Universal models for large-scale coherent flow structures in turbulence

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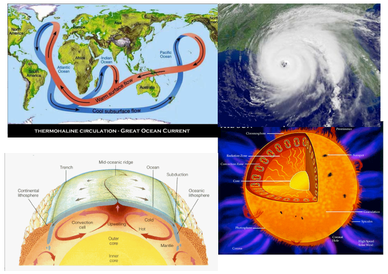
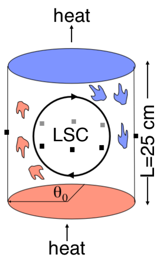
Motivation: Many turbulent astrophysical and geophysical flows are dominated by coherent structures on the largest scales, such as convection rolls in the atmosphere. A major challenge is to predict the dynamics of these largest-scale flow structures. Such structures often can be described with relatively few variables, suggesting they could potentially be described using low-dimensional models. I present an approach using empirical results for the largest-scale structures as approximate solutions to Navier-Stokes.



- Challenge: there are a wide variety of different flow structures and dynamics depending on flow geometry and boundary conditions

Model experimental system: Rayleigh-Benard convection

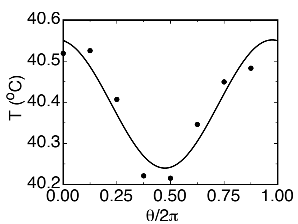


Observed Large-Scale Circulation (LSC) dynamics in a cylindrical cell:

- meandering of the orientation θ_0
- cessations of the LSC (Brown & Ahlers PRL 2005, JFM 2006)
- twisting & sloshing oscillation of the LSC structure (Brown & Ahlers JFM 2009)
- orientation aligns with asymmetries: tilt, heating, Coriolis force (Brown & Ahlers JFM 2006, PoF 2008b)
- Coriolis force causes net azimuthal rotation (Brown & Ahlers PoF 2006)

Obtaining flow structure information with sparse data

LSC parameters can be measured with an array of thermistors (black squares in diagram above)

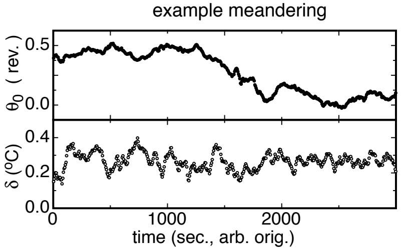


- fit LSC profile to measured temperatures to obtain dynamic parameters (i.e. θ_0):

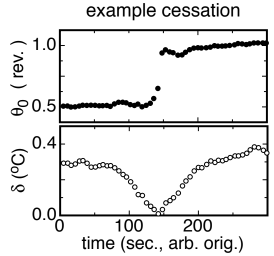
$$T = T_0 + \delta\cos(\theta_0 - \theta)$$

- The ability to make use of sparse data can be applied to field data

example meandering



example cessation

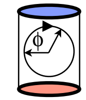


- we observe an average frequency of cessations ω_c = 1.5 per day
- orientation change during cessation is random (reversals are not preferred)

Stochastic ordinary differential equation (ODE) model

The model assumes the known mean flow structure to reduce the Navier-Stokes equations to ODEs:

Example: Navier-Stokes in ϕ-coordinate (partial differential equation):



$$\dot{u}_\phi = g\alpha(T-T_0) + \nu\nabla^2 u_\phi - \vec{u}\cdot\nabla u_\phi - \nabla P/\rho$$

plug in $u_\phi=2Ur/L$ for LSC: ➡ ODE

$$\frac{2\dot{U}}{3} = \frac{2g\alpha\delta}{3\pi} - \frac{12\nu^{1/2}U^{3/2}}{L^{3/2}}$$

U = mean velocity near boundary layer (scalar)

buoyancy; damping (boundary layer); turbulence (stochastic)

Assume $\delta \propto U$: (both characterize LSC strength)

$$\dot{\delta} = \frac{\delta}{\tau_\delta} - \frac{\delta^{3/2}}{\tau_\delta\sqrt{\delta_0}} + f_\delta(t)$$

$-\nabla_\delta V_\delta$

Equation for θ-coordinate:

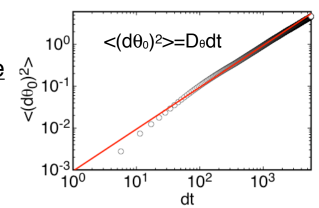
viscous damping; pressure from boundary; turbulence (stochastic)

$$\ddot{\theta}_0 = -\frac{\dot{\theta}_0}{\tau_{\dot{\theta}}} - \nabla_\theta V_g + f_{\dot{\theta}}(t)$$

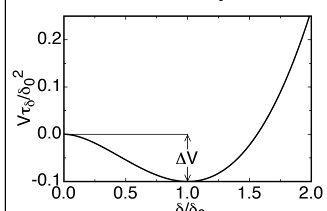
ω_ϕ = circulation frequency
δ_0 = stable fixed point of δ
τ_δ = damping timescale of δ
$\tau_{\dot{\theta}}$ = damping timescale of θ_0
V_g = boundary-dependent potential

Turbulent fluctuations are diffusive

- Stochastic terms $f_\theta(t)$ and $f_\delta(t)$ can be modeled as Gaussian white noise with diffusivity D_θ



Stochastic dynamics in a potential for δ



fixed points ($\dot{\delta}=0$)
$\delta=0$: unstable
$\delta=\delta_0$: stable

cessations:
rate of diffusion over potential barrier
frequency $\omega_c = (2\pi\tau_\delta)^{-1}\exp(-2\Delta V/D_\delta)$
predicted: 0.5 per day
measured: 1.5 per day

- cessations due to turbulent fluctuations stochastically overcoming potential barrier
- large orientation change during cessation possible because damping in θ_0 reduced as $\delta \to 0$

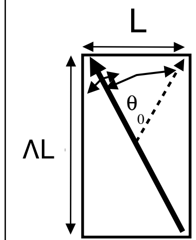
Other model results for cylindrical geometries

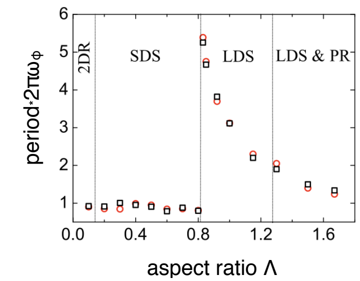
- Additional forcing terms can be added to the equations of motion to account for the effects of the Coriolis force, asymmetric heating, or tilting the cell (Brown & Ahlers PoF 2006, PoF 2008b)
- The twisting and sloshing oscillation are coupled by advection, and the slosh has a restoring force from the pressure term (Brown & Ahlers JFM 2009).

Conclusion:

A model consisting of stochastic ordinary differential equations can accurately describe several dynamical modes of convection rolls under different boundary conditions and flow geometries. This success suggests the method is promising as a general approach to simply model turbulent flows dominated by large-scale coherent flow structures.

What about different geometries? Example with a rectangular horizontal cross-section





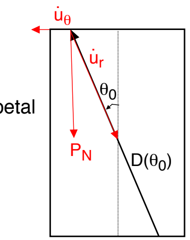
The different LSC dynamics observed in rectangular cross-section include:
- alignment of θ_0 with diagonals of container
- periodic oscillations around a corner
- SDS/LDS: oscillations from corner to corner
- 2DR: 2-dimensional rotation at extreme aspect ratios

[Song, Brown, Hawkins, & Tong, JFM 2014]

Boundary geometry can be accounted for generally with the pressure term

- pressure P_N from boundary provides both a centripetal force for LSC and an azimuthal forcing
- Potential V_g a simple function of diameter $D(\theta_0)$
- General result for any single-roll geometry!

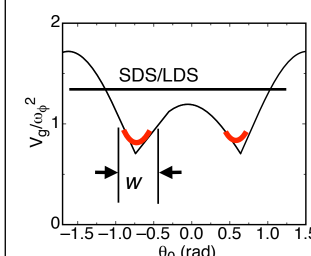
$$V_g = \frac{\omega_\phi^2 L^2}{2D(\theta_0)^2}$$

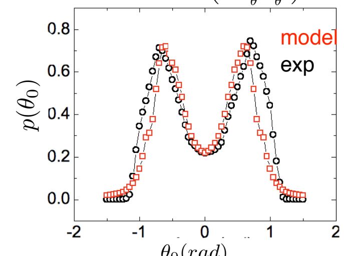


The probability distribution $p(\theta_0)$ and oscillation modes

$p(\theta_0)$ can be calculated from V_g by solving a Fokker-Planck equation

$$p(\theta_0) \propto \exp\left(-\frac{V_g(\theta_0)}{\tau_{\dot{\theta}} D_{\dot{\theta}}}\right)$$



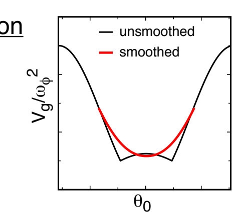


- peaks in $p(\theta_0)$ correspond to lowest potential (corners)
- averaging over width *w* of LSC smooths potential near sharp corners (red lines)
- quadratic potential near corners results in a damped driven linear harmonic oscillator equation -> drives oscillation mode around corner
- SDS/LDS: Oscillation between corners is a solution of stochastic motion in a 2-well potential for high D_θ (kinetic energy exceeds the lower potential barrier)

Transition to 2-dimensional rotation

critical aspect ratio Λ_c occurs when corners separated by $< w$
$\Rightarrow \Lambda_c = \tan\frac{w}{2} \approx 0.16$ for $w = \frac{\pi}{10}$

matches experimentally observed transition at Λ_c = 0.16



This work is supported by the Hong Kong Research Grants Council under Grant No. HKUST-604310 (P.T.) and the United States National Science Foundation CAREER program under grant no. CBET-1255541 (E.B.).