Santa Barbara Meeting, KITP, 27 March 2014

Jets: a roller-coaster ride from Earth to Jupiter and back, *or:*

A tale of three species, *or:*

Is homogeneous turbulence theory a dangerous idea?

Michael E. McIntyre,
Dept of Applied Mathematics & Theoretical Physics,
University of Cambridge, UK

1: Very brief look at standard jet mechanisms and the case of earthly **strong jets**, following Rosenbluth and Haurwitz lectures

2. Hot off the press: results from a new idealized model of **Jupiter's jets** – strange new territory!!
3. Even hotter (not to say hasty and preliminary) surprises from the **extended Hasegawa-Mima equation** including a `damn fool experiment' ...

For more detail, websearch **"lucidity principles"**
then back to my home page at "Encyclopedia", "Rosenbluth", "Haurwitz".

Our life support system

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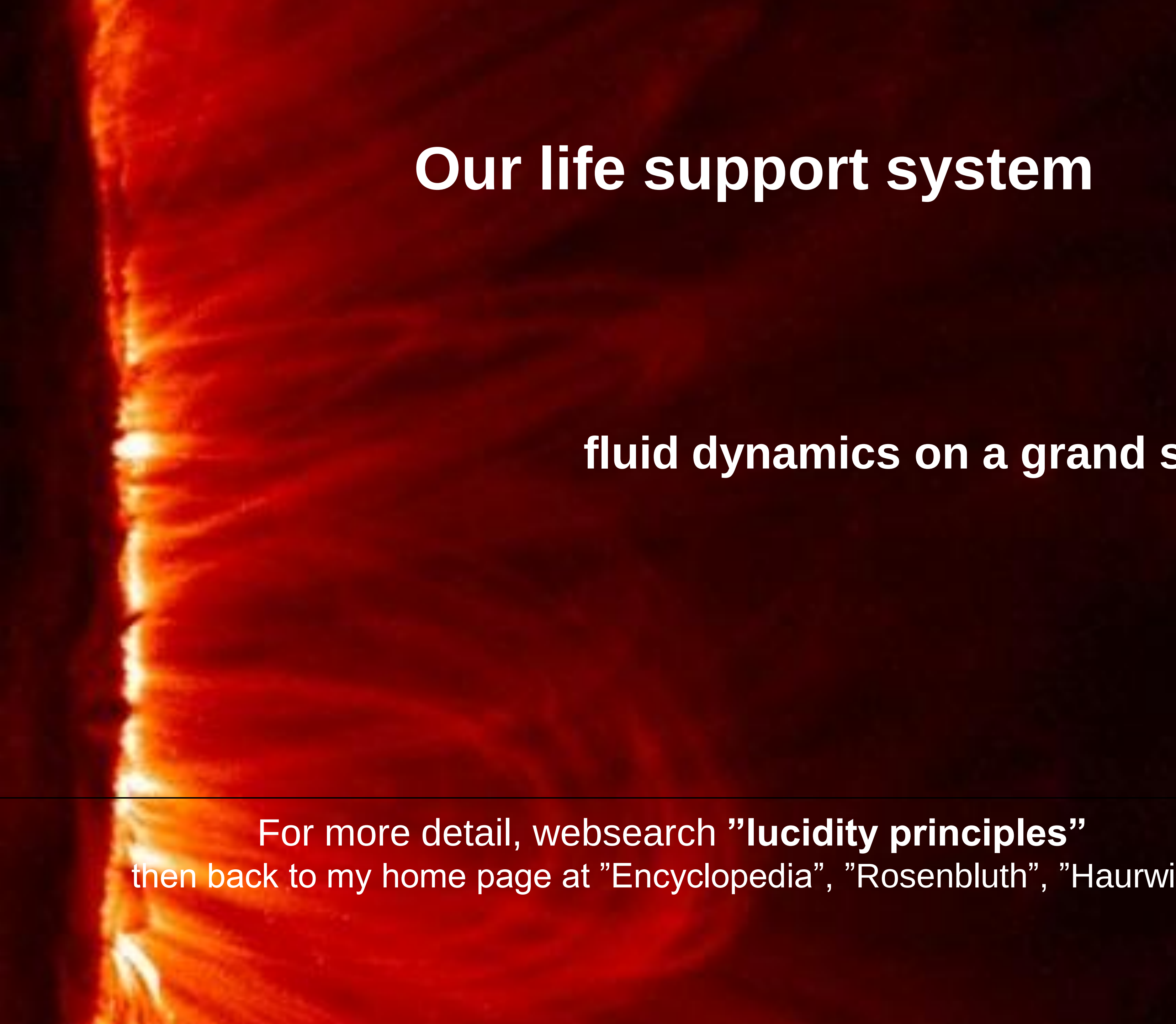

Our life support system

fluid dynamics on a grand scale…

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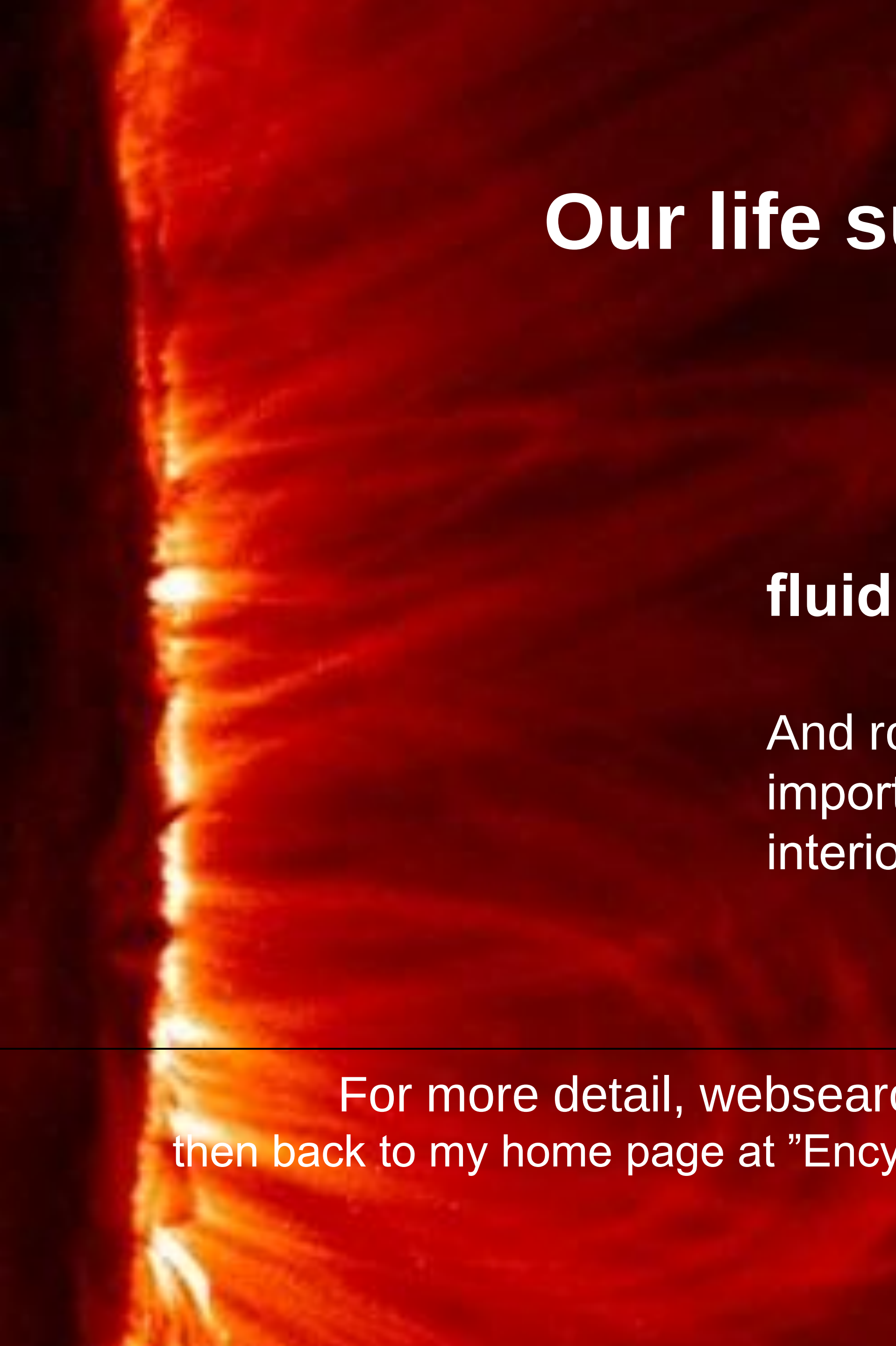

Our life support system

fluid dynamics on a grand scale…

And rotation and stable stratification are important in many cases, including the Sun's interior…

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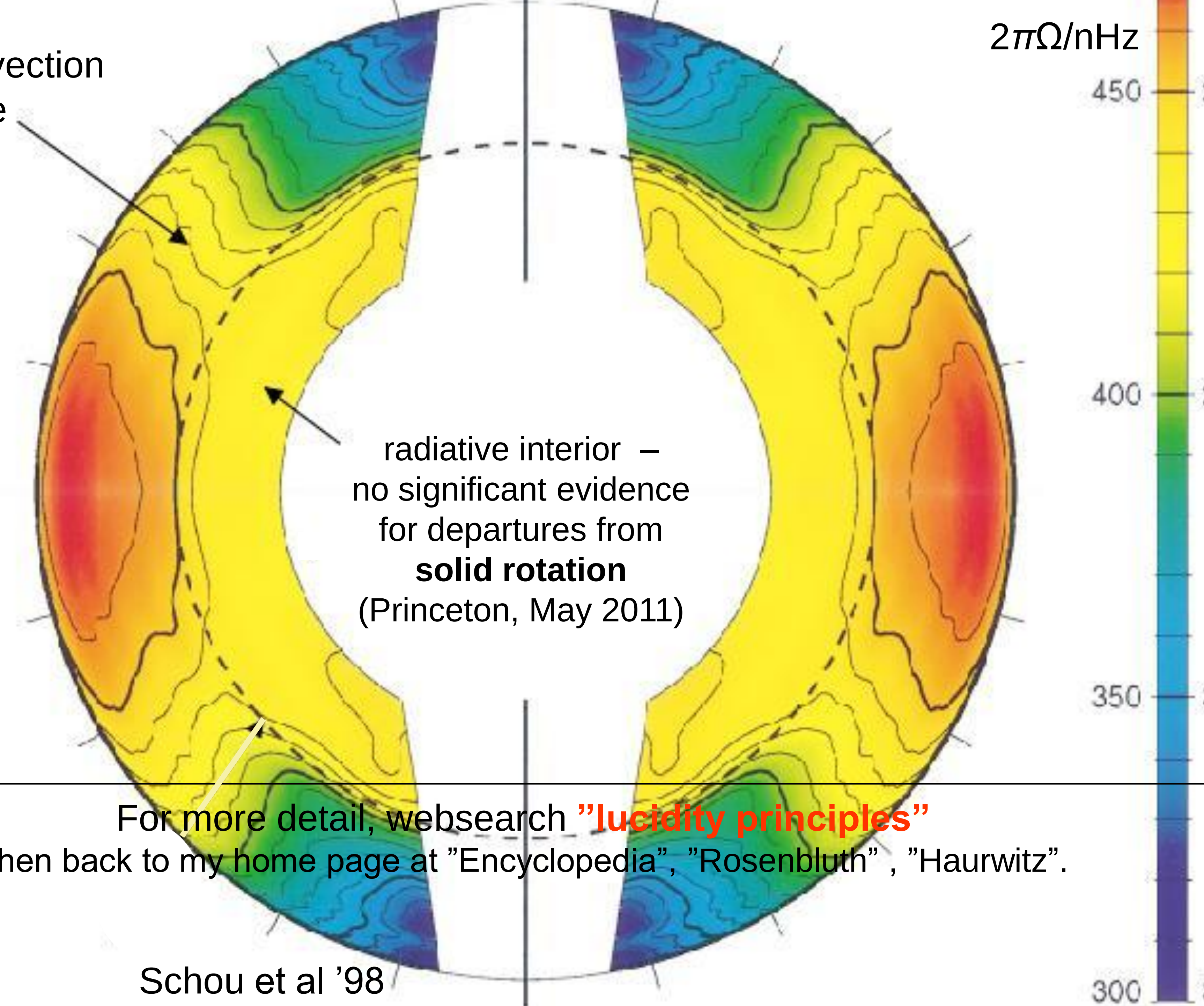

convection
zone

radiative interior –
no significant evidence
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(Princeton, May 2011)

Schou et al ’98

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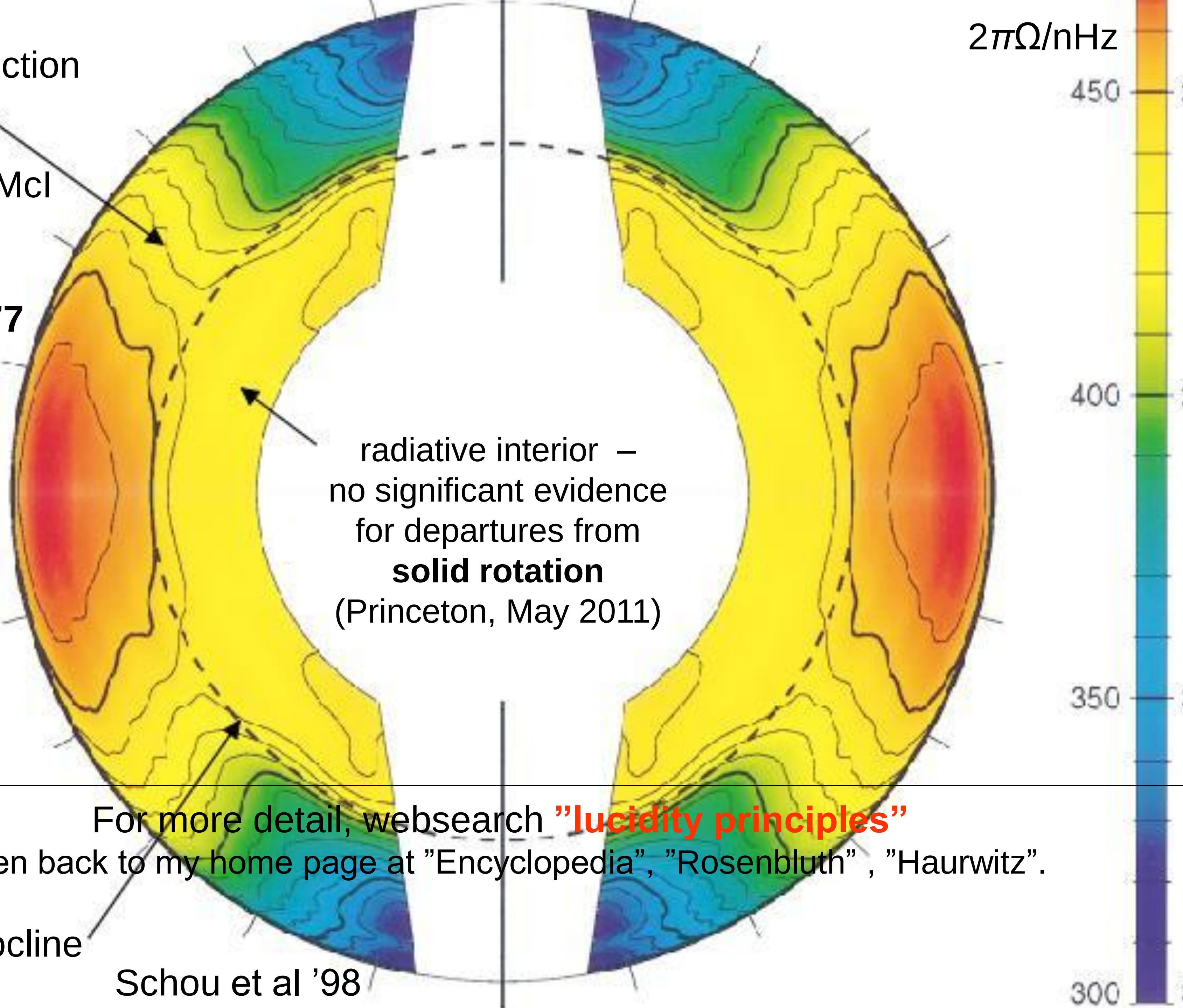

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Wood & McI
(2011),
J. Fluid
Mech. **677**

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tachocline

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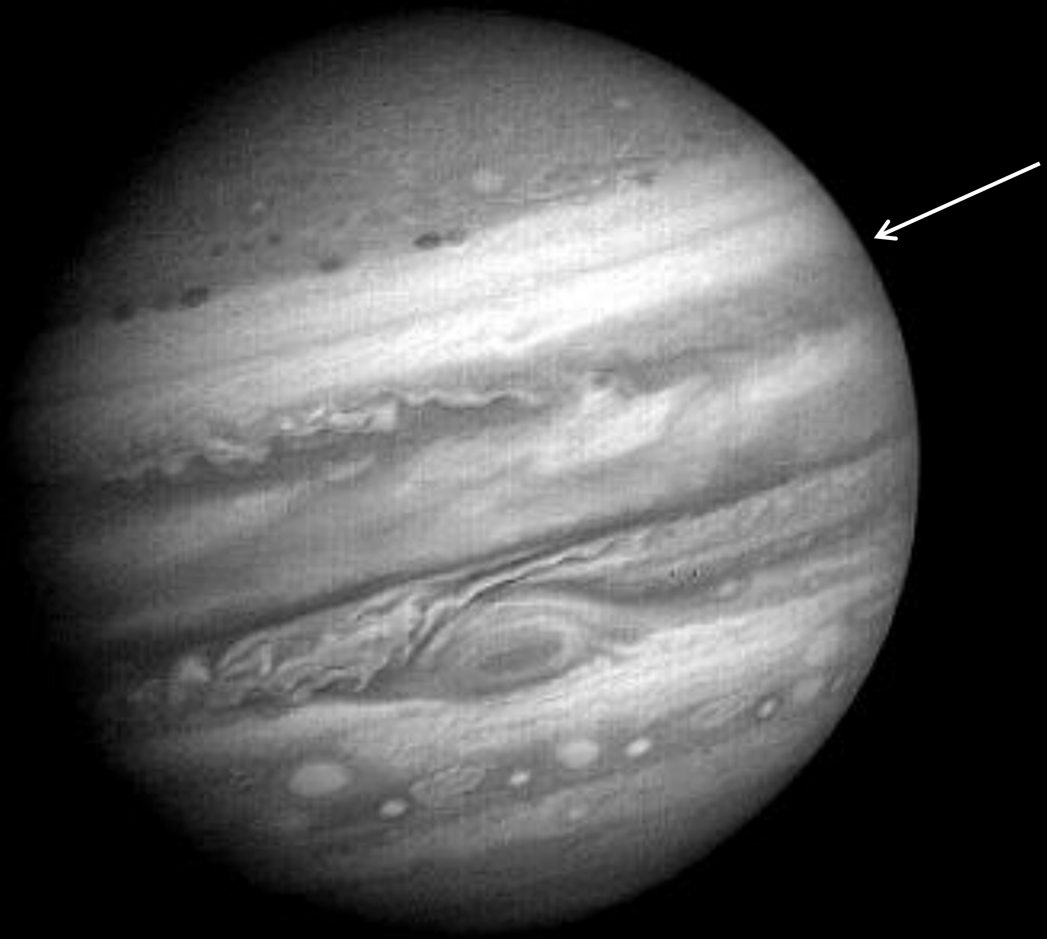
1979: Voyager 1 approaching (60 Jupiter days) – unearthly!



folded
filamentary
region

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Rotating, stratified
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Here’s the most beautiful definition of the **exact PV:**

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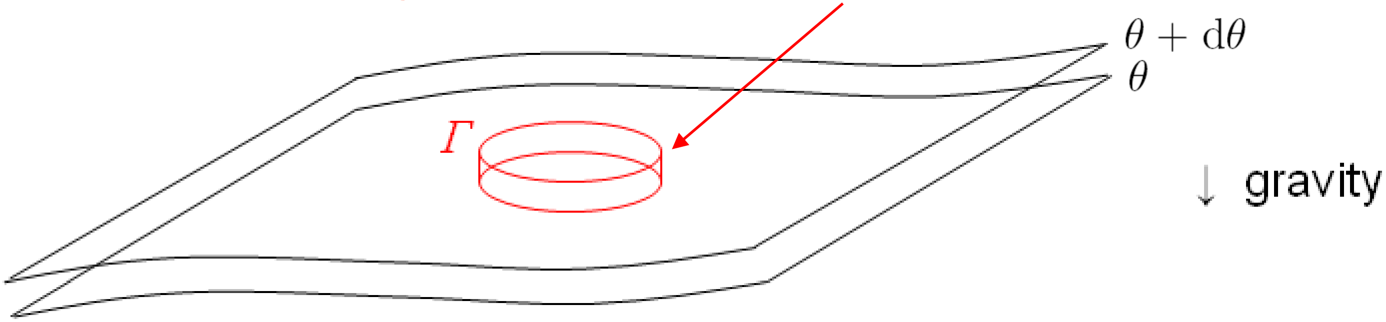

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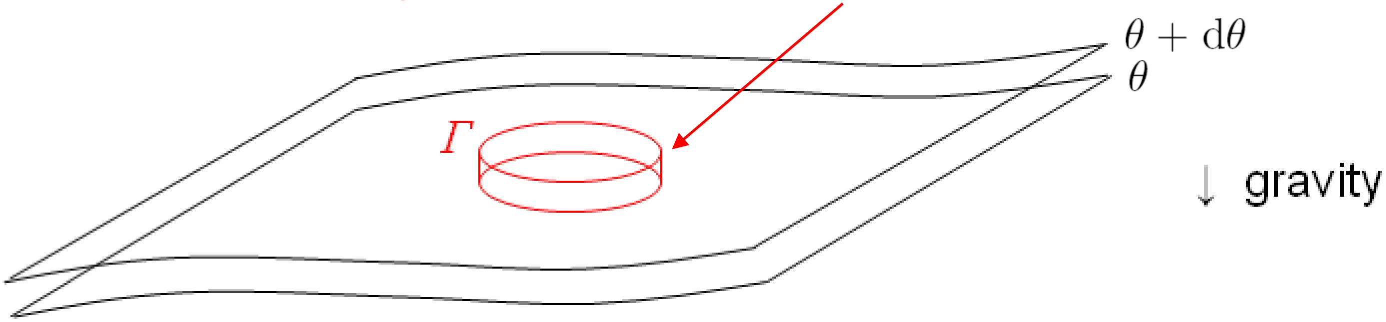

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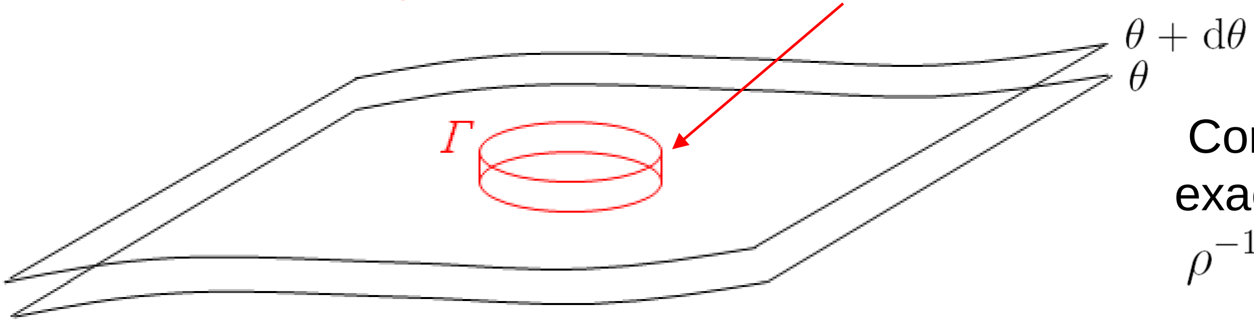

Corollary:
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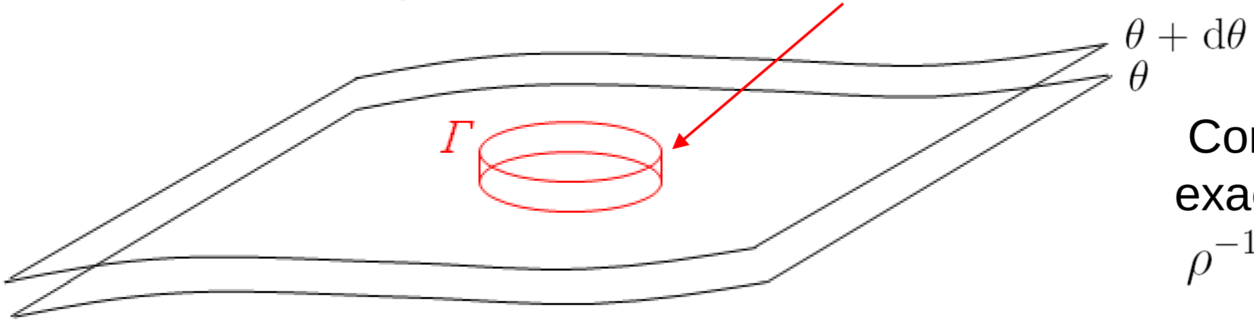

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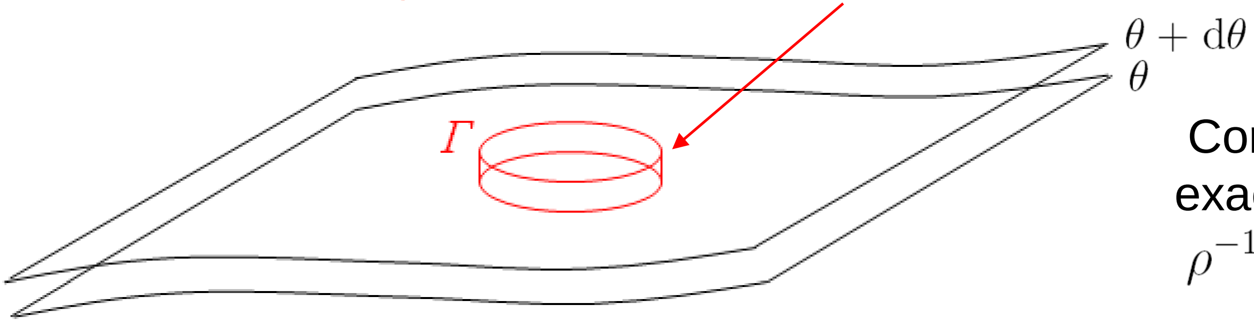

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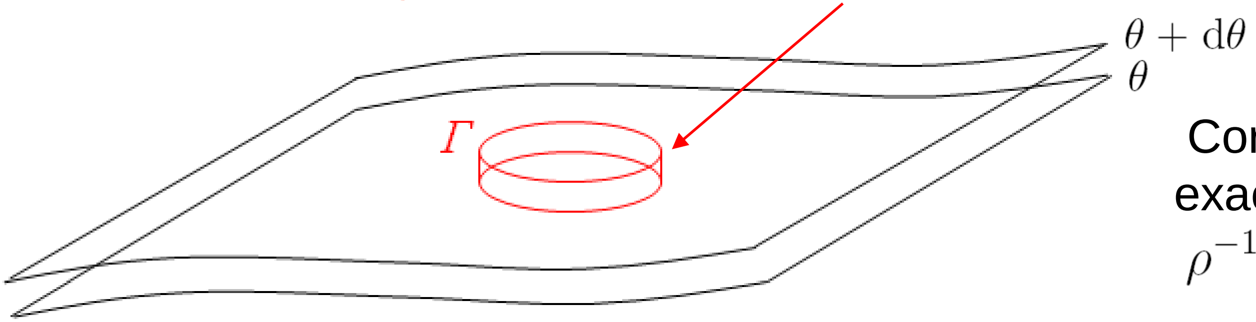

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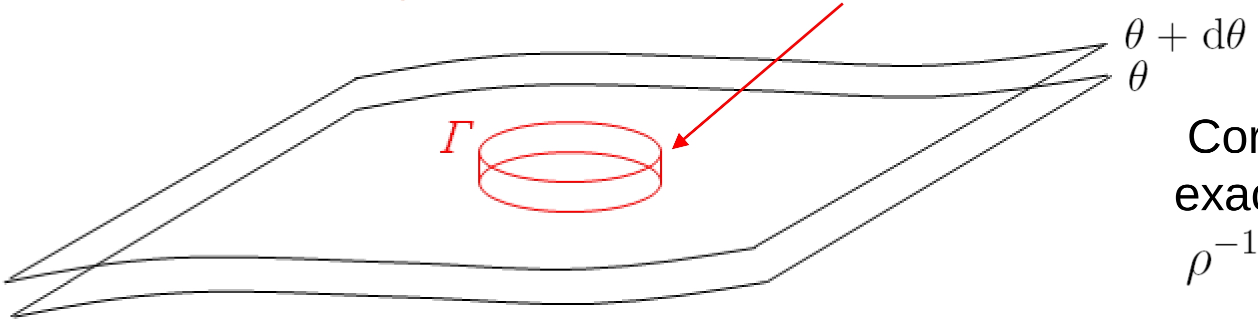

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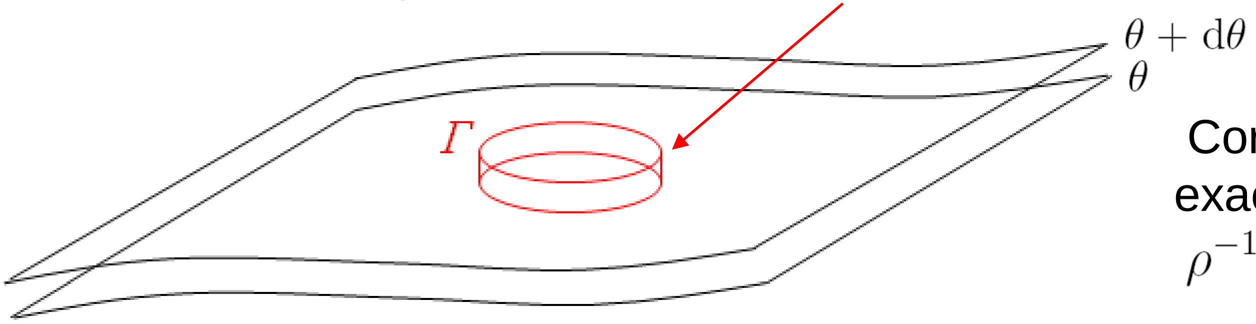

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Note **scale effect:** small-scale PV anomalies → weak velocities.

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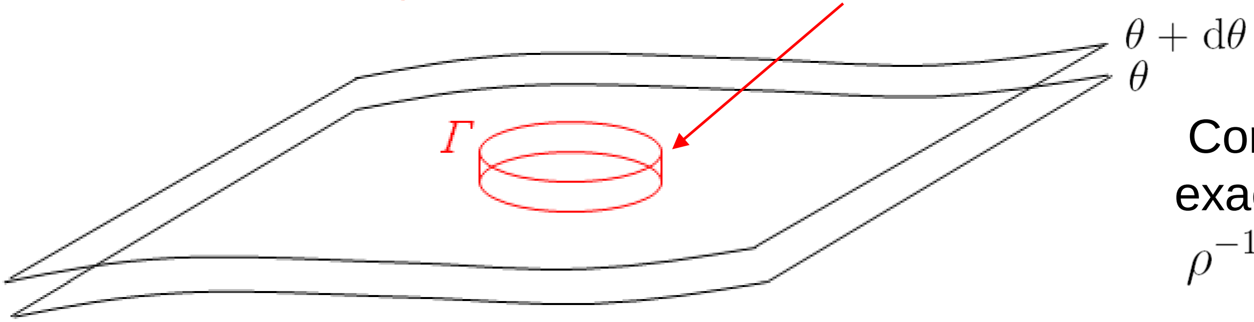

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What do **real PV fields** look like? Here’s an example (nostalgic for me):

Courtesy Dr A J Simmons,
European Centre for Medium
Range Weather Forecasts:

McIntyre and Palmer (1983), revisited

PV on the 850K stratification surface:

Breaking planetary waves in the stratosphere

M. E. McIntyre* **& T. N. Palmer**†

* Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, UK
† Meteorological Office, Bracknell, Berks RG12 2SZ, UK

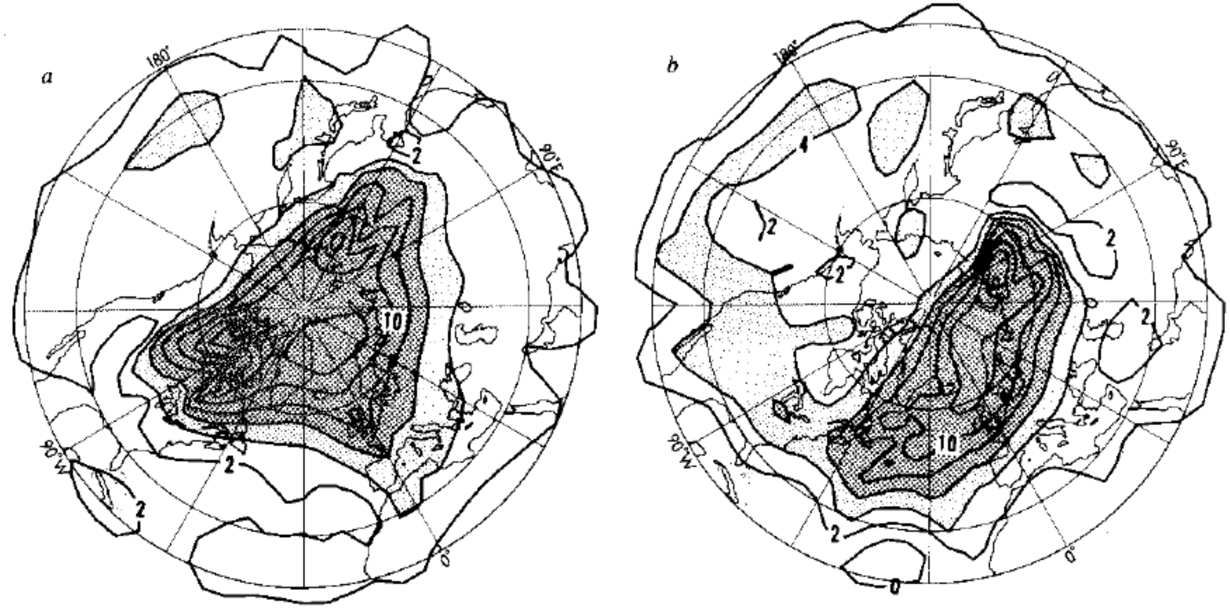
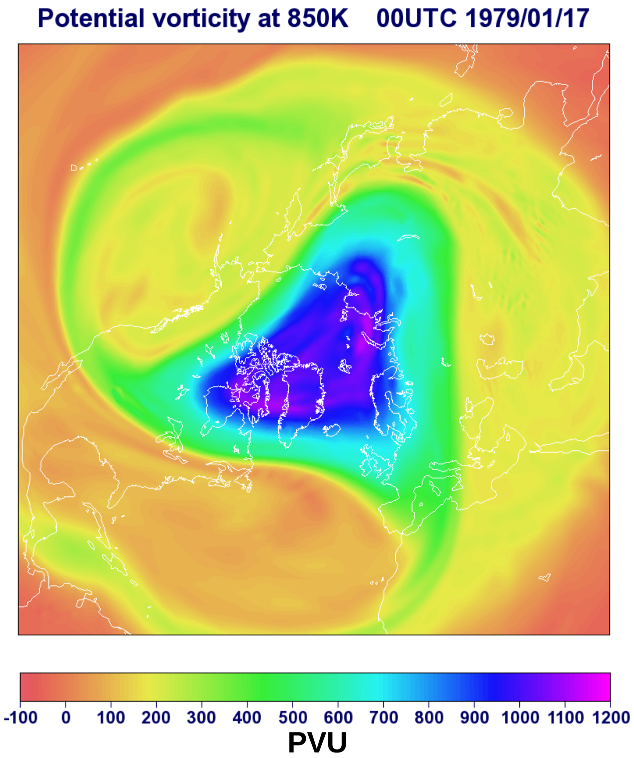


Fig. 2 Coarse-grain estimates of Ertel's potential vorticity Q on the 850 K isentropic surface (near the 10-mbar isobaric surface) on 17 (a) and 27 (b) January 1979, at 00 h GMT. The southernmost latitude circle shown is 20° N; the others are 30° N and 60° N. Map projection is polar stereographic. For units see equation (5) onwards. Contour interval is 2 units. Values greater than 4 units are lightly shaded, and greater than 6 units heavily shaded.

Initial state



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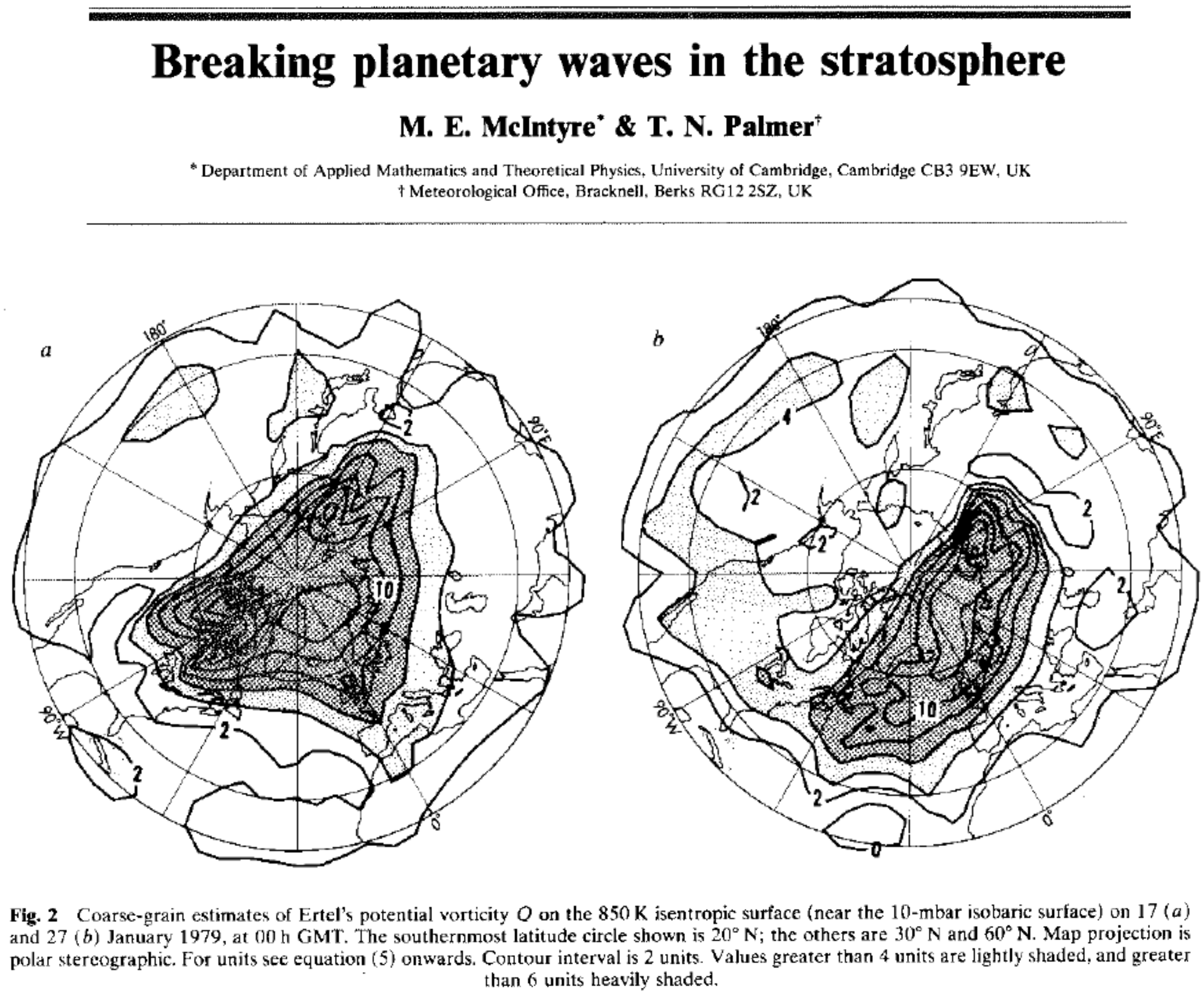

Breaking planetary waves in the stratosphere

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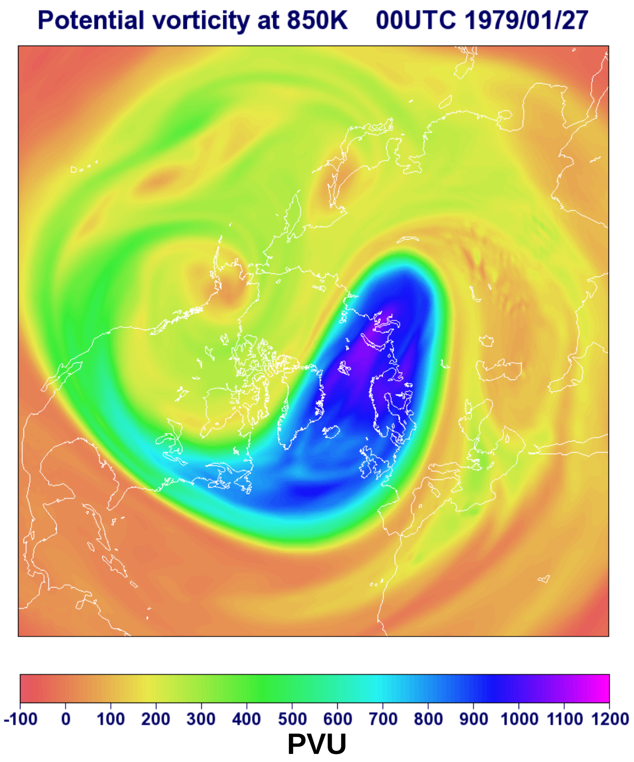

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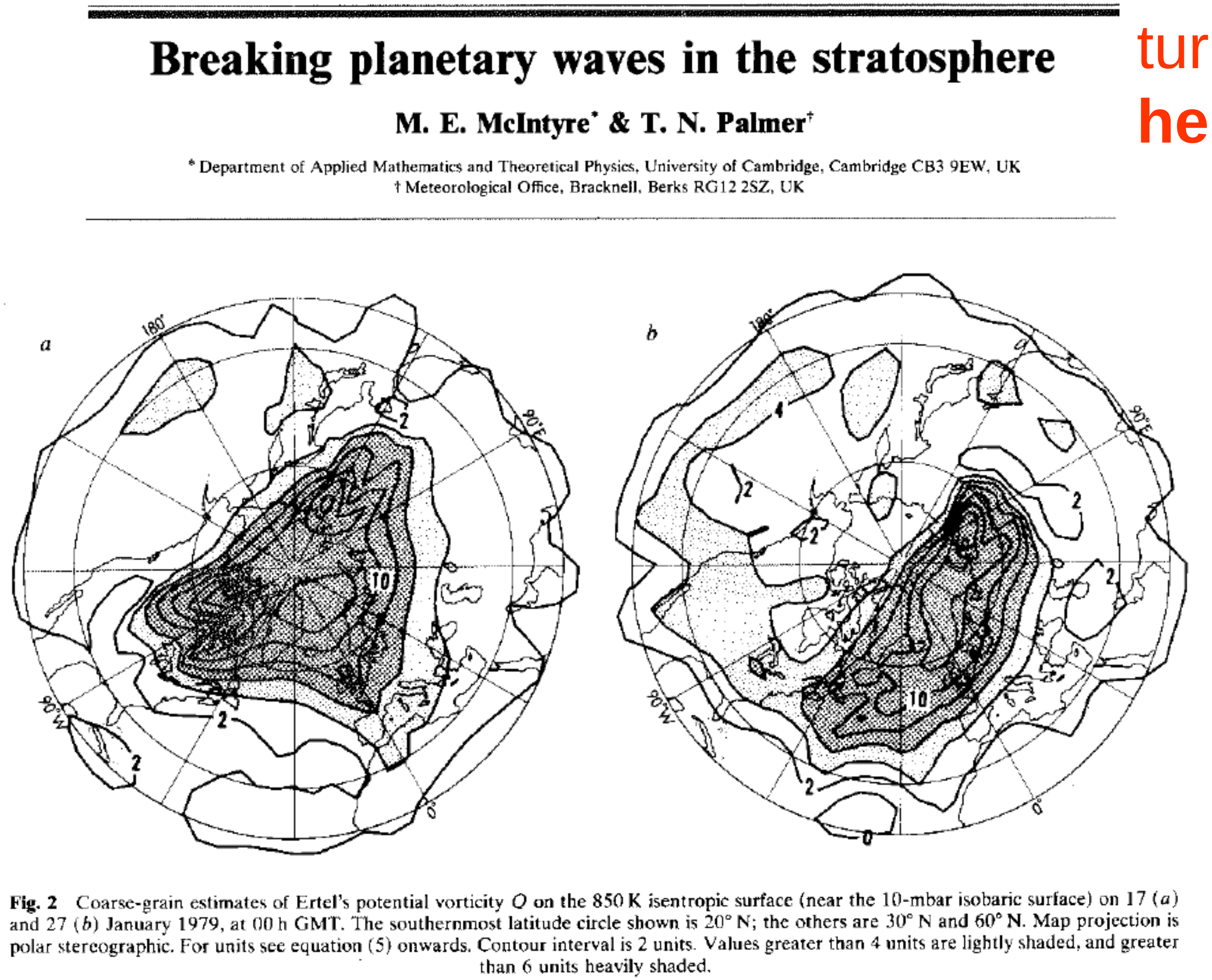

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turbulent
here

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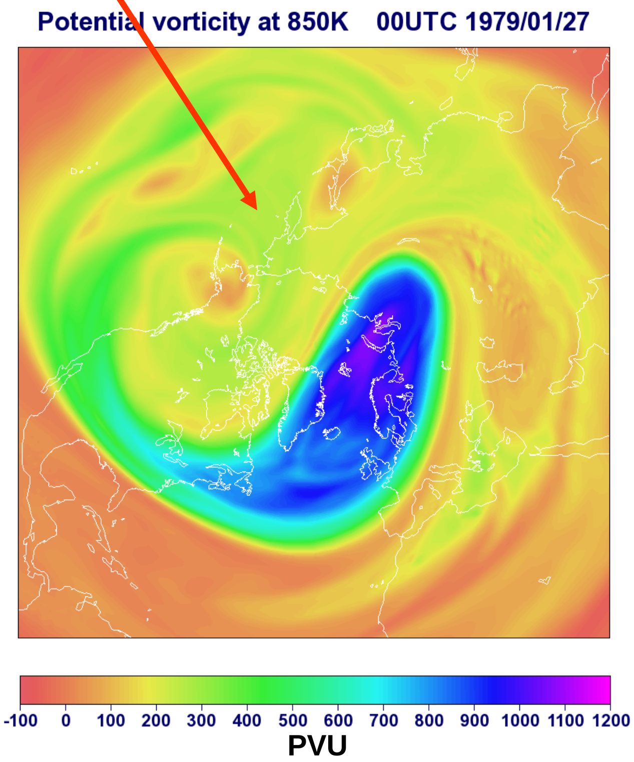

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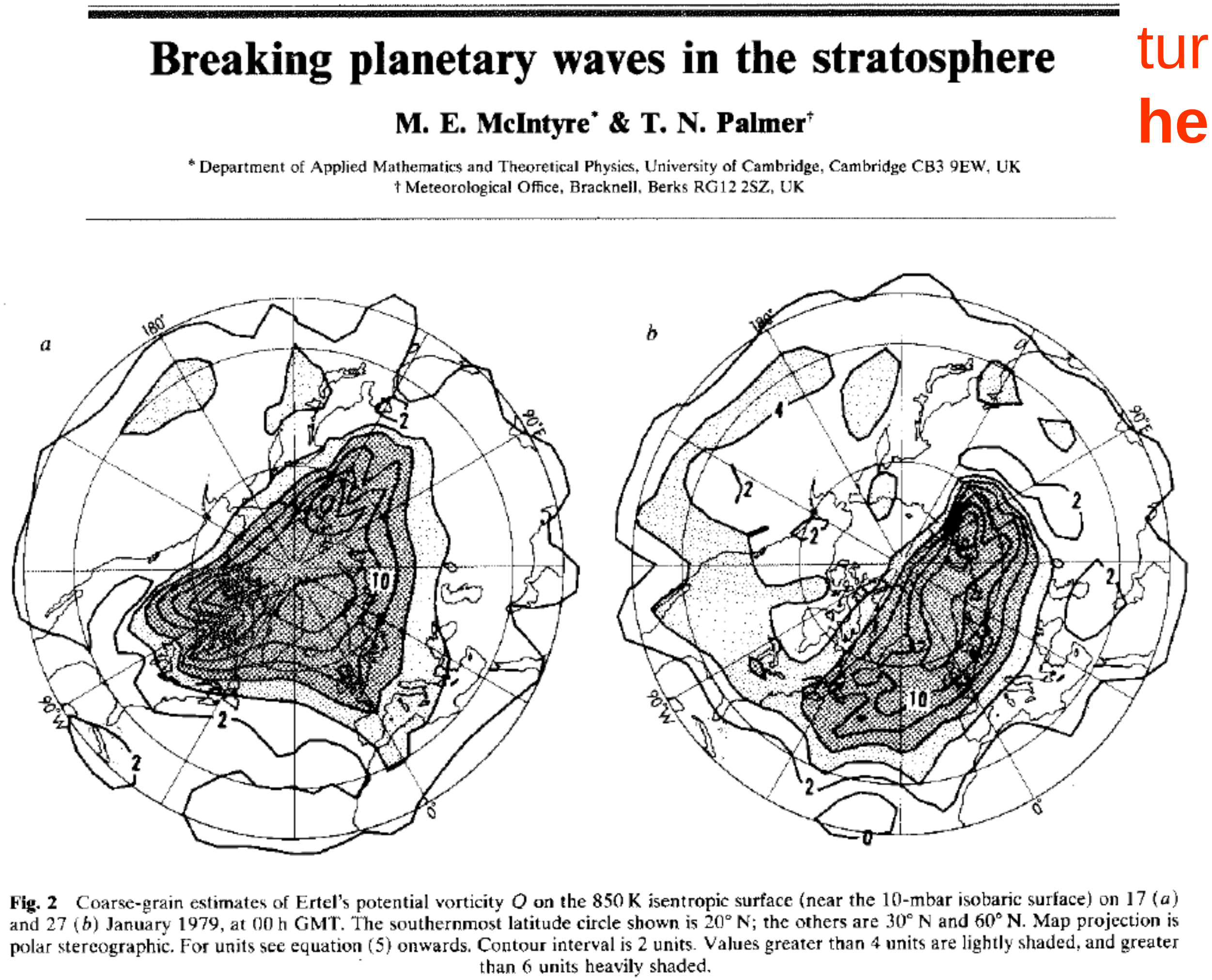

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turbulent
here

wavelike
here

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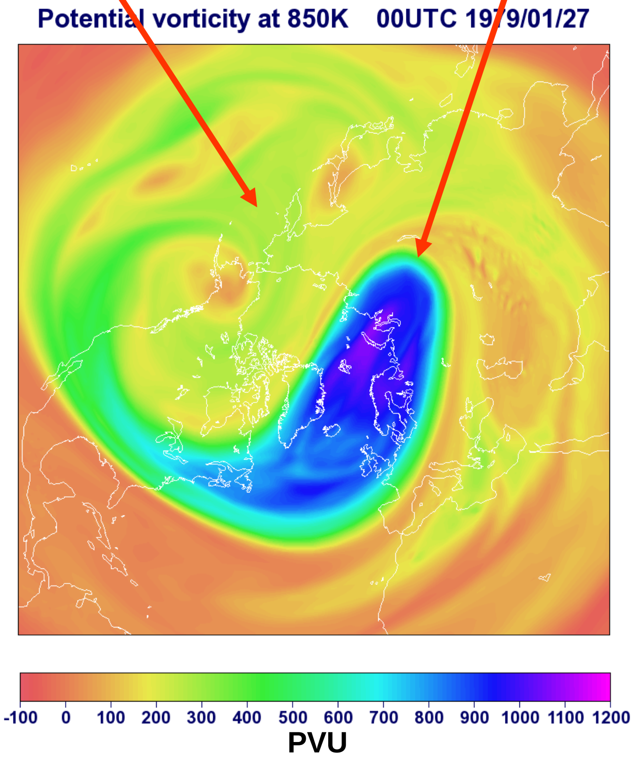

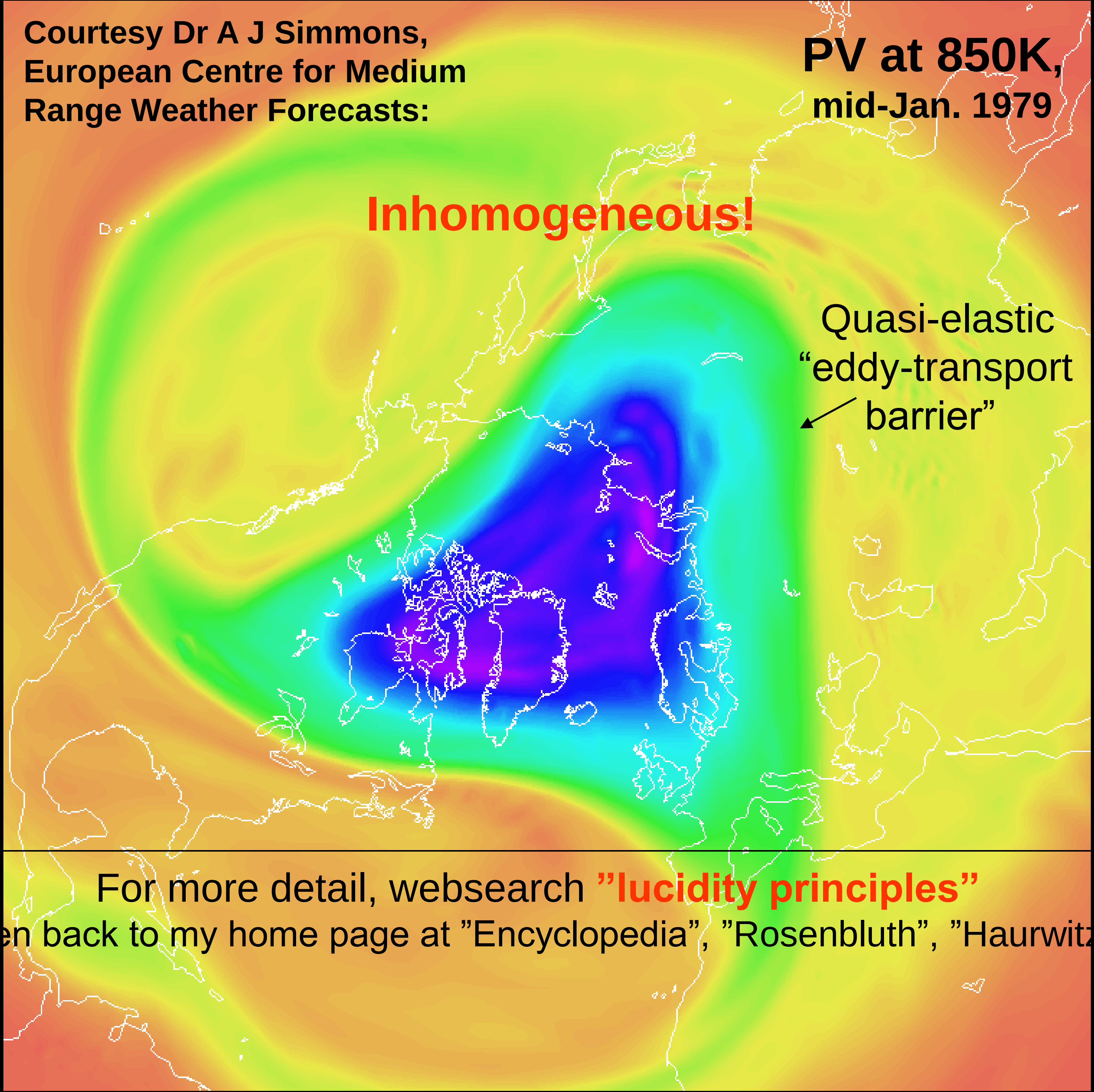

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PV at 850K,
mid-Jan. 1979

Inhomogeneous!

Quasi-elastic
“eddy-transport
barrier”

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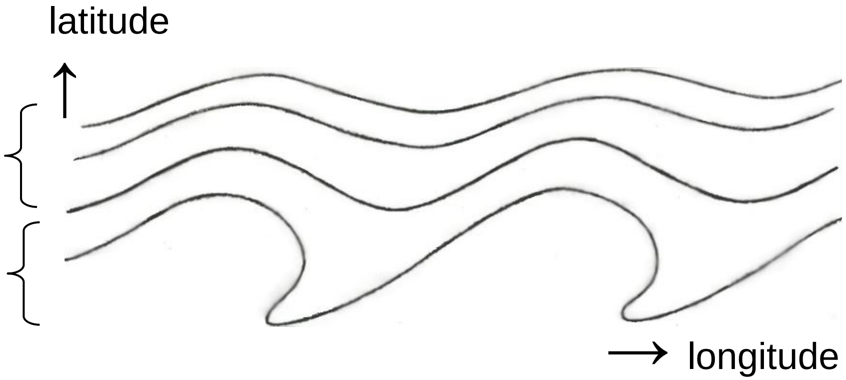

PV contours undulate reversibly:

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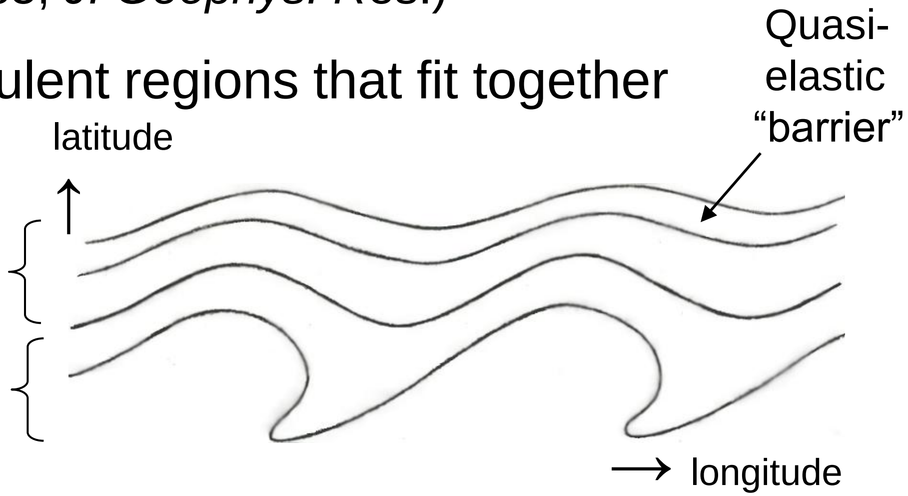

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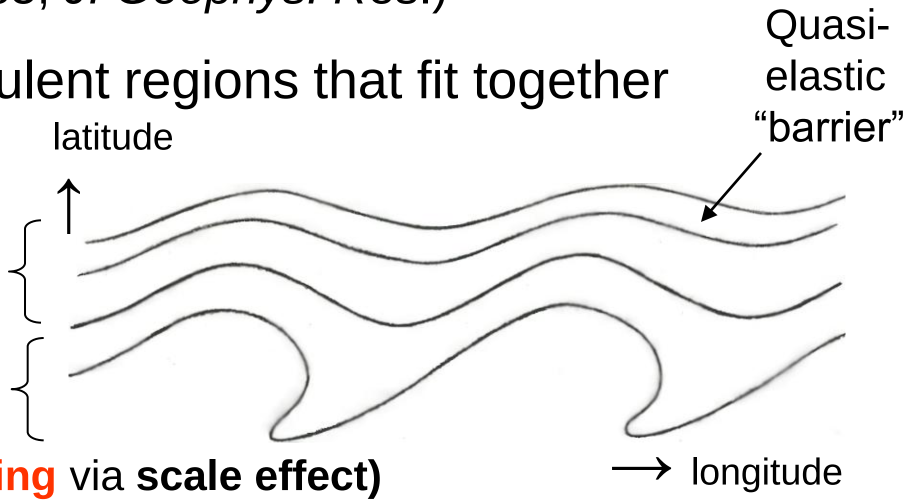

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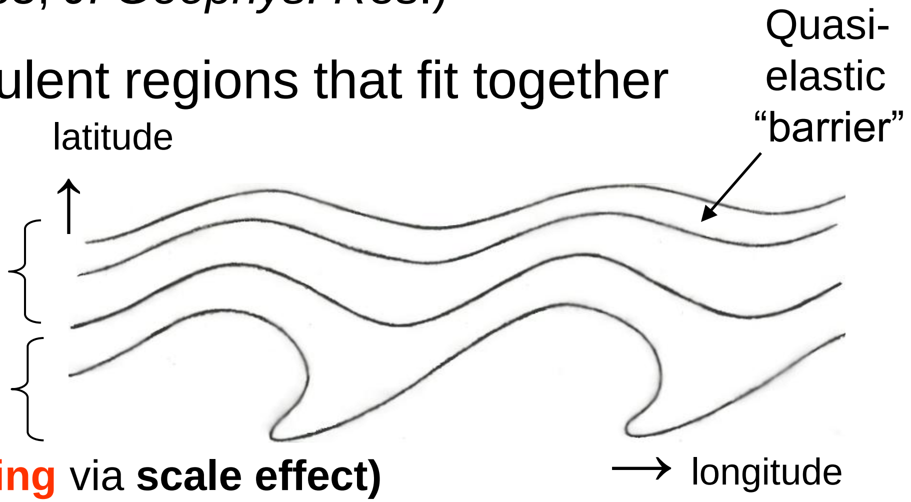

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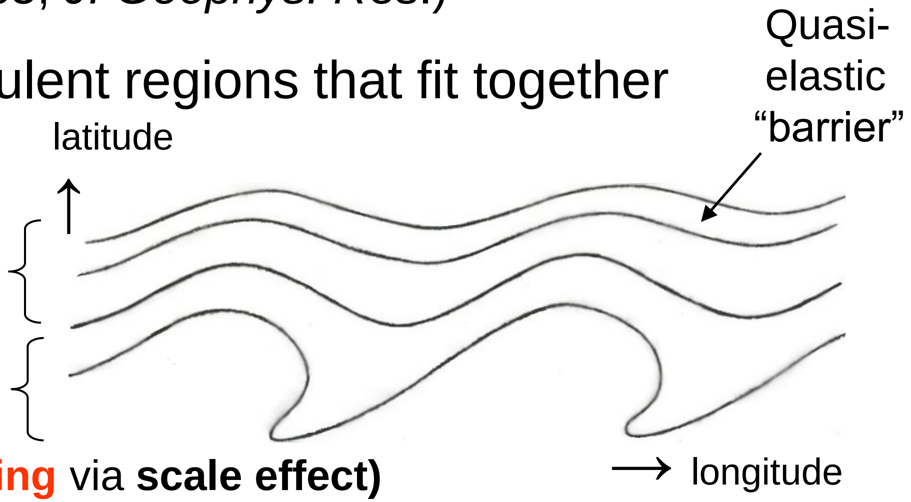

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The simplest **fully self-consistent** model of the structure is the **Stewartson-Warn-Warn (1978) nonlinear critical-layer theory.** And the essence of it was first recognized by Dickinson (1969).

The wave-turbulence jigsaw structure is often associated with **jet-self-sharpening scenarios**. (**Negative viscosity** or **anti-friction** again.)

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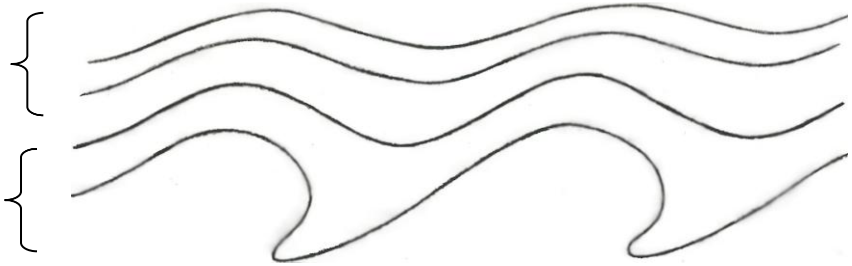

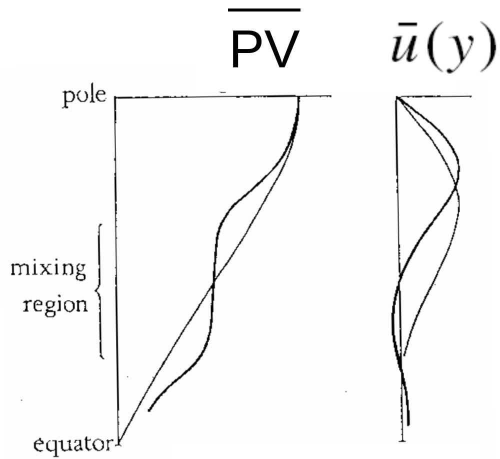

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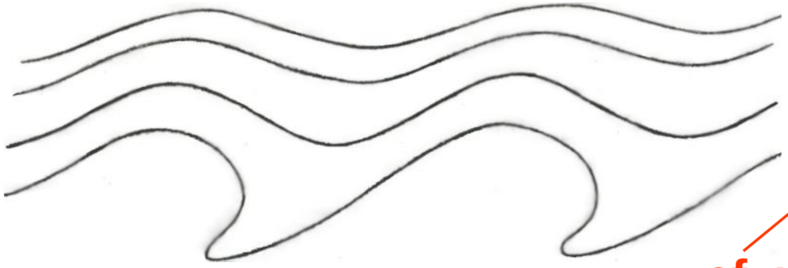

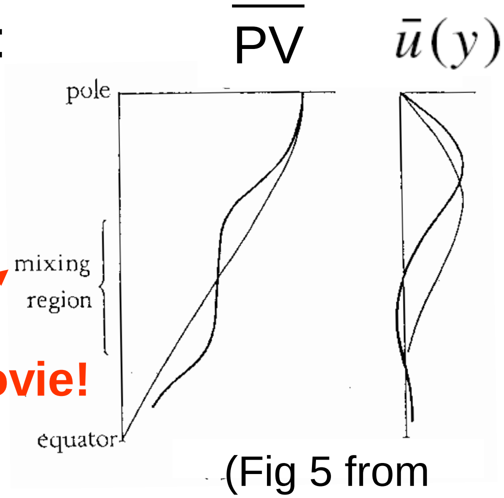

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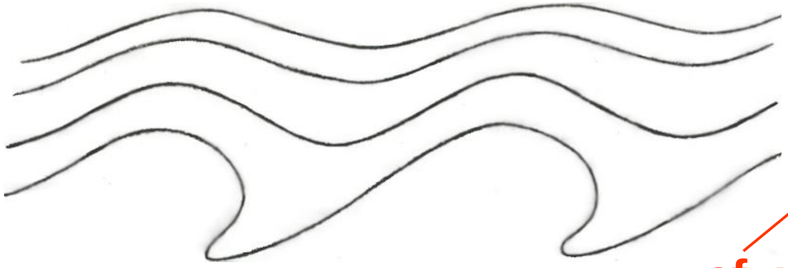

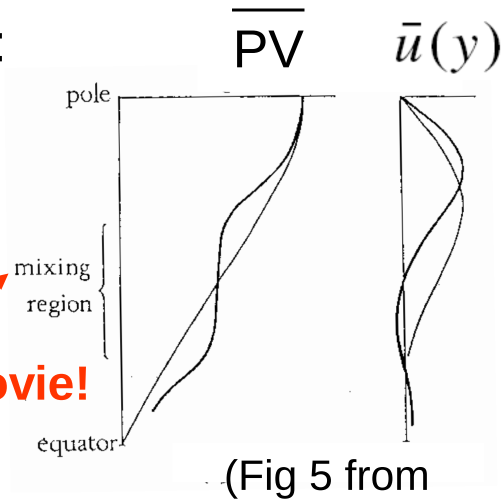

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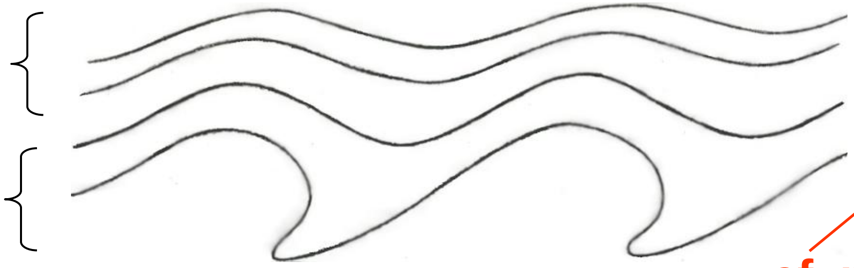

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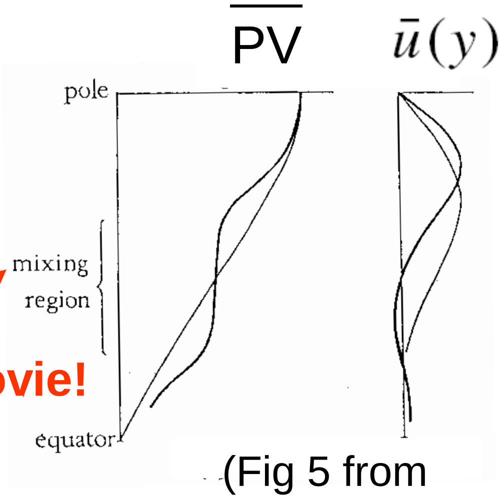

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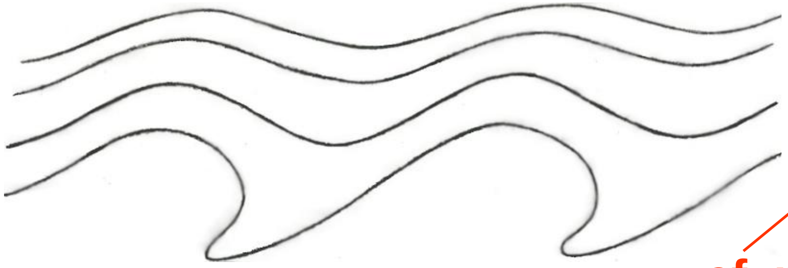

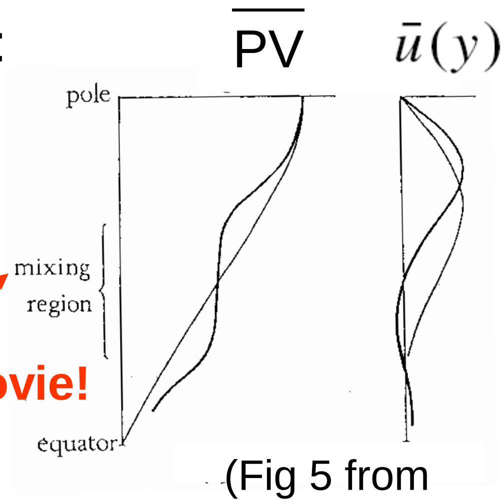

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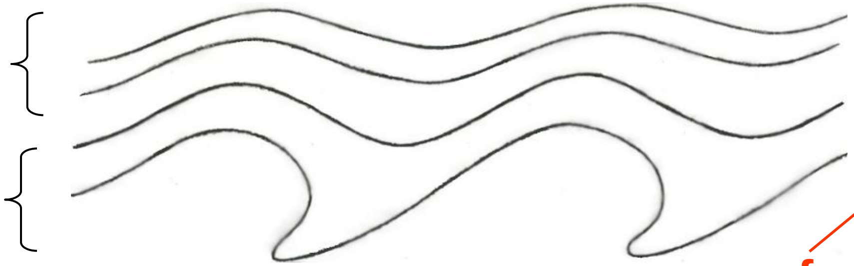

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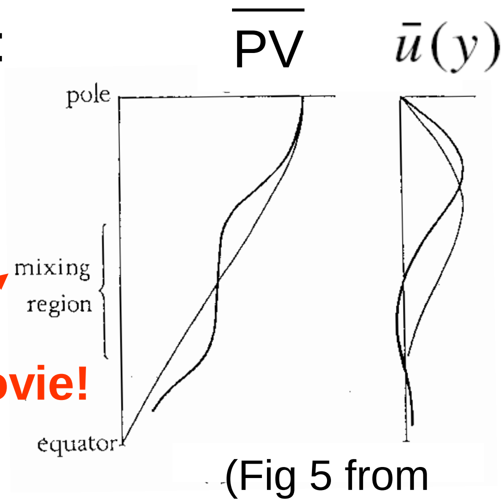

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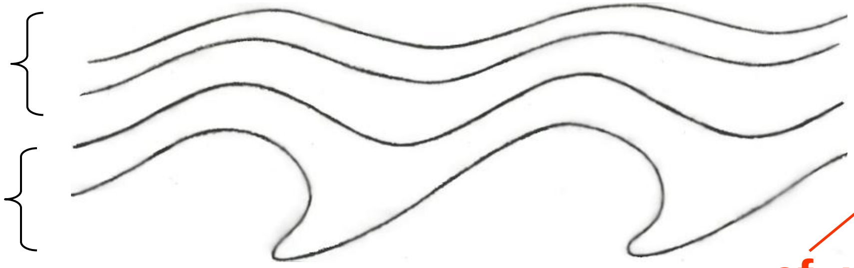

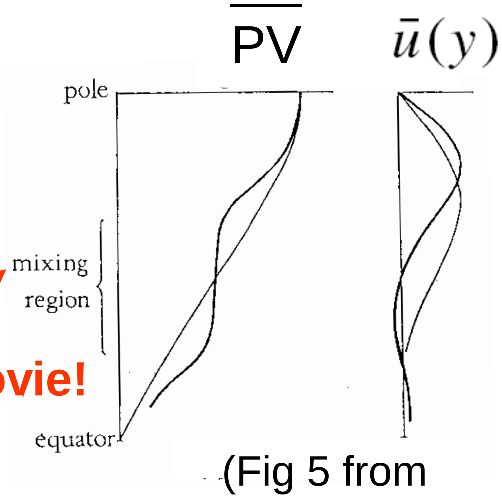

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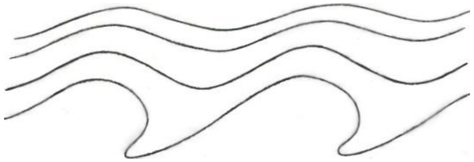

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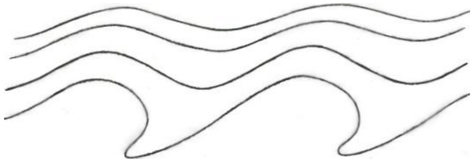

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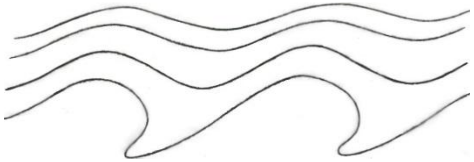

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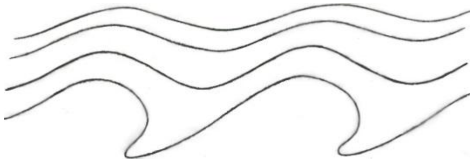

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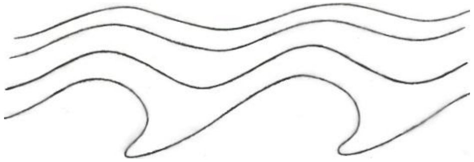

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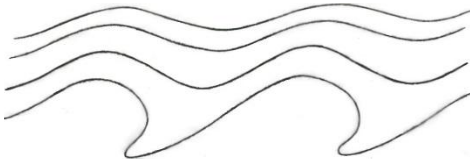

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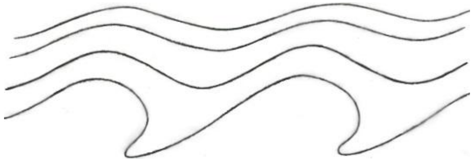

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O. M. Phillips (1972 *Deep Sea Res*).

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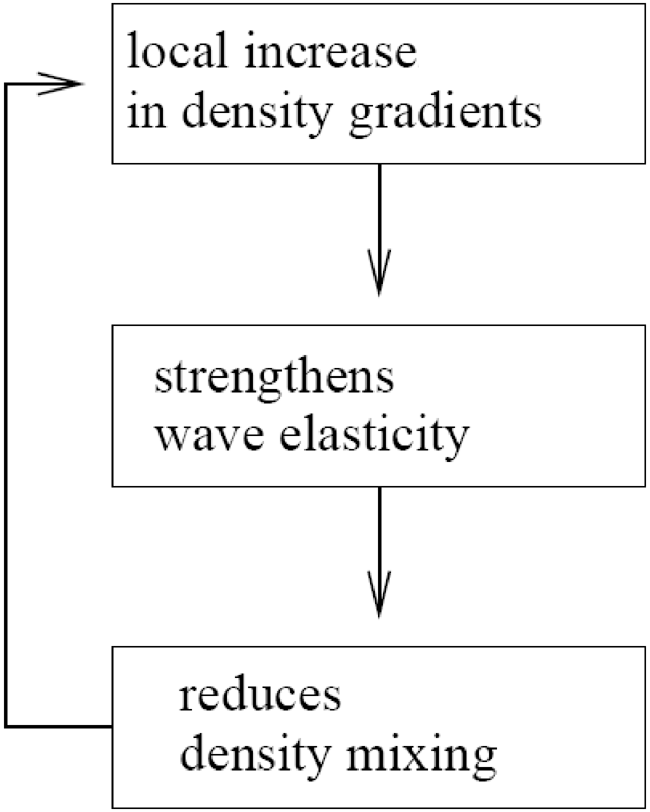

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“staircase” on my home page

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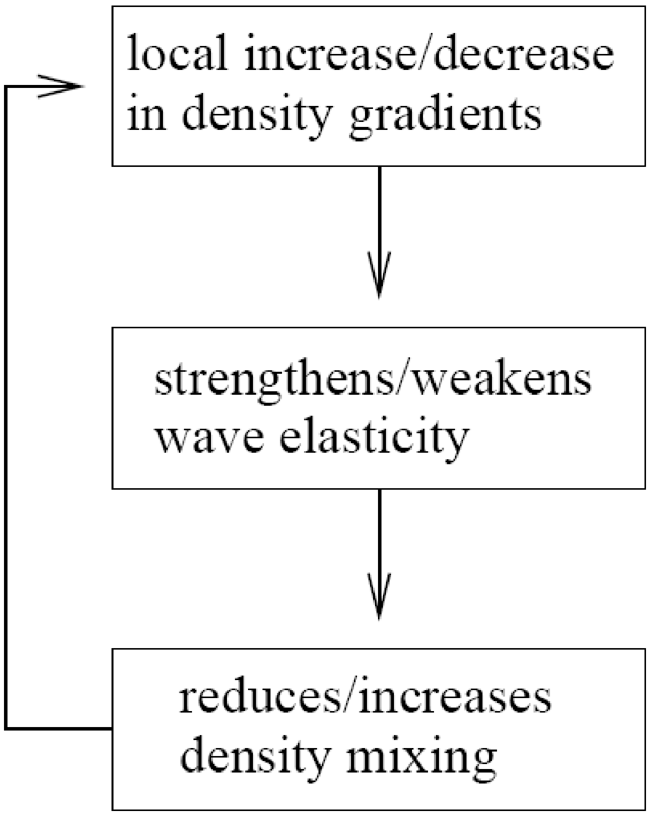

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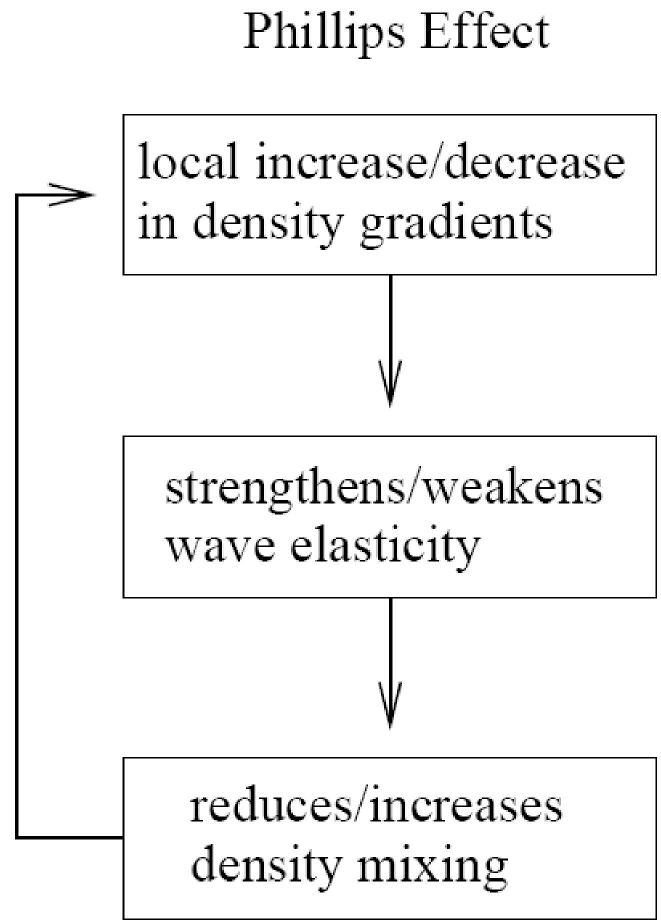

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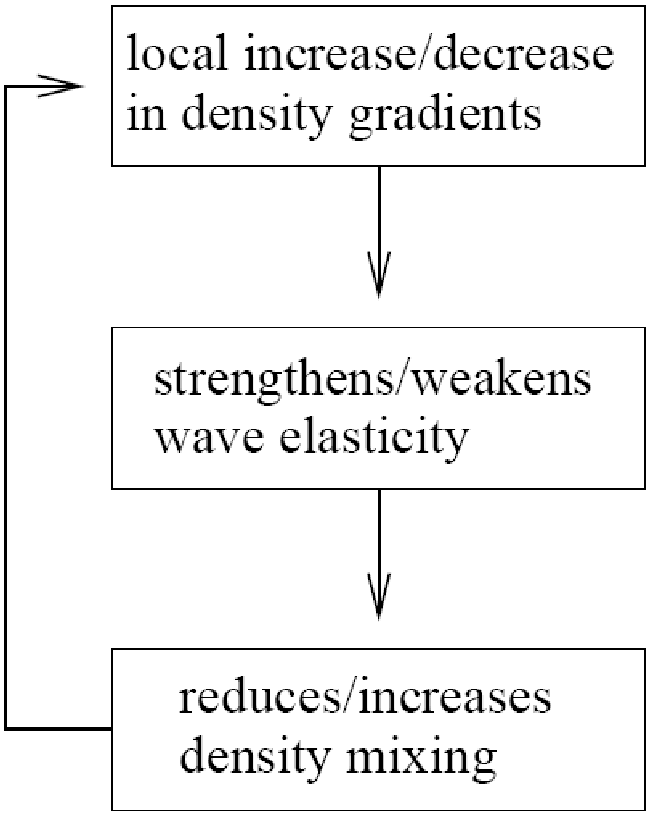

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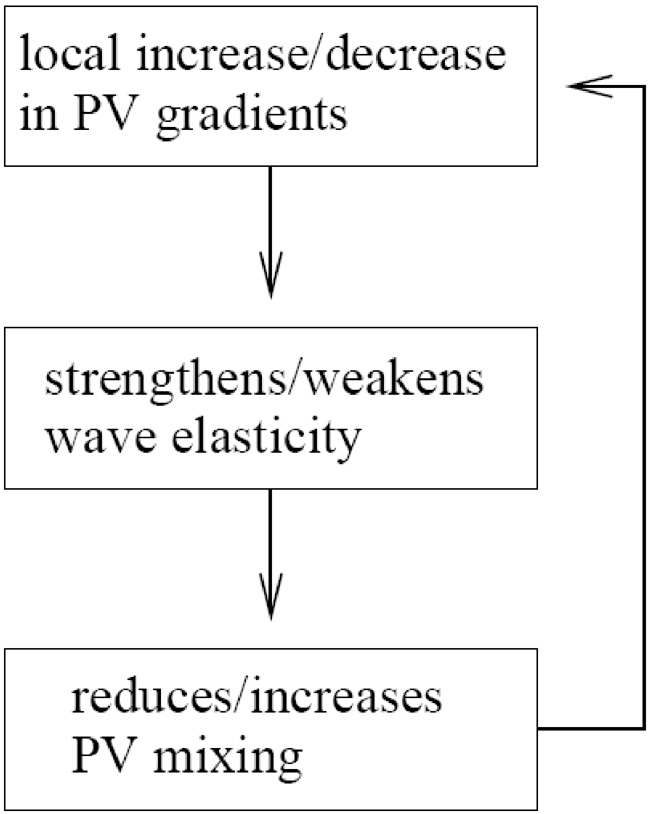

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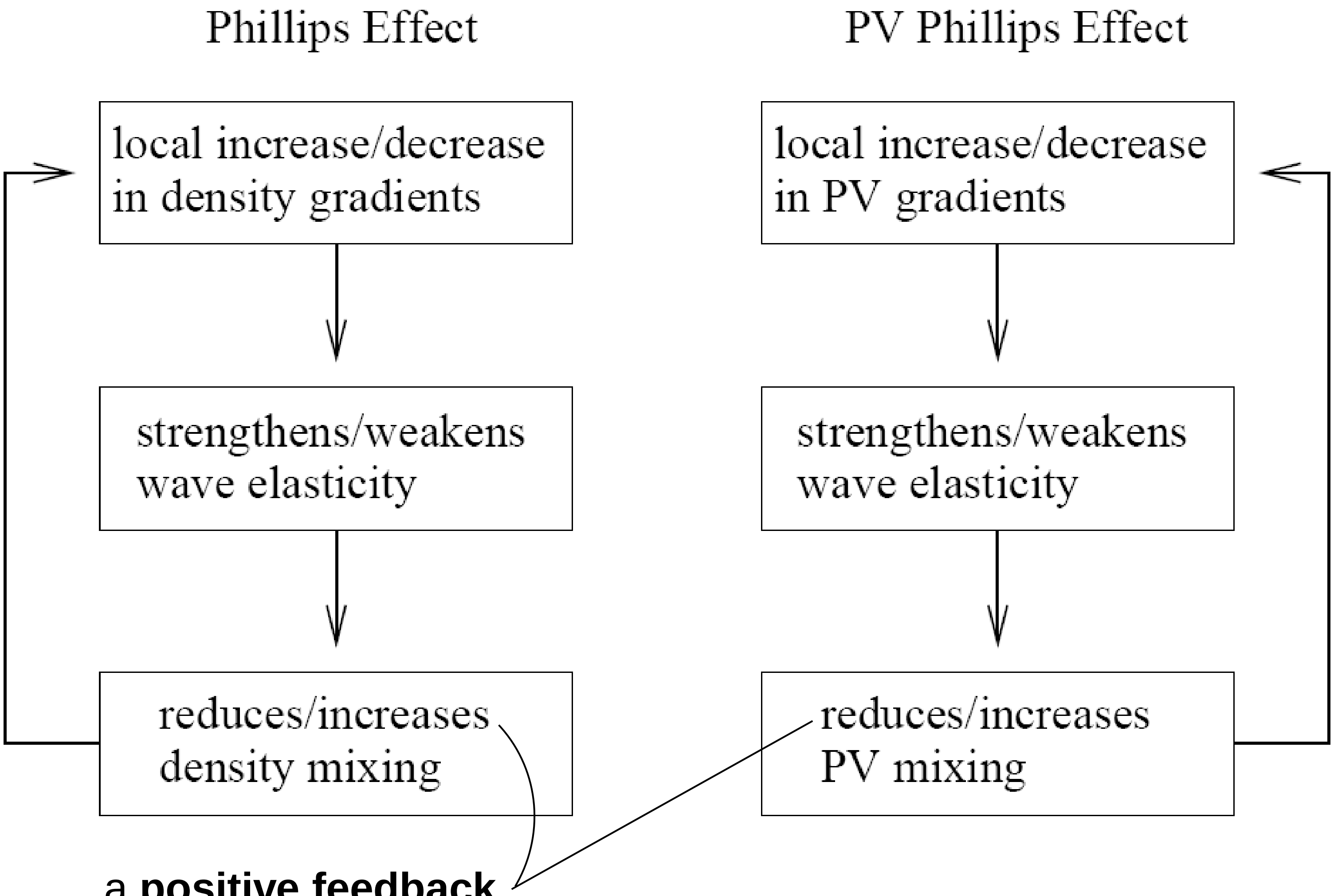

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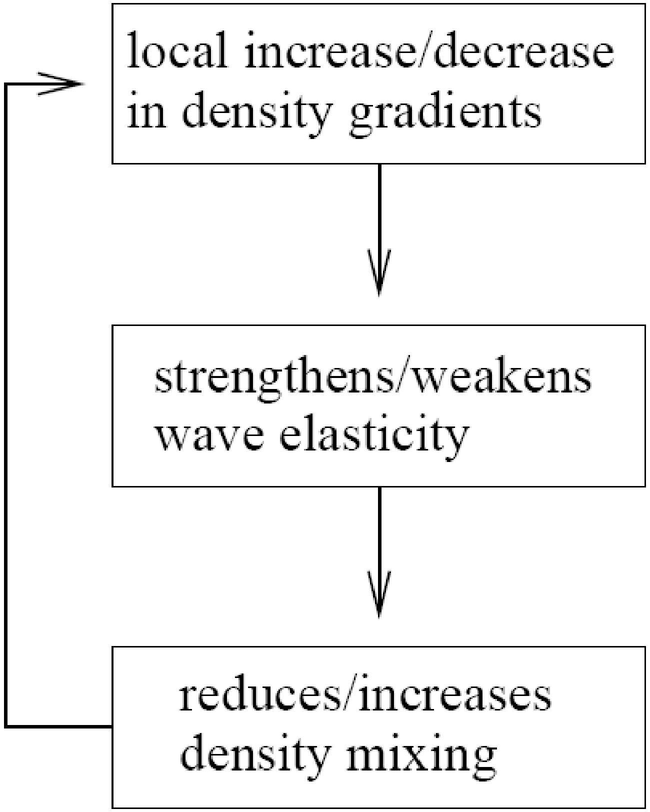

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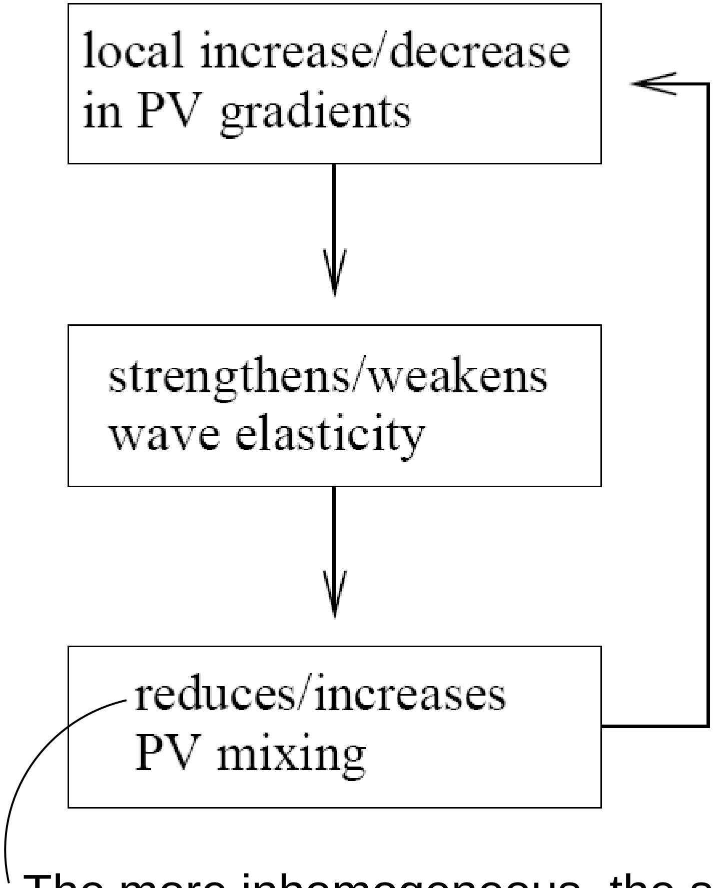

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The more inhomogeneous, the stronger the feedback, bringing in the **shear** effects (Juckes & M, *Nature* 1987).

"staircase" on my home page

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Simplest quasi-geostrophic **strong-jet model** (Rosenbluth Lecture & refs.)**.**
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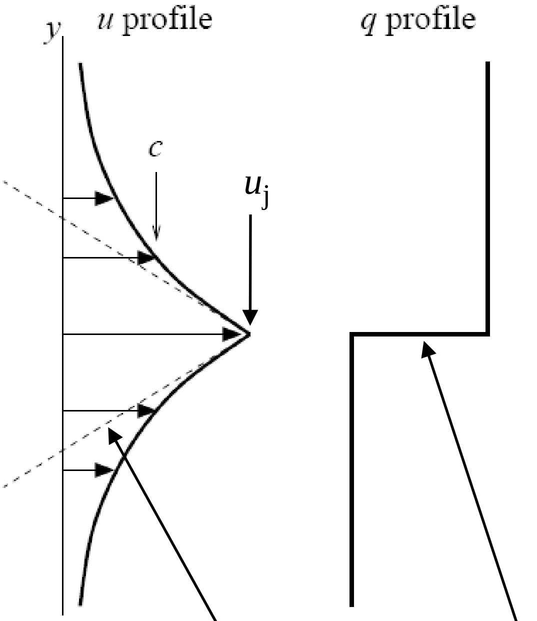

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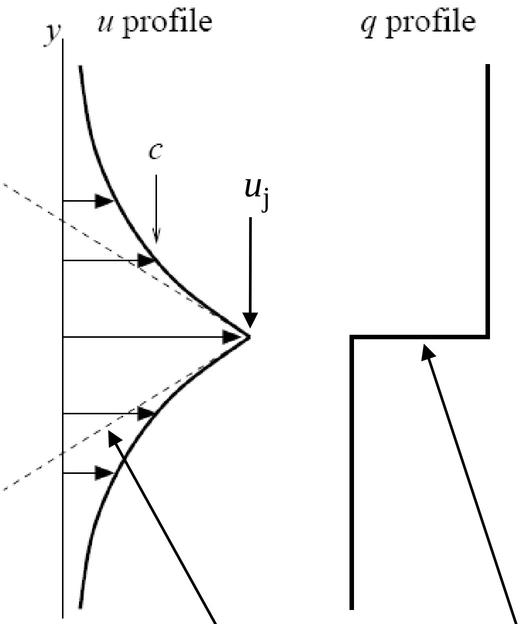

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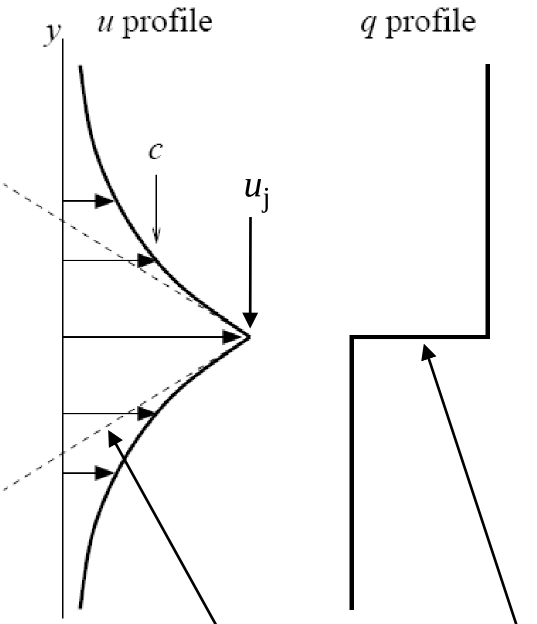

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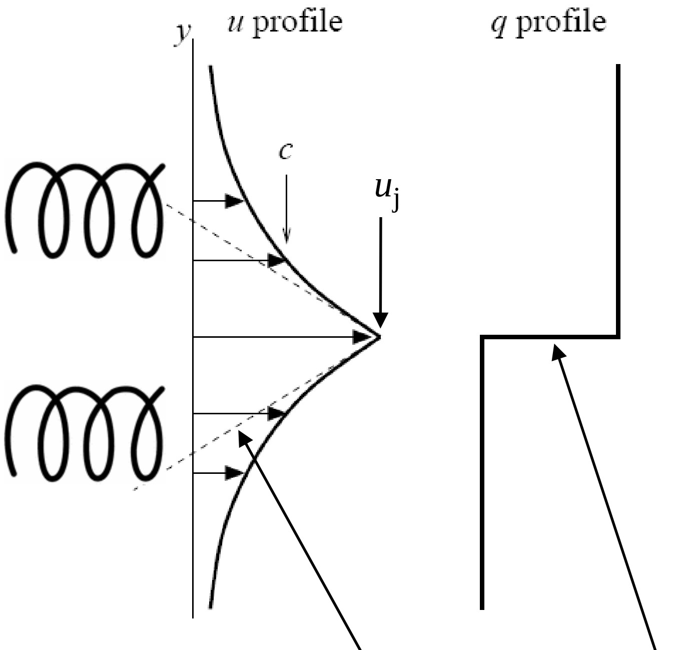

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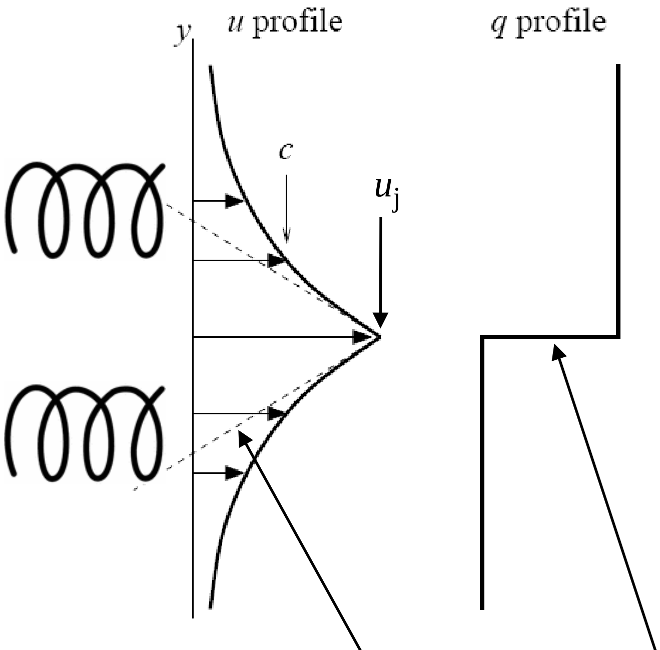

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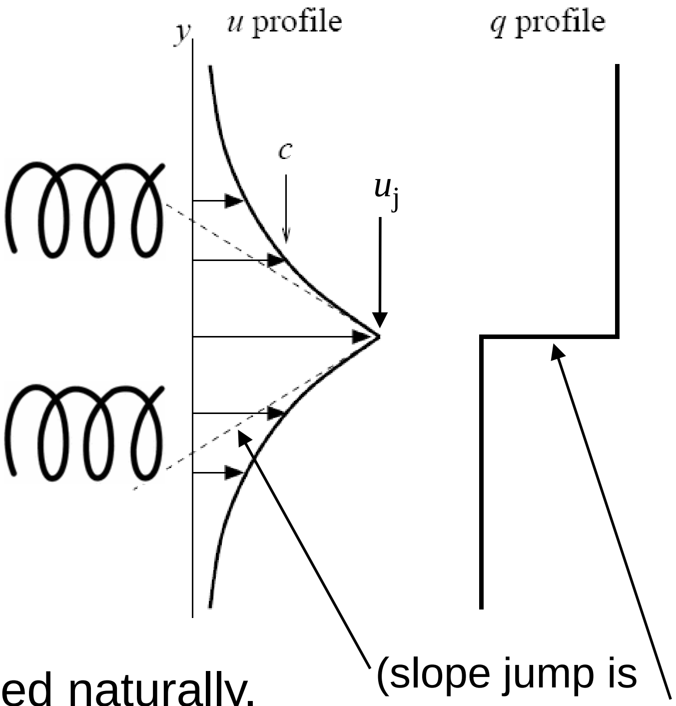

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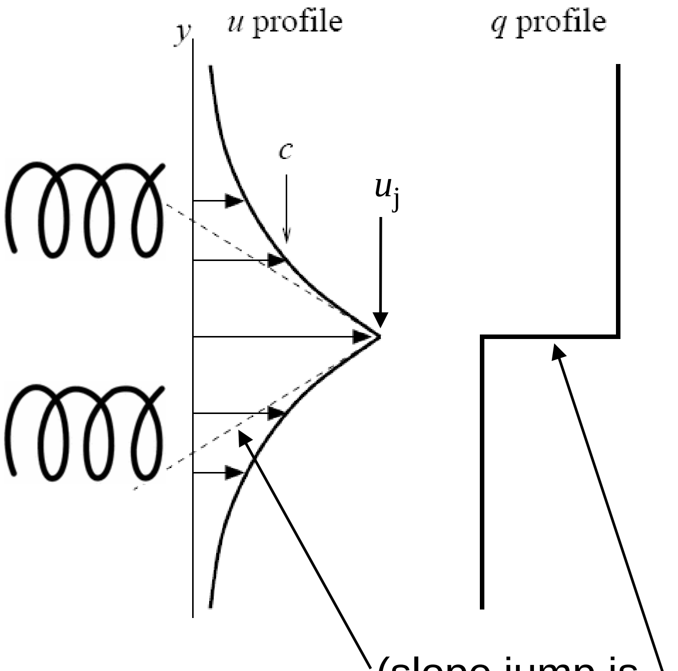

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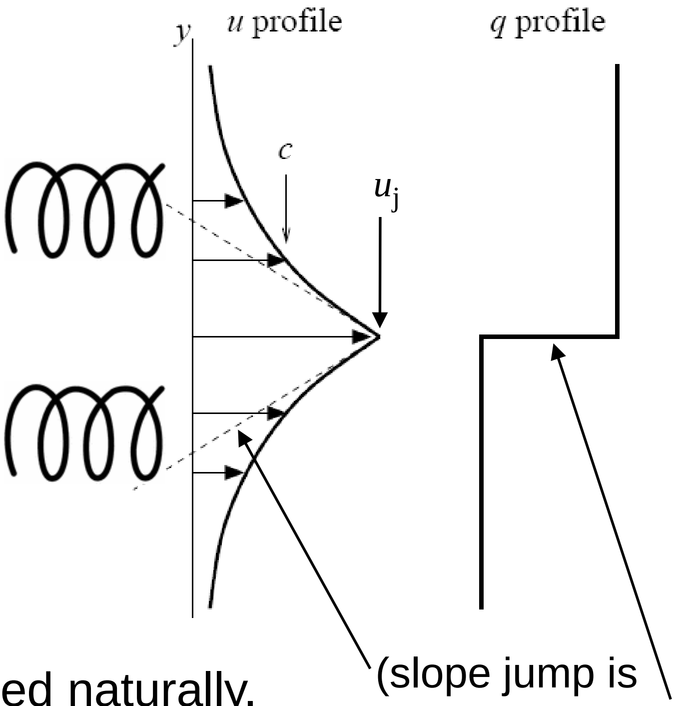

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Strong support comes from numerical and laboratory experiments, e.g., Scott and Dritschel (2012, *J. Fluid Mech.* **711**, 576) – **forced-dissipative** but clarifying how to reach the **more “natural”** low-excitation, low-dissipation parameter regimes.

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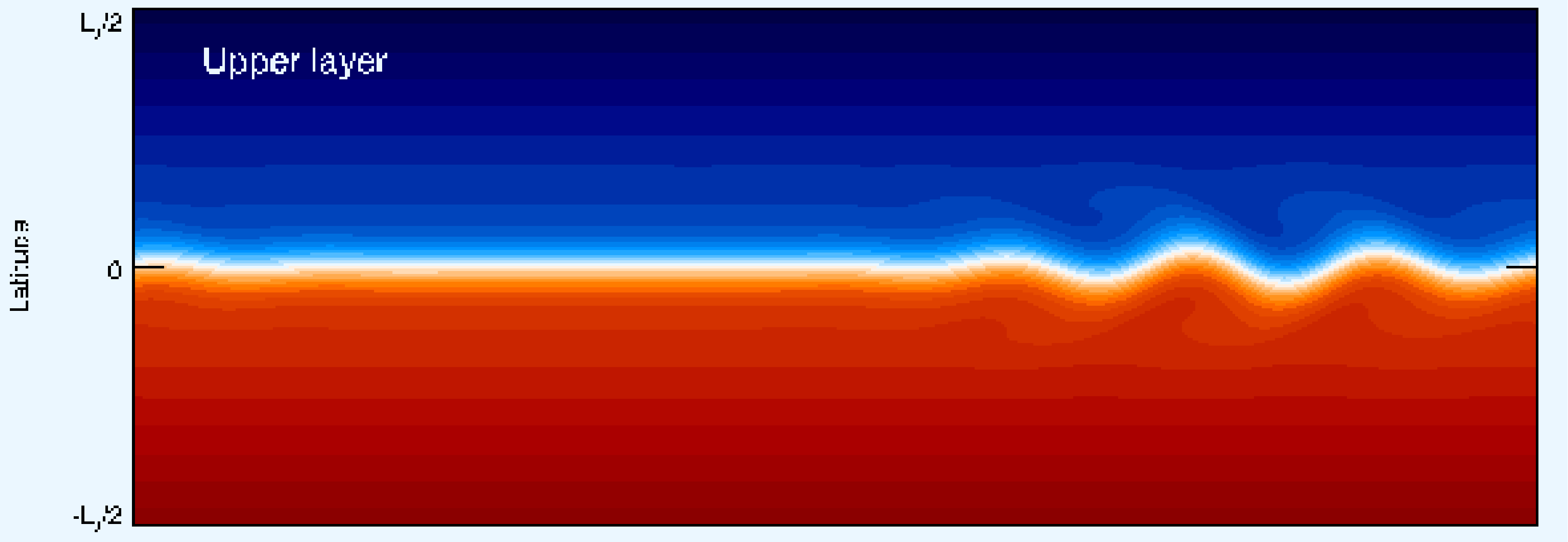
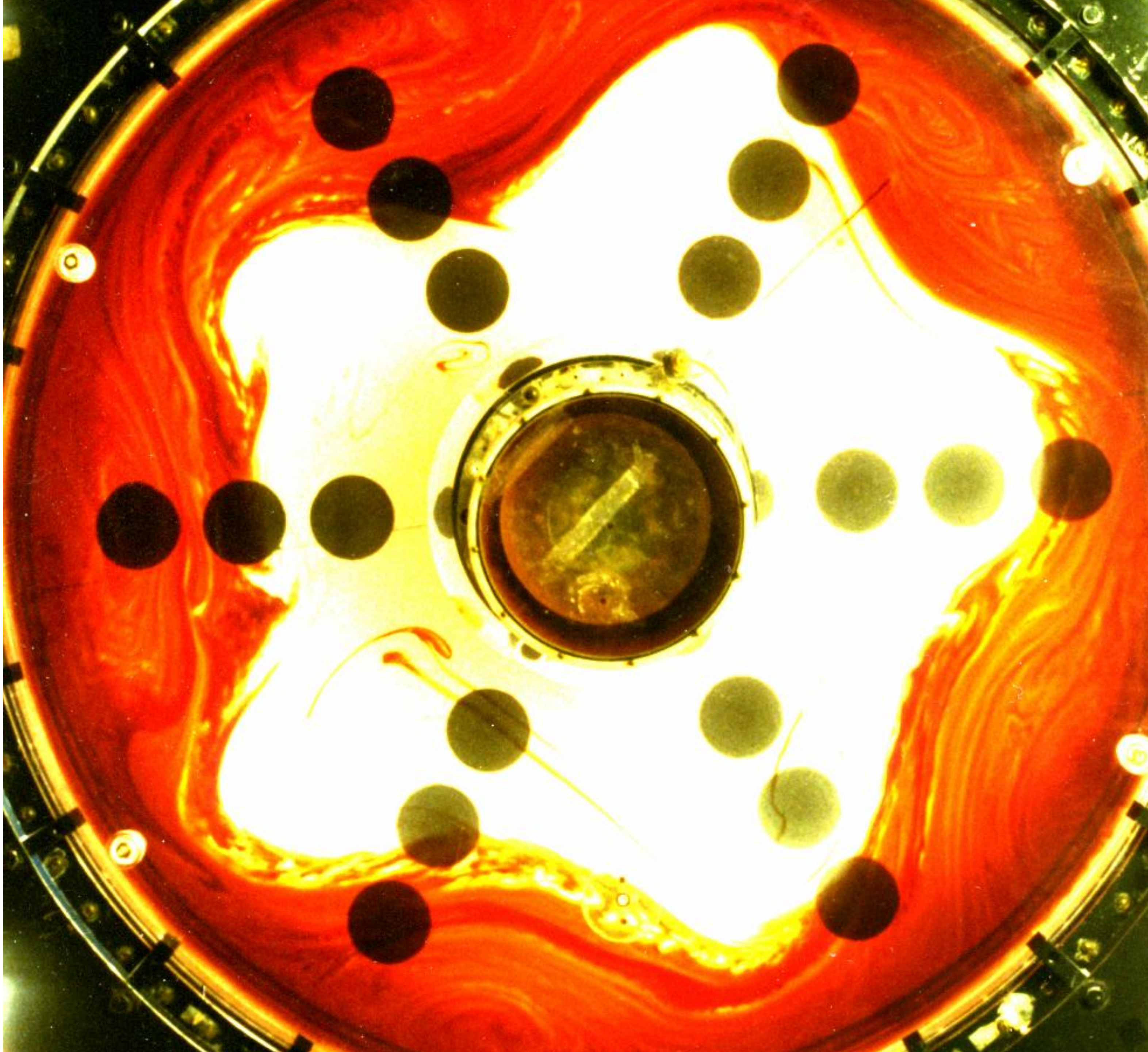
Here the Taylor identity is satisfied via a **form stress** exerted from below:



Next is a classic lab experiment conveying the same message:

Sommeria, Myers, and Swinney,
Nature 1989
86.4 cm dia.;
3 revs/sec (!)



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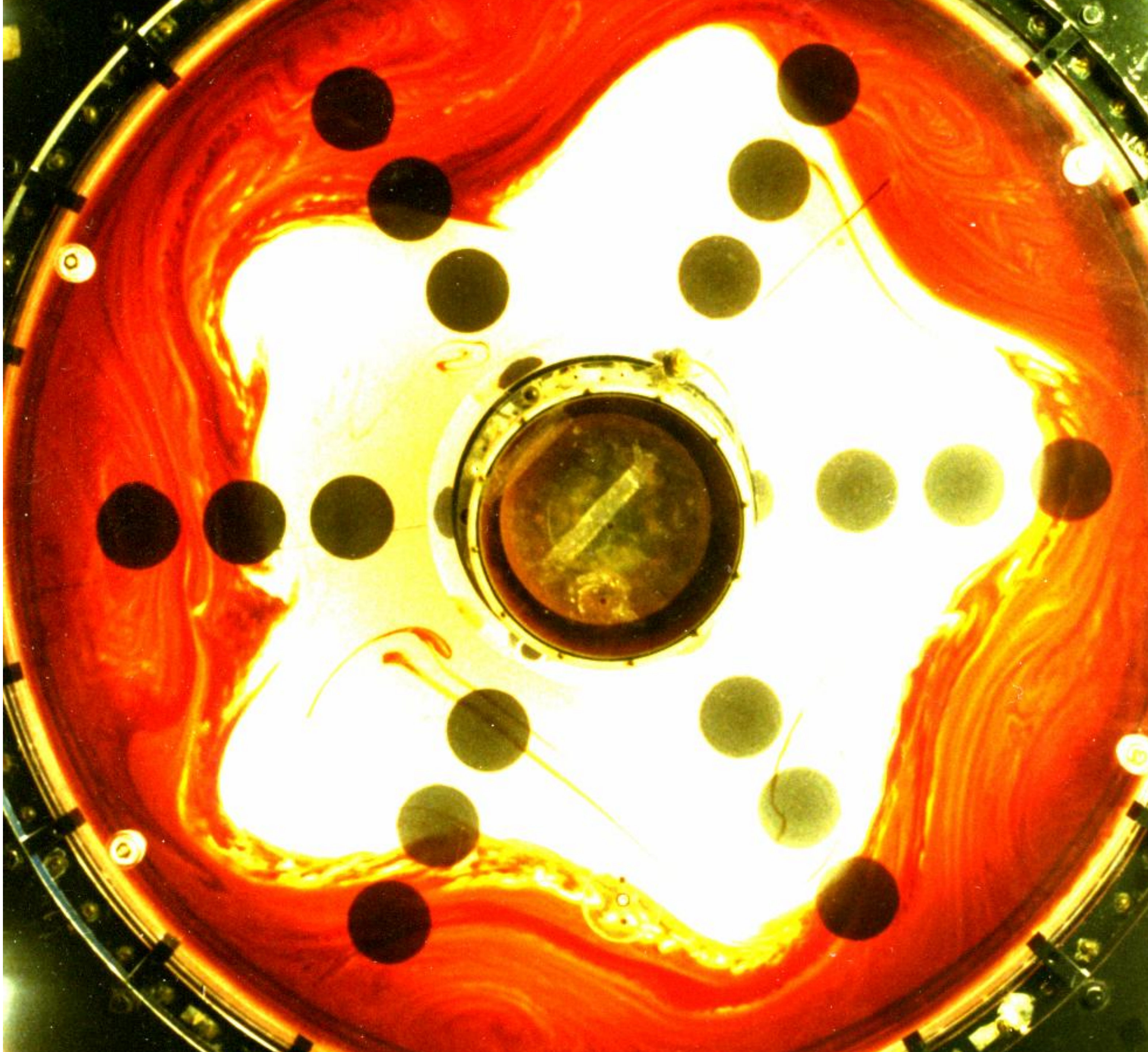

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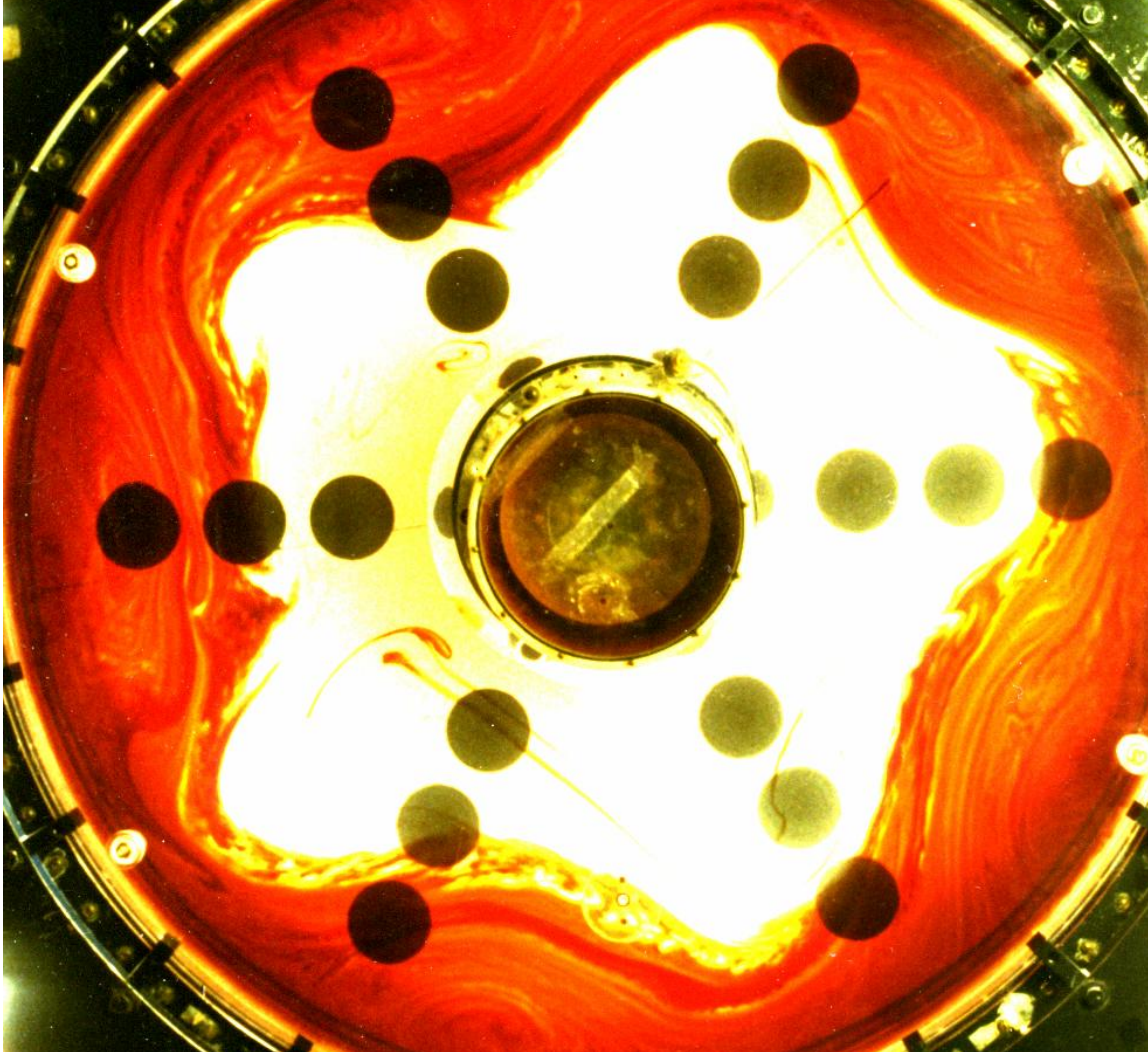

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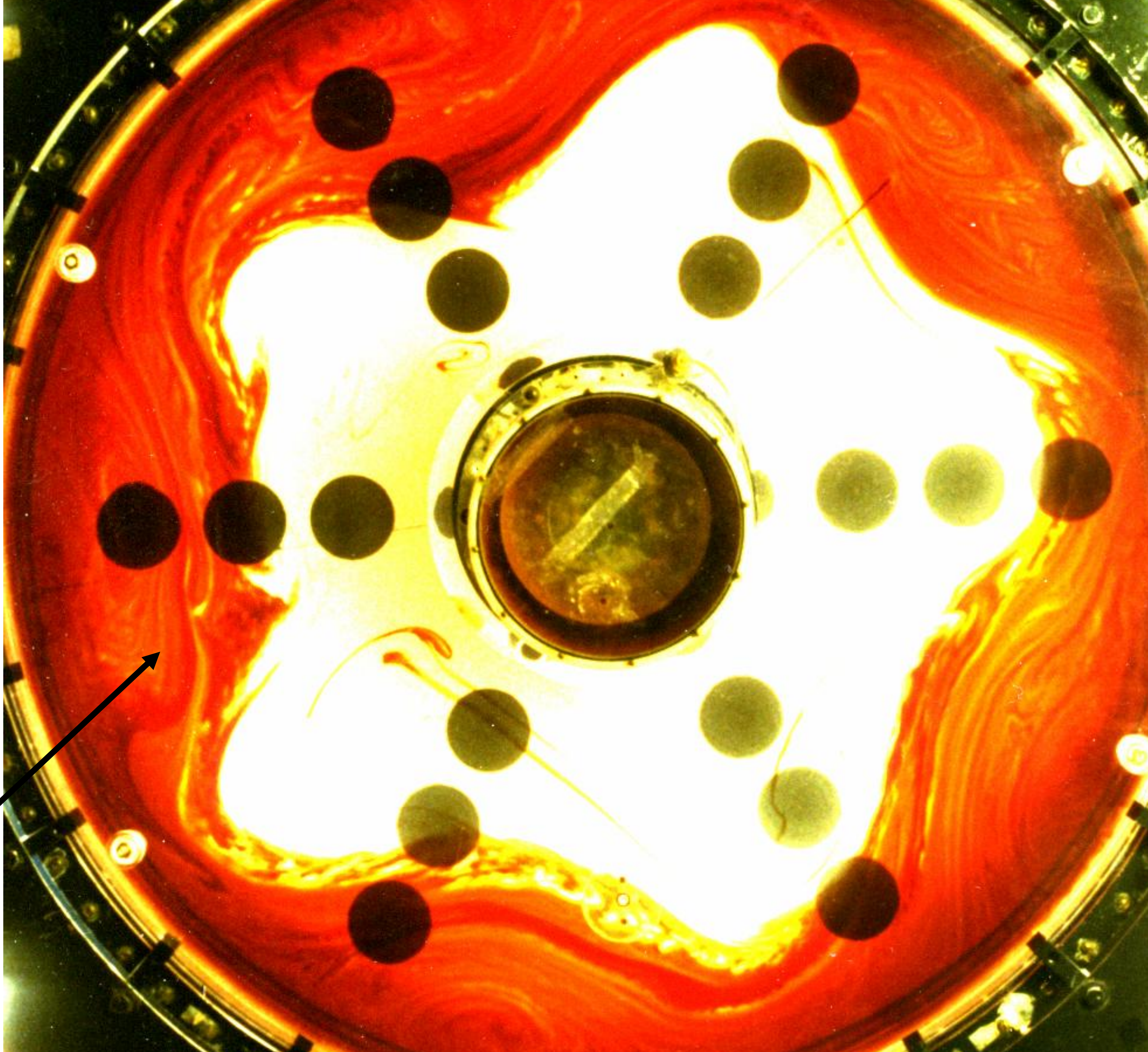

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Dye put in **> 500 revs.** earlier.

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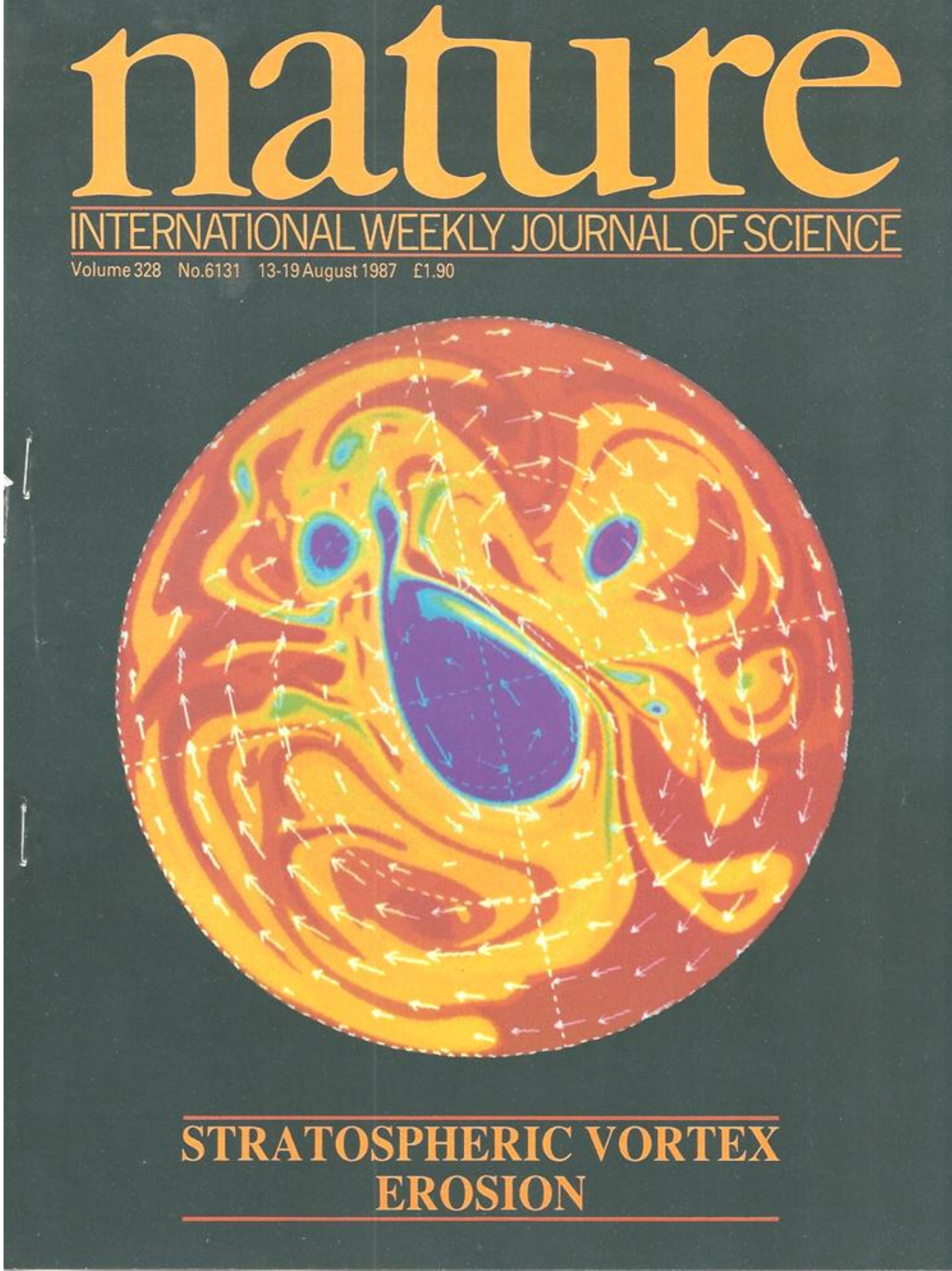

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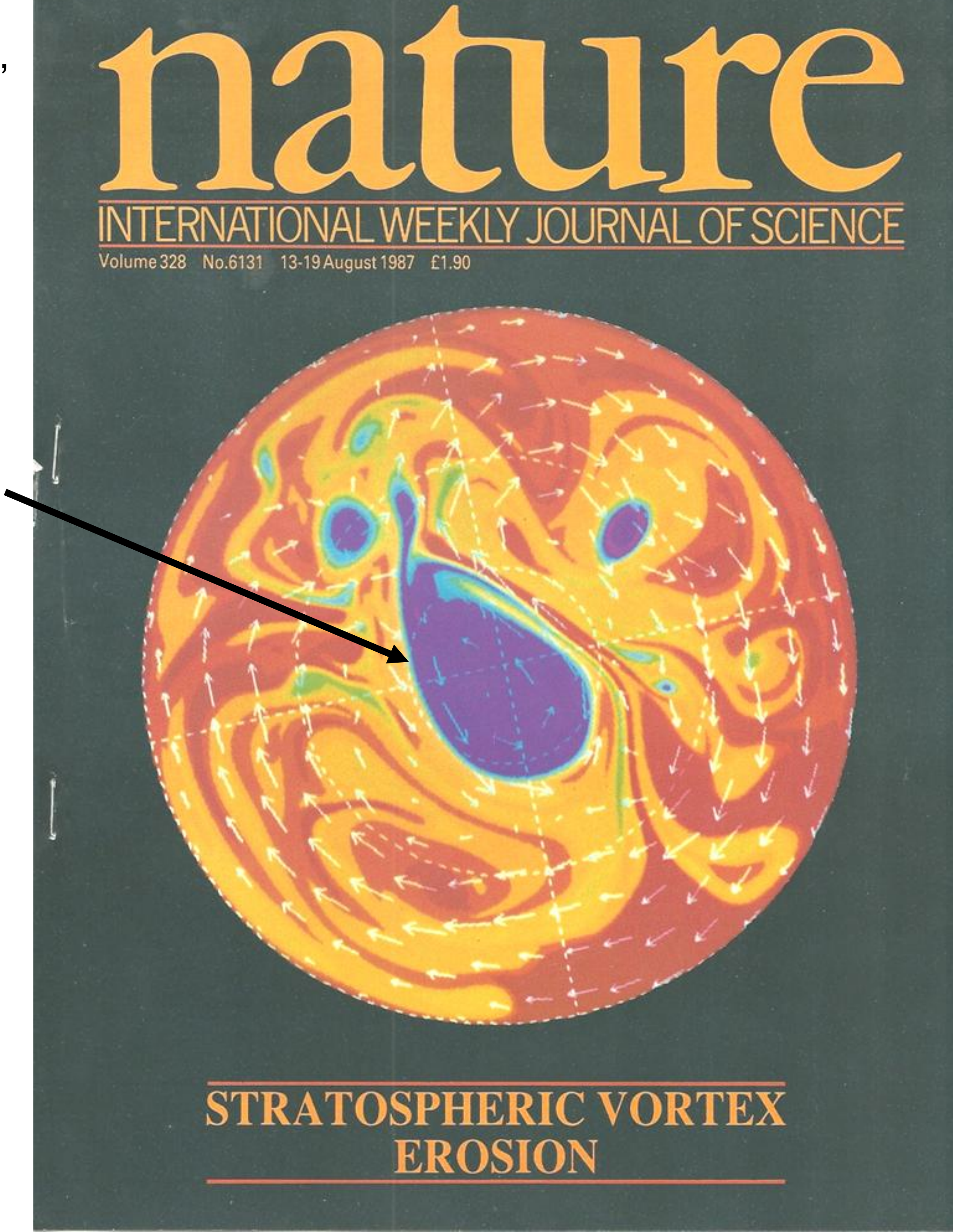

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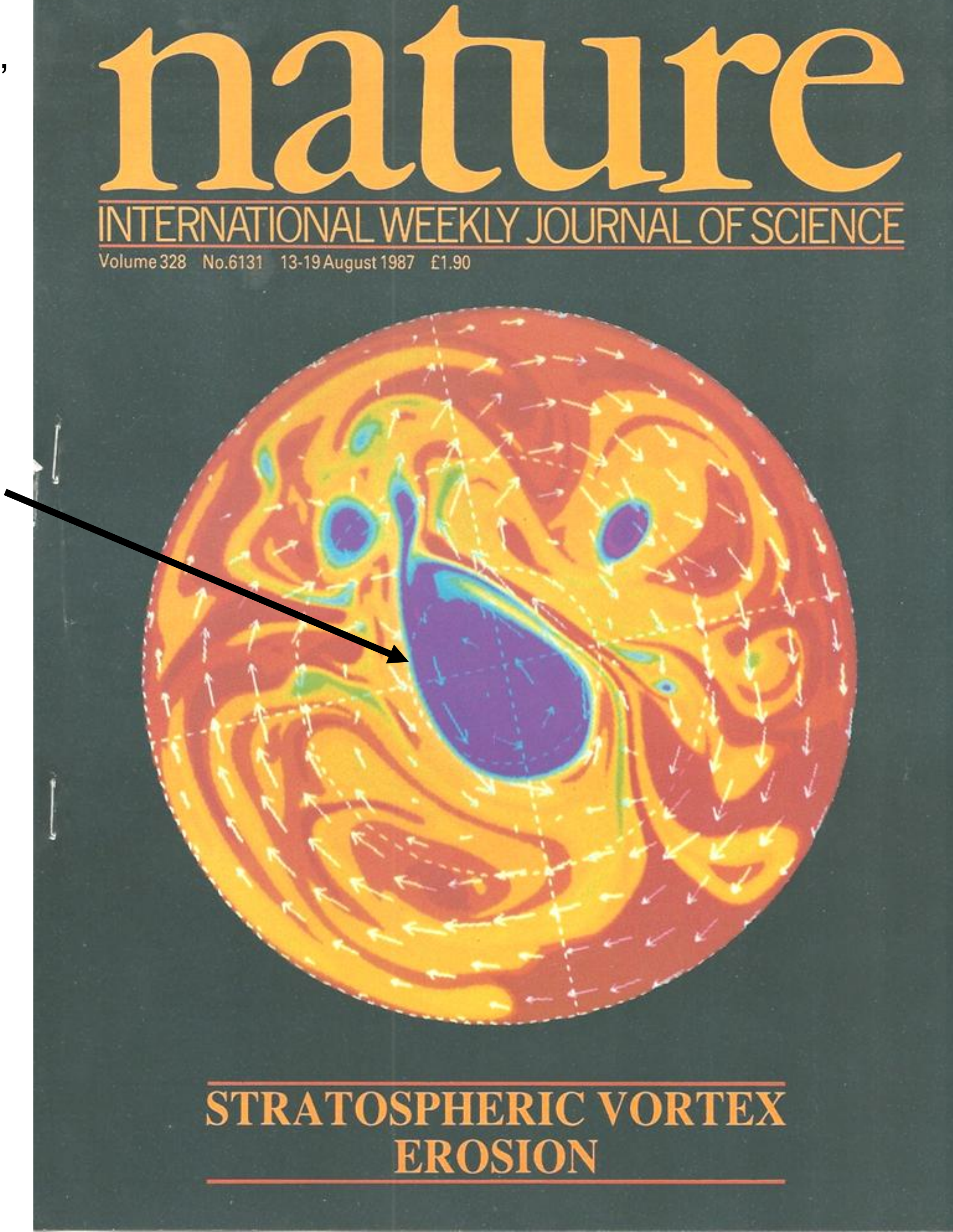

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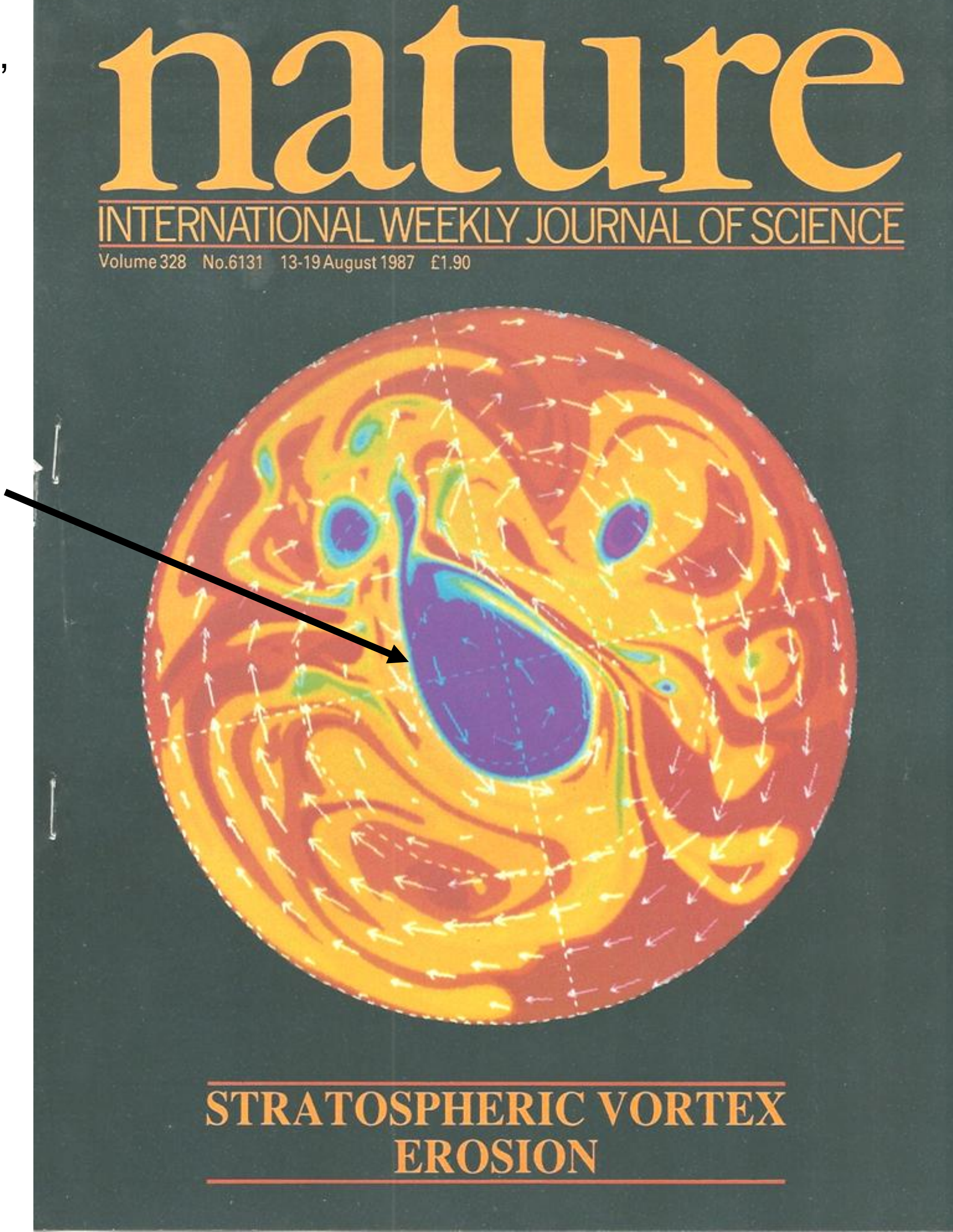
Many other examples (e.g. nice observational work in Huw Davies’ group. So here’s the bottom line:



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The paradigm is ***not*,** of course, the Answer to Everything:

- PV **doesn’t** always mix (e.g., **vortex merging**)
- **Not all** jets are strong jets (e.g., **“ghost jets”** in the Pacific Ocean)
- And there are, of course, **other nonlinear mechanisms** that show up in different thought-experiments:

Jet dynamics – a **complex conceptual landscape** wth a 2-level hierarchy of ideas:

1. Generic ideas:

PV Phillips effect
PV invertibility

Taylor identity:
$\overline{v'q'} =$ – div (eddy momentum flux)
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(vii) **PV-biased forcing:** critical in the **idealized Jupiter weather-layer model** being studied by Stephen Thomson and myself.

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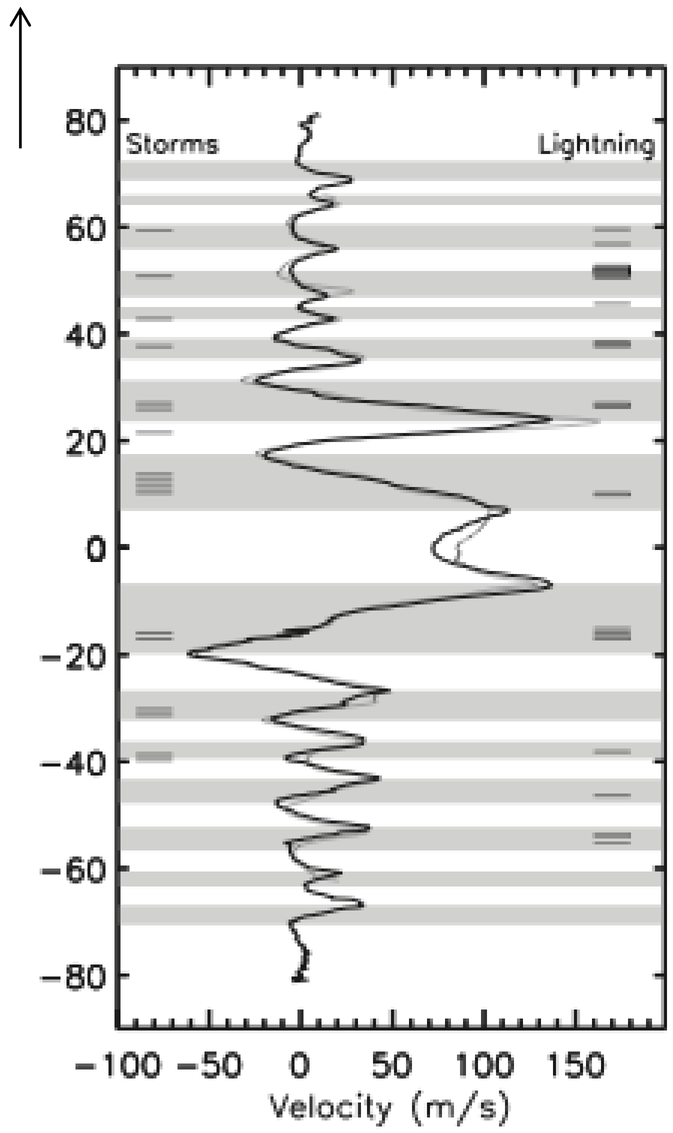

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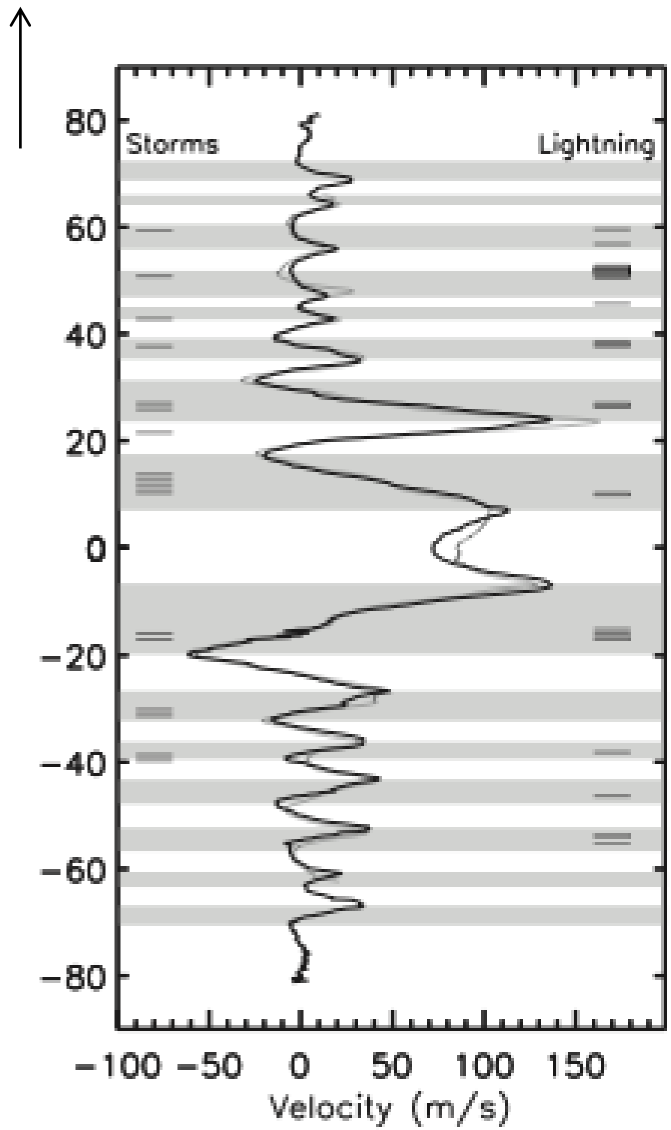

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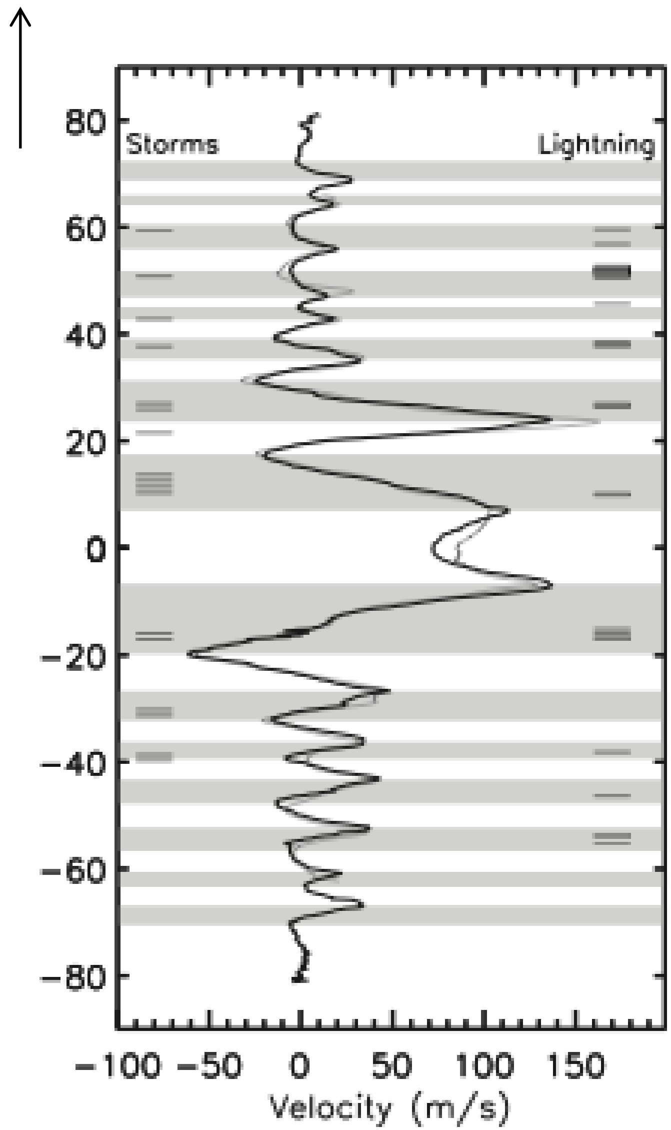

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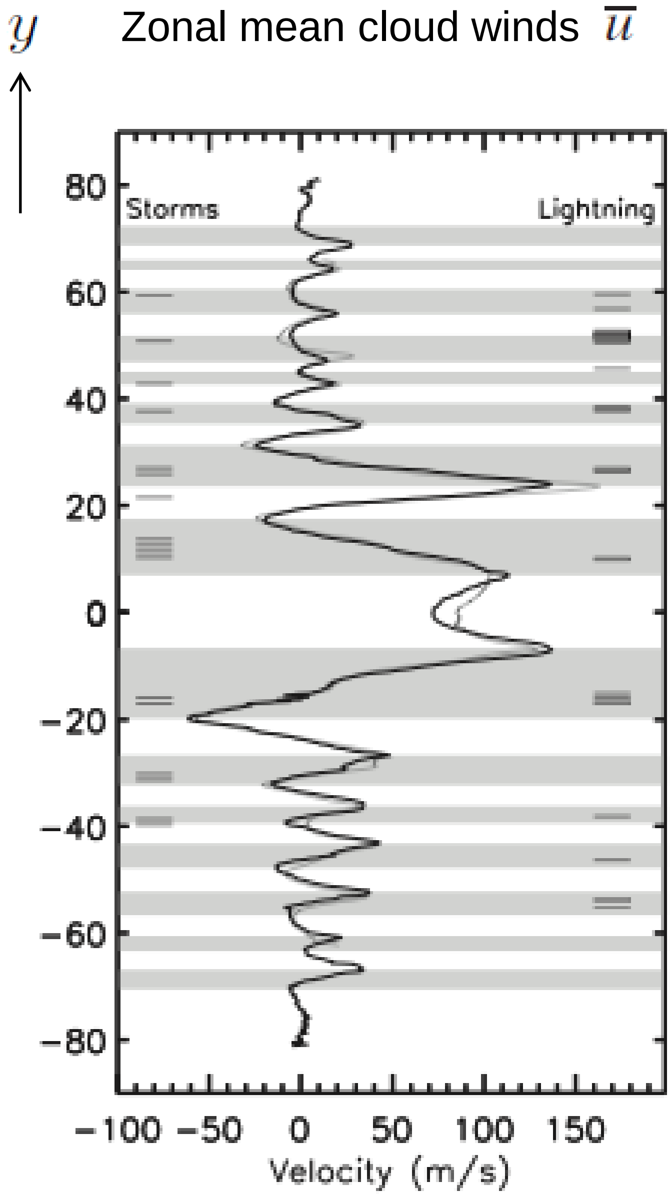

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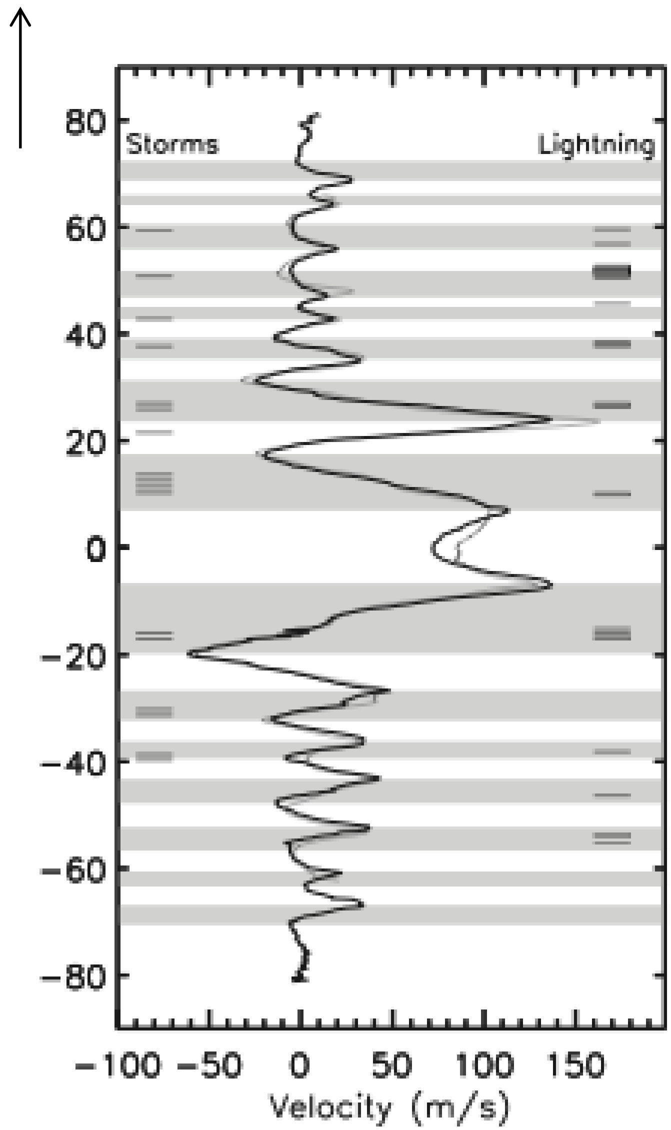

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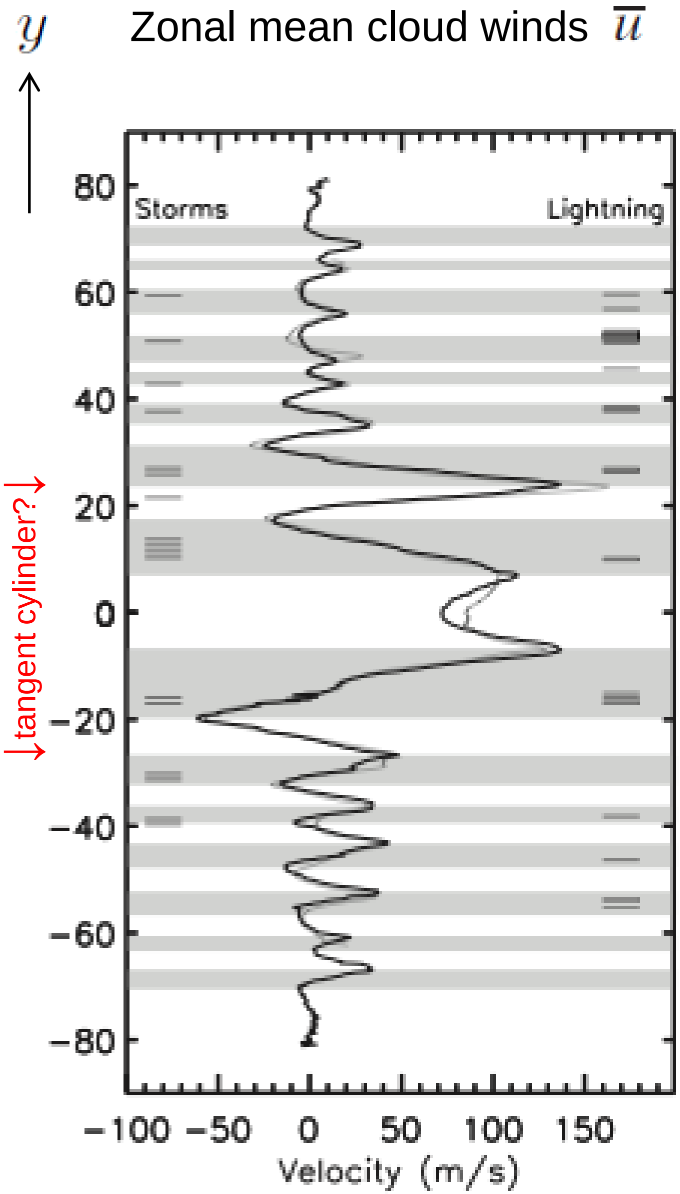

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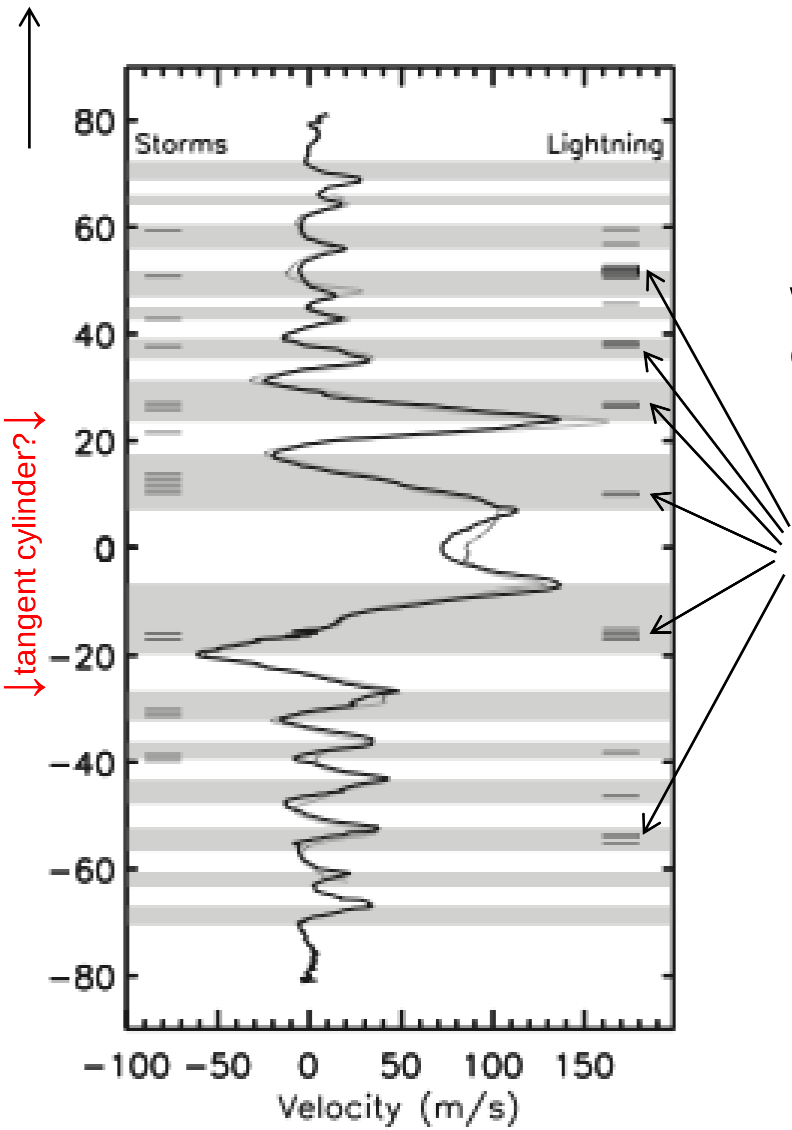

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Moist convection may be the dominant coupling: stronger in ‘belts’ (cyclonic, shaded) than in ‘zones’ (anticyclonic, unshaded). R.h. bars show lightning.

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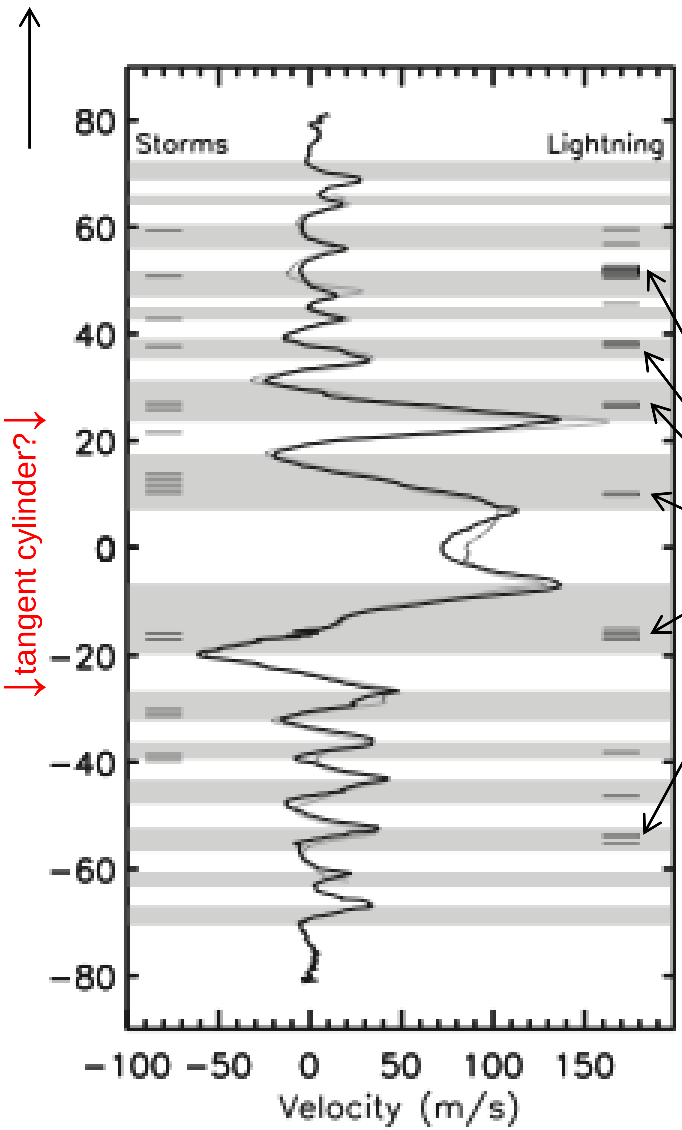

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Jets extremely straight, especially the prograde jets – unearthly !! Unlike weather-layer-only models.

Weather layer coupled to deep dry-convection layer offers best hope of fluid-dynamical understanding (Ingersoll-Cuong 1981, Dowling-Ingersoll 1989,…) (And NB: there is only one GRS.)

(rejecting an old false dichotomy)

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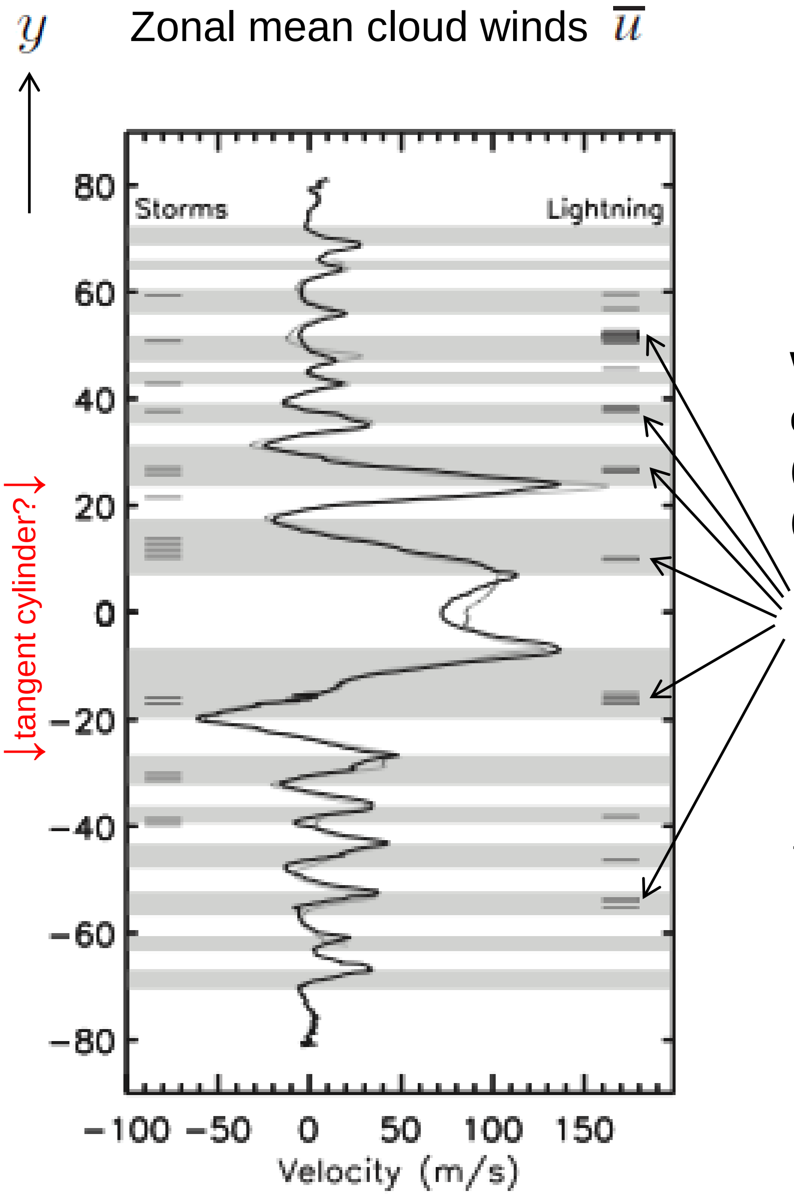

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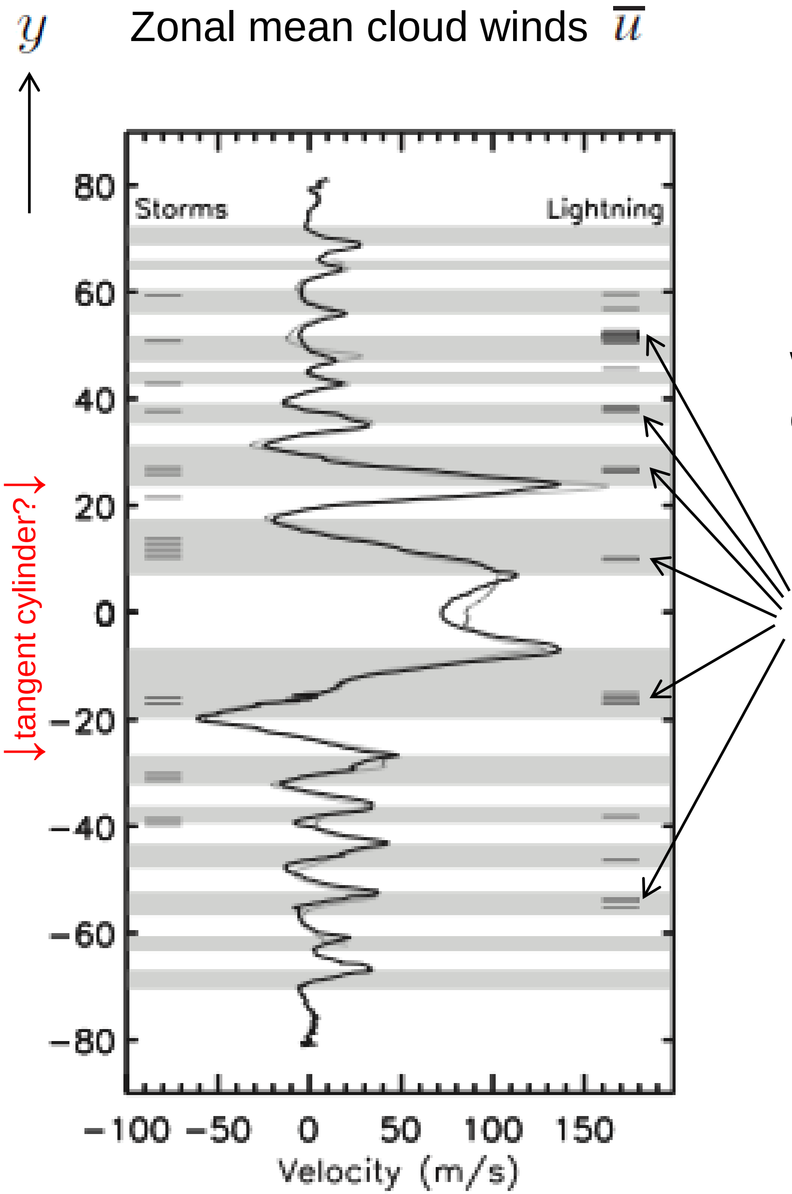

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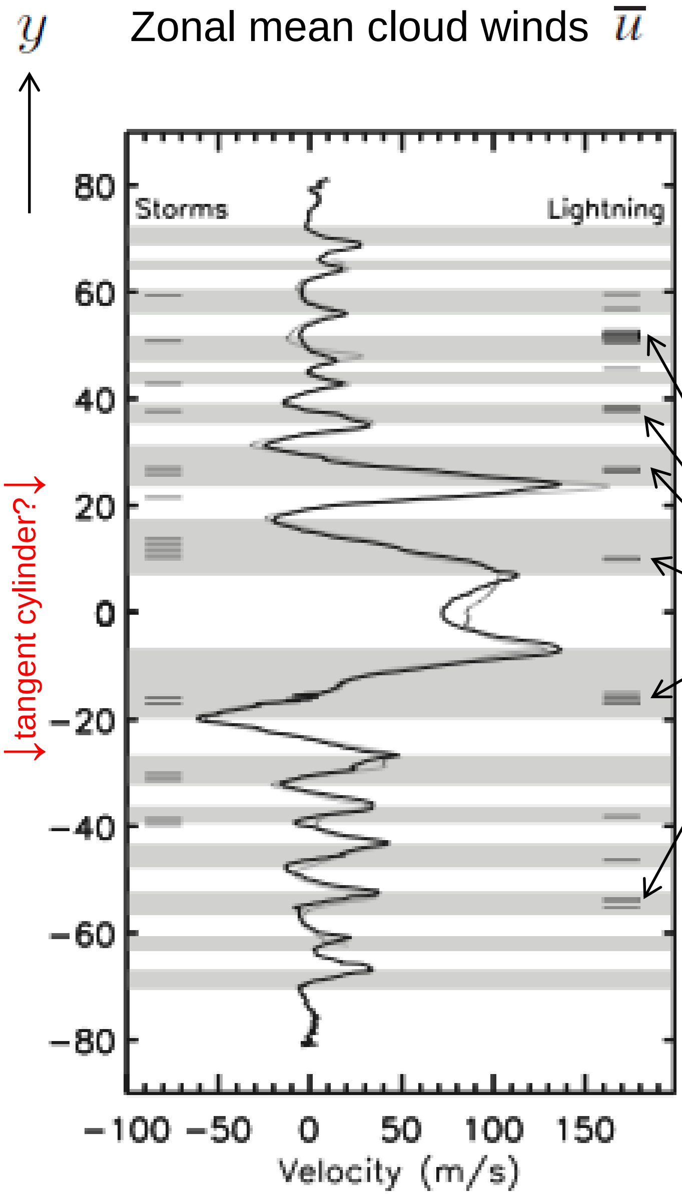

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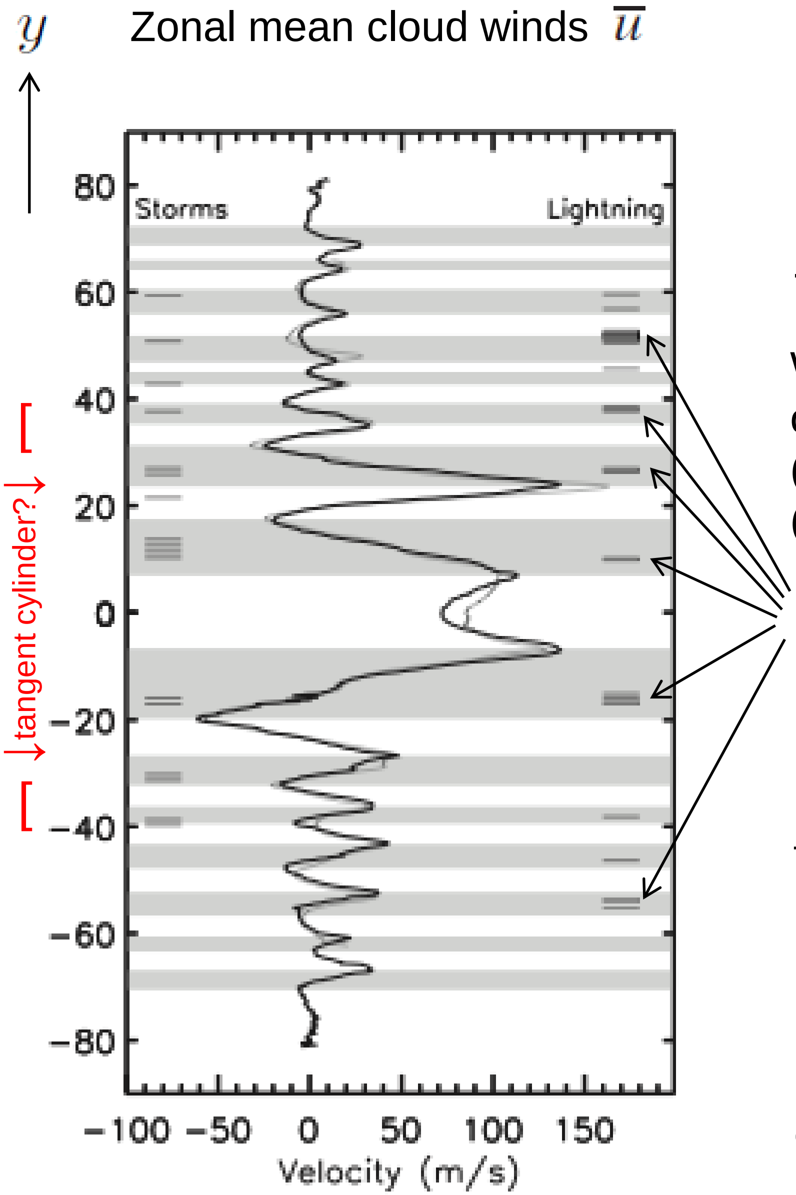

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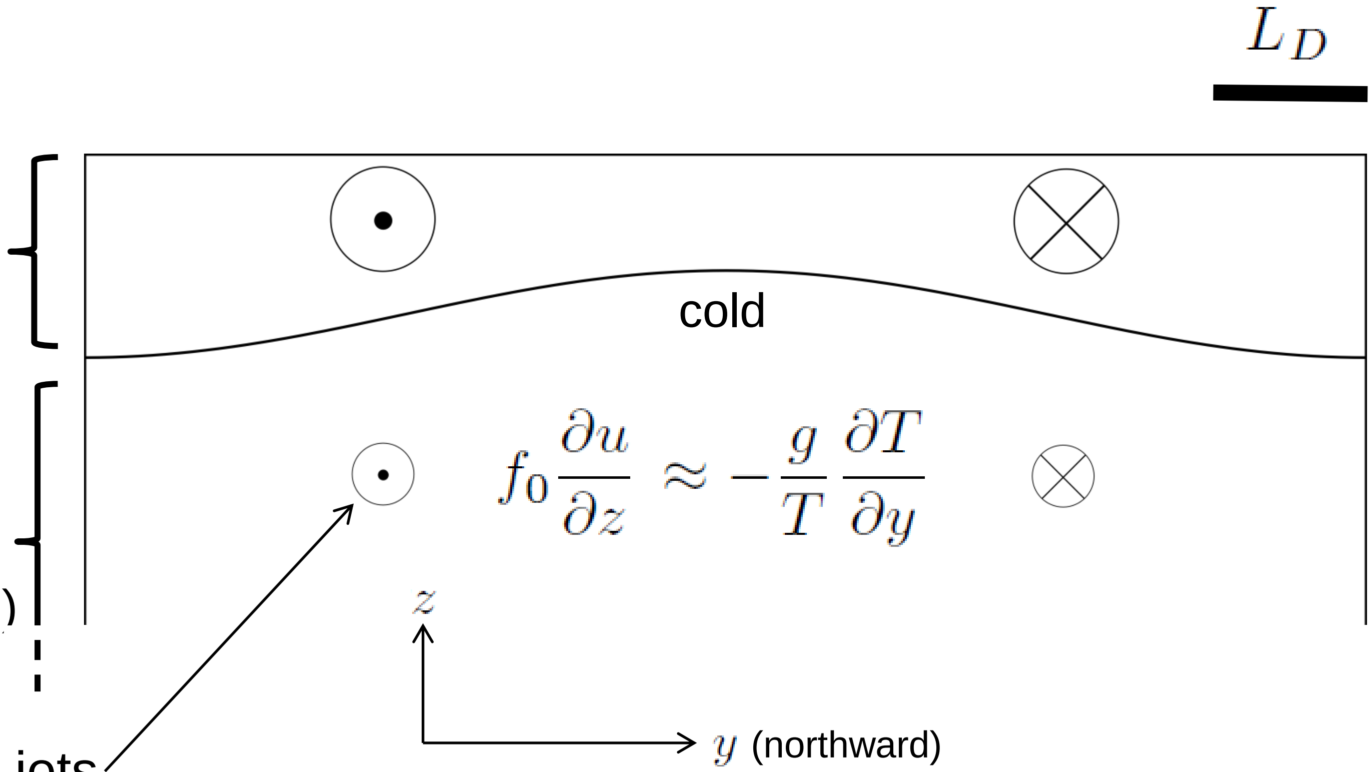

weather layer (1)

dry-convec’n layer (2) (massive)

$$f_0 \frac{\partial u}{\partial z} \approx -\frac{g}{T}\frac{\partial T}{\partial y}$$

deep jets are fixed.

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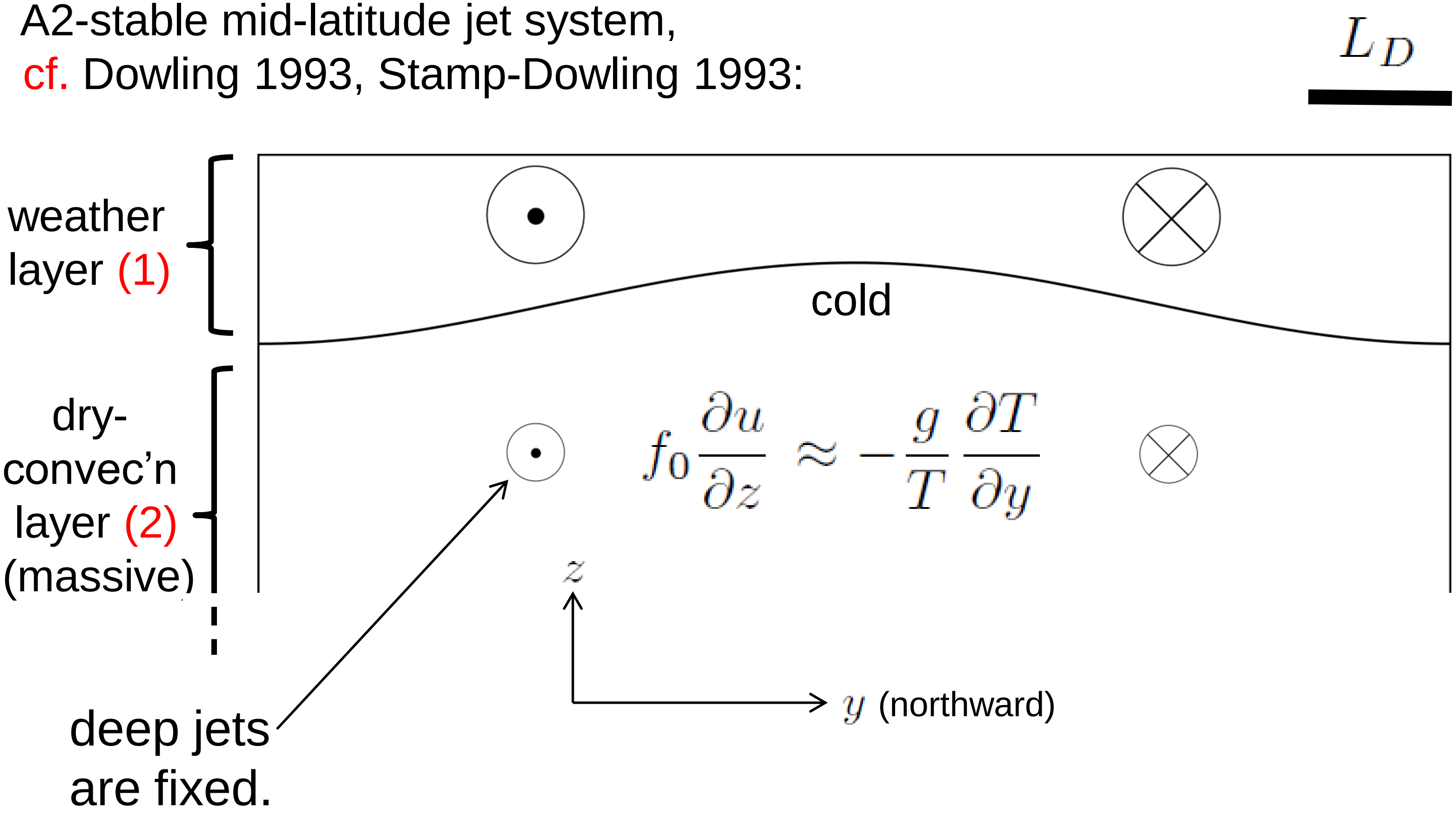

L_D

weather layer (1)

cold

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z

y (northward)

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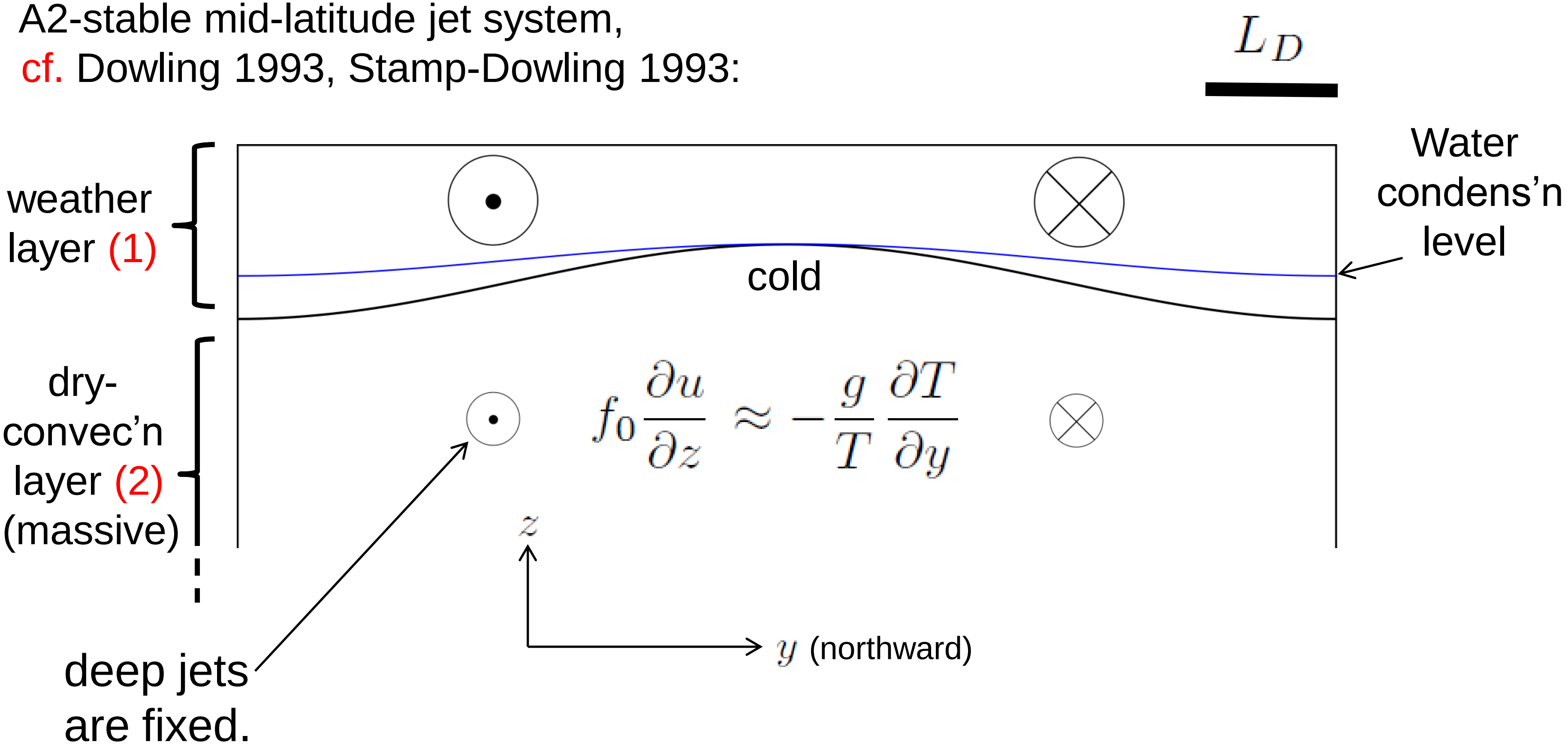

L_D

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Water condens’n level

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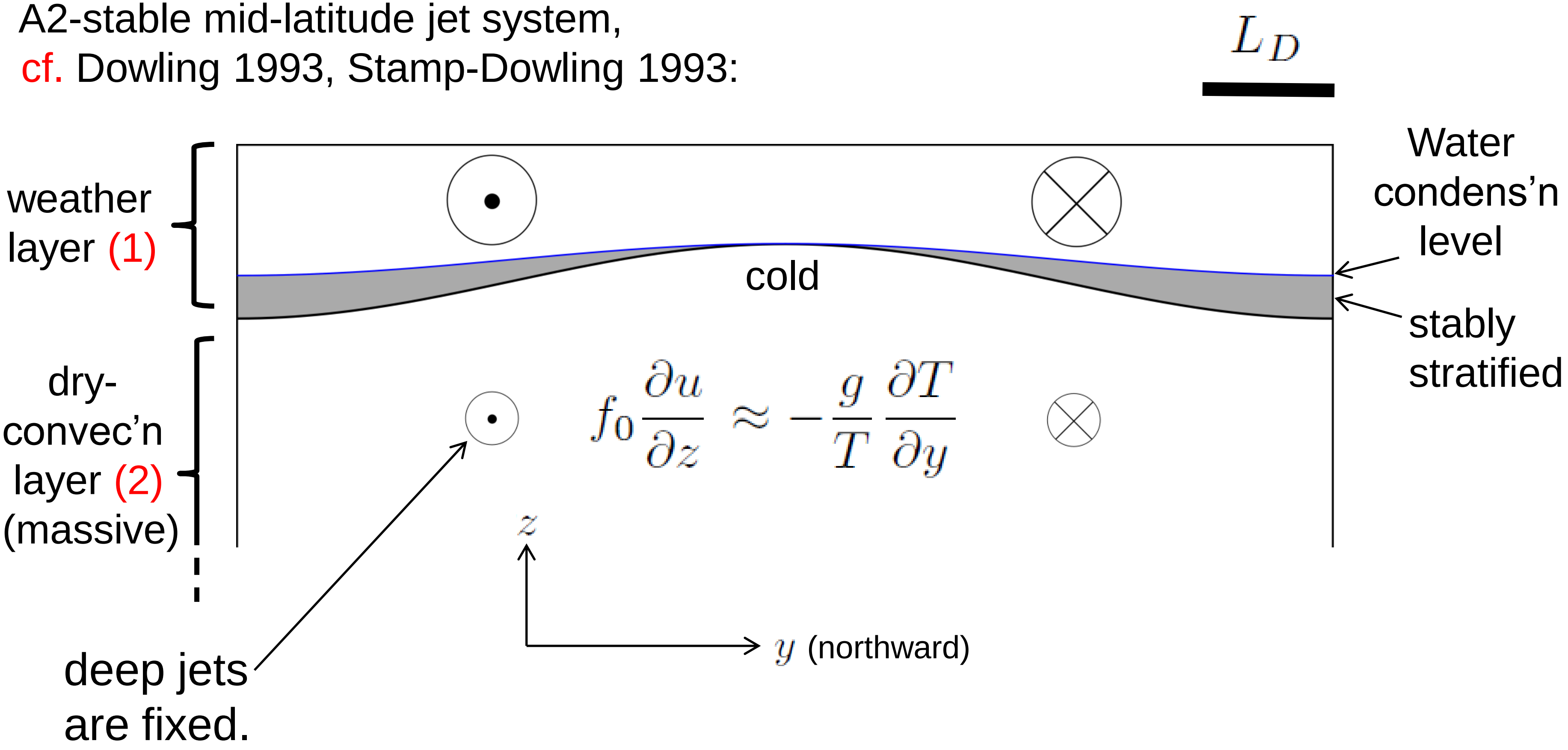

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stably stratified

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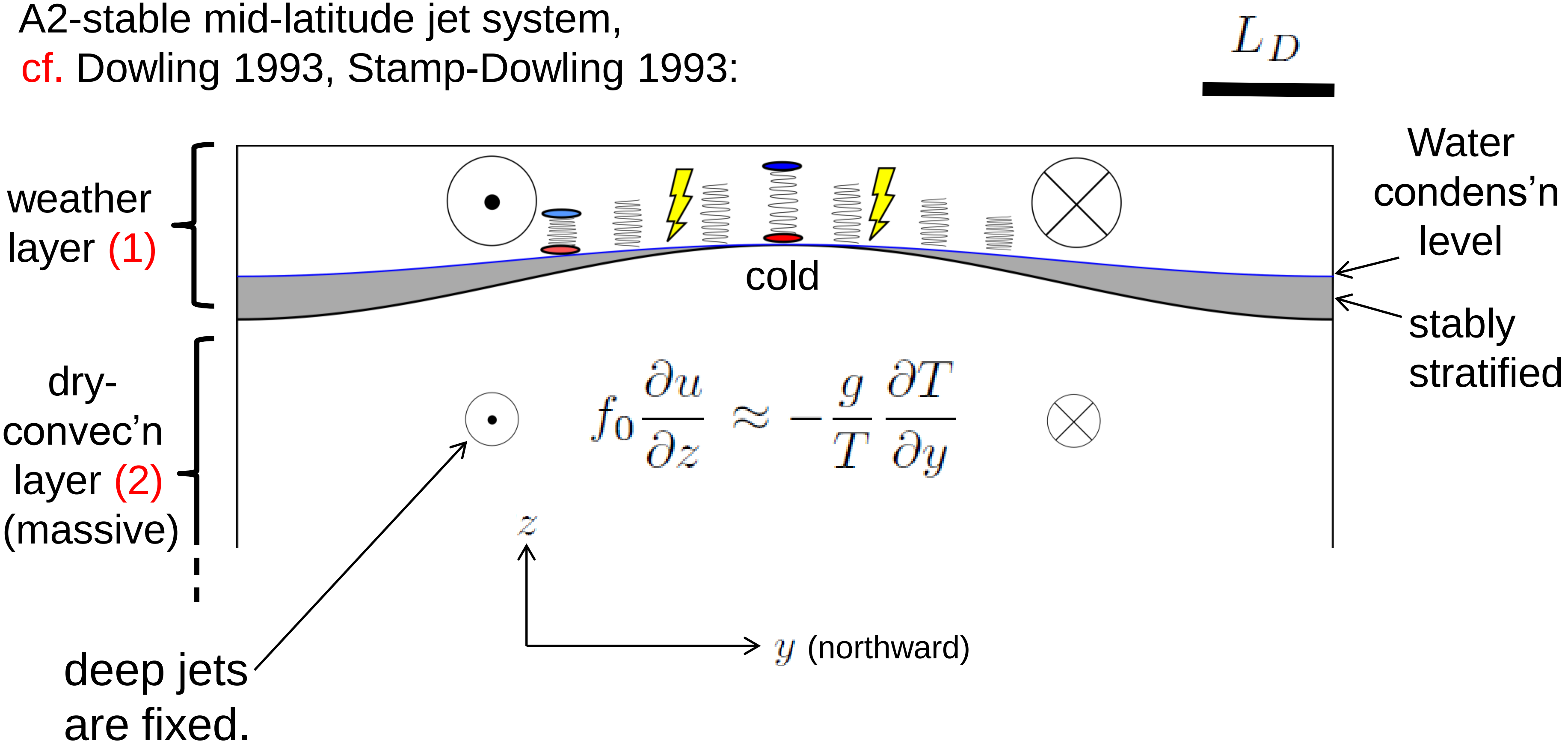

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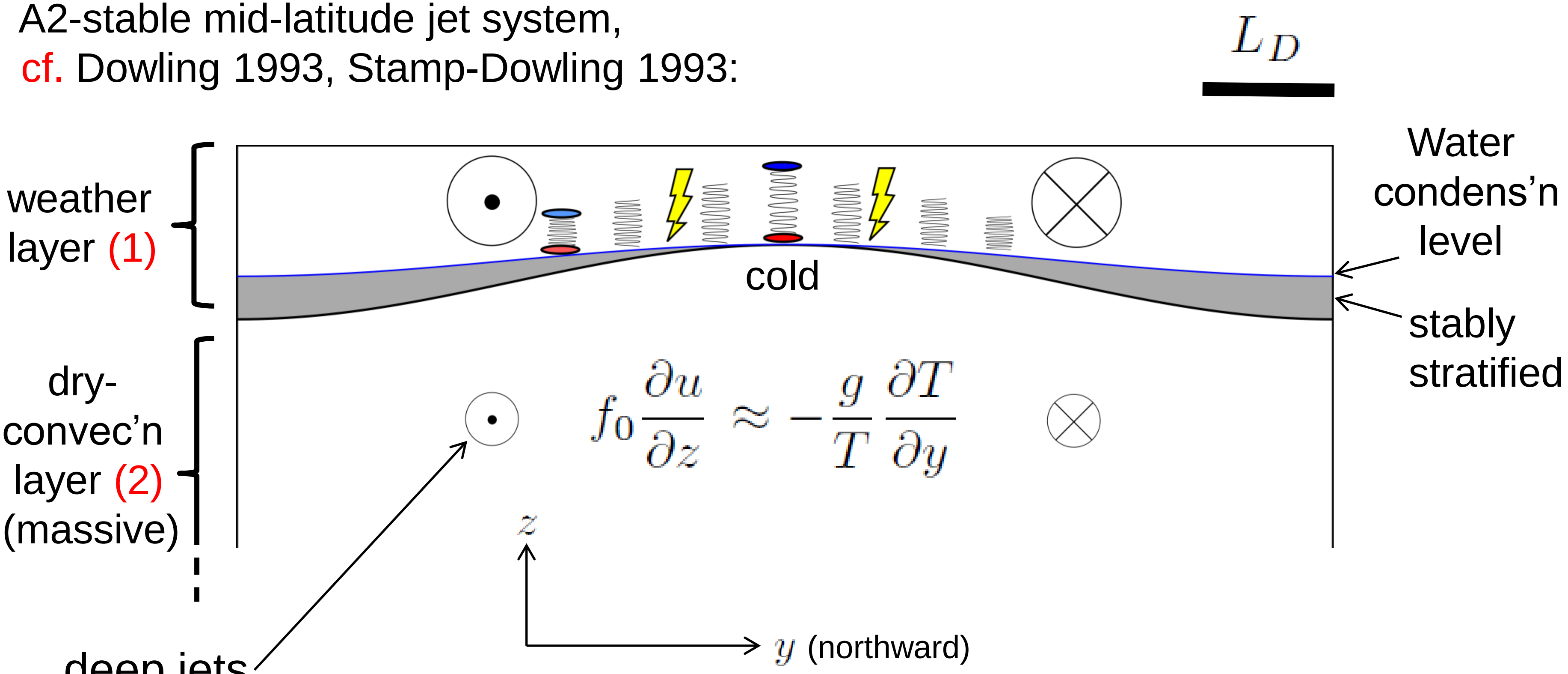

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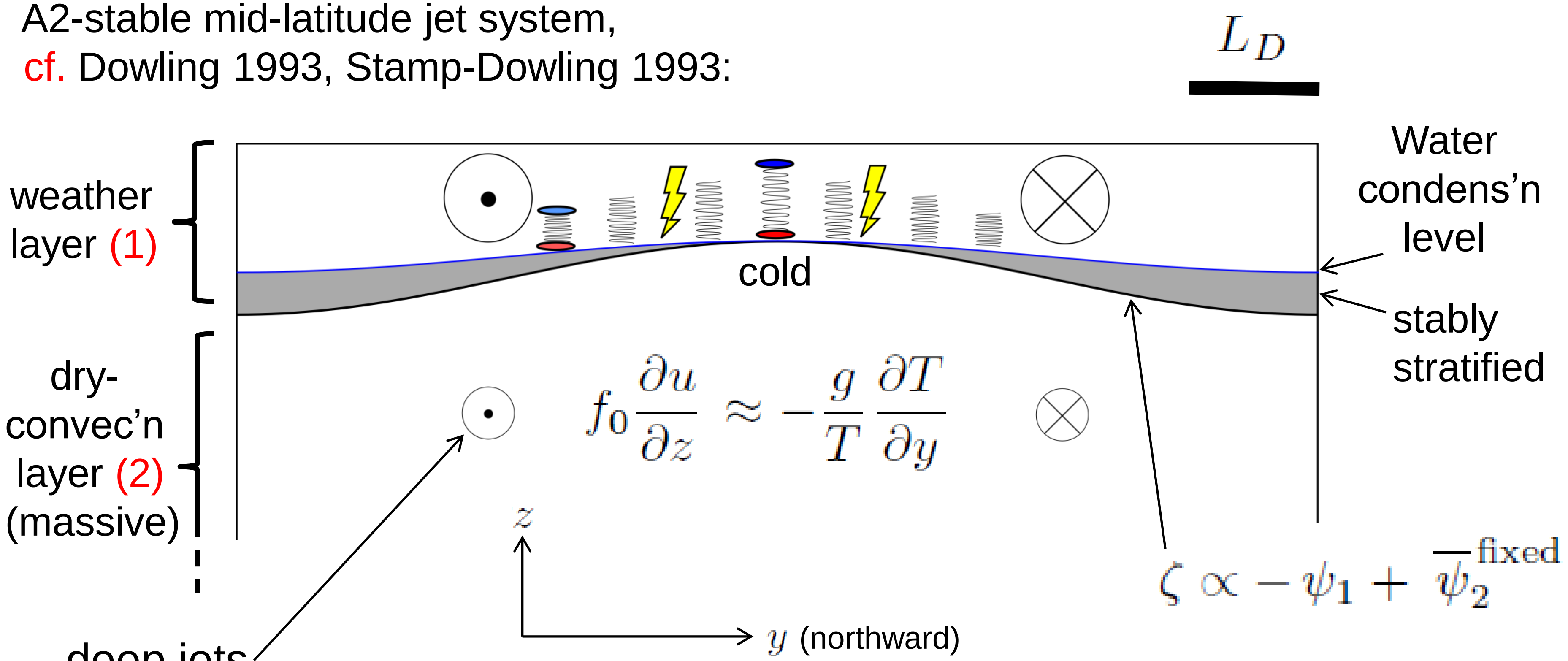

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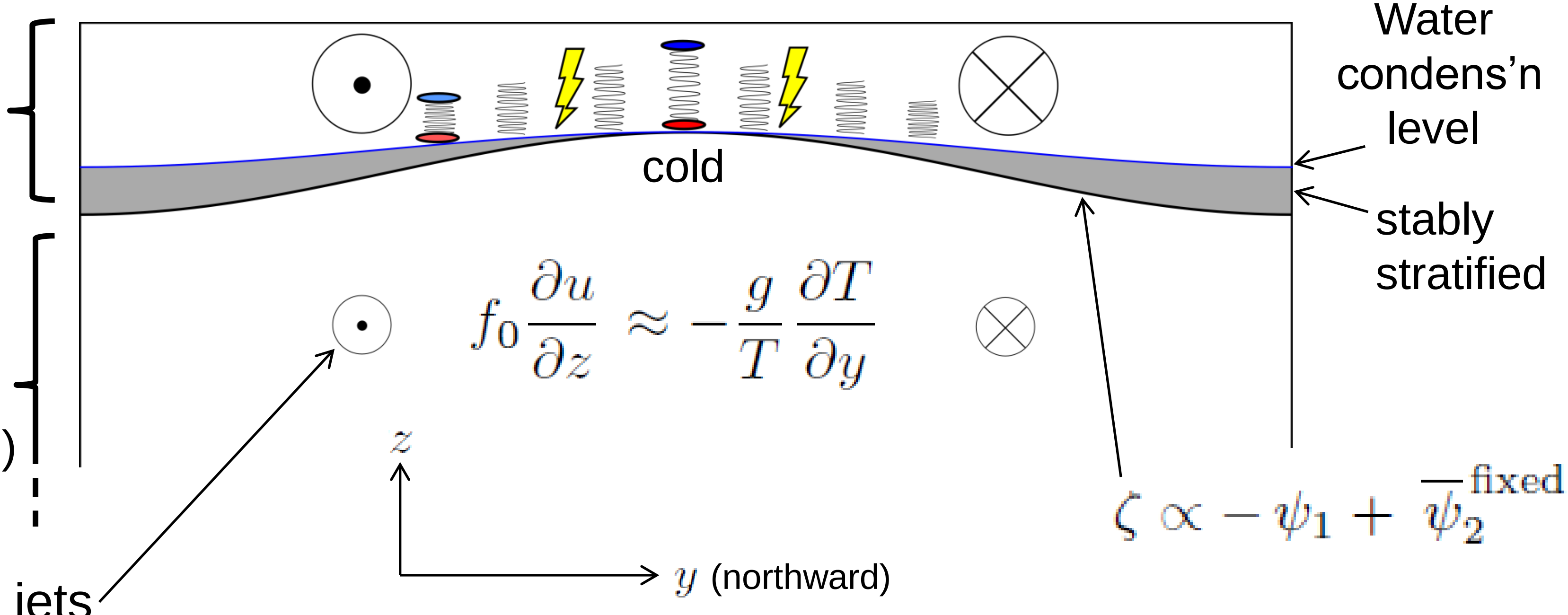
“Large” will mean by comparison with L_D



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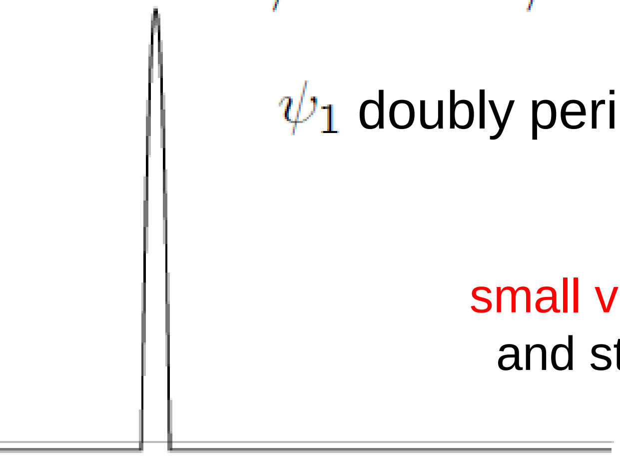

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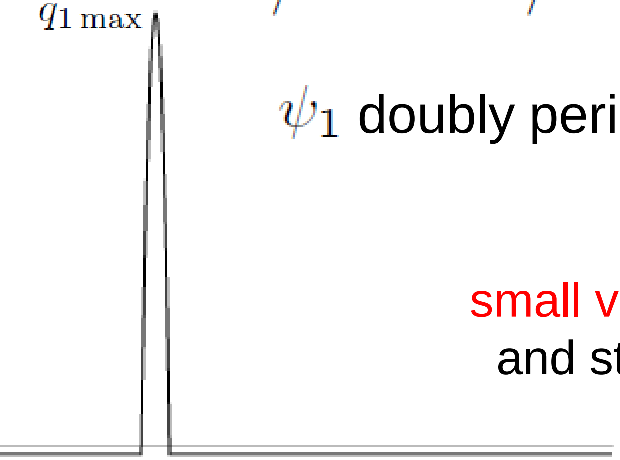

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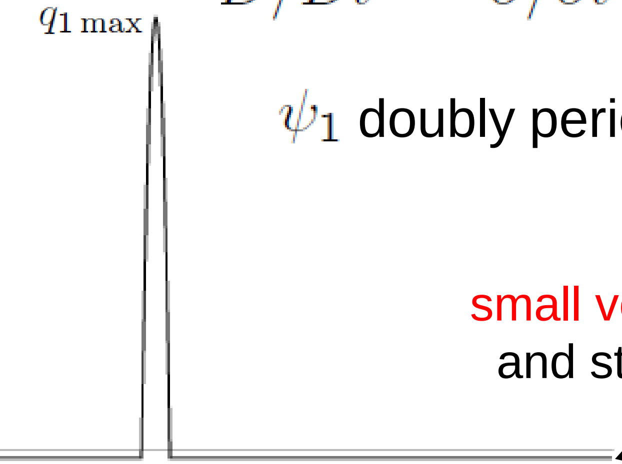

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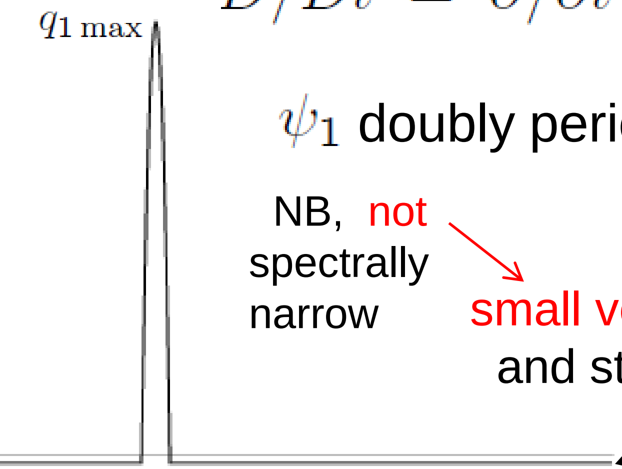

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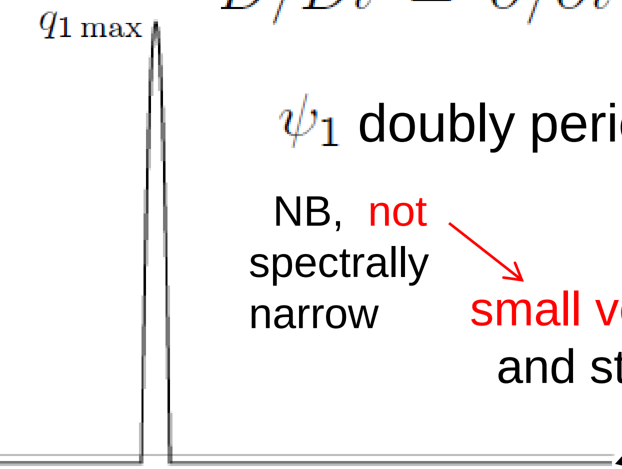

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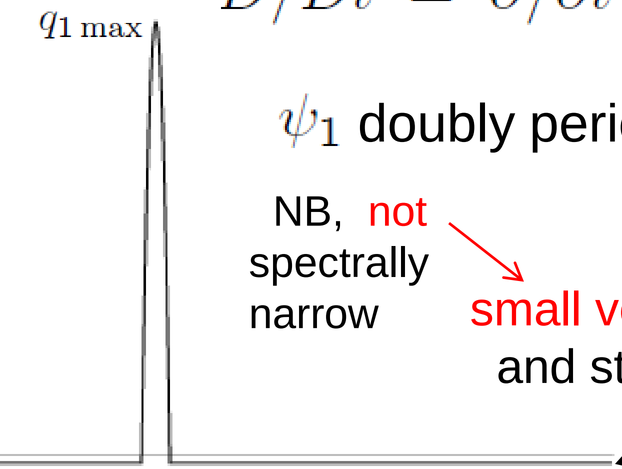

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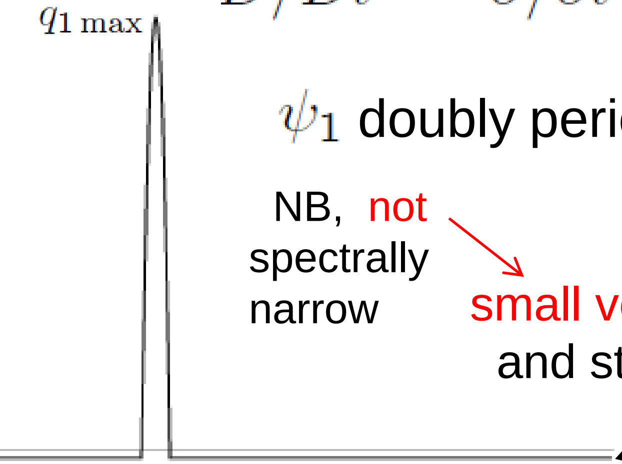

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$$\frac{\partial}{\partial t}\iint\left(\tfrac{1}{2}|\nabla\psi_1|^2 + \tfrac{1}{2}\frac{\psi_1^2}{L_D^2}\right)dxdy = -\iint \psi_1 S\, dxdy$$

Can have either sign

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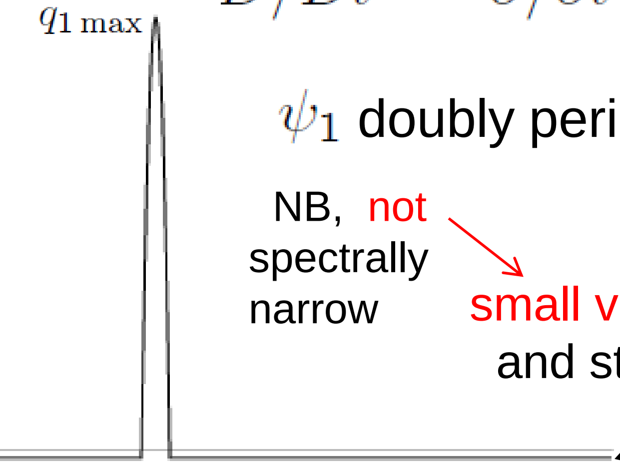

ψ_1 doubly periodic, $\displaystyle\iint \psi_1\,dx\,dy = 0\ ,$ similarly S and $\xcancel{\mathcal{D}}$

cf. exact impermeability theorem

NB, not spectrally narrow

randomly-injected E-W pairs of small vortices, with anticyclonic PV-bias, and stronger wherever base colder, & v.v.

quasi-hyperdiffusive. Ignore from here on.

Note domain-wide “complementary forcing”

Key point: PV-bias plus base-temperature dependence can replace large-scale friction! Statistical steadiness possible!!

$$\frac{\partial}{\partial t}\iint\left(\tfrac{1}{2}|\nabla\psi_1|^2 + \tfrac{1}{2}\frac{\psi_1^2}{L_D^2}\right)dx\,dy = -\iint \psi_1 S\,dx\,dy$$

Can have either sign

No need for radiative damping either.

Standard 1½-layer quasi-geostrophic beta-plane model:

$$\frac{Dq_1}{Dt} = S(x,y,t) + \mathcal{D}\,,$$ where:

PV inversion: given q_1, solve for ψ_1.

$$q_1 = \nabla^2\psi_1 + \beta y - k_D^2(\psi_1 - \overset{\text{fixed}}{\overline{\psi}_2})\,, \quad k_D = 1/L_D = 1/\rho_s$$

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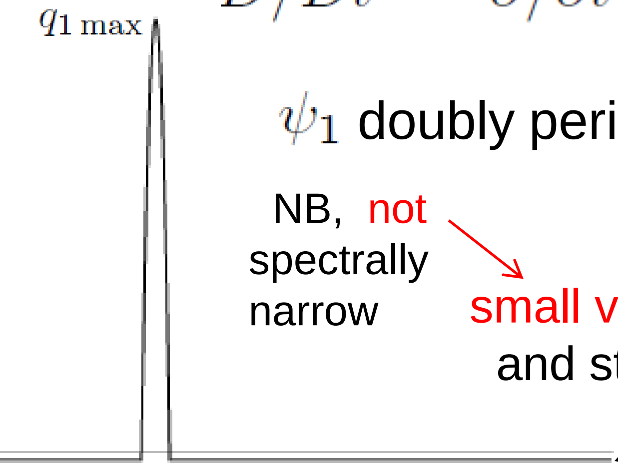

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No need for radiative damping either.
(Realistic radiative damping timescales? *Or* is concept irrelevant?)

In particular, in the zonally-averaged PV equation

$$\frac{\partial \overline{q}_1}{\partial t} = -\frac{\partial (\overline{v_1' q_1'})}{\partial y} + \overline{S}$$

$\overline{S}$ acts qualitatively like a Rayleigh drag toward the deep-jet velocity profile.

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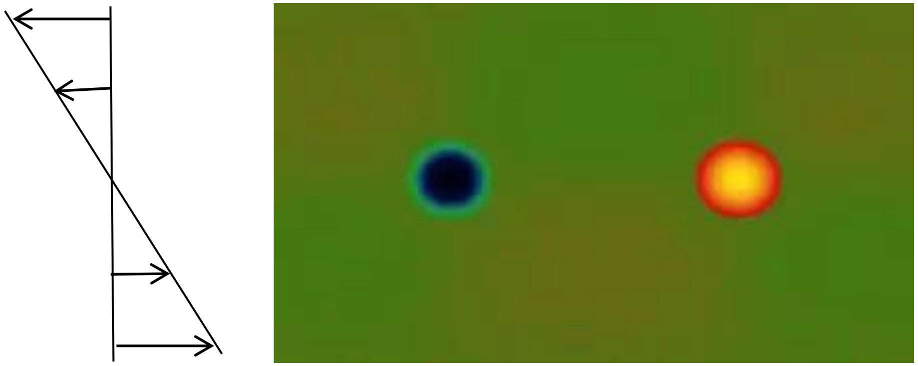
What happens is sensitive to $\overline{v_1' q_1'}$ & hence to the strength of S:

Forcing strengths $|q_{1\,\max}|$,

cf. Shigeo Kida (1981):

Strong injections $|q_{1\,\max}|$ = 16:

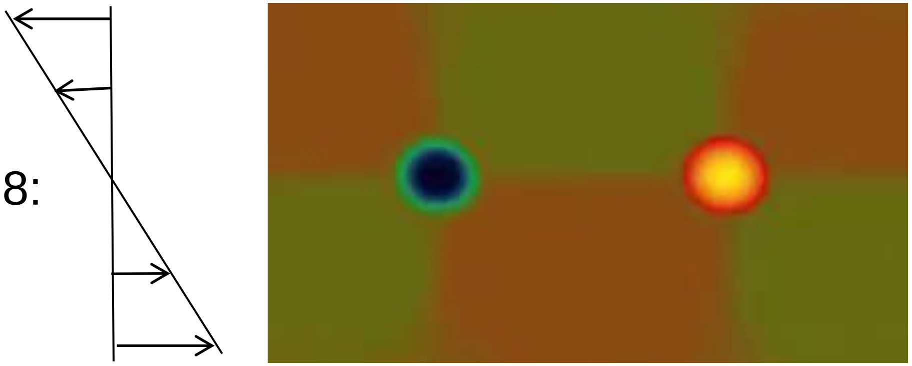
in units of background shear



Recall

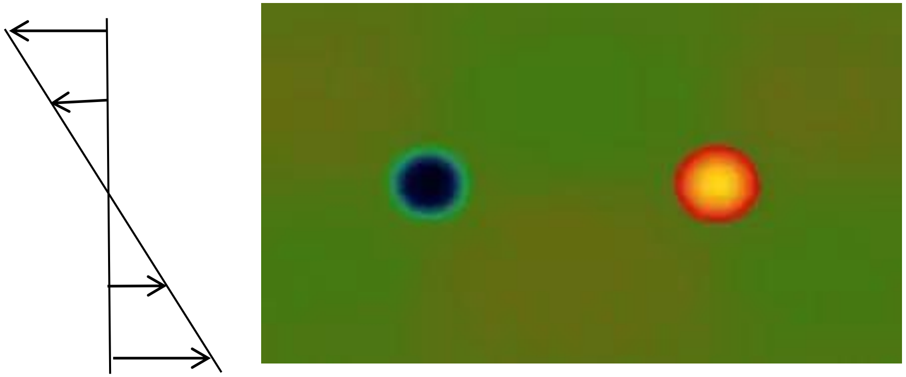
$$\frac{\partial \overline{q}_1}{\partial t} = -\frac{\partial\left(\overline{v_1' q_1'}\right)}{\partial y} + \overline{S}$$

Semi-strong injec’ns $|q_{1\,\max}|$ = 8:

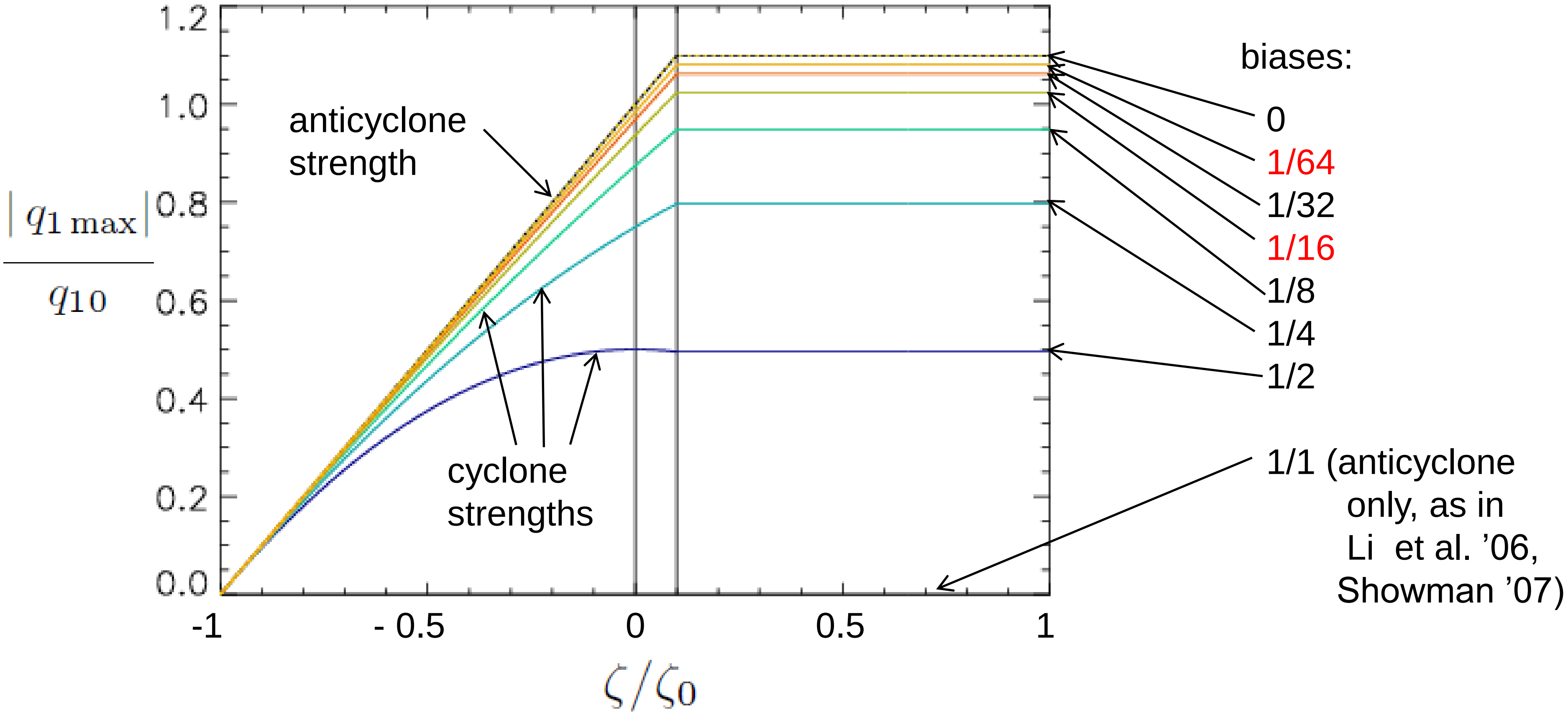


Weak injections $|q_{1\,\max}|$ = 2:

(as required for passive Kelvin shearing)



At present we use the following injection-pair algorithm, with “saturation”:



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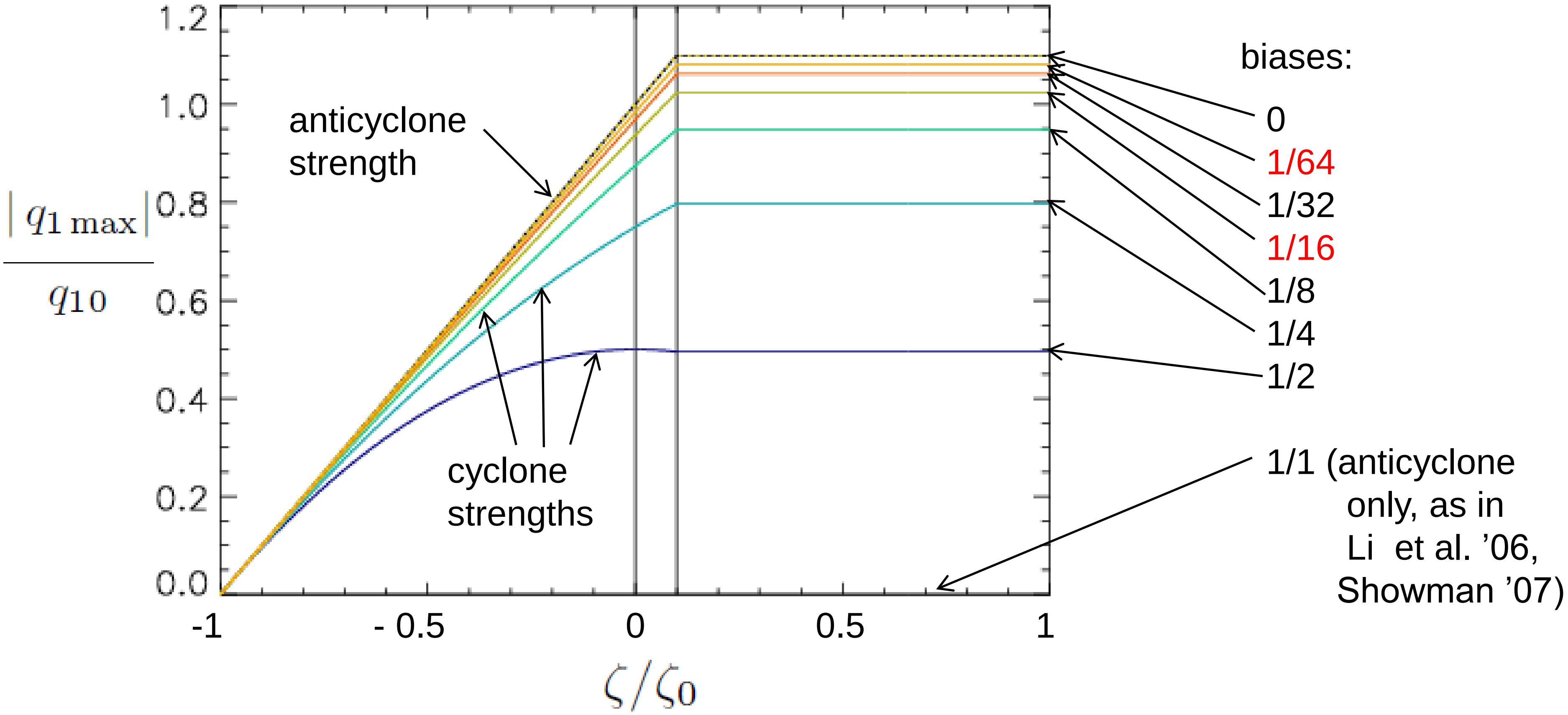

anticyclone strength

cyclone strengths

biases:

0

1/64

1/32

1/16

1/8

1/4

1/2

1/1 (anticyclone only, as in Li et al. ’06, Showman ’07)

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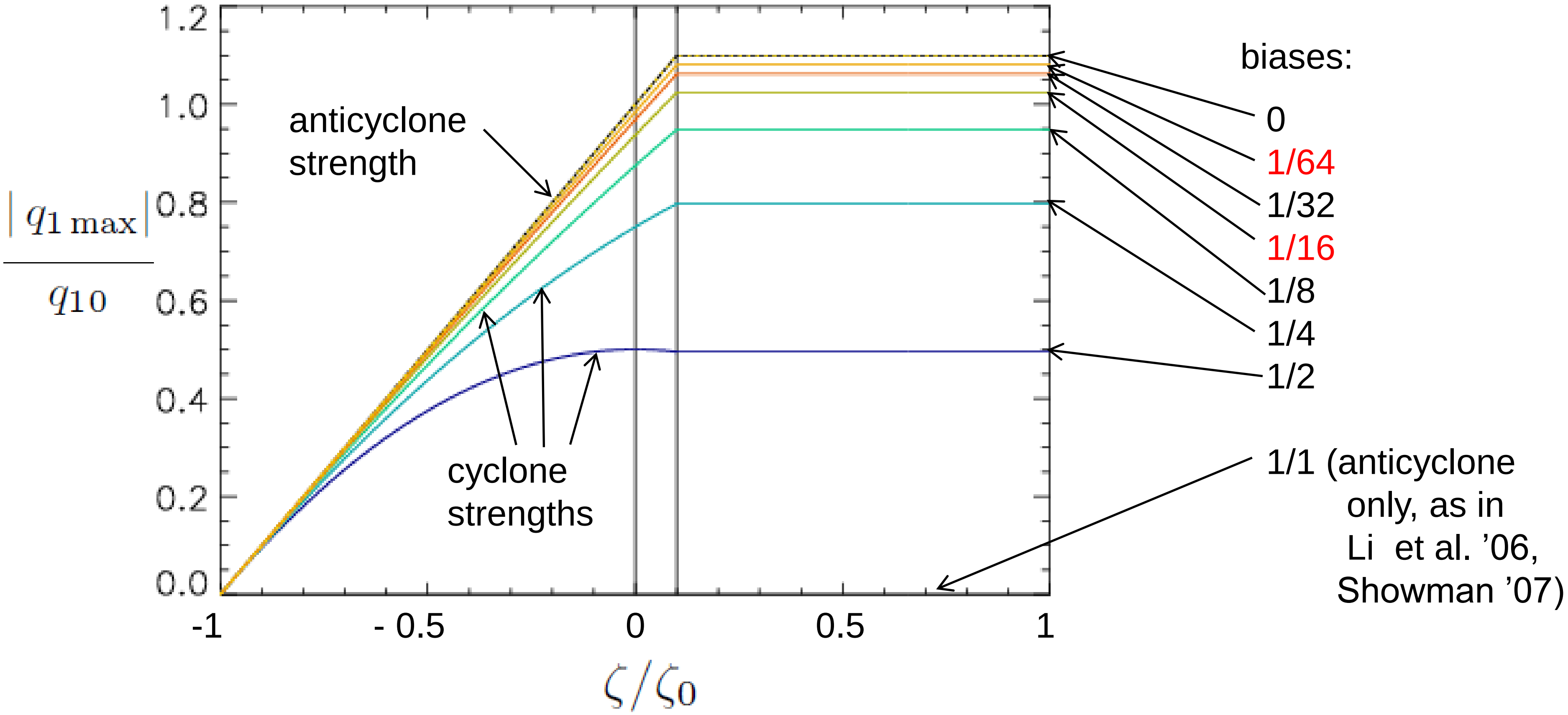

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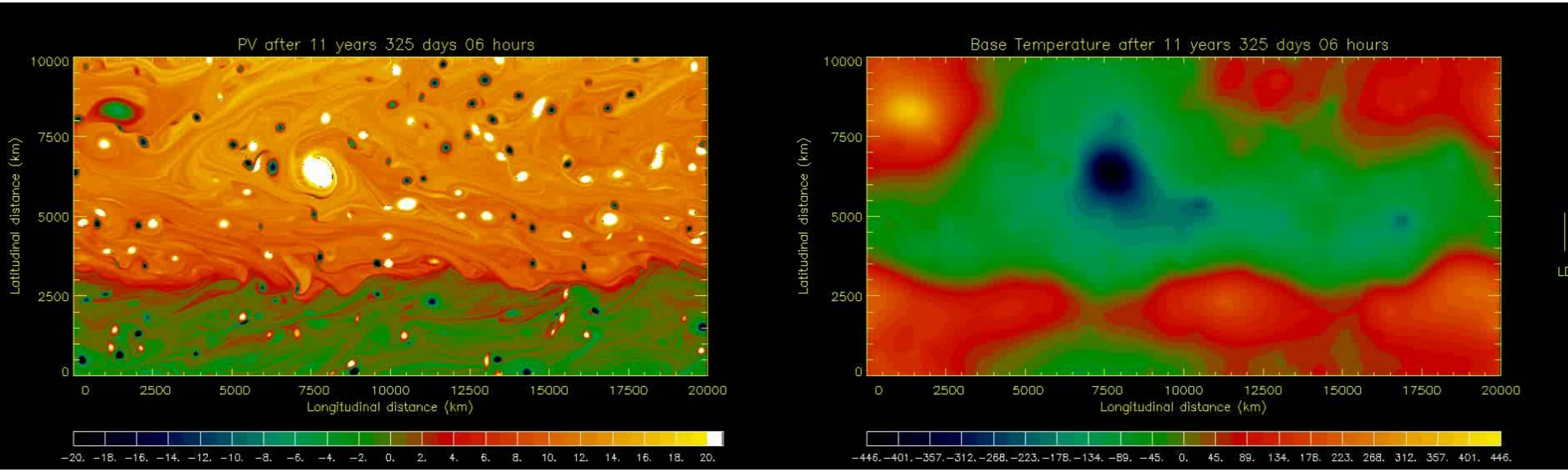
Illustrate for strength q_{10} = 16 and β = 4 times nominal Jovian value:

q_1

Early mature, bias 1/16:

ζ



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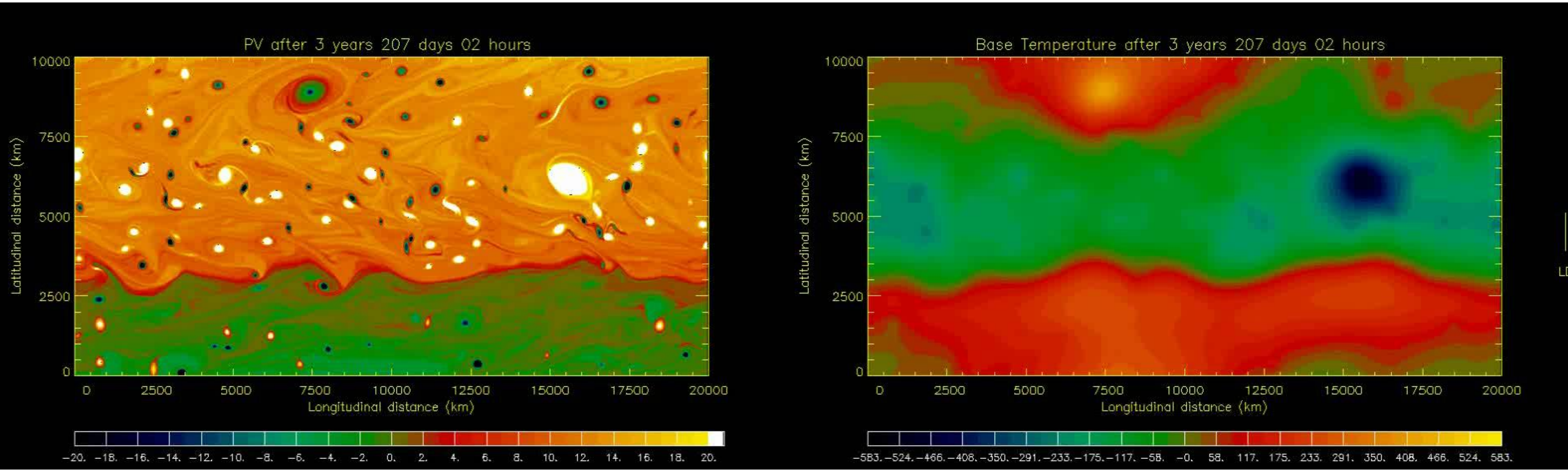
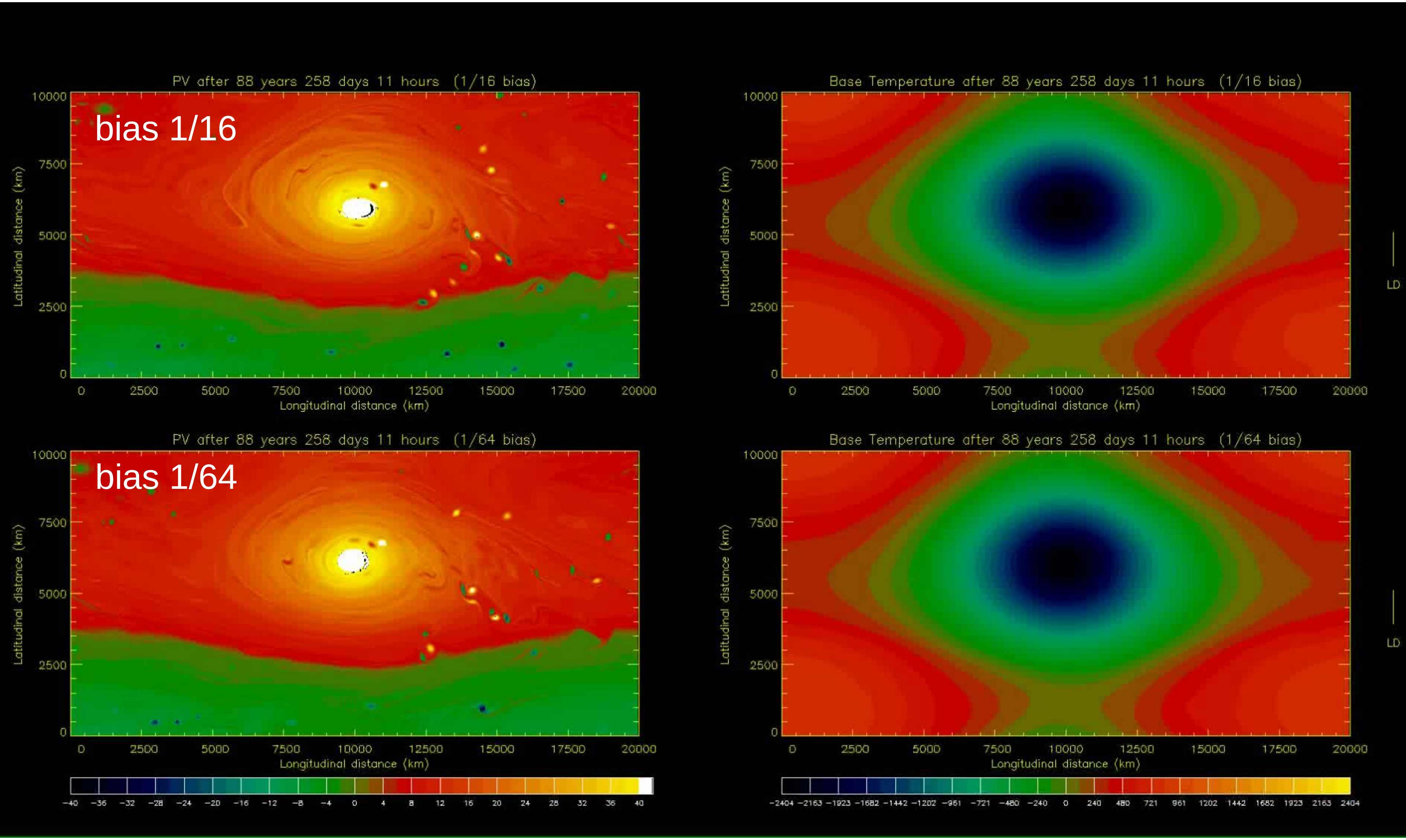


Bias 1/64 is too weak to stop the large cyclone from growing further:

q_1

Skipping forward, ~ 80 Earth-years later:

ζ



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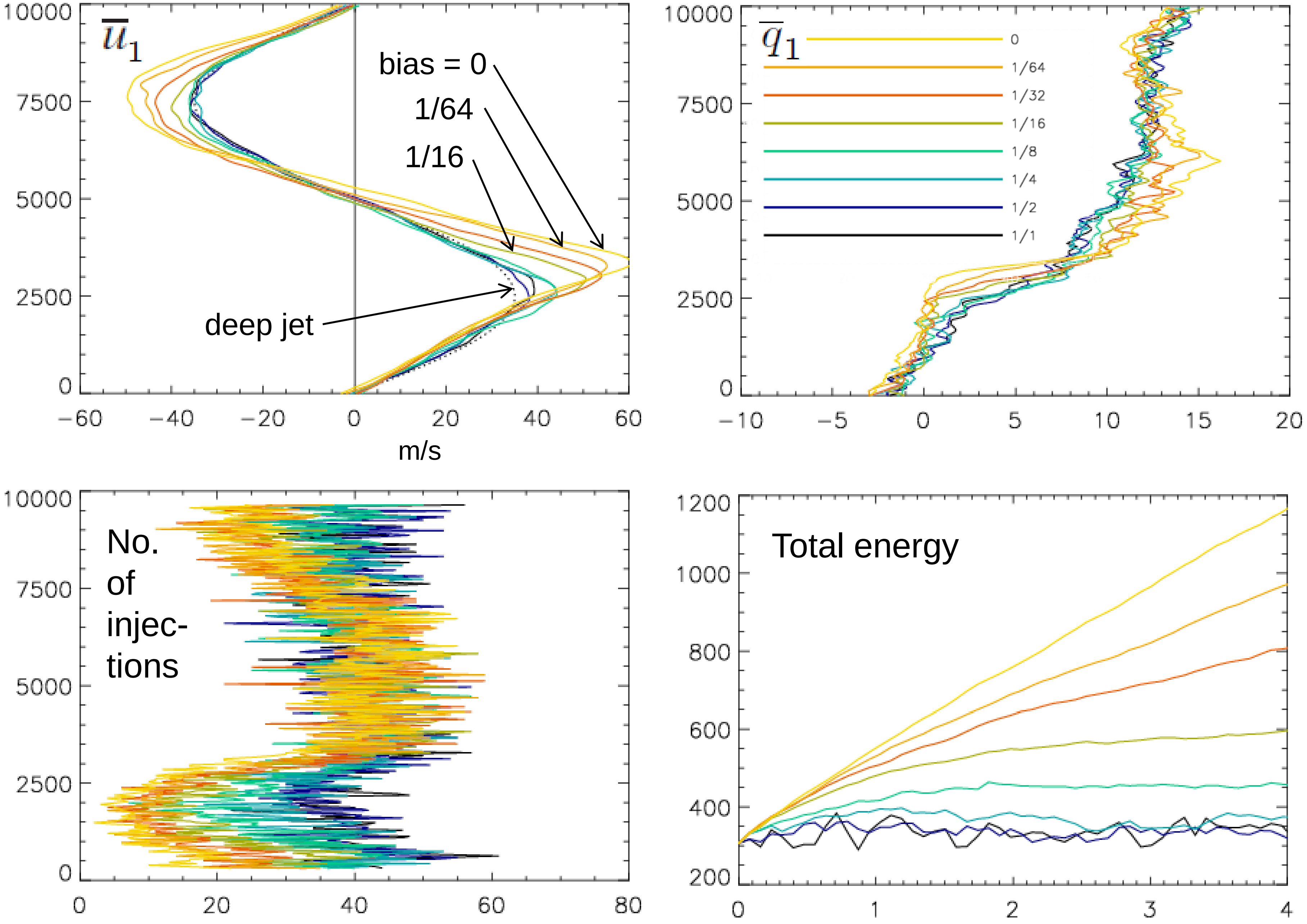
Early mature stage, 4 Earth years, strength still 16:



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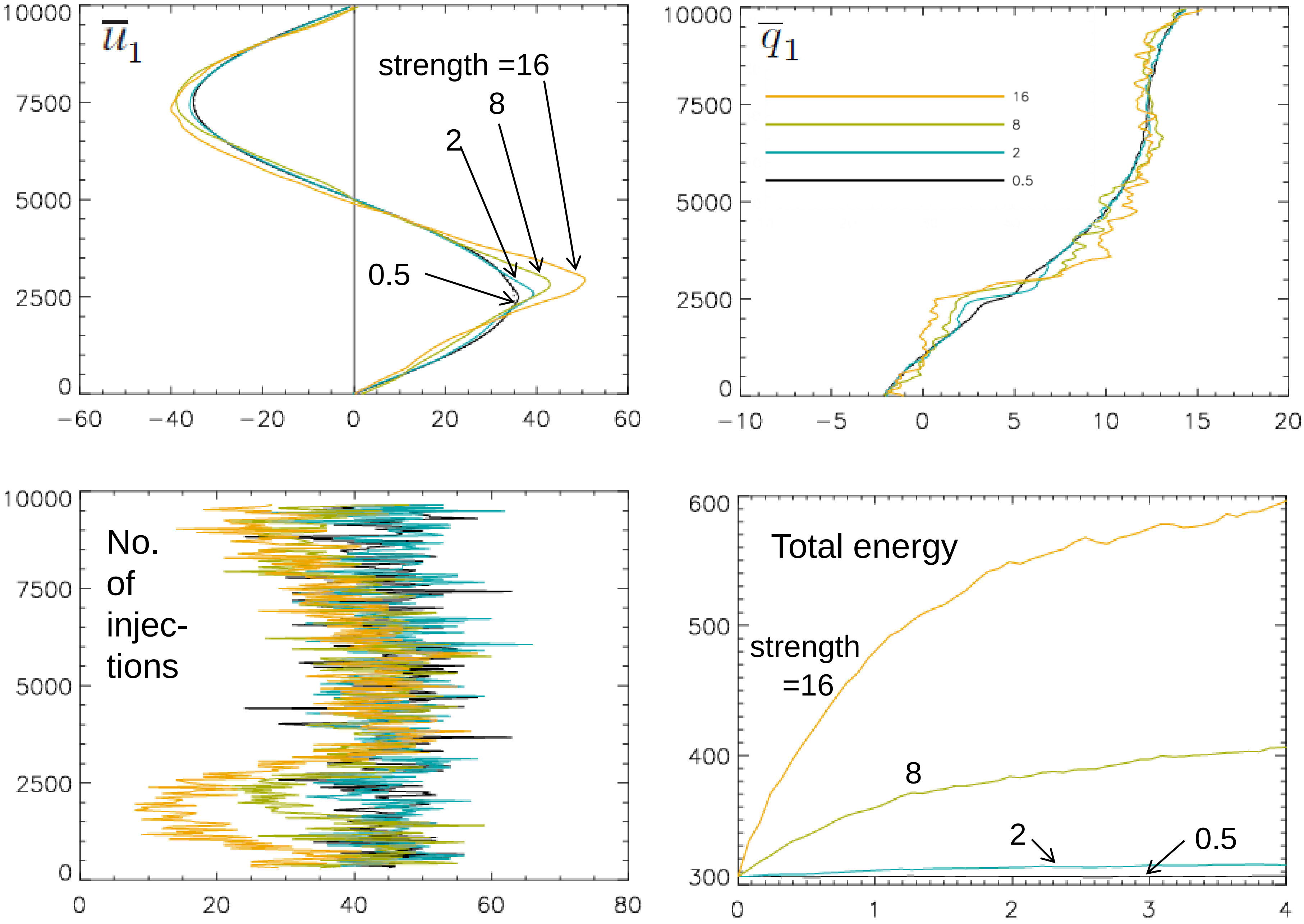

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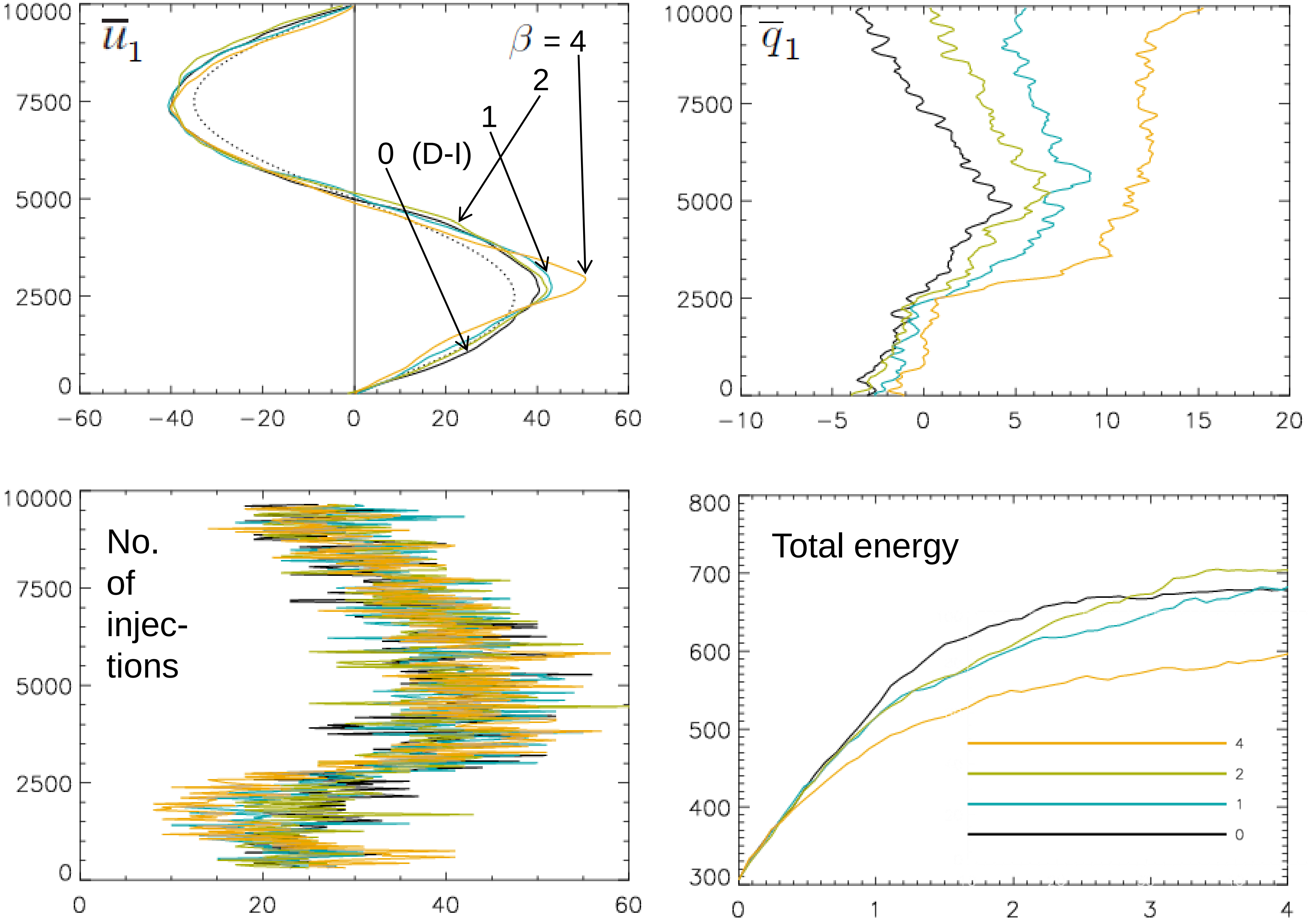

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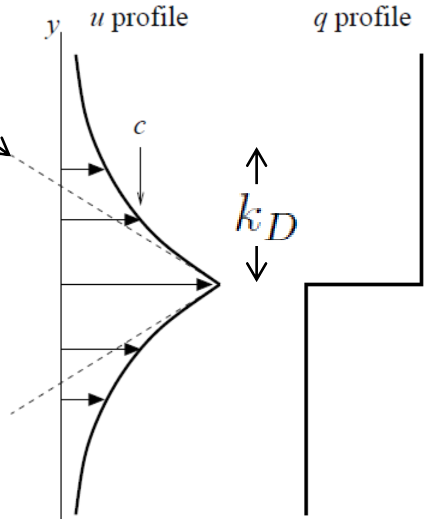
Implication: strong jet with given PV contrast has far greater velocity contrast:



Rosenbluth Lecture, Fig. 1.8

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constant nominal **growth rate**

constant **Rayleigh frictions** (more cheating)
with μ' >> $\bar{\mu}$

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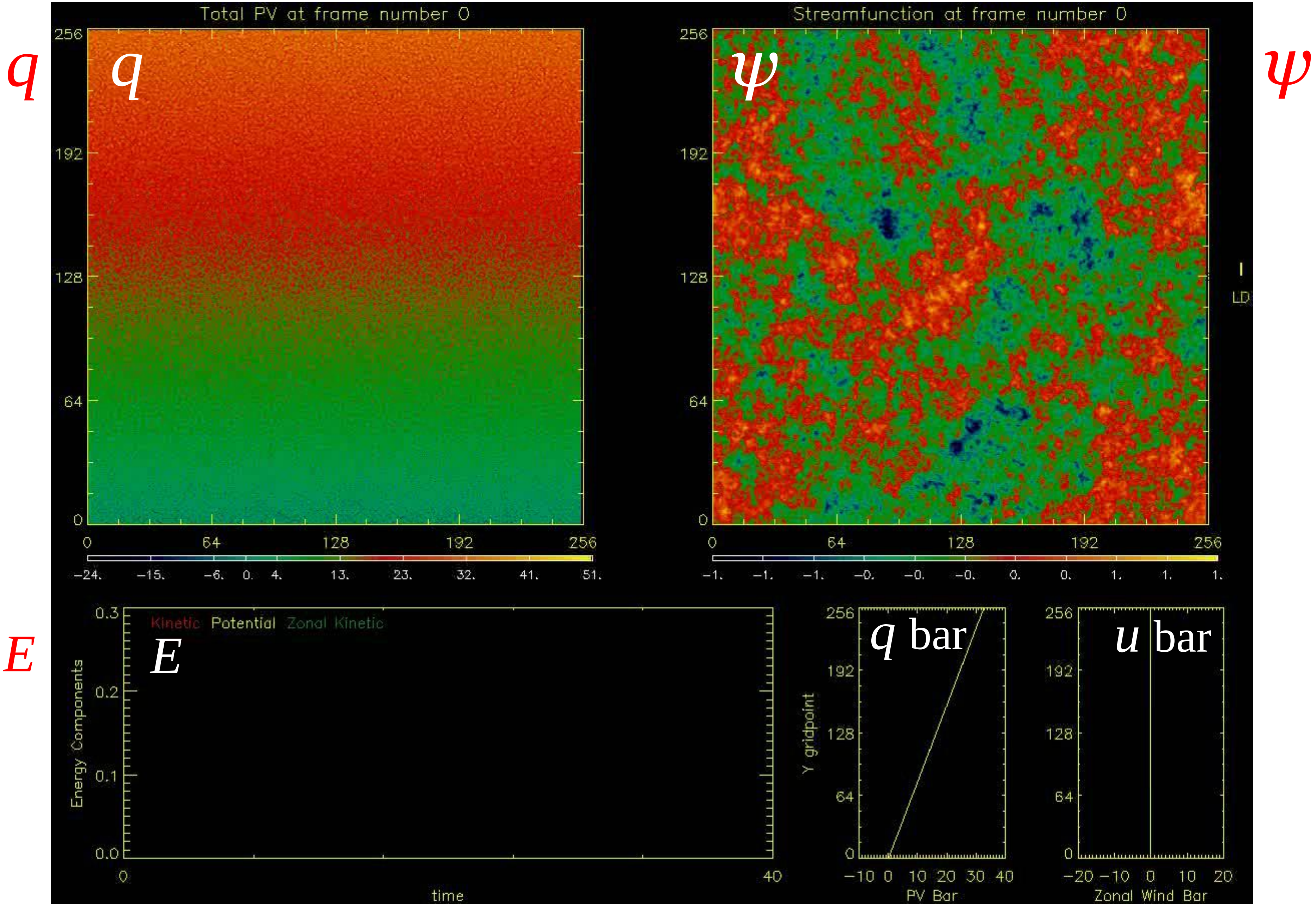
Get **predator-prey-like situation**, with **hyper-staircasing** of the $\bar{q}$ profile. (My Rosenbluth speculation was **WRONG!!**) *Not* weakly forced/dissipating, PV-mixing-dominated, à la Scott-Dritschel *JFM* 2012.


q
q
Total PV at frame number 0
256
192
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64
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256
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64
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PV Bar
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Zonal Wind Bar

“Erasmus Darwin held that every so often you should
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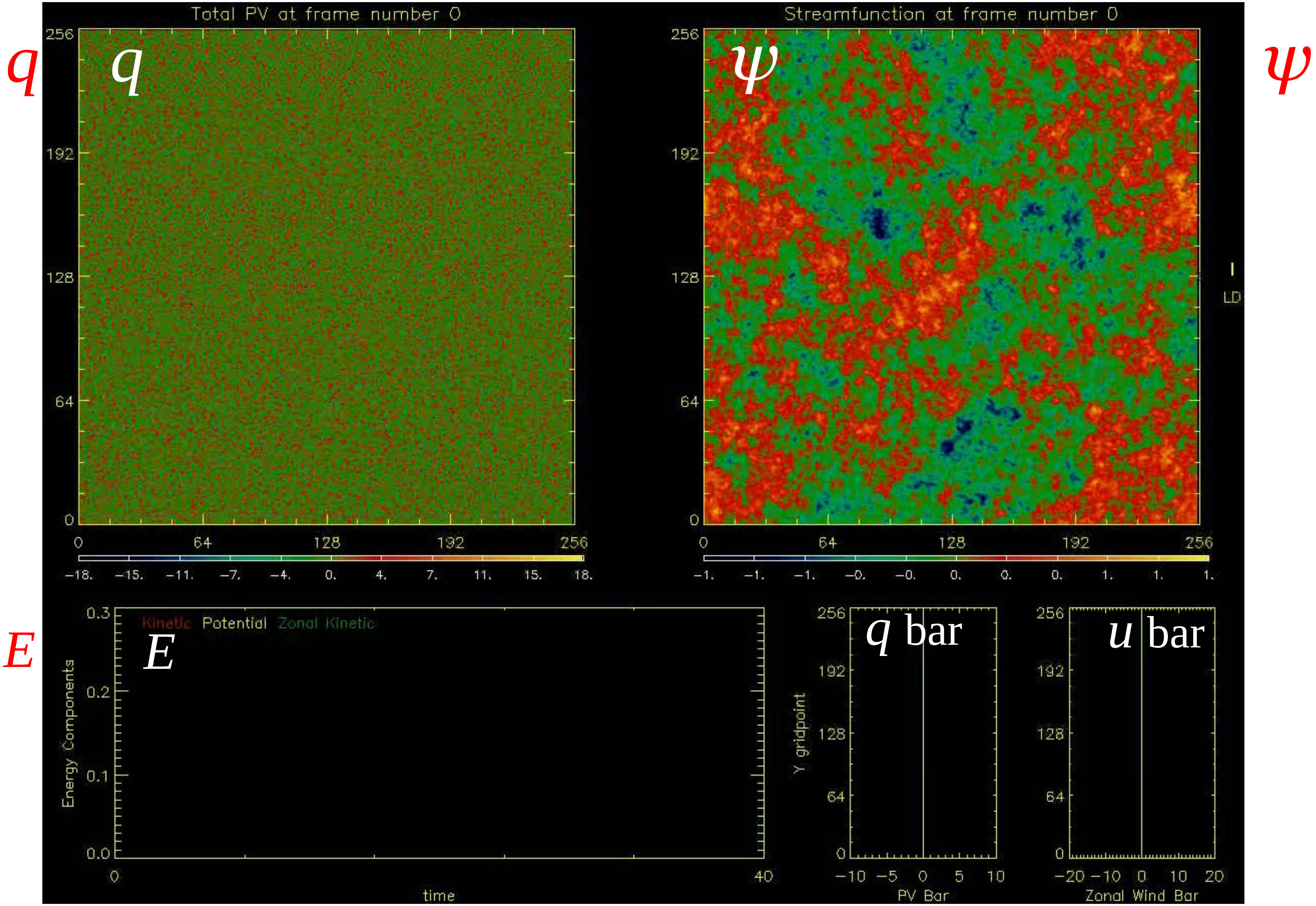

q
q
Total PV at frame number 0
256
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−15.
−11.
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Streamfunction at frame number 0
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Potential
Zonal Kinetic
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Y gridpoint
−10
−5
0
5
10
PV Bar
u bar
−20
−10
0
10
20
Zonal Wind Bar

Conclusions re new work (Jupiter and extended Hasegawa-Mima:

- Jupiter model has **no** large-scale friction and **no** radiative damping.
- Despite that, PV bias and zeta-dependence **allow statistically-steady states** – but closely tied to the deep jets. **Deep jets now essential!!**
- Other significant mechanisms in our model include monopole migration, PV mixing (esp. when background beta strong), and cyclone attrition, as well as vortex merging/cannibalism. (Relevance to **GRS** etc. ???)
- Passive Kelvin (SSST, CE2) almost **vanishingly weak** ***despite*** favourable forcing-anisotropy (Srinivasan & Young).
- EHM results robustly suggest the opposite: Kelvin mech. probably **dominant,** thanks to **perfectly unbiased self-excitation**, **but** easily **killed by shear**. Reason is the **enormously stronger and more extensive shear** arising from $k_D = 0$ zonal-mean PV inversion. ("Killed": **easy extension of Kelvin 1887**.)

For more detail, websearch **"lucidity principles"**
then back to my home page at "Encyclopedia", "Rosenbluth", "Haurwitz".