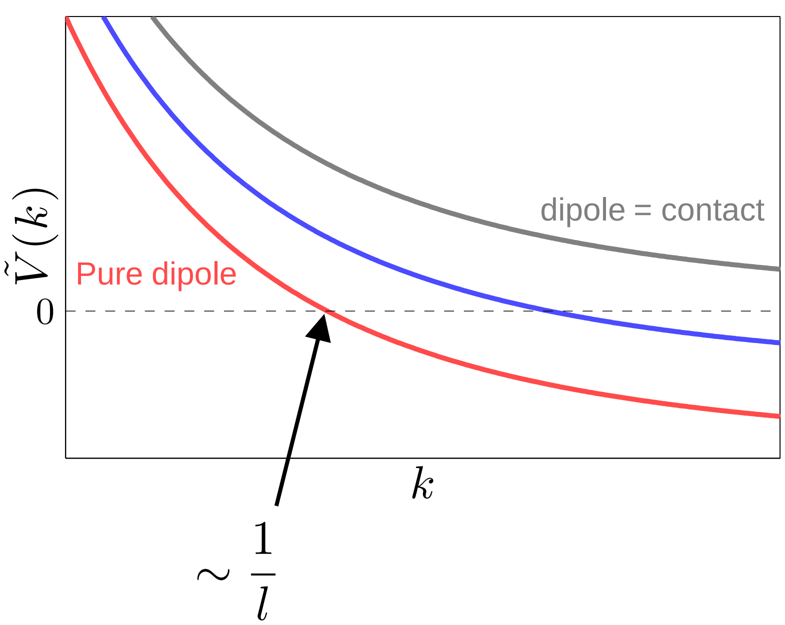
Magnetic dipolar atoms (high-spin)

- ^{52}Cr (2005, Pfau, Stuttgart) PRL **94** 160401 (2005)
- ^{164}Dy (2011, Lev Lab, Illinois/Stanford) PRL **107** 190401 (2011)
- ^{168}Er (2012, Ferlaino, Innsbruck) PRL **108** 210401 (2012)
- ^{160}Dy & ^{162}Dy (2014, Lev Lab, Stanford) arXiv:1311.3069 (2014)

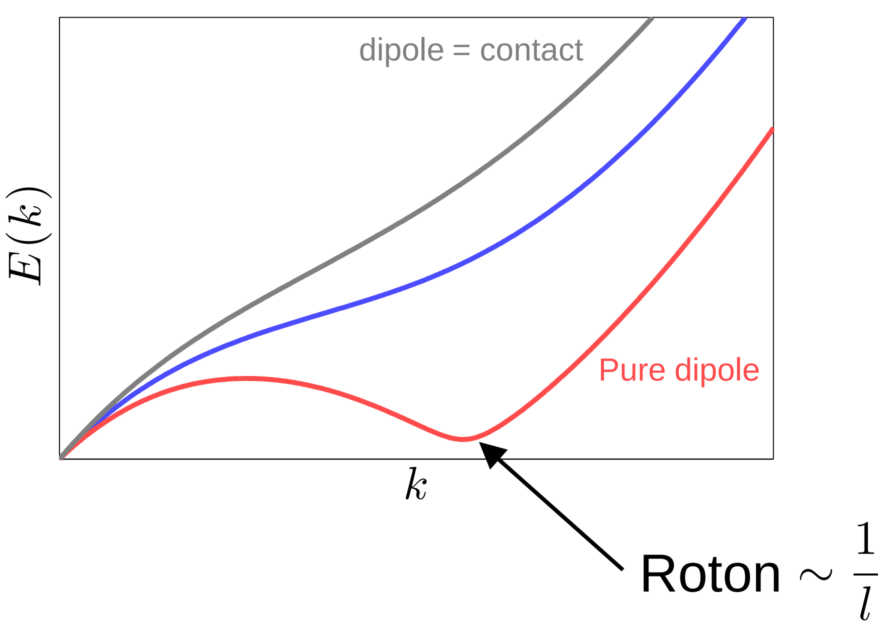




k-space interaction potential



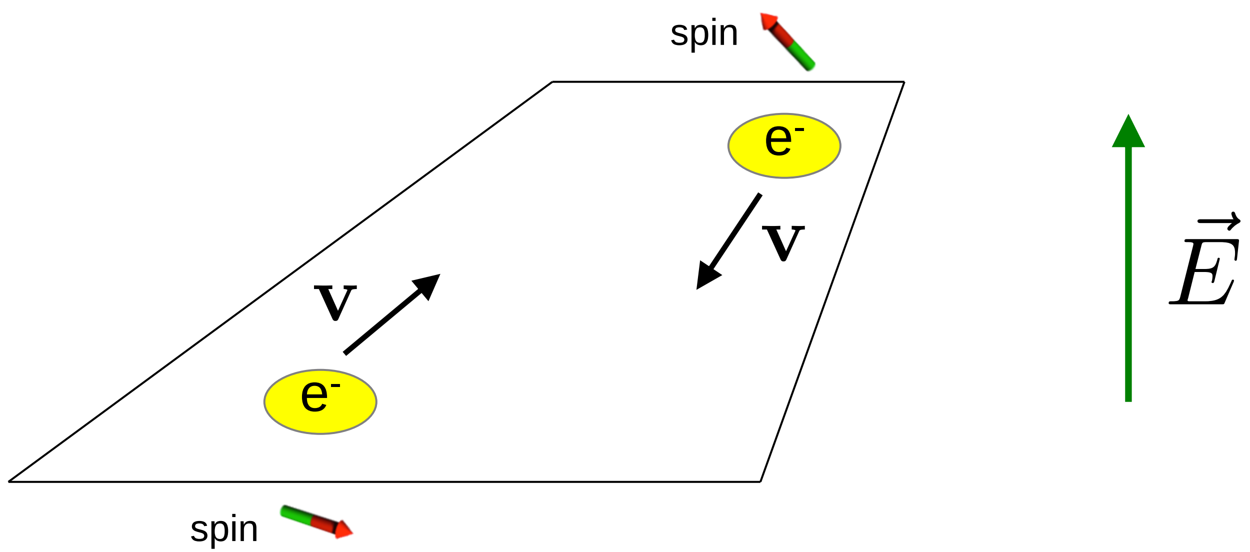
Dispersion of quantum fluctuations



Rashba spin-orbit coupling

Electrons in 2D…

(idea: shift dispersion minimum away from k=0)

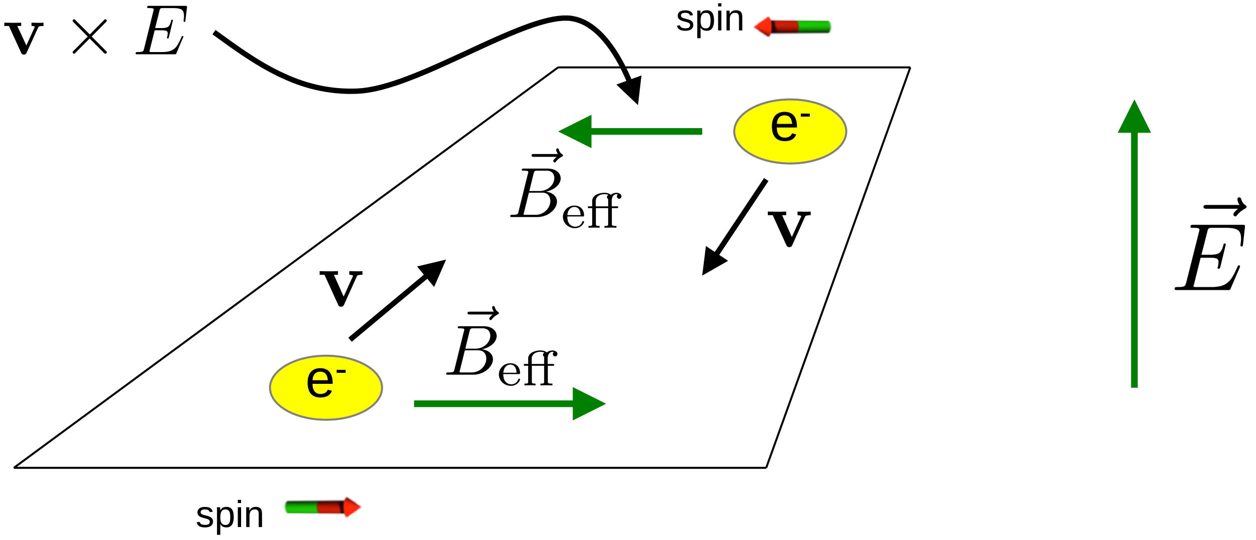


Rashba spin-orbit coupling

Electrons in 2D…

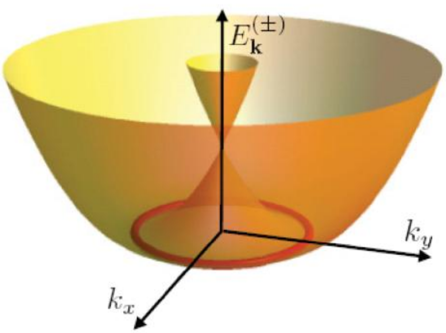
$$\hat{\mathcal{H}} \sim -\boldsymbol{\sigma} \cdot \vec{B}_{\text{eff}}$$

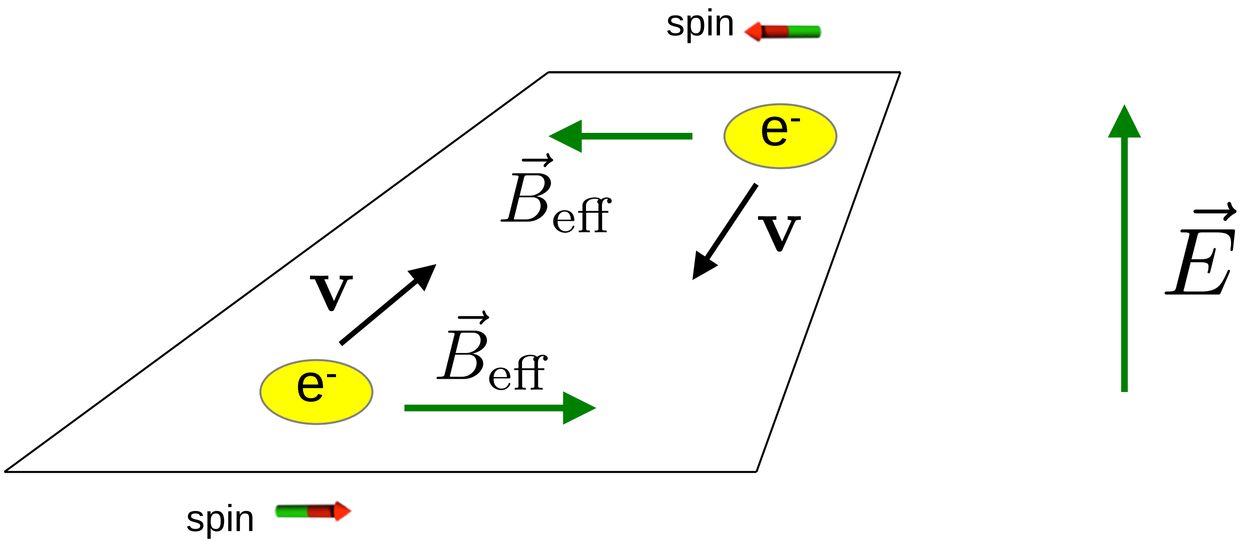
$$\vec{B}_{\text{eff}} \propto \mathbf{v} \times \vec{E}$$



Rashba spin-orbit coupling

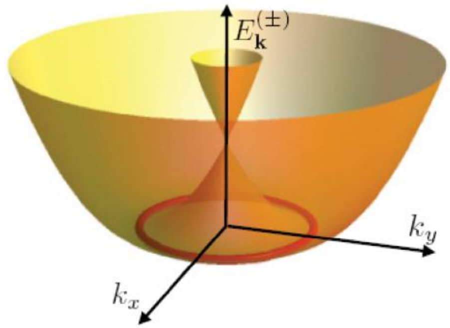
Rashba Hamiltonian: $$\hat{\mathcal{H}}_{\rm so} = k_{\rm so}(\mathbf{p} \times \boldsymbol{\sigma}) \cdot \hat{z}$$



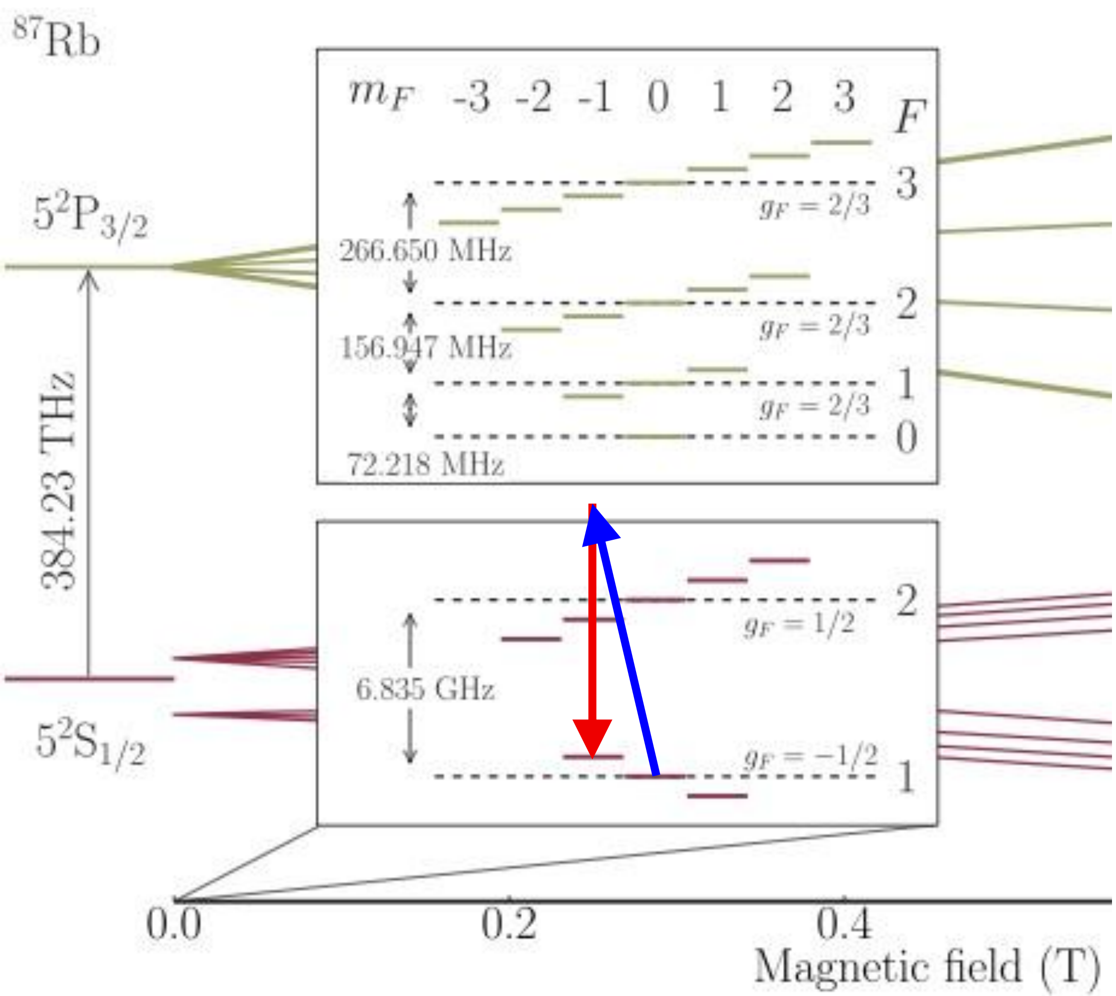


Rashba spin-orbit coupling

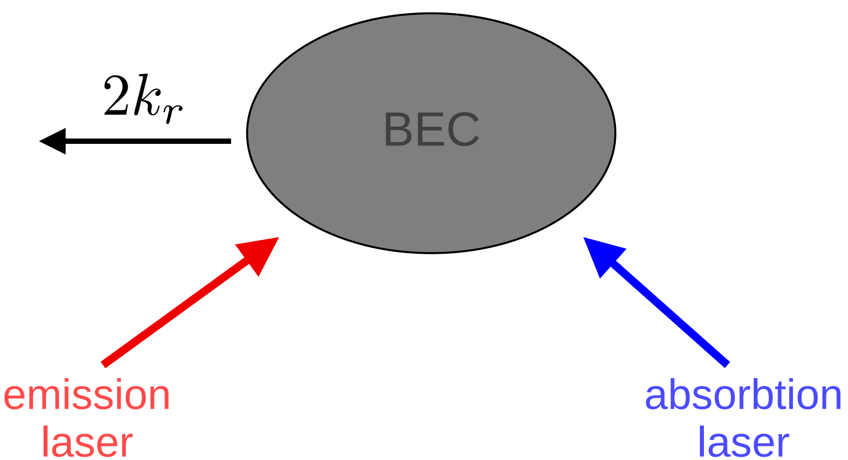
Rashba Hamiltonian: $$\hat{\mathcal{H}}_{\mathrm{so}} = k_{\mathrm{so}}(\mathbf{p} \times \boldsymbol{\sigma}) \cdot \hat{z}$$



Engineering in cold atoms…

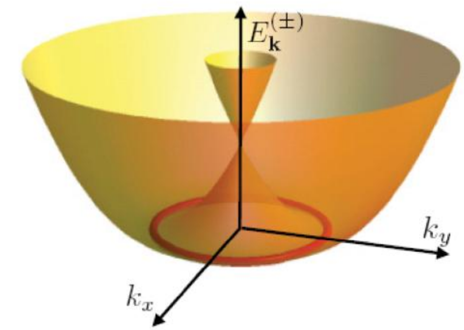


$$|\downarrow\rangle \rightarrow |\uparrow\rangle e^{2ik_r x}$$

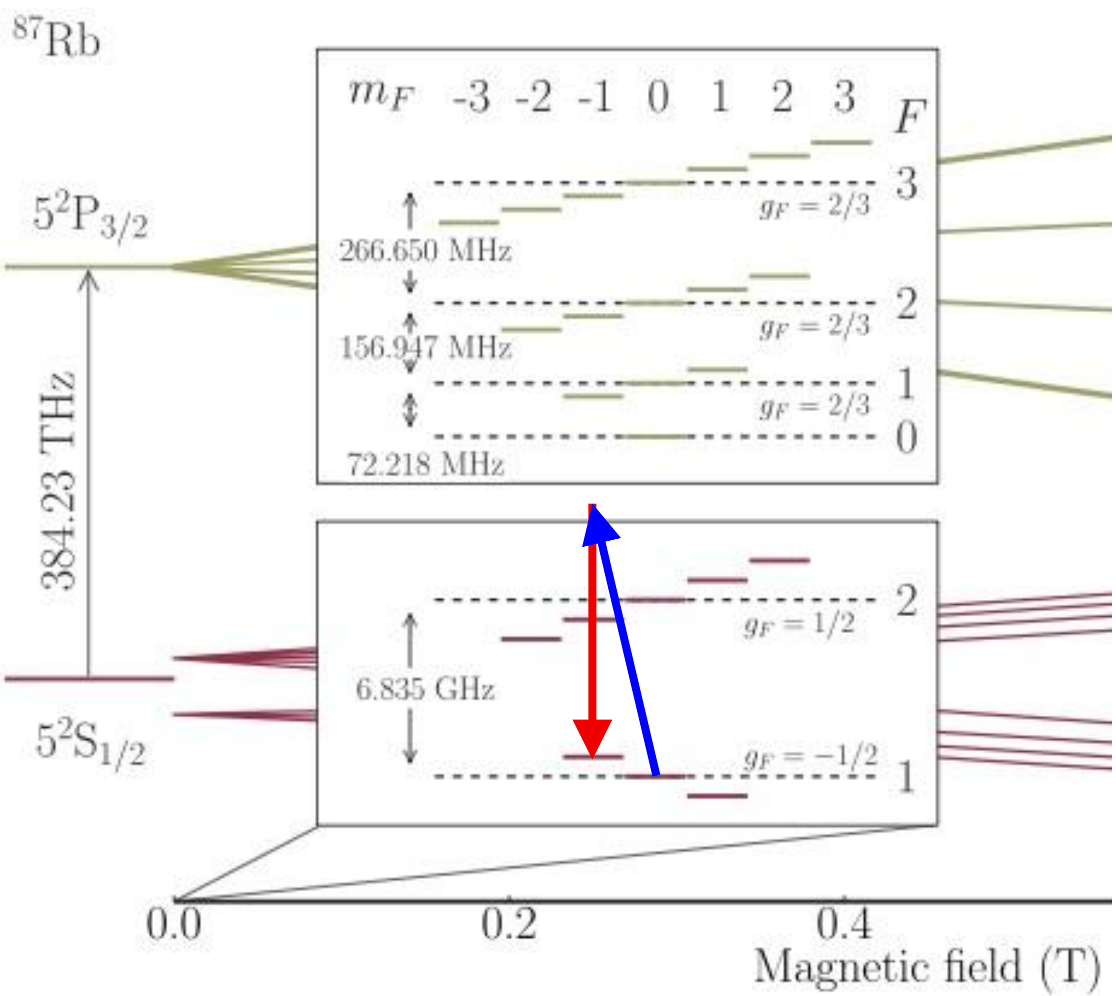


Rashba spin-orbit coupling

Rashba Hamiltonian: $$\hat{\mathcal{H}}_{\mathrm{so}} = k_{\mathrm{so}}(\mathbf{p} \times \boldsymbol{\sigma}) \cdot \hat{z}$$

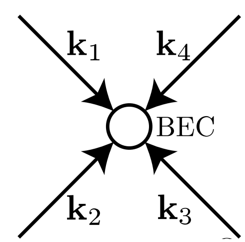


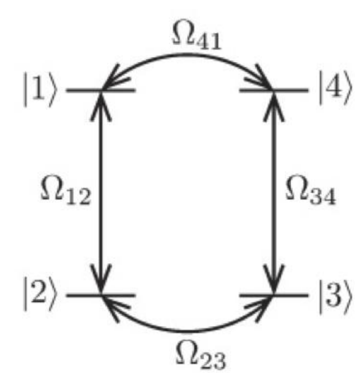
Engineering in cold atoms…



Energetically isolate 3 or 4 hyperfine states

Cyclically couple with Raman beams

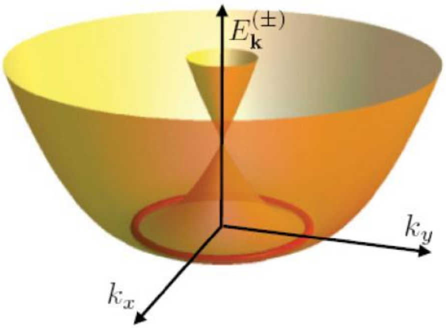




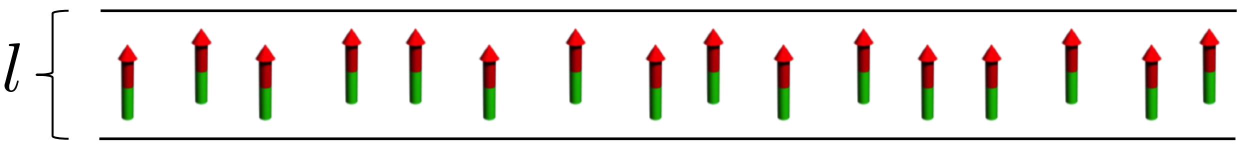
Campbell *et al.* PRA **84** 025602 (2011)

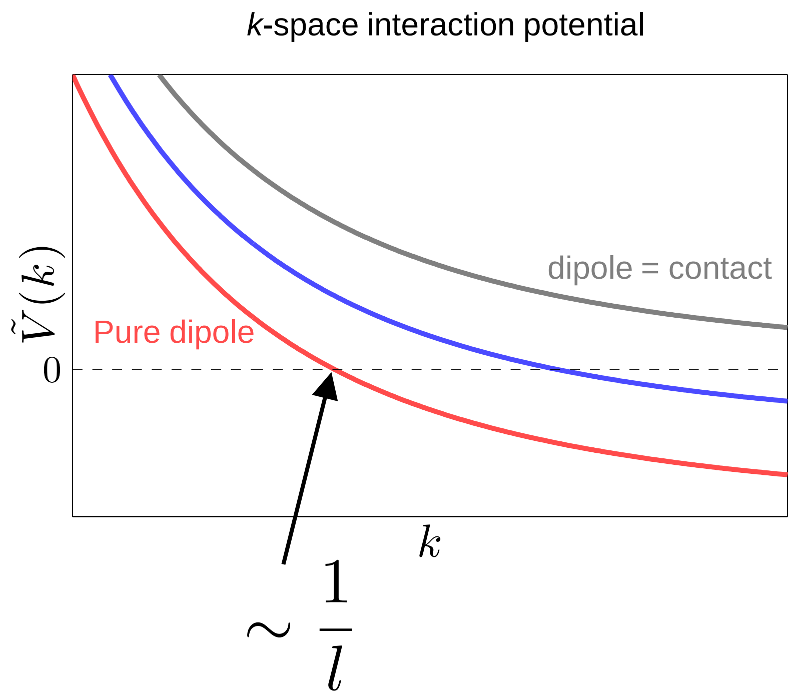
Rashba spin-orbit coupled dipolar BEC

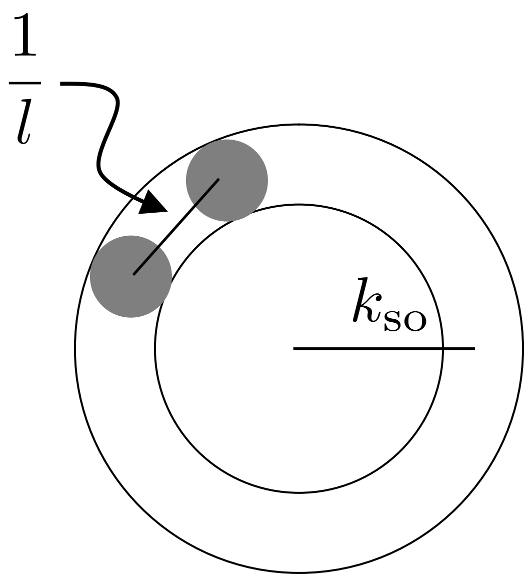
Rashba Hamiltonian: $$\hat{\mathcal{H}}_{\rm so} = k_{\rm so}(\mathbf{p} \times \boldsymbol{\sigma}) \cdot \hat{z}$$



Engineering in cold atoms…



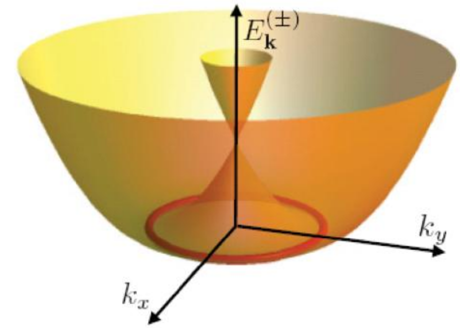




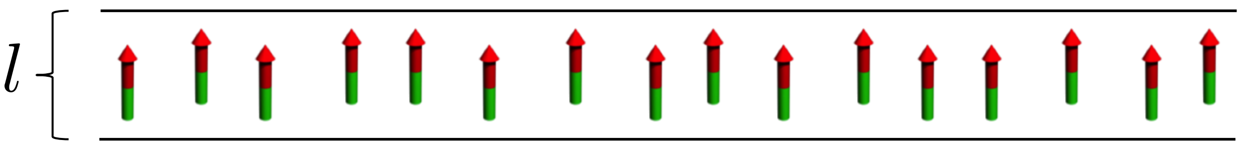
$$\frac{1}{l} < k_{\rm so}$$

Rashba spin-orbit coupled dipolar BEC

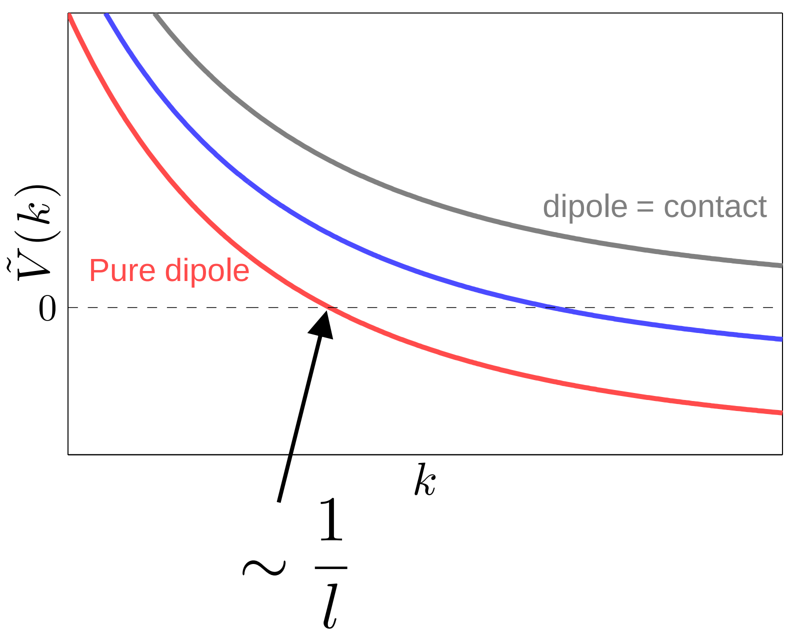
Rashba Hamiltonian: $\hat{\mathcal{H}}_{\mathrm{so}} = k_{\mathrm{so}}(\mathbf{p} \times \boldsymbol{\sigma}) \cdot \hat{z}$

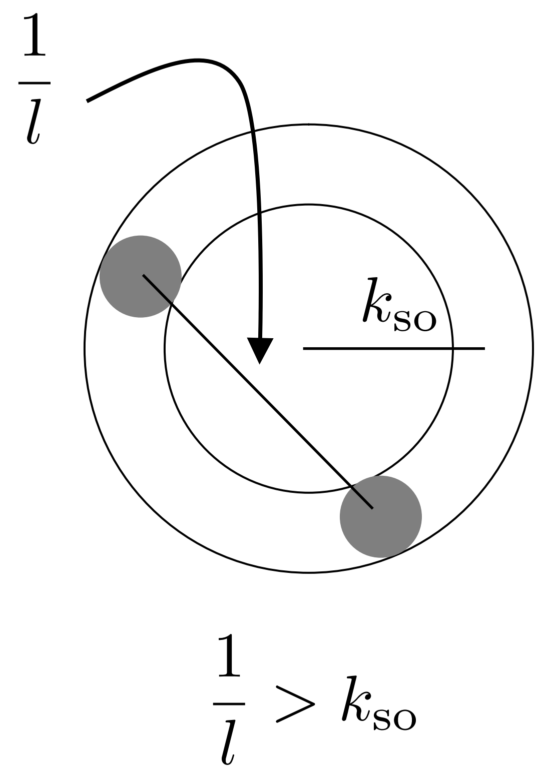


Engineering in cold atoms…

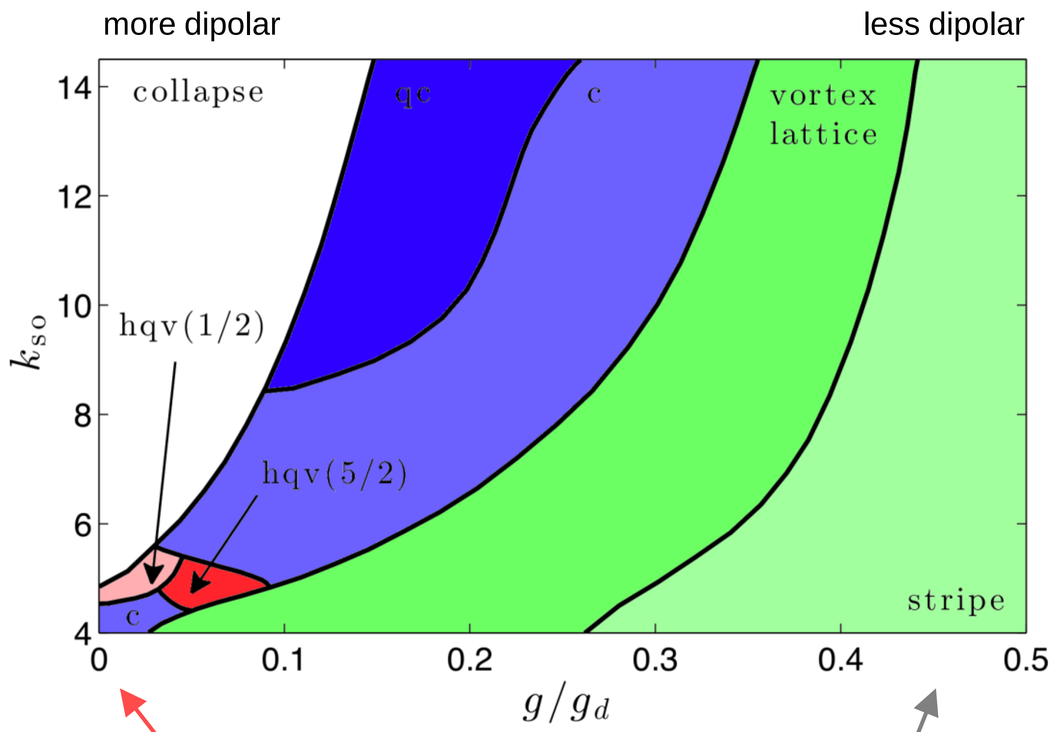


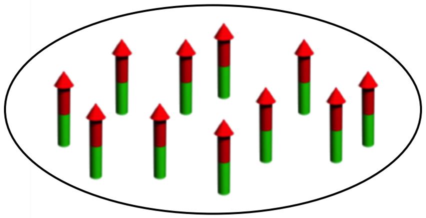
k-space interaction potential

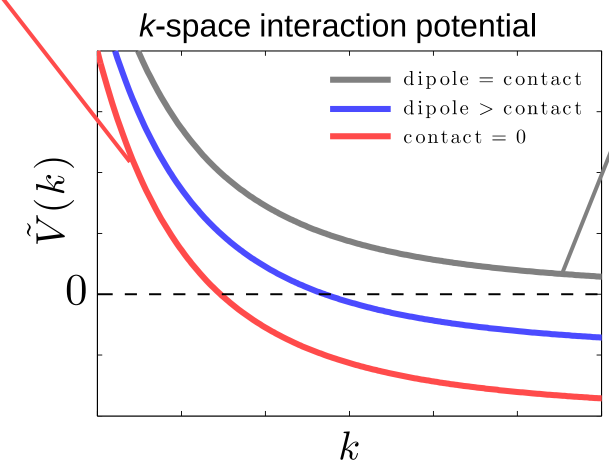




Rashba spin-orbit coupled dipolar BEC

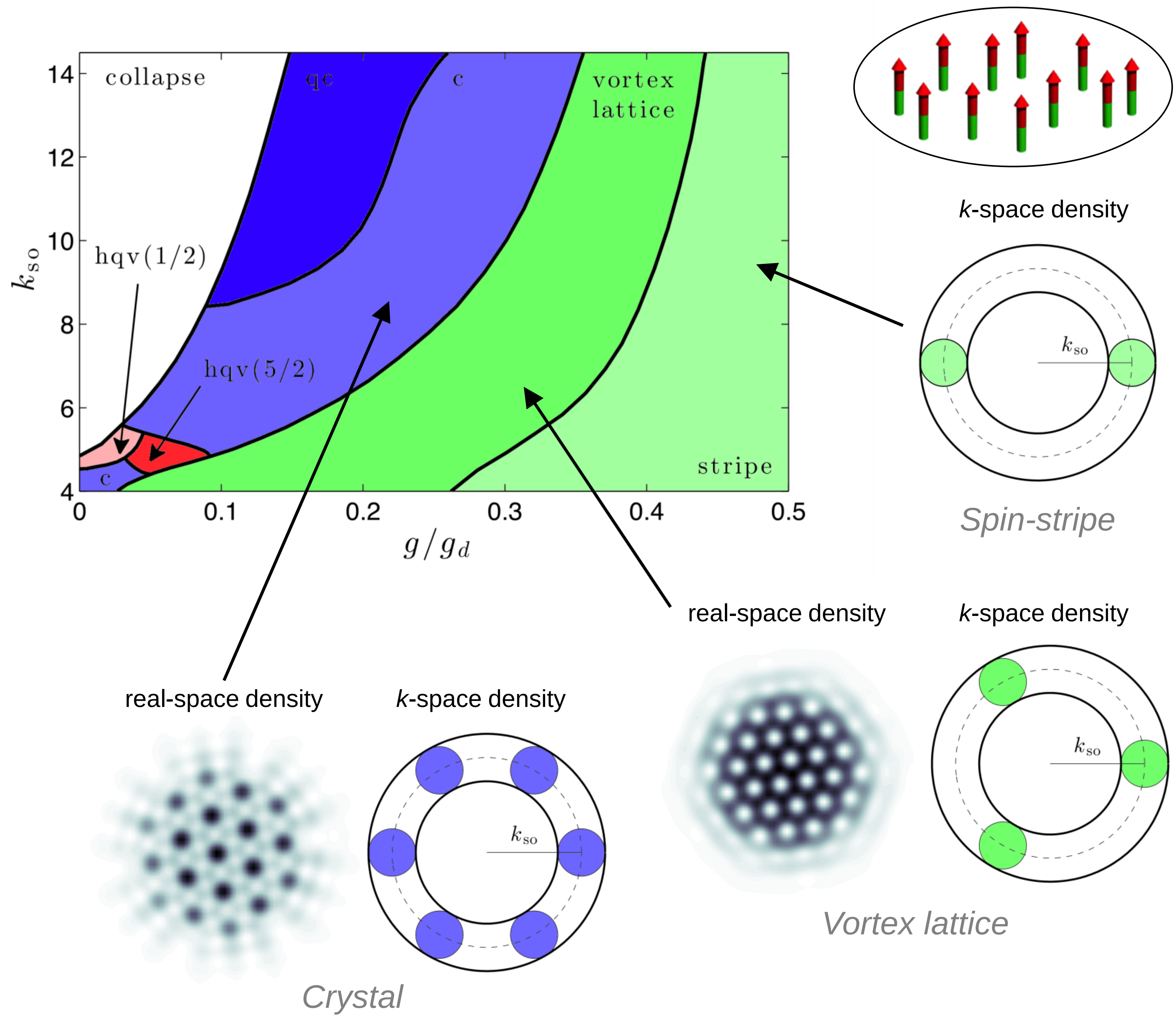






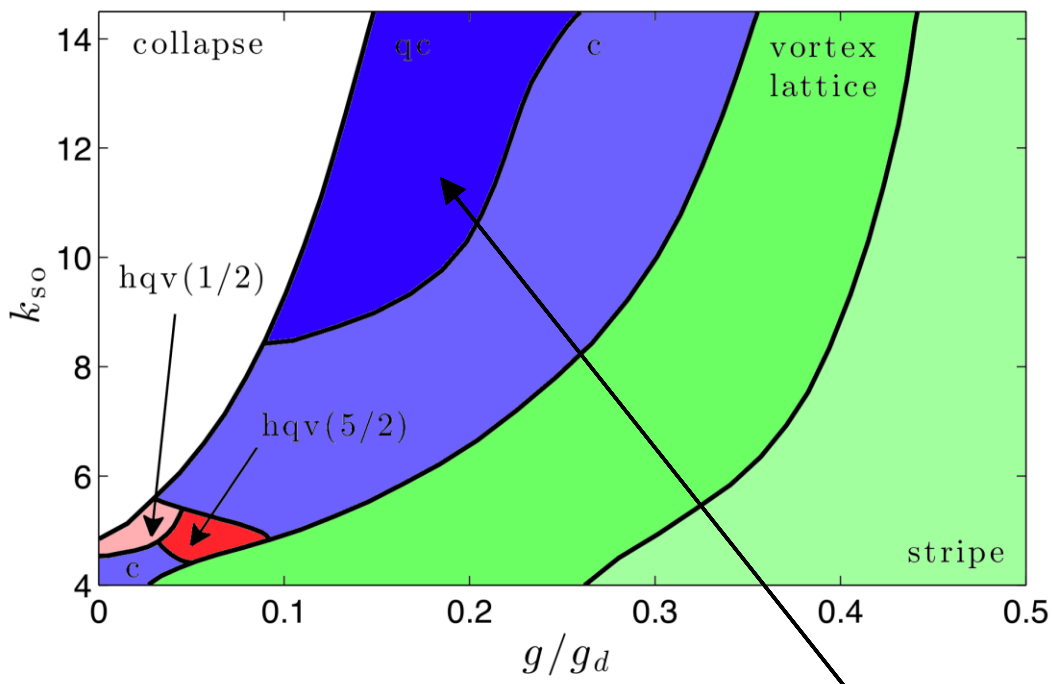
RW, S. Gopalakrishnan, in prep. (2015)

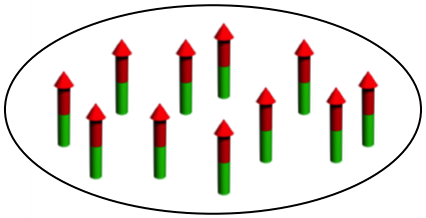
Rashba spin-orbit coupled dipolar BEC



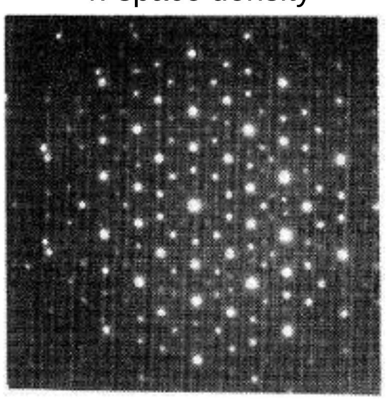
RW, S. Gopalakrishnan, in prep. (2015)

Rashba spin-orbit coupled dipolar BEC

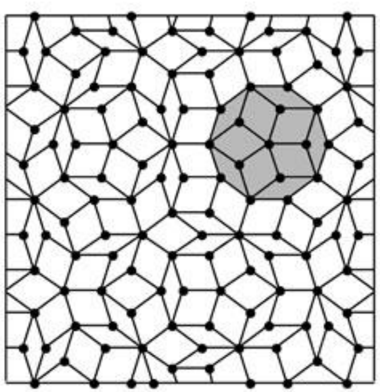




k-space density

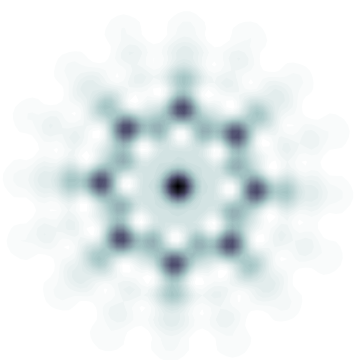


real-space density

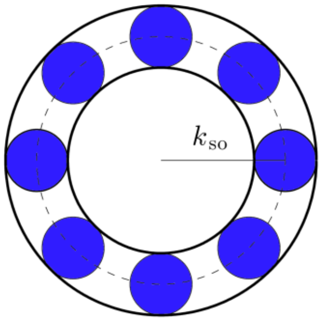


Shechtman, 1984
Nobel, Chemistry, 2011

real-space density



k-space density

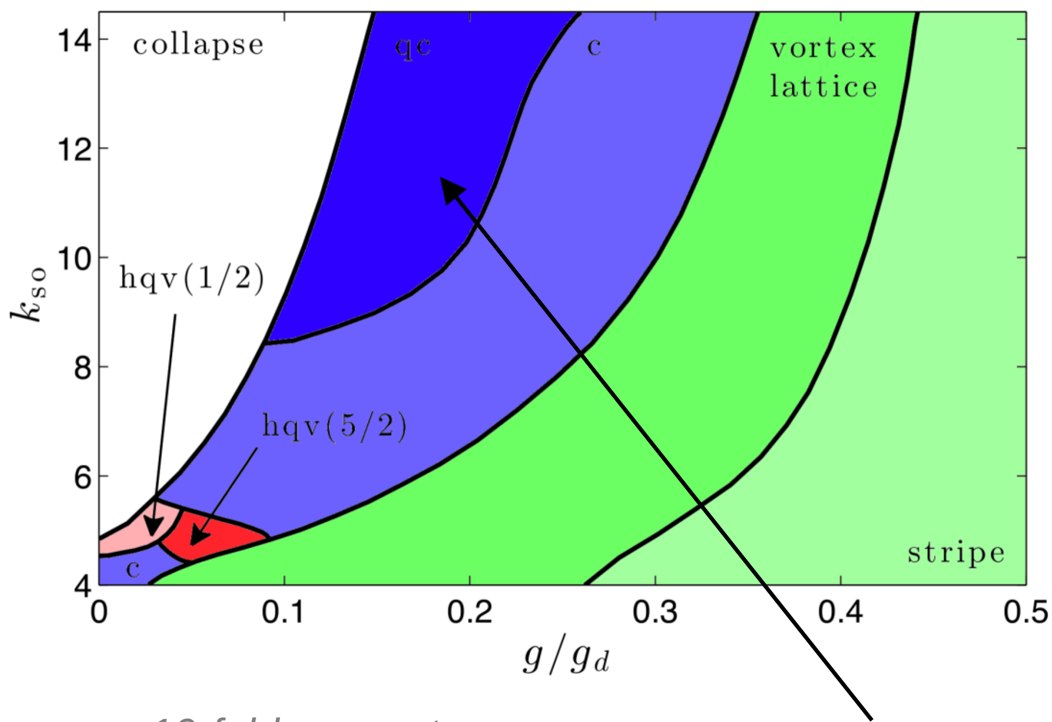


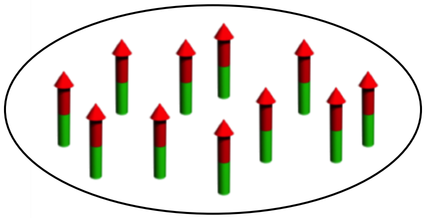
8-fold symmetry
In k-space

Quasicrystals

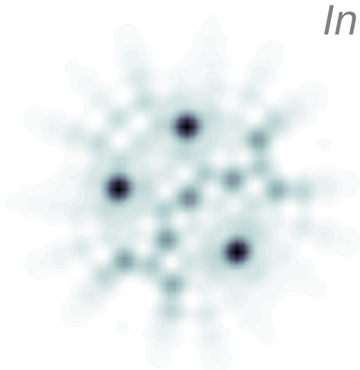
RW, S. Gopalakrishnan, in prep. (2015)

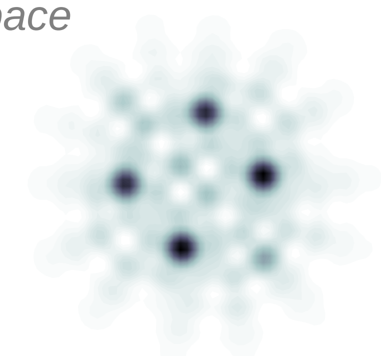
Rashba spin-orbit coupled dipolar BEC



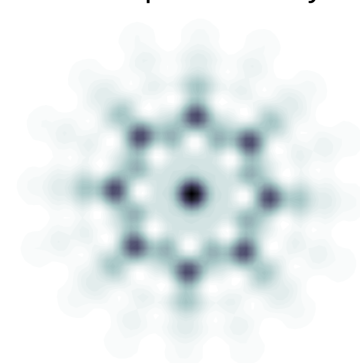


10-fold symmetry
In k-space

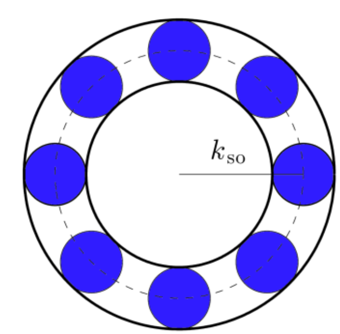




real-space density



k-space density



8-fold symmetry
In k-space

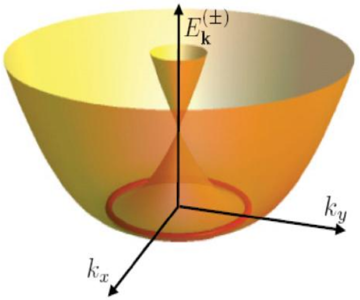
Quasicrystals

RW, S. Gopalakrishnan, in prep. (2015)

Synthetic Quantum Matter with Atoms and Photons

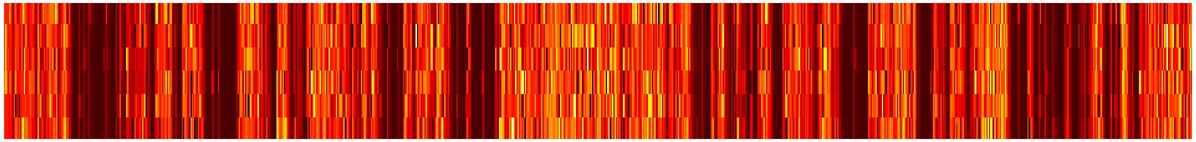
Dipolar BEC in synthetic gauge field (spin-orbit coupling)

Ground state ordering



Driven-dissipative array of nonlinear optical cavities

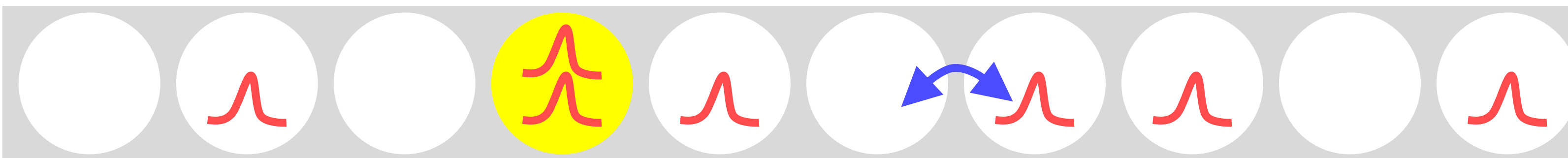
Emergence in open quantum systems



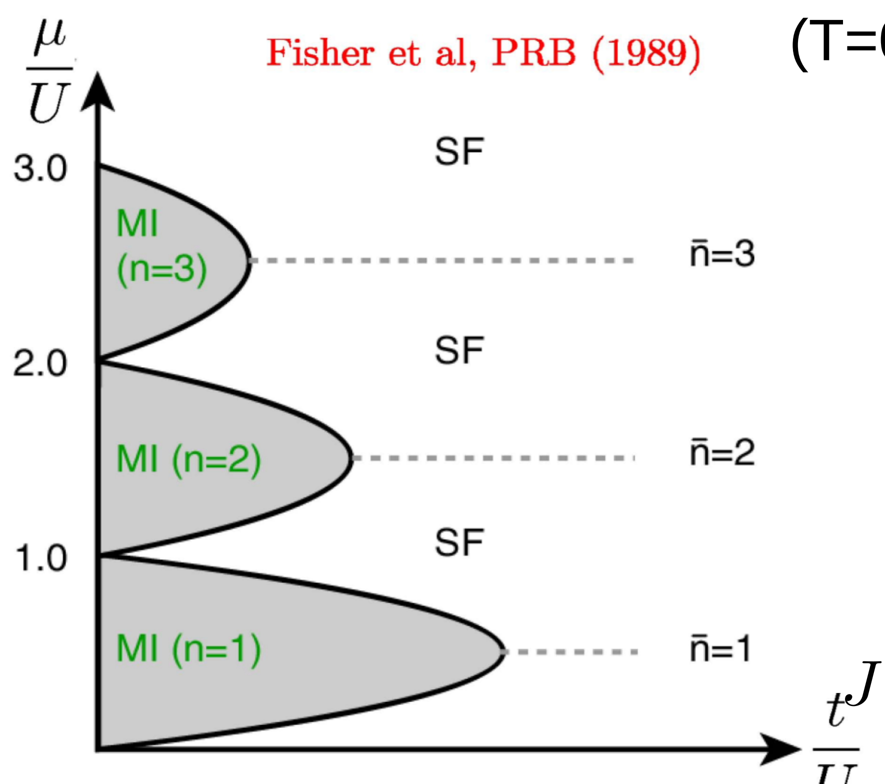
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{a}_j^\dagger\hat{a}_i - \mu\sum_i \hat{n}_i + U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

(Bose-Hubbard)



(T=0, high-dim.)



Fisher *et al.*, PRB, 1989

Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle} \hat{a}_j^\dagger \hat{a}_i - \mu\sum_i \hat{n}_i + U\sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega\sum_i \left(\hat{a}_i + \hat{a}_i^\dagger\right)$$

System = cavities + photon bath (Markovian), trace out bath

$$\dot{\hat{\rho}} = -i\left[\hat{\mathcal{H}}, \hat{\rho}\right] + \frac{\gamma}{2}\sum_i \left(2\hat{a}_i\hat{\rho}\hat{a}_i^\dagger - \hat{n}_i\hat{\rho} - \hat{\rho}\hat{n}_i\right)$$

(Lindblad equation)

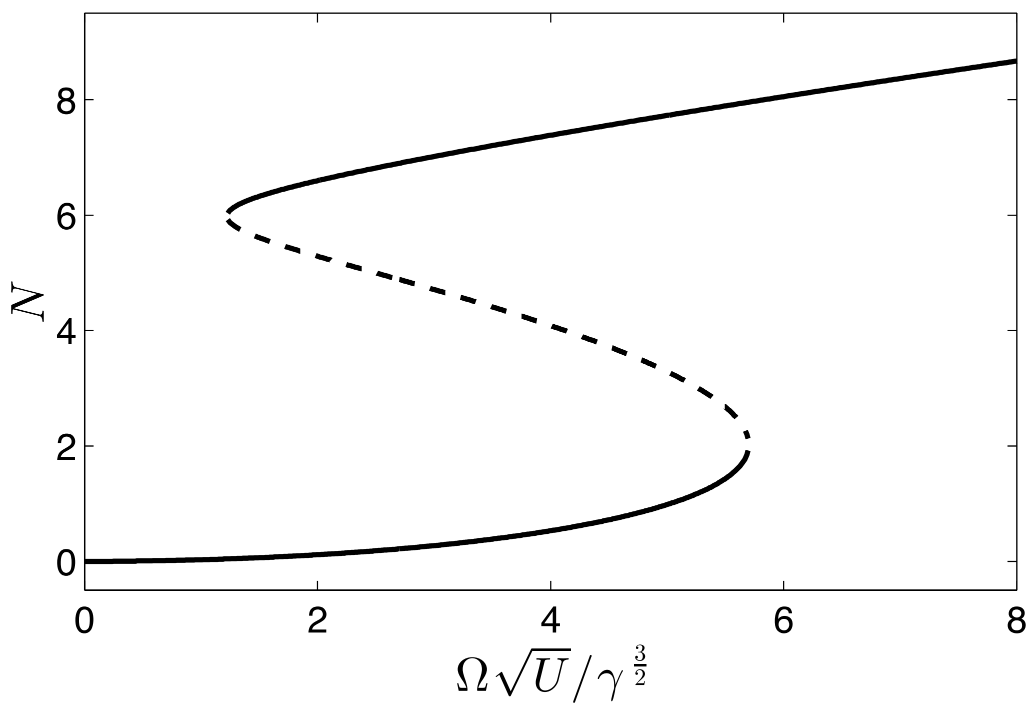
- *U(1)* symmetry broken by coherent driving (superfluid gone)
- Mott insulator phase is gone
- Energy & number not conserved (no ground state)
- Phases characterized by **steady state**

Nonlinear Optical Cavity

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{a}_j^\dagger \hat{a}_i - \mu \sum_i \hat{n}_i + U \sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega \sum_i \left(\hat{a}_i + \hat{a}_i^\dagger\right)$$

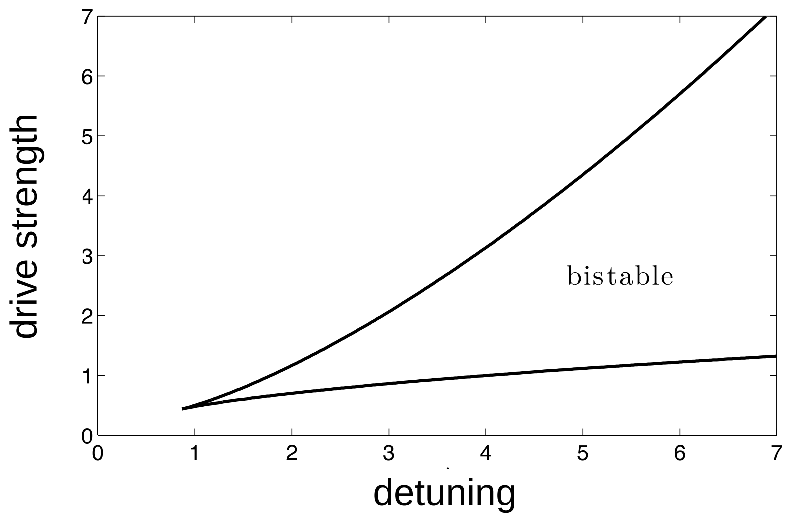
Single cavity, mean-field $\alpha = \langle \hat{a} \rangle$ $\qquad \dot{\alpha} = \mathrm{tr}\left[\dot{\hat{\rho}}\hat{a}\right]$

Optical bistability



Two stable steady states

1st order phase transition

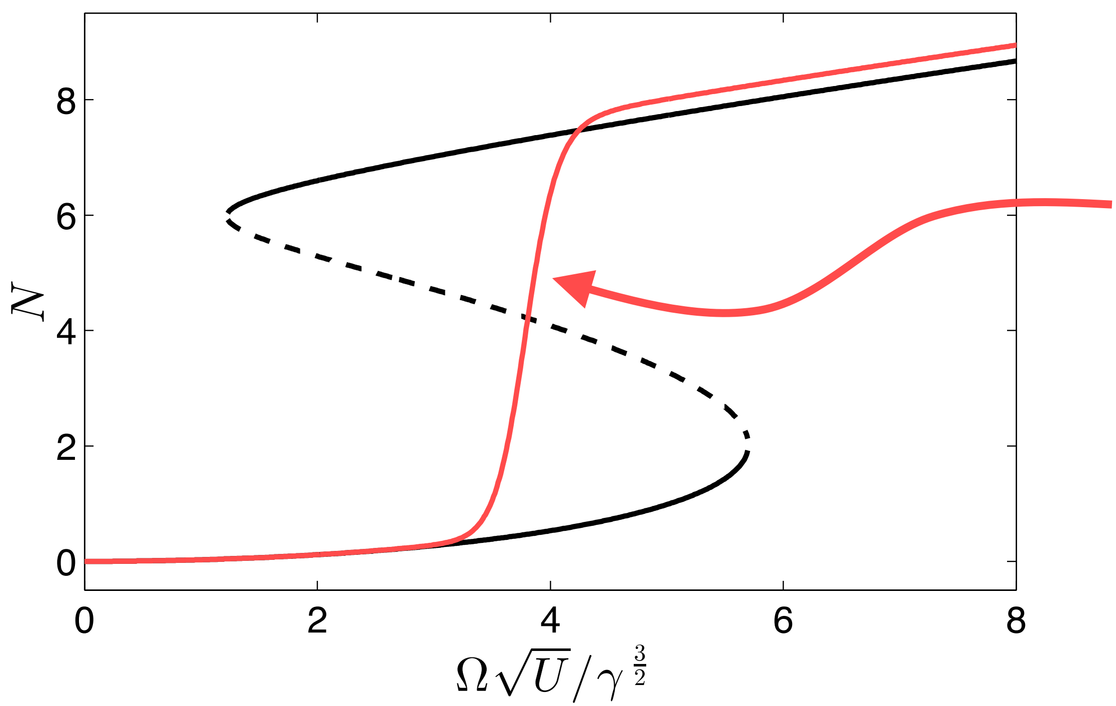


Nonlinear Optical Cavity

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{a}_j^\dagger\hat{a}_i - \mu\sum_i \hat{n}_i + U\sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega\sum_i\left(\hat{a}_i + \hat{a}_i^\dagger\right)$$

Single cavity, exact solution

Optical bistability



~~Two stable steady states~~

~~1^{st} order phase transition~~

Exact solution **not bistable**

Steady state is unique

Corresponds to ensemble average of many measurements

Continuous (crossover, not a phase transition)

Nonlinear Optical Cavity

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{a}_j^\dagger \hat{a}_i - \mu \sum_i \hat{n}_i + U \sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega \sum_i \left(\hat{a}_i + \hat{a}_i^\dagger \right)$$

Wigner representation of density matrix: $\chi_W(\alpha, \alpha^*) = \mathrm{tr}\left[\hat{\rho} e^{\alpha \hat{a}^\dagger - \hat{a}\alpha^*}\right]$

Stochastic nonlinear equation: $i\dot{\alpha} = -\mu\alpha + U|\alpha|^2\alpha + \Omega - i\frac{\gamma}{2} + d\dot{W}$

(Gross-Pitaevskii)

Wiener process (Brownian motion)

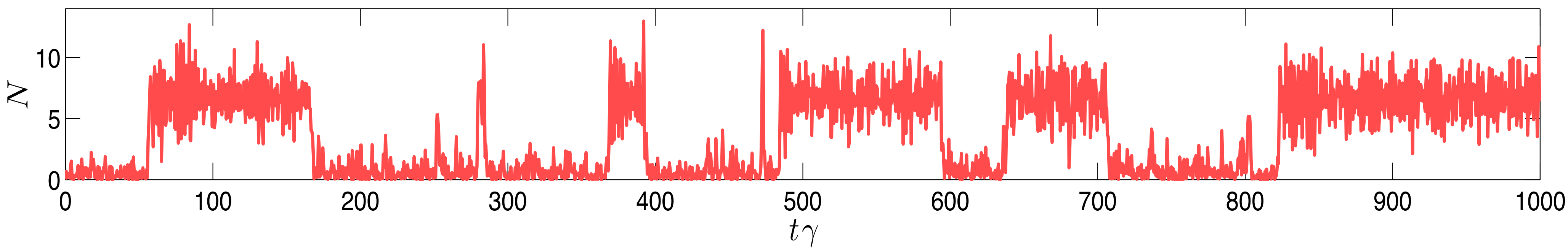
Nonlinear Optical Cavity

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{a}_j^\dagger \hat{a}_i - \mu \sum_i \hat{n}_i + U \sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega \sum_i \left(\hat{a}_i + \hat{a}_i^\dagger\right)$$

Wigner representation of density matrix: $\chi_W(\alpha, \alpha^*) = \mathrm{tr}\left[\hat{\rho} e^{\alpha \hat{a}^\dagger - \hat{a}\alpha^*}\right]$

Stochastic nonlinear equation: $i\dot{\alpha} = -\mu\alpha + U|\alpha|^2\alpha + \Omega - i\frac{\gamma}{2} + d\dot{W}$

Wiener process (Brownian motion)



(Perturbative) quantum treatment exhibits switching between classical steady states

Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{a}_j^{\dagger}\hat{a}_i - \mu\sum_i \hat{n}_i + U\sum_i \hat{n}_i(\hat{n}_i - 1) + \Omega\sum_i\left(\hat{a}_i + \hat{a}_i^{\dagger}\right)$$

Hardcore bosons map to dissipative XXZ spin model: $\hat{a}_i \rightarrow \hat{\sigma}_i^-$

No bistability for single spin

Assymetric simpl3e exclusion process

Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Gutzwiller appx: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle = \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (no entanglement)

Uniform phases? $\left(\langle \hat{\sigma}_i^\alpha \rangle = \langle \hat{\sigma}_j^\alpha \rangle \right)$

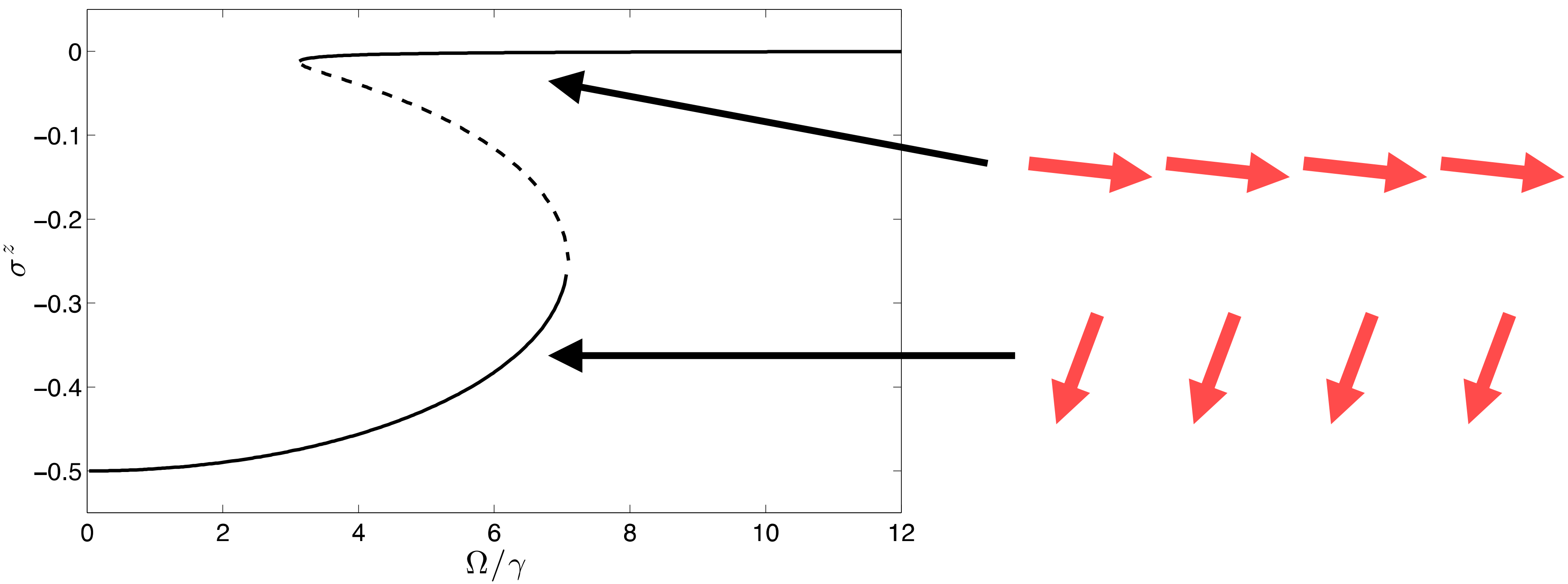
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{\sigma}_j^+\hat{\sigma}_i^- - \mu\sum_i \hat{\sigma}_i^+\hat{\sigma}_i^- + \Omega\sum_i\left(\hat{\sigma}_i^+ + \hat{\sigma}_i^-\right)$$

Gutzwiller appx: $\langle\hat{\sigma}_j^\alpha\hat{\sigma}_i^\alpha\rangle = \langle\hat{\sigma}_j^\alpha\rangle\langle\hat{\sigma}_i^\alpha\rangle$ (no entanglement)

Uniform phases? $(\langle\hat{\sigma}_i^\alpha\rangle = \langle\hat{\sigma}_j^\alpha\rangle)$

Collective bistability



$J/\gamma = 10$

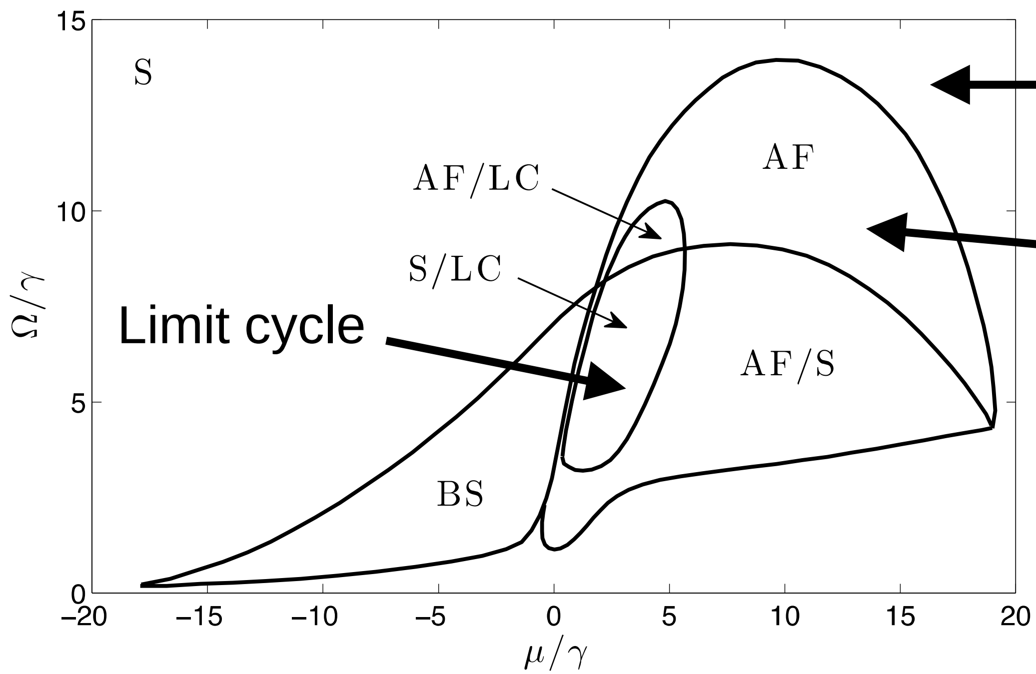
Nonlinear Optical Cavities

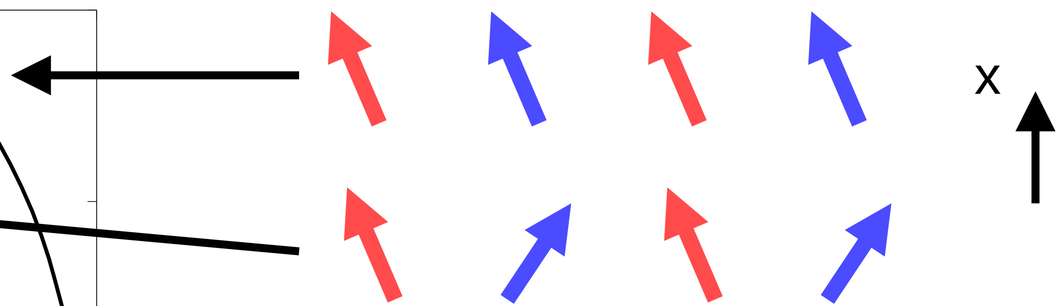
$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Gutzwiller appx: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle = \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (no entanglement)

Non-uniform phases have **AB** sublattice (AF) symmetry

Semiclassical/Gutzwiller phase diagram





$J/\gamma = 10$

RW *et al.*, in prep. (2015)

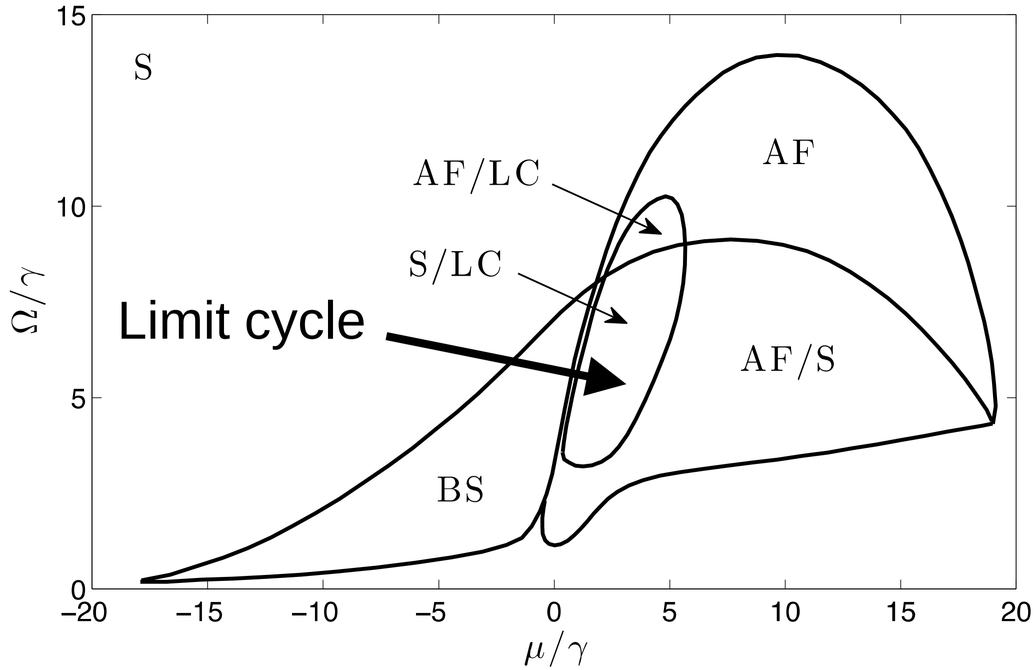
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

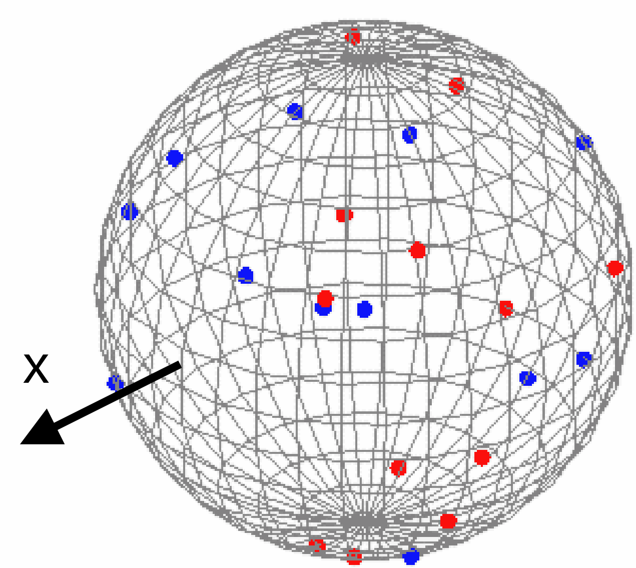
Gutzwiller appx: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle = \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (no entanglement)

Non-uniform phases have **AB** sublattice (AF) symmetry

Semiclassical/Gutzwiller phase diagram



Bloch sphere



$J/\gamma = 10$

RW *et al.*, in prep. (2015)

Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Exact solutions: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle \neq \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (entanglement)

For N spins, Hilbert space scales as 2^N, ρ has 2^{2N} elements

Exact solutions with quantum trajectories of wave function

$$\dot{\hat{\rho}} = -i \left[\hat{\mathcal{H}}, \hat{\rho} \right] + \frac{\gamma}{2} \sum_i \left(2\hat{\sigma}_i^- \hat{\rho} \hat{\sigma}_i^+ - \{ \hat{\sigma}_i^+ \hat{\sigma}_i^-, \hat{\rho} \} \right)$$

Stochastic applications of $\hat{\sigma}_i^-$ non-Hermitian evolution

Ensemble averaging corresponds to **exact** solution

RW *et al.*, in prep. (2015)

Nonlinear Optical Cavities

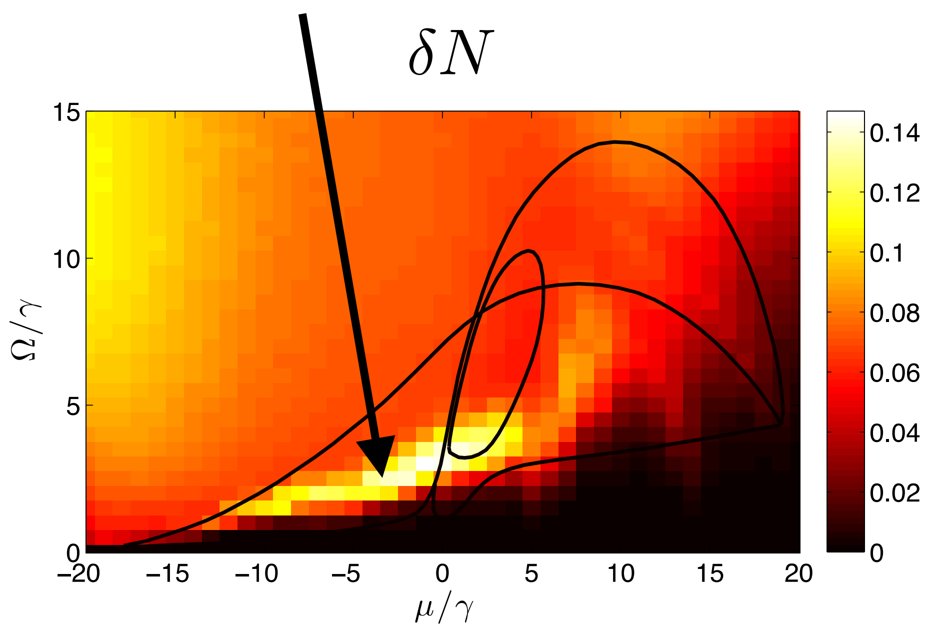
$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

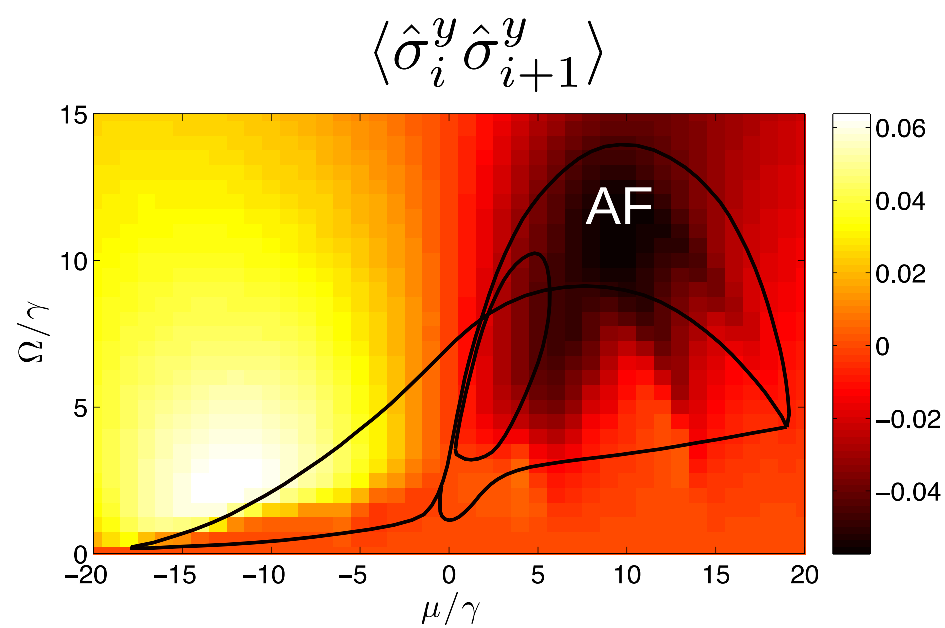
Exact solutions: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle \neq \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (entanglement)

For N spins, Hilbert space scales as 2^N, ρ has 2^{2N} elements

Exact solutions with quantum trajectories of wave function

Enhanced number fluctuations in BS region





16 spins

RW *et al.*, in prep. (2015)

Nonlinear Optical Cavities

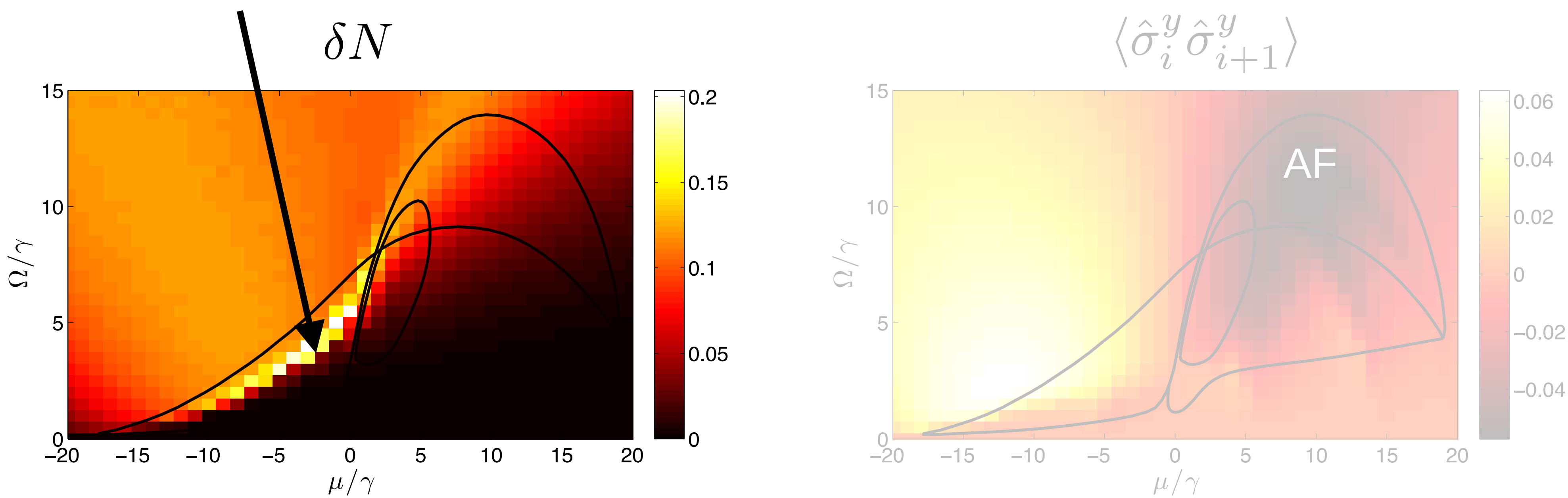
$$\hat{\mathcal{H}} = -J\sum_{\langle i,j\rangle}\hat{\sigma}_j^+\hat{\sigma}_i^- - \mu\sum_i \hat{\sigma}_i^+\hat{\sigma}_i^- + \Omega\sum_i\left(\hat{\sigma}_i^+ + \hat{\sigma}_i^-\right)$$

Exact solutions: $\langle\hat{\sigma}_j^\alpha\hat{\sigma}_i^\alpha\rangle \neq \langle\hat{\sigma}_j^\alpha\rangle\langle\hat{\sigma}_i^\alpha\rangle$ (entanglement)

For N spins, Hilbert space scales as 2^N, ρ has 2^{2N} elements

All-to-all coupling approaches mean-field limit

Enhanced number fluctuations in BS region



16 spins

RW *et al.*, in prep. (2015)

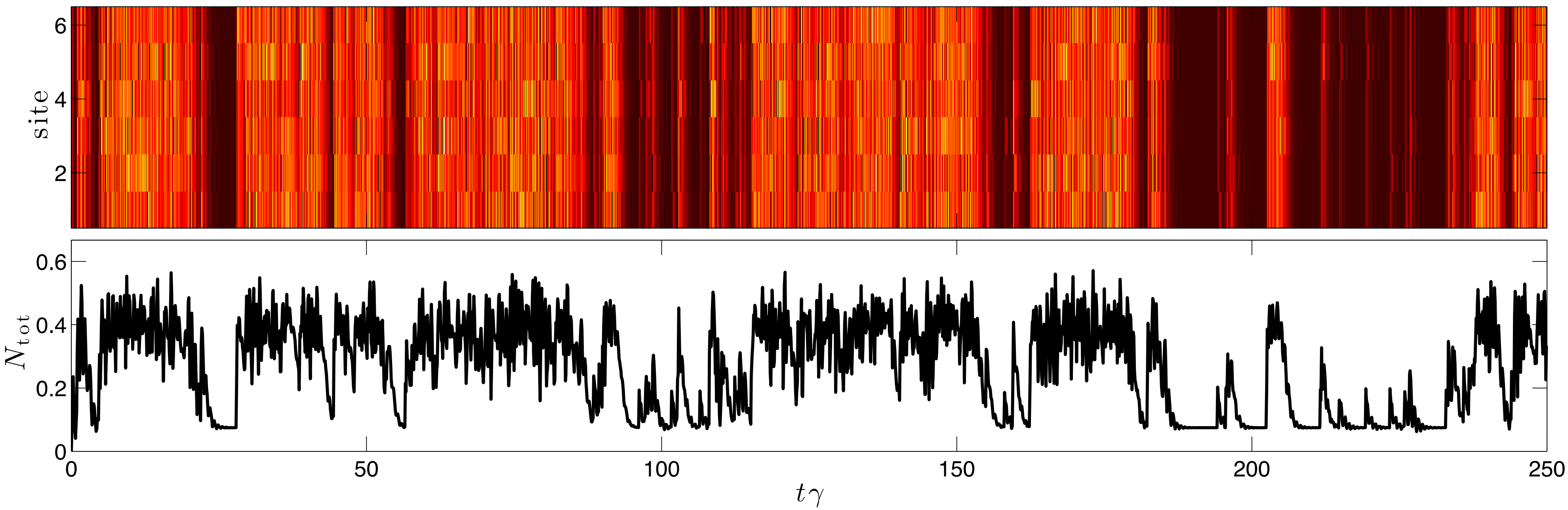
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Exact solutions: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle \neq \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (entanglement)

For N spins, Hilbert space scales as 2^N

Exact solutions with quantum trajectories of wave function



Trajectory exhibits *collective* switching in bistable region

RW *et al.*, in prep. (2015)

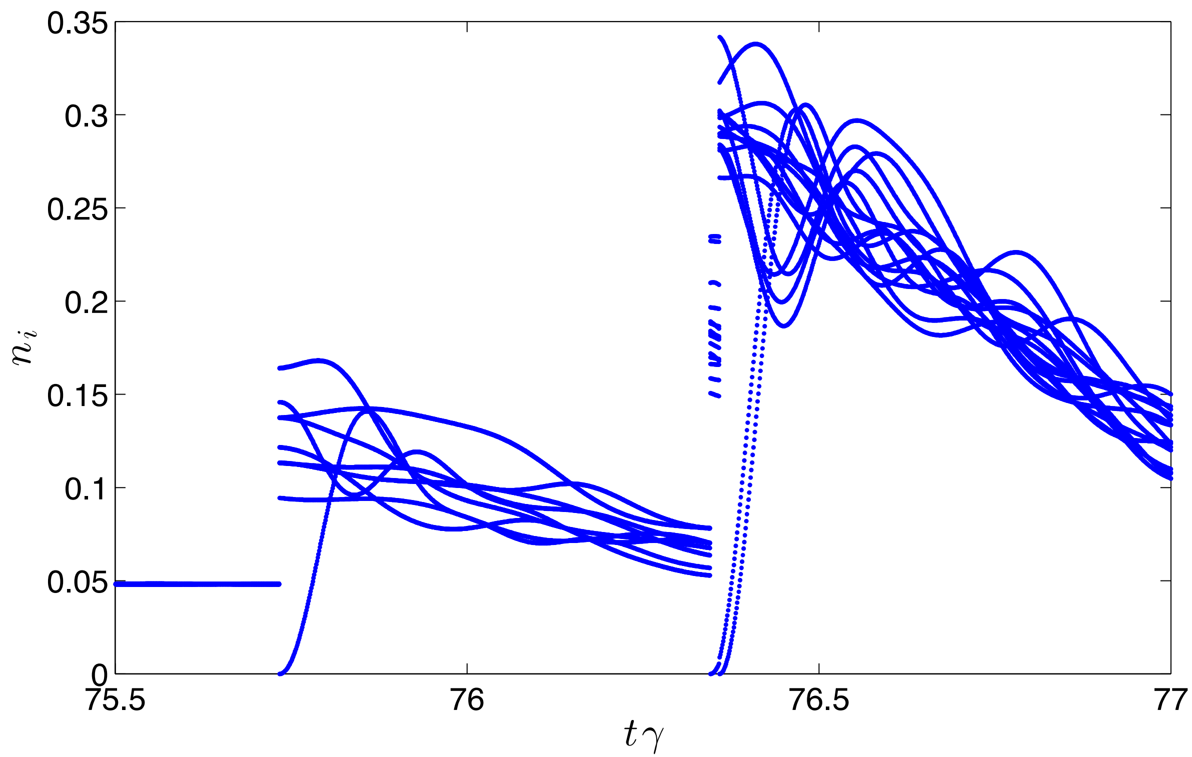
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Exact solutions: $\langle \hat{\sigma}_j^\alpha \hat{\sigma}_i^\alpha \rangle \neq \langle \hat{\sigma}_j^\alpha \rangle \langle \hat{\sigma}_i^\alpha \rangle$ (entanglement)

For N spins, Hilbert space scales as 2^N

Exact solutions with quantum trajectories of wave function



16 spins

RW *et al.*, in prep. (2015)

Nonlinear Optical Cavities

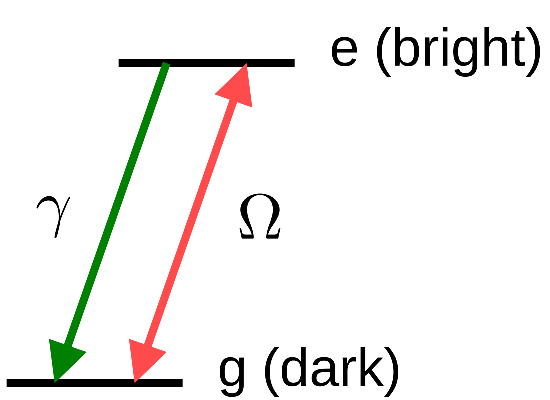
$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Two-state toy model

$$\dot{\rho}_g = -\Omega \rho_g + (\Omega + \gamma) \rho_e$$

$$\dot{\rho}_e = \Omega \rho_g - (\Omega + \gamma) \rho_e$$

Gap: $\Delta = \frac{1}{T_g} + \frac{1}{T_e}$



RW *et al.*, in prep. (2015)

Nonlinear Optical Cavities

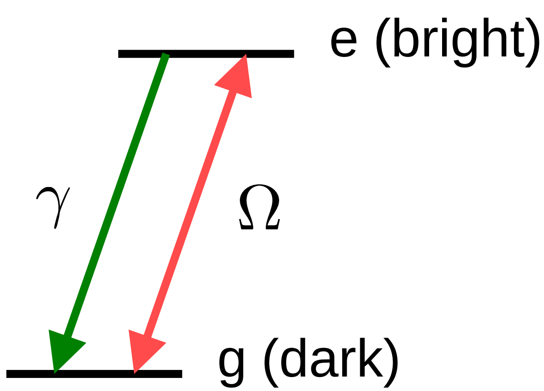
$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

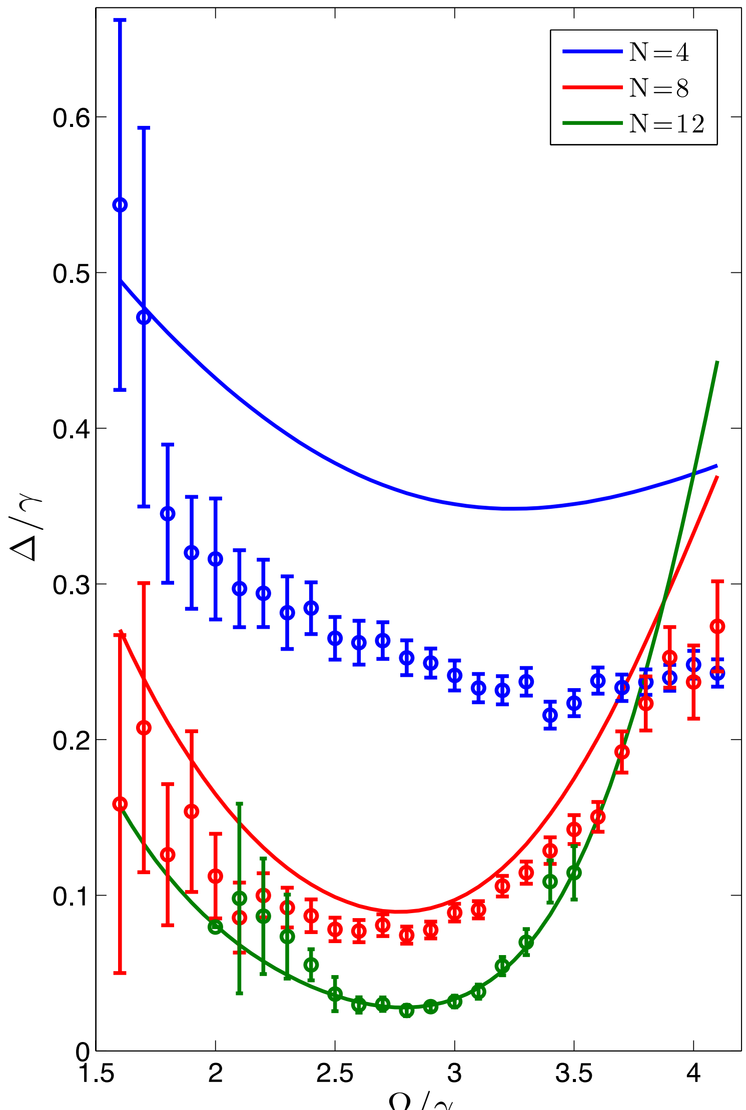
Two-state toy model

$$\dot{\rho}_g = -\Omega \rho_g + (\Omega + \gamma) \rho_e$$

$$\dot{\rho}_e = \Omega \rho_g - (\Omega + \gamma) \rho_e$$

Gap: $\Delta = \dfrac{1}{T_g} + \dfrac{1}{T_e}$





all-to-all coupling

RW *et al.*, in prep. (2015)

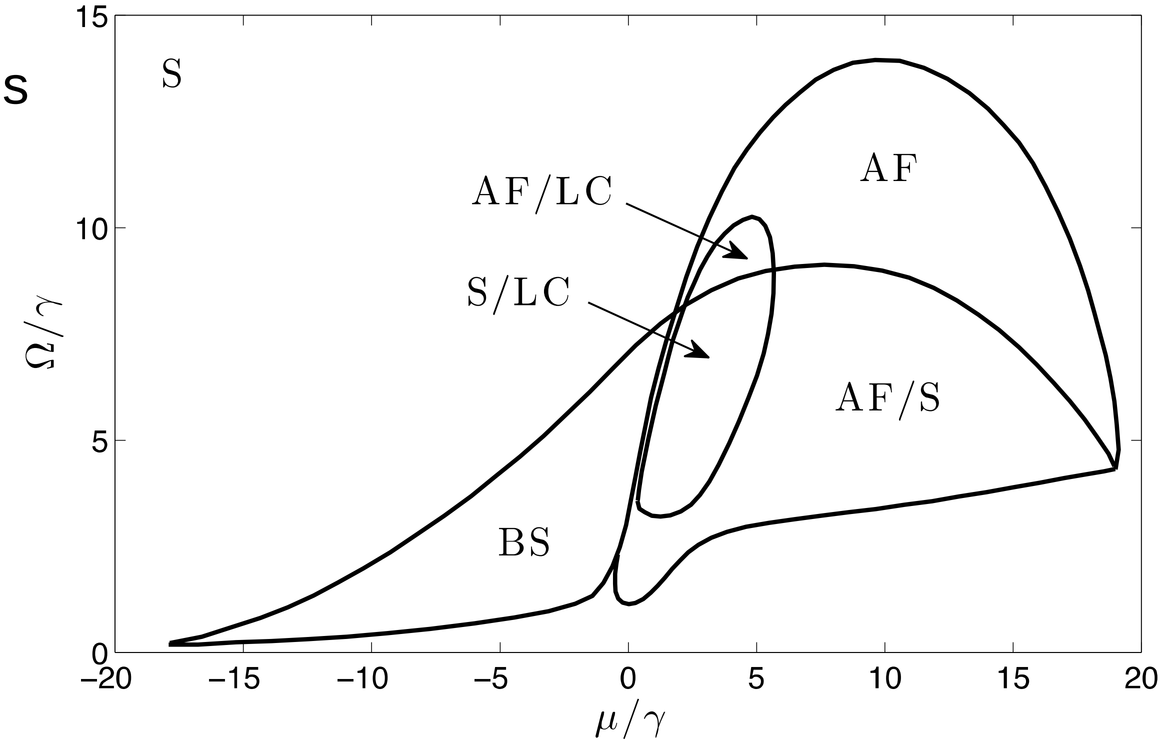
Nonlinear Optical Cavities

$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Semiclassical phase diagram
- 1^{st} order transition, limit cycles
- When is it recovered?

Quantum trajectories
w/ long-range couplings

Stochastic GP simulations?
- no entanglement
- classical correlations
- good for small U, large N



Nonlinear Optical Cavities

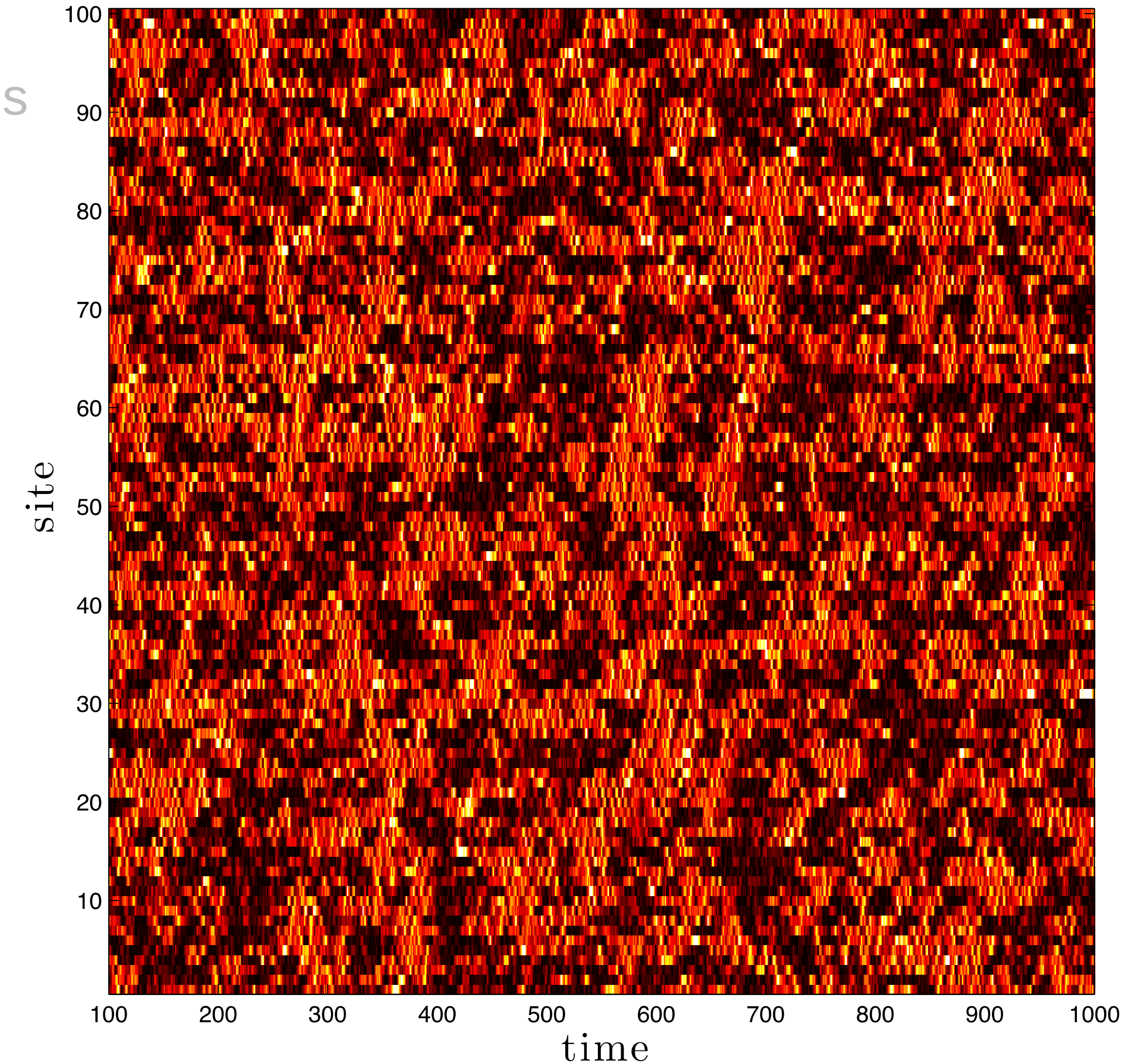
$$\hat{\mathcal{H}} = -J \sum_{\langle i,j \rangle} \hat{\sigma}_j^+ \hat{\sigma}_i^- - \mu \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + \Omega \sum_i \left(\hat{\sigma}_i^+ + \hat{\sigma}_i^- \right)$$

Semiclassical phase diagram
- 1st order transition, limit cycles
- When is it recovered?

Quantum trajectories
w/ long-range couplings

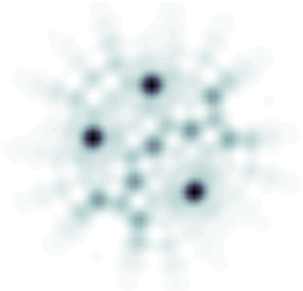
Stochastic GP simulations?
- no entanglement
- classical correlations
- good for small U, large N

domains in 1D



Conclusion

T=0 (BEC) ground states of dilute Bose gases



Interplay of dipolar interactions and spin-orbit coupling

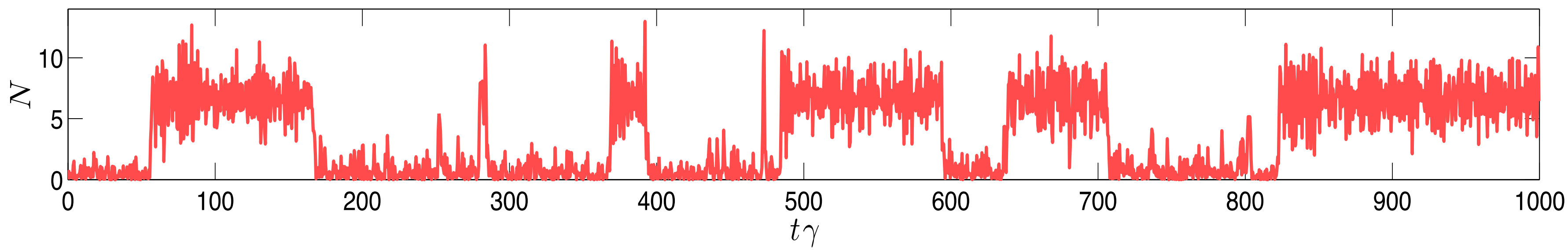
Rich phase diagram: vortex lattices, crystals, quasicrystals

Driven-dissipative nonlinear optical cavities

Semiclassical steady states include bistability (1st order), limit cycles, AF order

Quantum trajectories show collective bistable switching and AF correlations

Limit cycle, 1st order transition emerge with increasing dimensionality



Other

Dipolar fermions, large spin Bose gases (S=1), atoms in optical lattices

NSF (RUI) to study quantum gases of diatomic molecules