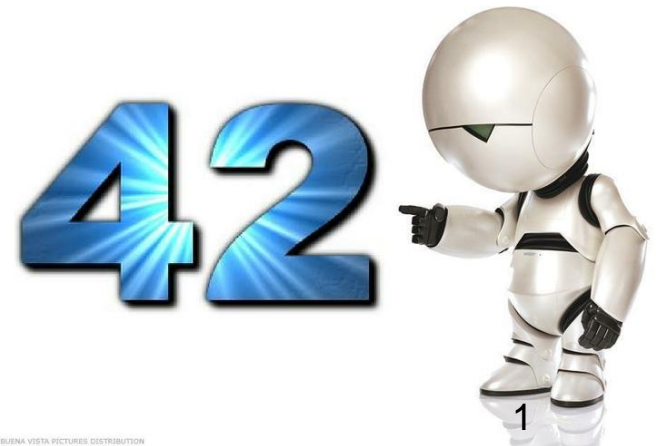


Zonal Jets

Editors: Boris Galperin and Peter Read
CUP [due to appear 2015?]

The answer to life, the Universe
and everything?



Multiple zonal jet formation in rotating, thermally-driven convection on a topographic β -plane



Peter Read

University of Oxford UK



+T. Jacoby, L.P.H.T. Rogberg, R. D. Wordsworth, Y.H. Yamazaki, K. Miki-Yamazaki, R.M.B. Young, J. Sommeria, H. Didelle, S. Viboud & B. Galperin



Motivation

- Phenomenology of anisotropic large-scale turbulence and “jets” in geophysical and planetary fluids
 - in oceans and in gas giant planet atmospheres,
- Questions:
 - (i) Why are jets prominent on gas giant planets but weak in the oceans?
 - (ii) energetics and energy flow (esp. on gas giants)?
 - large apparent $C(K_E, K_Z)$ conversion rates?
 - Relationship to more general (upscale?) energy cascades?
 - Local or non-local?
 - (iii) Potential Vorticity and configuration of zonal jets?
 - PV mixing (‘Phillips effect’)
 - PV staircases?
 - (iv) Passive tracer transport and mixing?
 - Zonation and transport barriers?

2D turbulence with rotation & sphericity

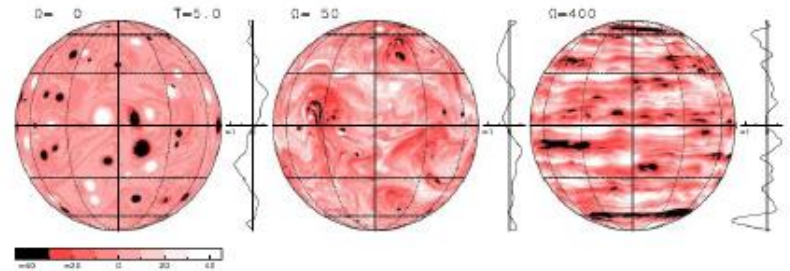
Cascade becomes anisotropic at a scale where Rossby waves become important

$$L_b \gg \sqrt[5]{e / b^3}; \text{ where } t_{RW} \gg L / U$$

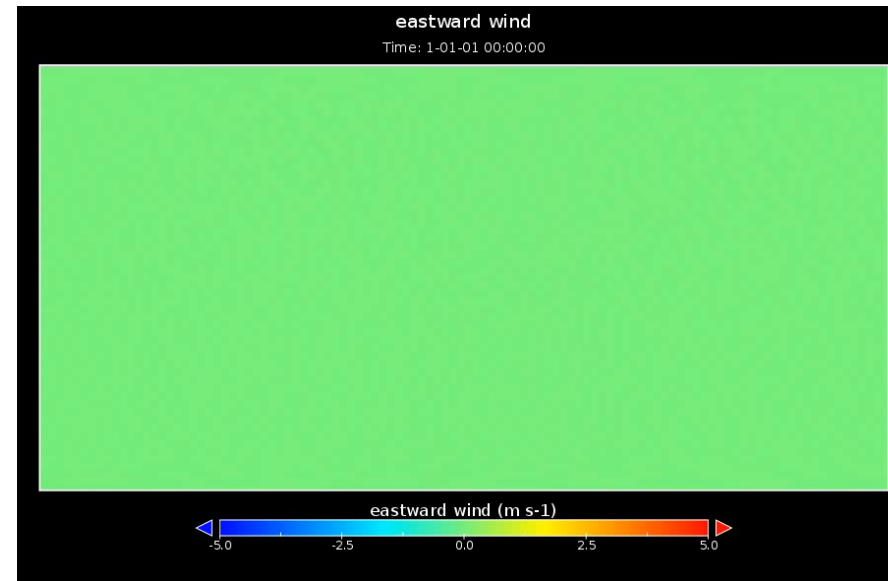
- $L < L_\beta$; Nonlinear effects dominate
- $L > L_\beta$; β -effect dominates
[Vallis & Maltrud 1993]

Kinetic energy removed at largest scales e.g. by bottom friction

$$L^3 L_{Rh} = \sqrt{2U_{rms} / b}; \text{ Rhines scale}$$



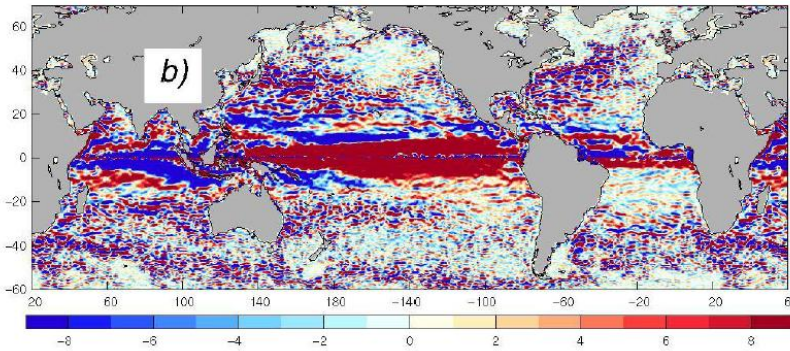
[Yoden et al., 1999]



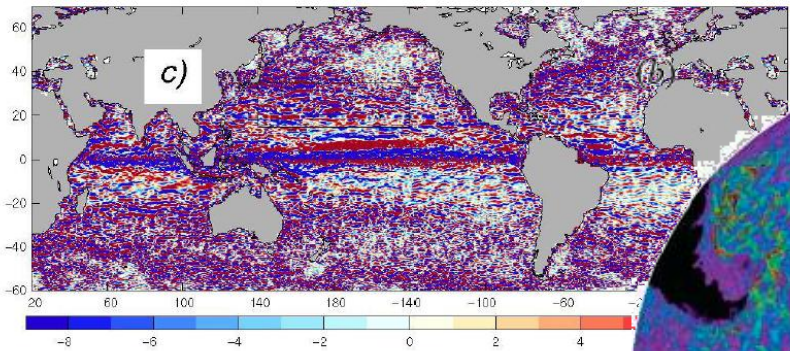
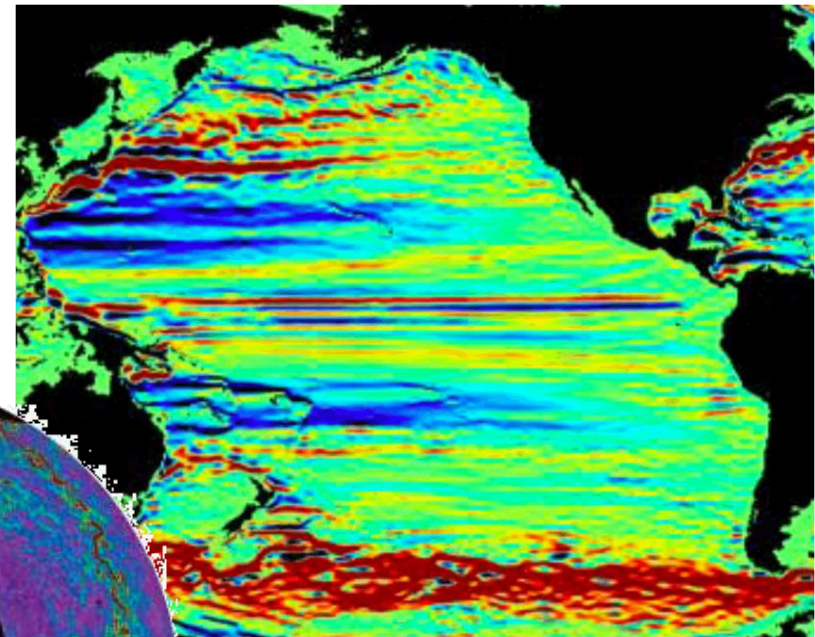
Wang & Read (2014 – in preparation)

'Zonation' in the Ocean

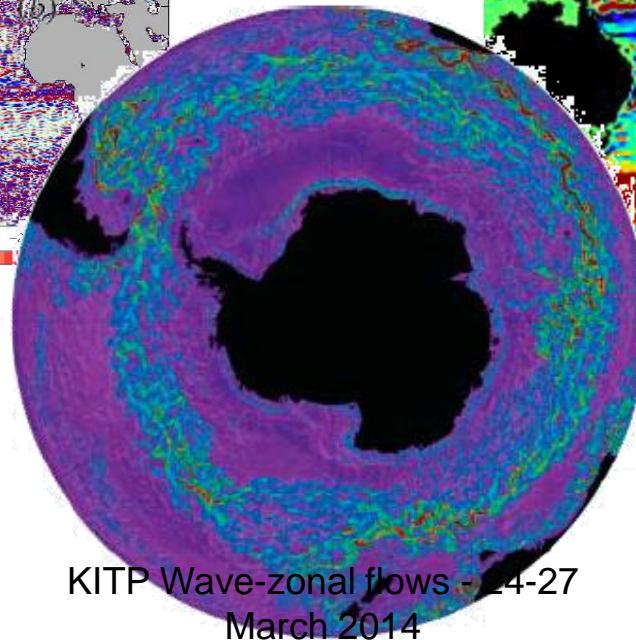
Pacific Ocean in observations & eddy-permitting numerical models



Thompson
(2008)



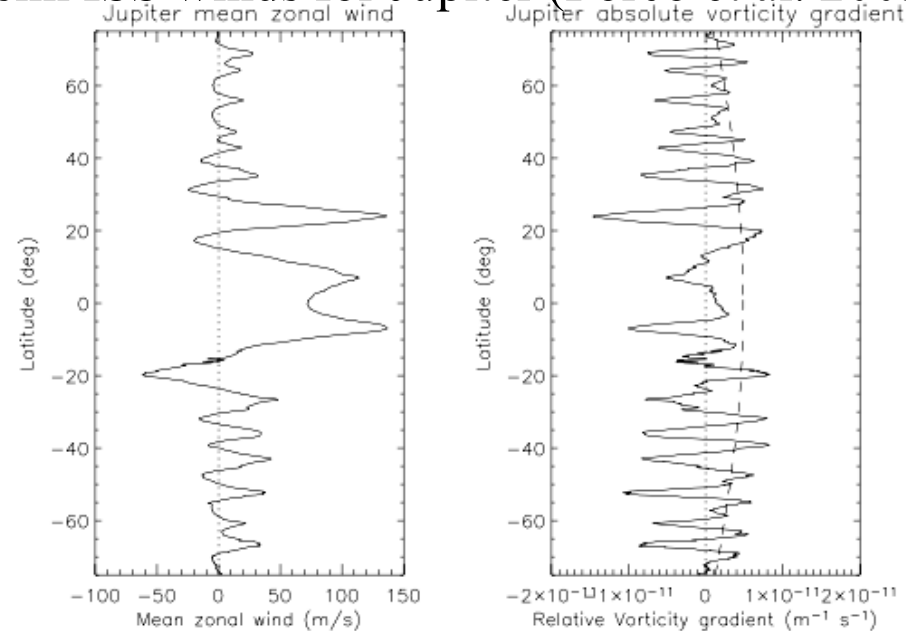
Maximenko et al.
(2005)



Richards et al.
(2006)

Planetary Zonal Jets

Cassini ISS winds for Jupiter (Porco et al. 2003)

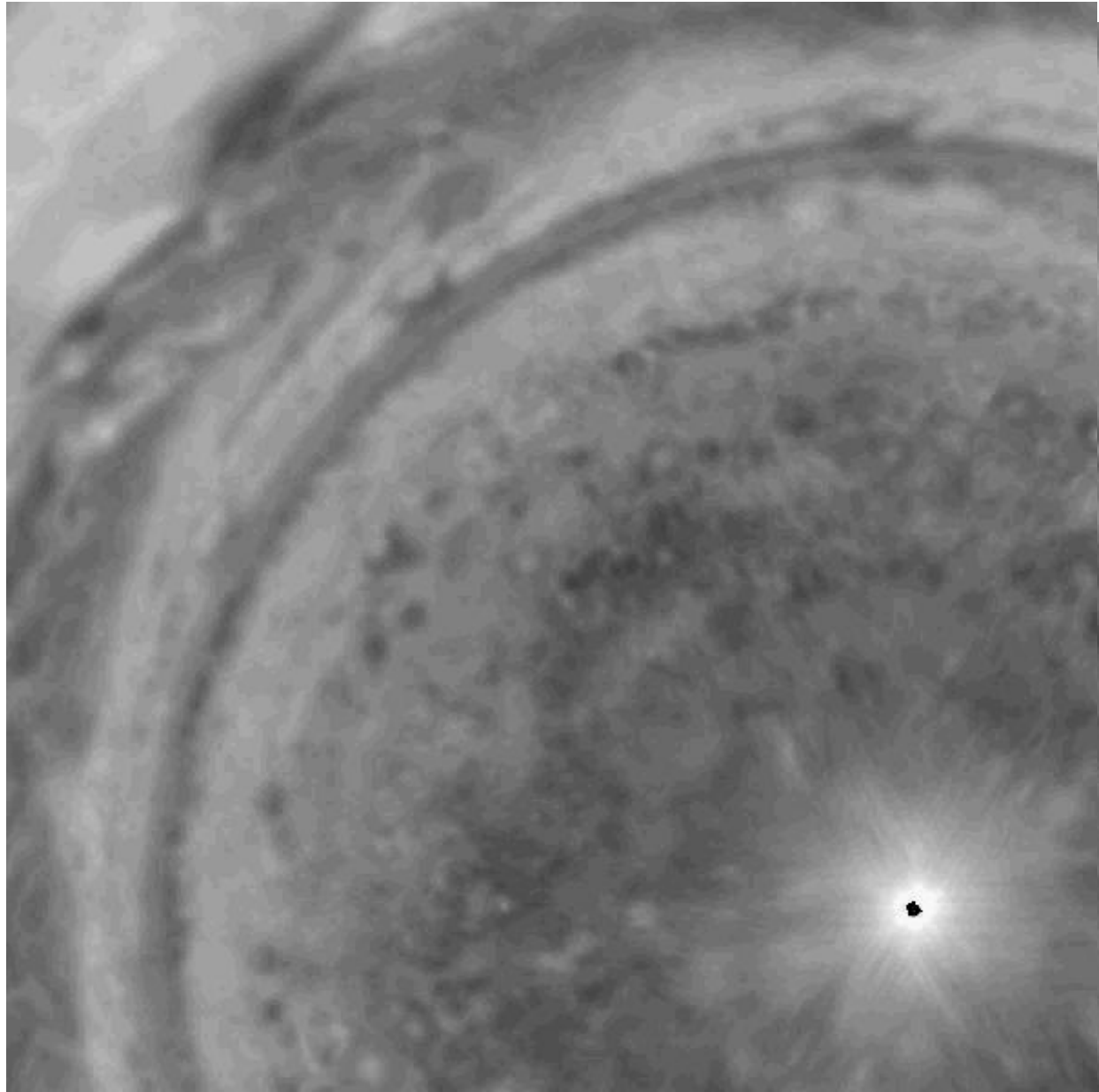


- Robust, long-lived and ~rectilinear?
- $q_y = \beta - u_{yy} < 0$ in easterly jets
- Jets maintained by horizontal eddy momentum fluxes (Reynolds stresses)?
 - $C(K_E, K_Z) \sim 3-12 \times 10^{-5} \text{ W kg}^{-1}$ [??]

Zonal jets on Jupiter (Cassini)

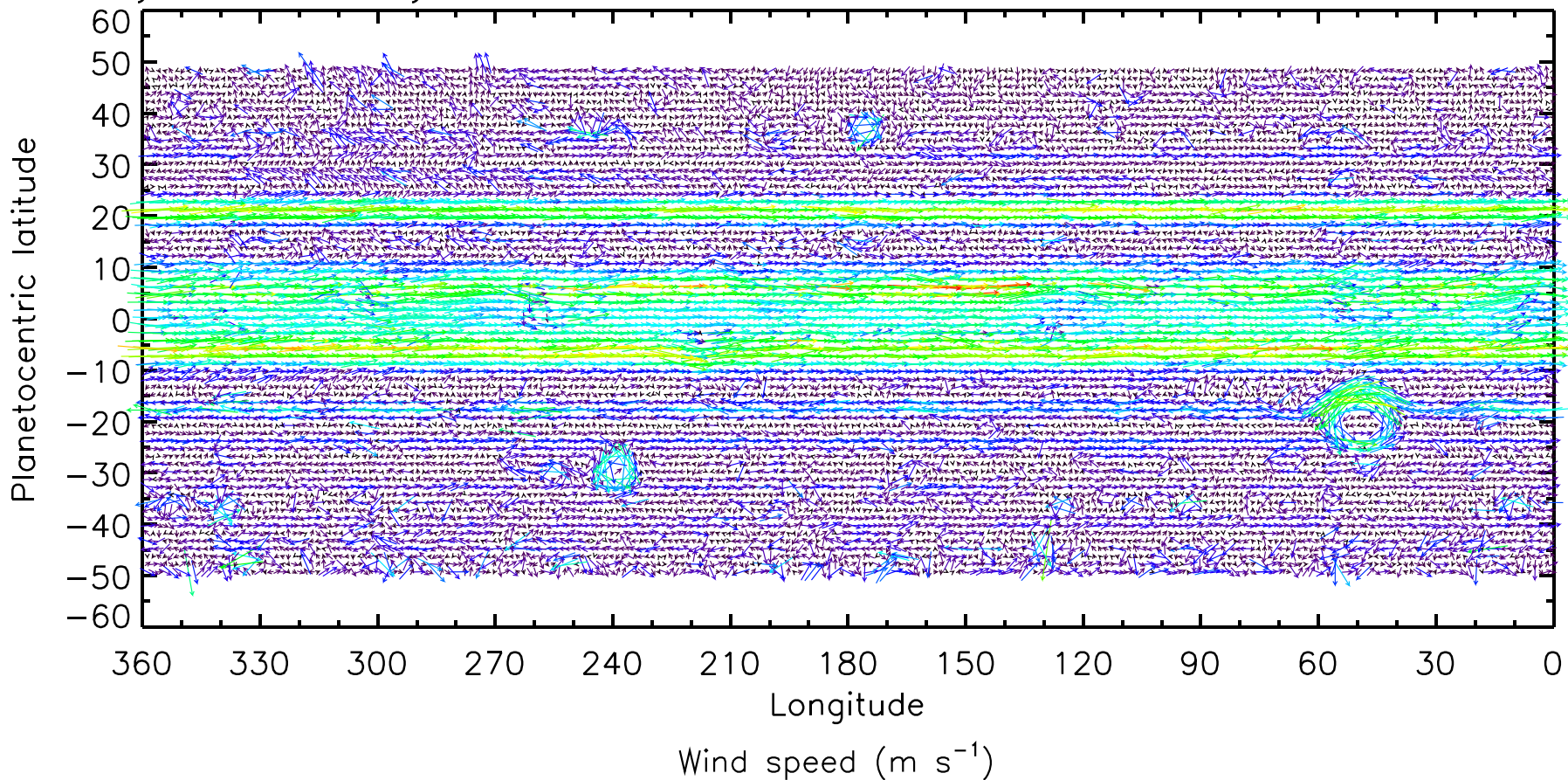
- Unique series of Jupiter images from the Cassini fly-by in December 2000
- Closest approach has resolution ~ 0.05 deg/px

*Credits:
Ashwin Vasavada
Cassini Orbiter
Imaging Team
(2001)*



Velocity fields

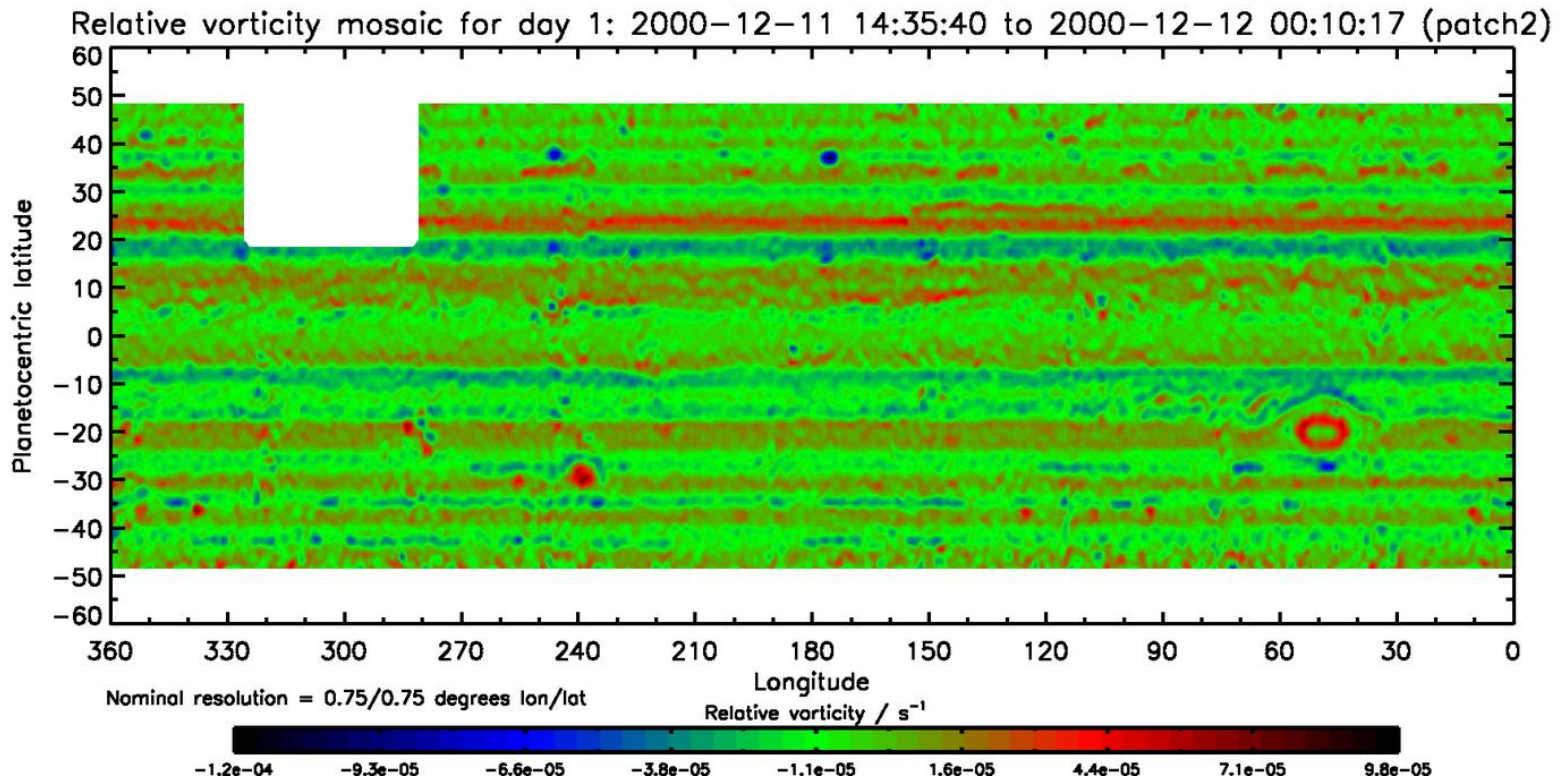
Velocity field for day 2: 2000-12-12 00:03:34 to 2000-12-12 09:38:12 (c)



0.0 23.8 47.7 71.5 95.4 119.2 143.0 166.9 190.7

Jupiter: relative vorticity

(Cassini ISS images - Galperin et al 2014)



Spectrally local vs nonlocal interactions?

- Decompose 2D KE spectrum

$E(k_x, k_y) dk_x dk_y$ into

- zonal mean

$$E_Z(n) dn = E(k_x = 0, k_y) dk_y; \quad n = [k_x^2 + k_y^2]^{1/2}$$

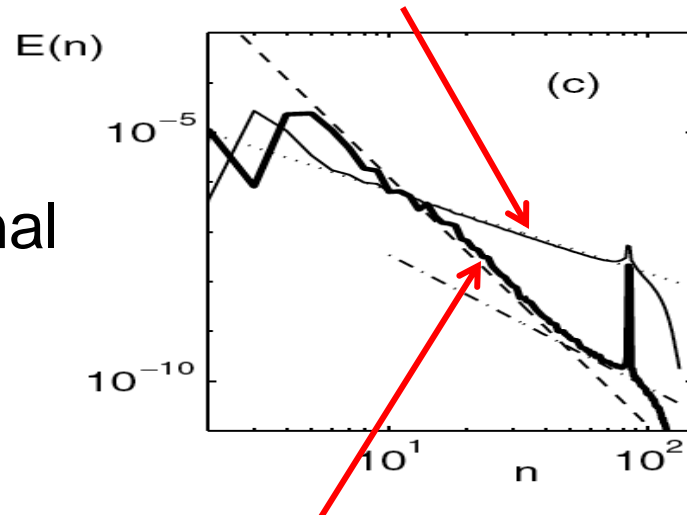
- and directionally-averaged non-zonal (residual) components

$$E_R(n) dn = \int_0^{2\pi} \int_0^{2\pi} E(k_x \neq 0, k_y) n d\theta d\phi dn :$$

such that

$$\int_0^\infty \int_0^\infty E(k_x, k_y) dk_x dk_y = \int_0^\infty [E_Z(n) + E_R(n)] dn$$

For ideal ‘zonostrophic flow’ - $E_R \sim n^{-5/3}$
[spectrally-local, isotropic inverse cascade]

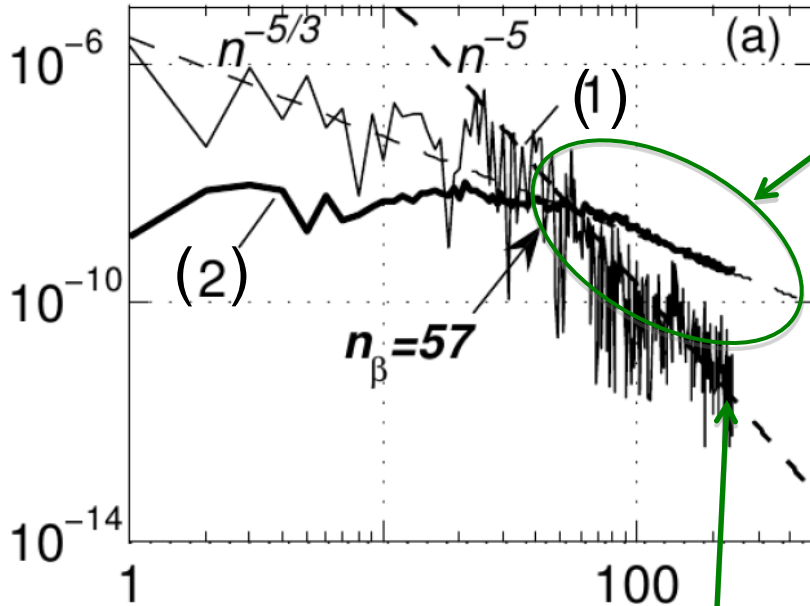


$E_Z \sim n^{-5}$
[non-local “cascade”]
e.g. Sukoriansky et al. (2002) PRL....

Jupiter's kinetic energy spectrum

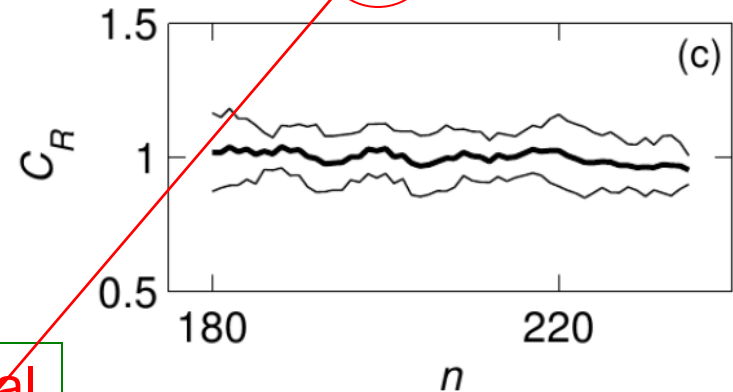
Zonostrophy index $R_\beta = L_R / L_\beta H^5$

Shallowing slope ($\sim n^{-5/3}$)
 - Upscale cascade?



$$E_Z = C_Z b^2 (n/a)^{-5} \quad (1)$$

$$E_R = C_R e^{2/3} (n/a)^{-5/3} \quad (2)$$



$n \sim n^{-5} \rightarrow$ non-local energy transfers

Thin: Zonal spectrum

Thick: Residual (eddy) spectrum

Compensated residual spectrum with $\pm\sigma$ error bars

$$e \gg 0.5 - 1 \sim 10^{-5} \text{ W kg}^{-1}$$

See Galperin et al. (2014)

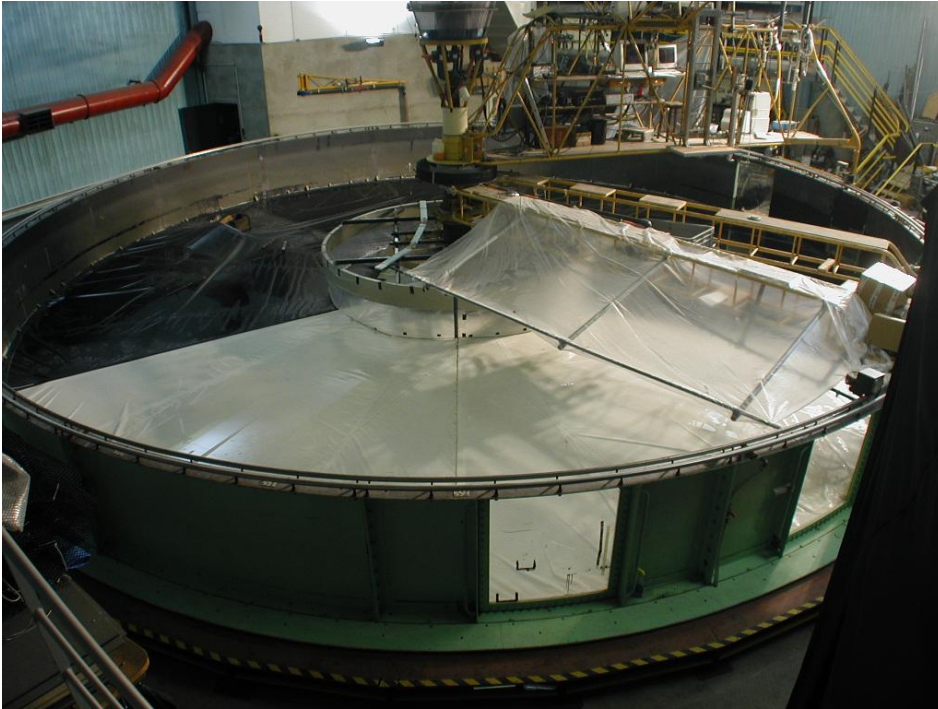
How to realise in Laboratory Experiments?

- Experimental requirements
 - Horizontal scale $L > L_{Rh} \sim \pi(2U_{rms}/\beta)^{1/2}$
 - Reynolds number $UL/\nu > 10^3$
 - Ekman number $(\nu/fD^2) \leq 10^{-5}$
 - Suitable forcing on a small scale ($\ll L$)
 - preferably not fixed in space...
 - Rapid rotation (small Rossby number)



LARGE-SCALE EXPERIMENT
(GRENOBLE 13 m dia. Rotating table)

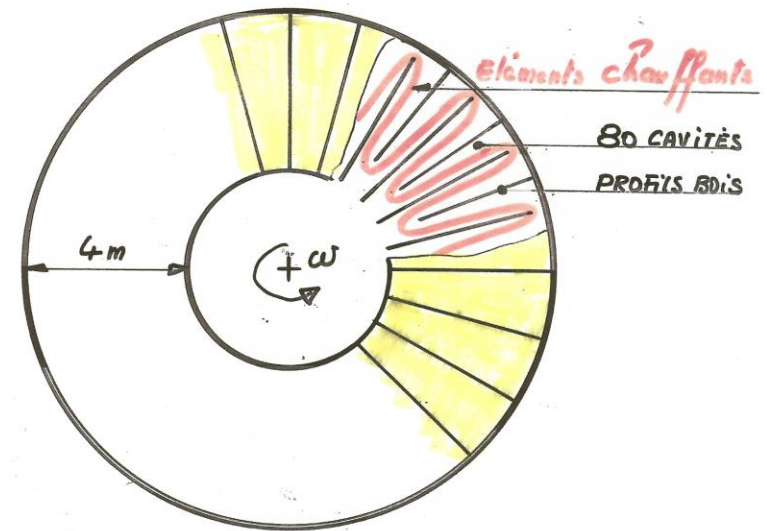
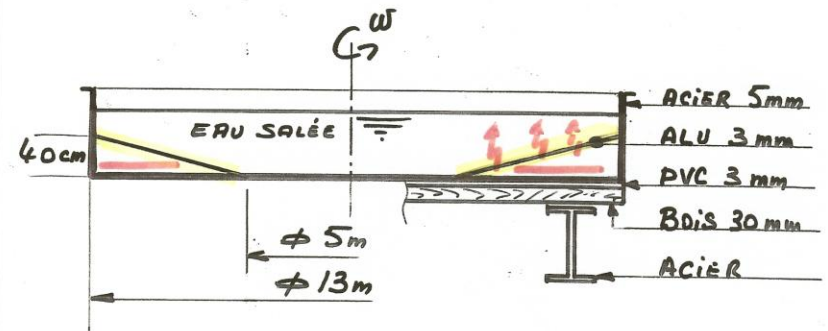
Experimental configuration 1: Salt-driven convection



- Overhead salt-water spray system
- Nozzles mounted on rotating, radial arm
- Spacing/orientation designed to supply uniform buoyancy flux
- Rotation, density and flow rate (buoyancy flux) controlled

Experimental configuration 2: THERMALLY-forced convection

- Coriolis Platform, Grenoble (France)
- ~3km of heating cable layed beneath hollow sloping bottom
- Smooth, rigid sloping bottom of segmented Al plates
- Upward slope with r at approx. 6°

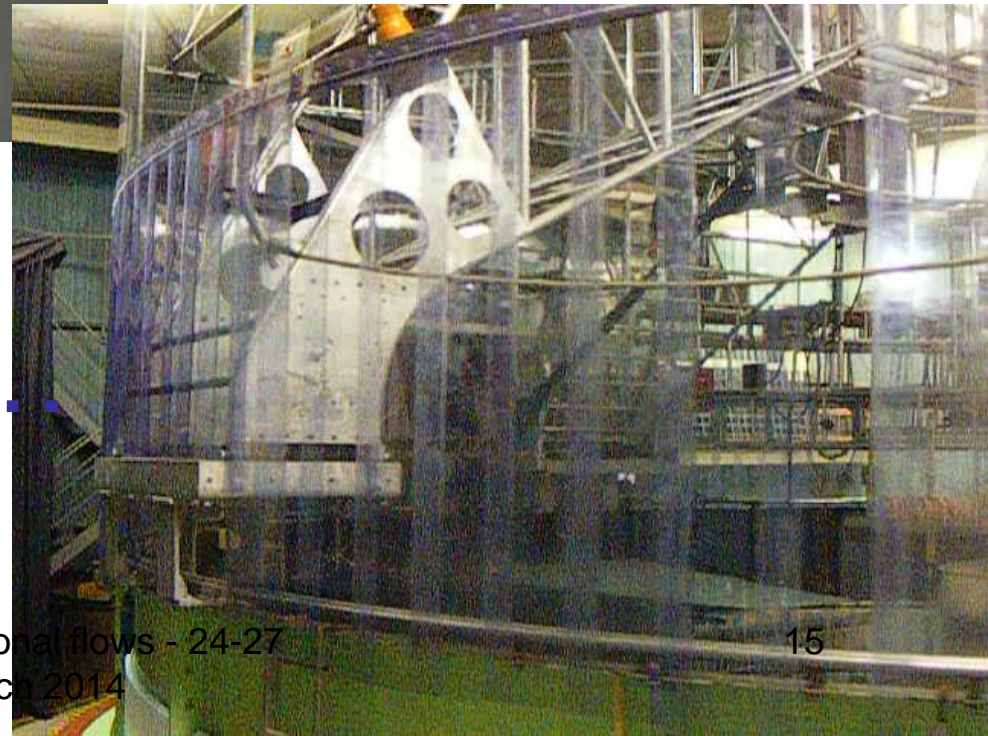


EXPERIENCE de CONVECTION FORLEE (P. Read).
LEGI. Coriolis 14 p. 5. 07

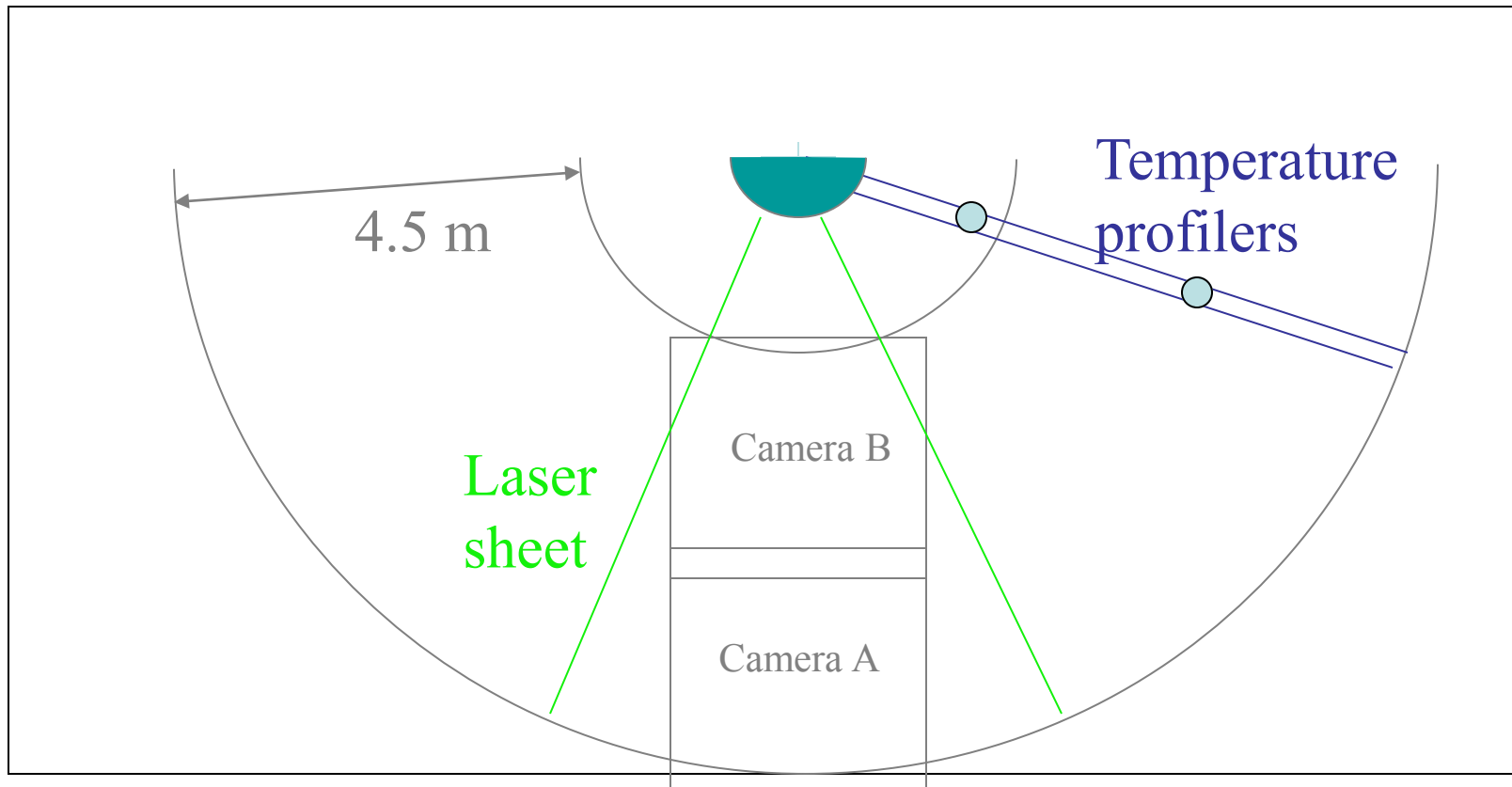
Upward-sloping bottom



Experiment running...

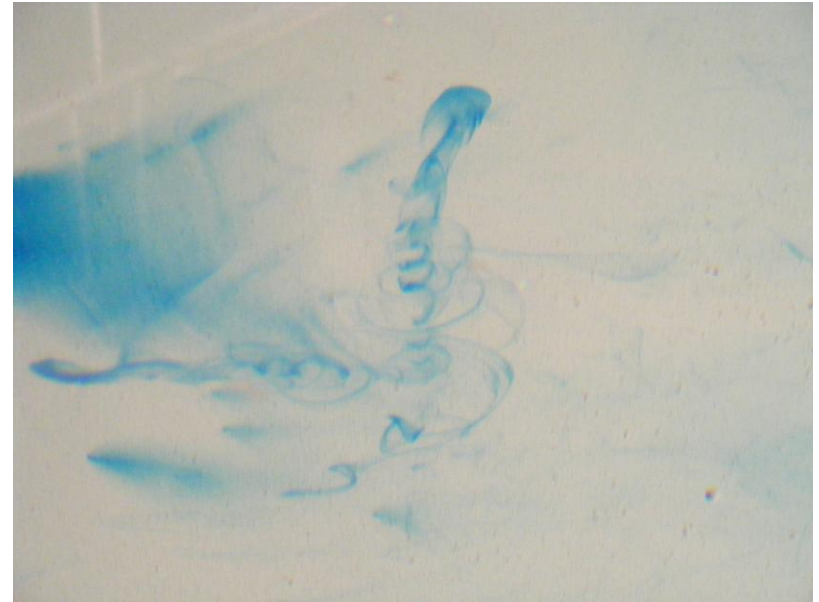


Measurement configuration



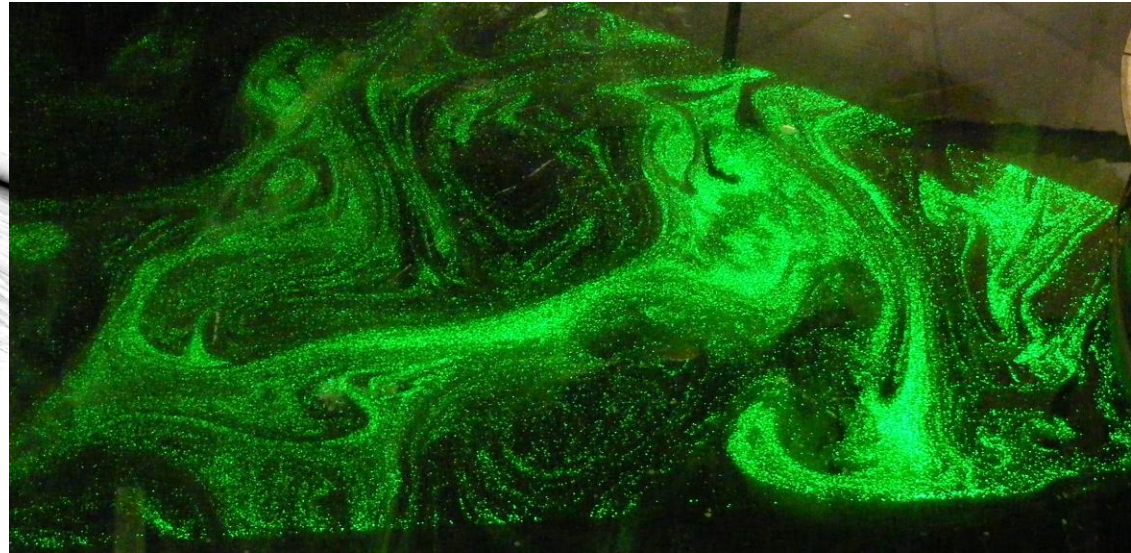
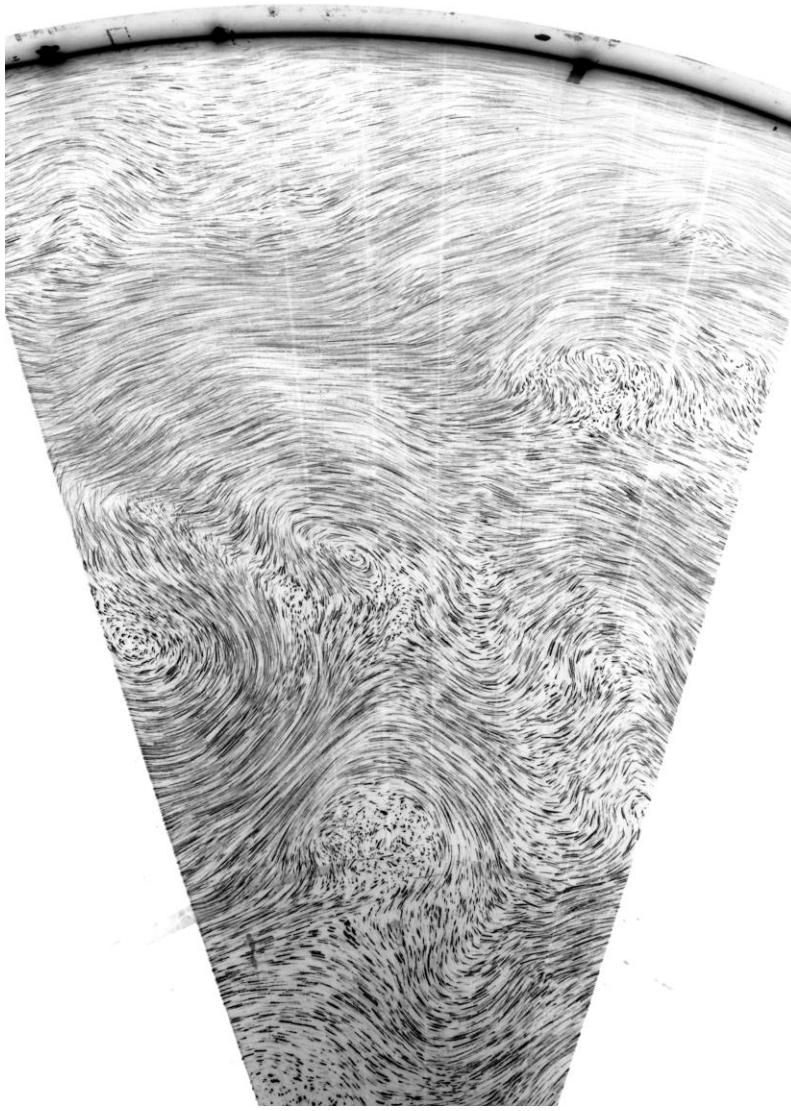
BUT views ~7% only of total area! – *keyhole diagnostics*

Small-scale convection?



- 11 kW of heating $\Rightarrow F_B \sim 5.8 \times 10^{-8} \text{ m}^2 \text{ s}^{-3}$
- Intermittent convective plumes form compact, intense cyclonic vortices
 - Around 5-50 cm diameter
 - Consistent with $l_{rot} \sim (Ro^*)^{1/2} h$; $Ro^* = (F_B / f^3 h^2)^{1/2} \sim 10^{-3} - 10^{-2}$
[Fernando et al. 1991]

Flow visualisation in laser sheet

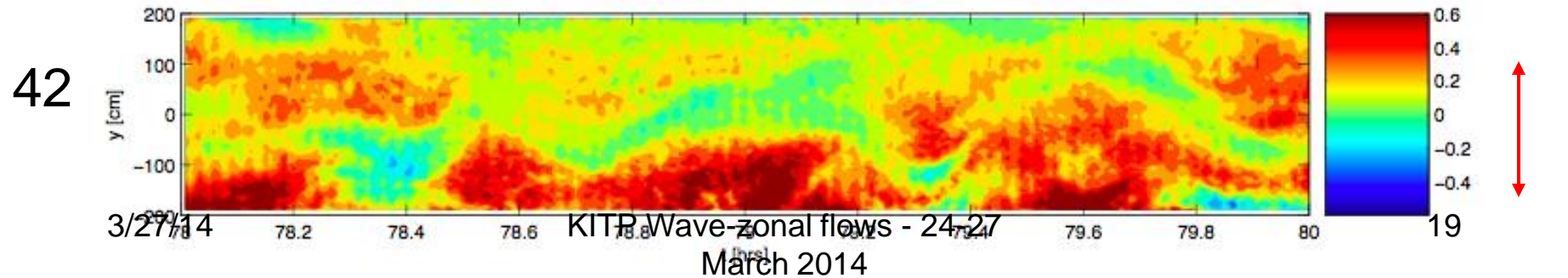
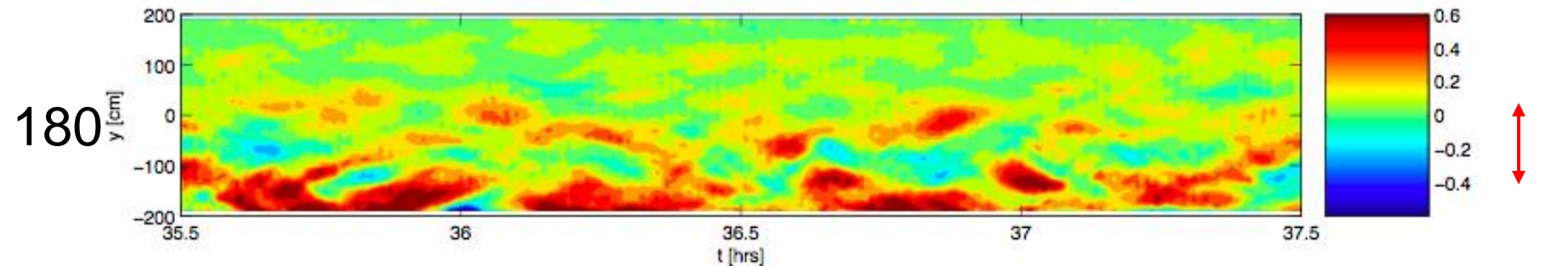
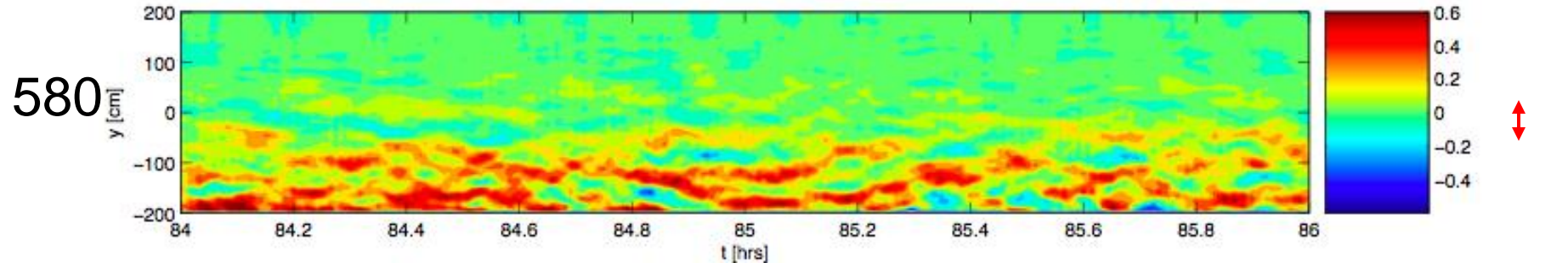


- Neutrally buoyant particles
 - Measure horizontal velocity using PIV/CIV
- Jets & vortices
- Baroclinic instabilities?

Radial scale of Azimuthal mean jets

Azimuthal mean azimuthal (zonal) velocity

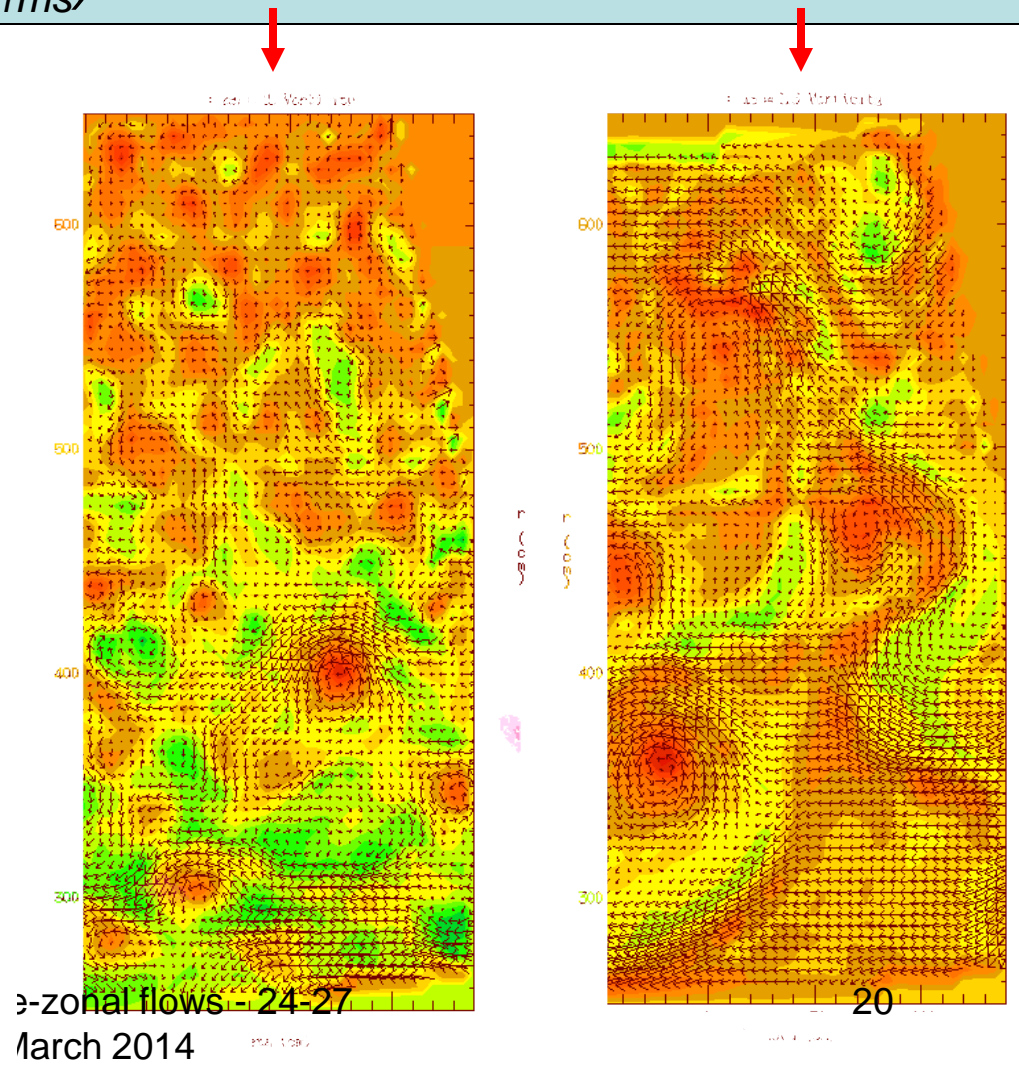
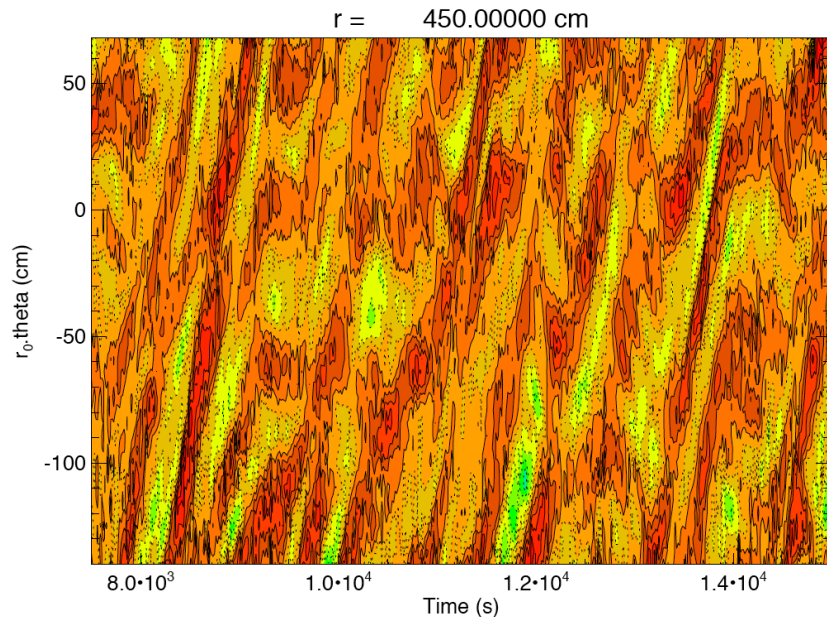
$$\beta L^2 / (2u_{rms})$$



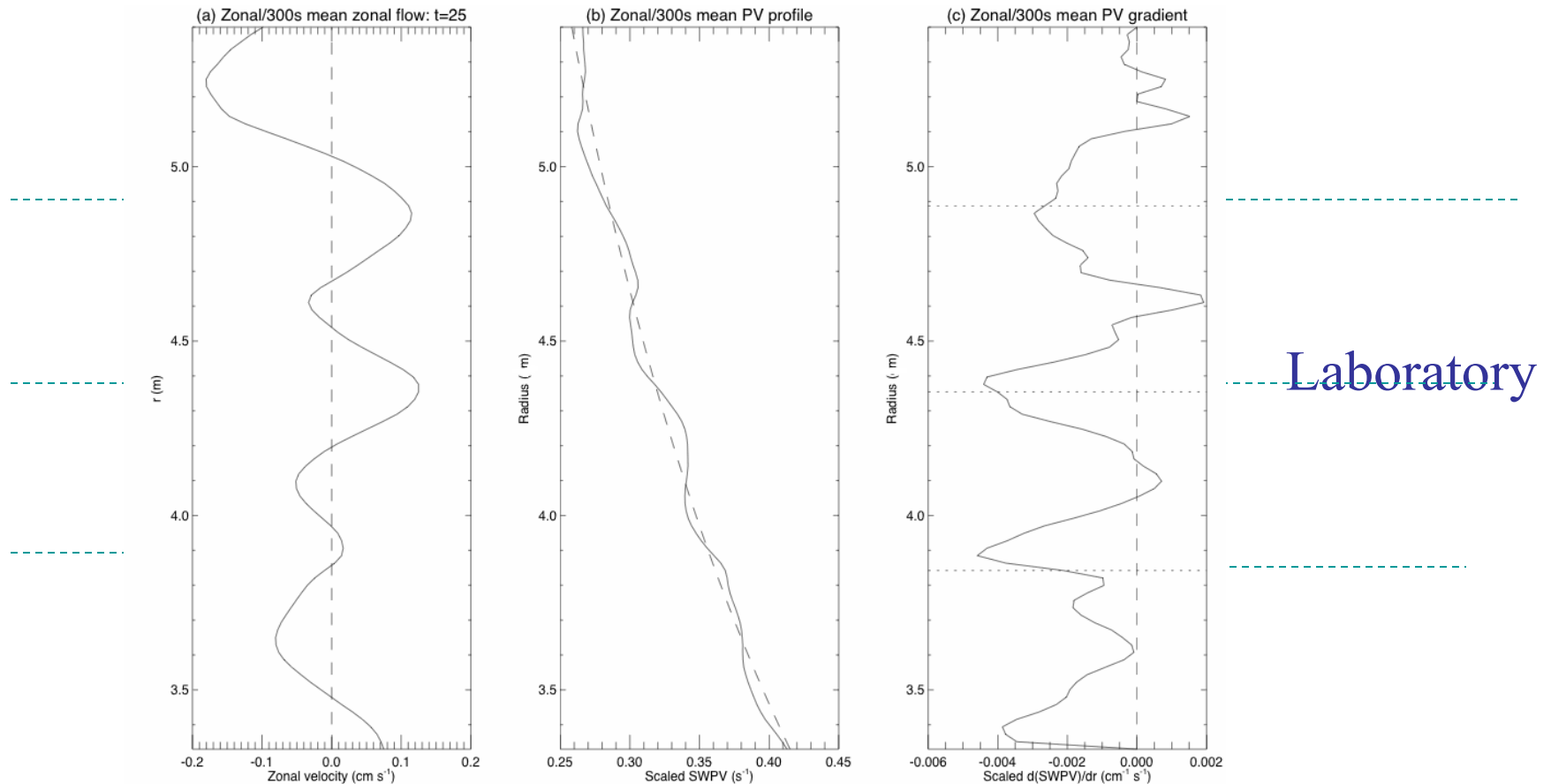
(SW) Potential Vorticity fields

E	$= 5 \times 10^{-6}$	2×10^{-5}
$\beta L^2 / (2u_{rms})$	$= 395$	91

- $q = (\zeta + 2\Omega)/h(r)$
- Complex vortex dynamics & waves
- ‘Eastward’ propagation of discrete vortices



Potential Vorticity “Staircases”:



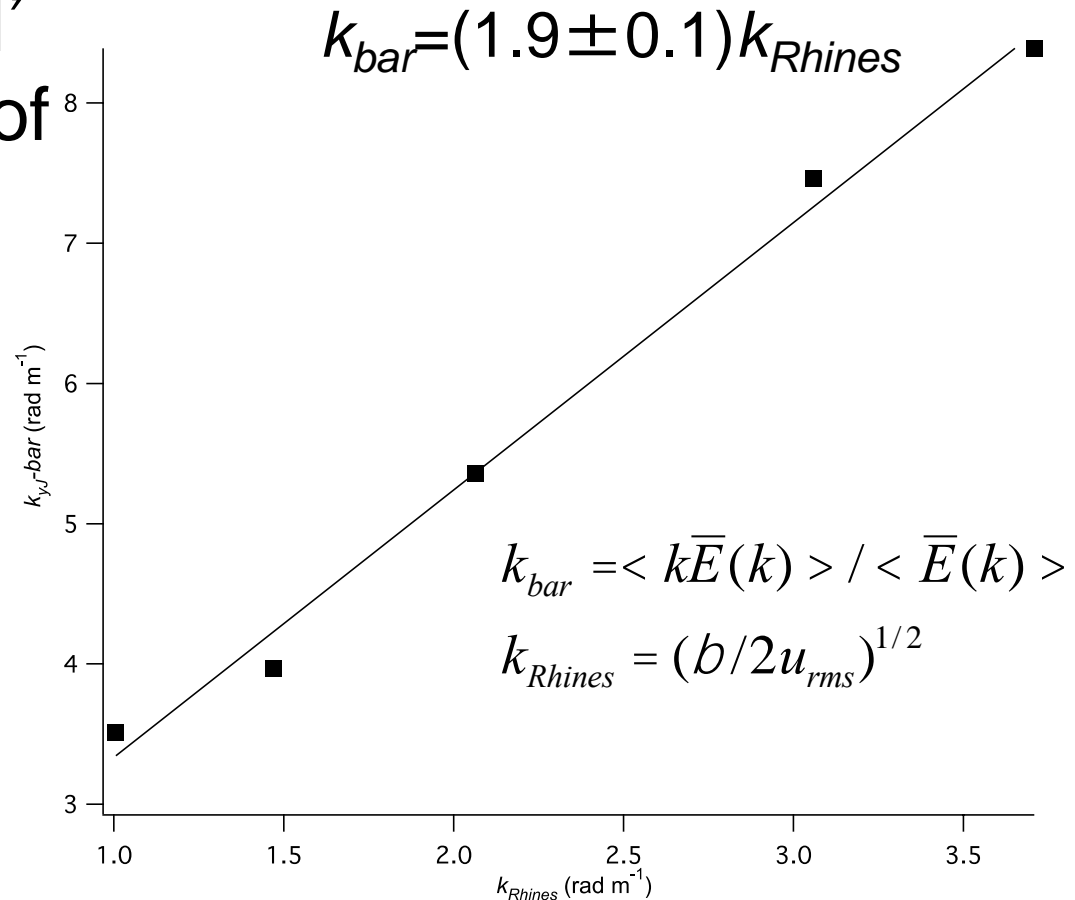
- Band-wise PV homogenisation in retrograde jets
- PV gradient still shows sustained reversals.....instability or forcing?

Scaling of jet separation

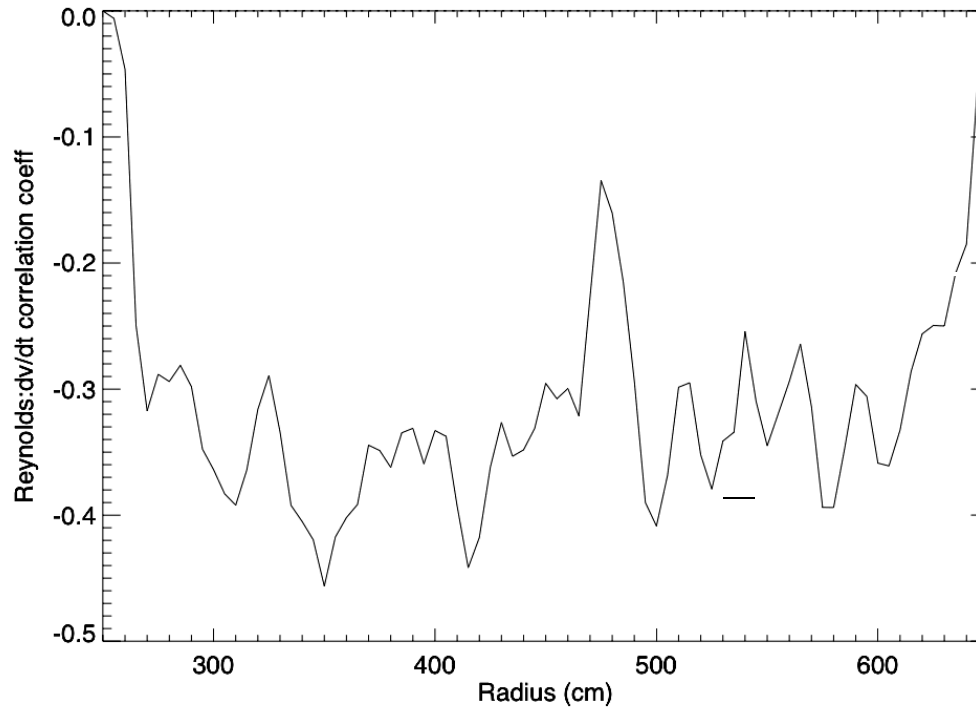
- Determine ‘centroid’ radial wavenumber of jets (k_{bar})
- ‘Pre-whiten’ zonal flow

$$\hat{u}(r) = (\bar{u} - \langle \bar{u} \rangle) \frac{\partial^3}{\partial r^3} \frac{1}{r_0}$$

- FFT to get $E(k)$
- Find k_{bar} vs k_{Rhines}

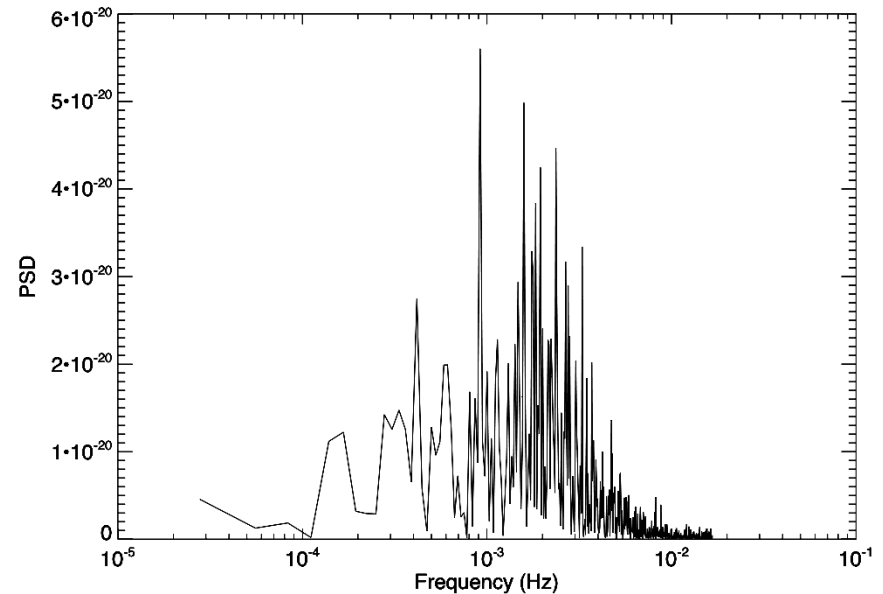
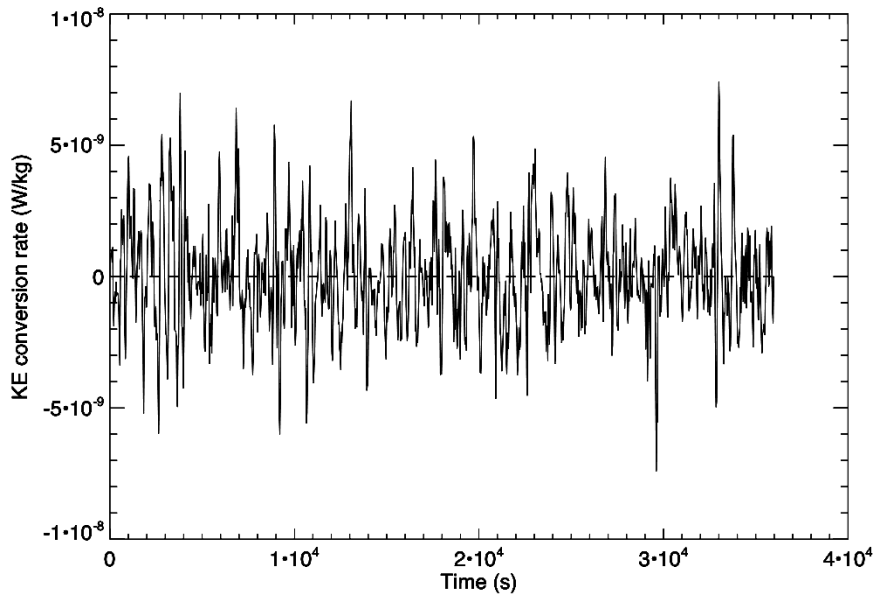


Eddy-zonal flow interactions: non-local spectral energy transfer?



- Separately compute $\overline{1/r \partial(ru'v')/\partial r}$ and $\partial \bar{u}/\partial t$
- Correlate in time
- Significant anti-correlation ($C \sim -0.4$) **across all radii:**
 - Systematic KE conversion from eddies \rightarrow mean flow

Eddy-zonal flow interactions: non-local spectral energy transfer?

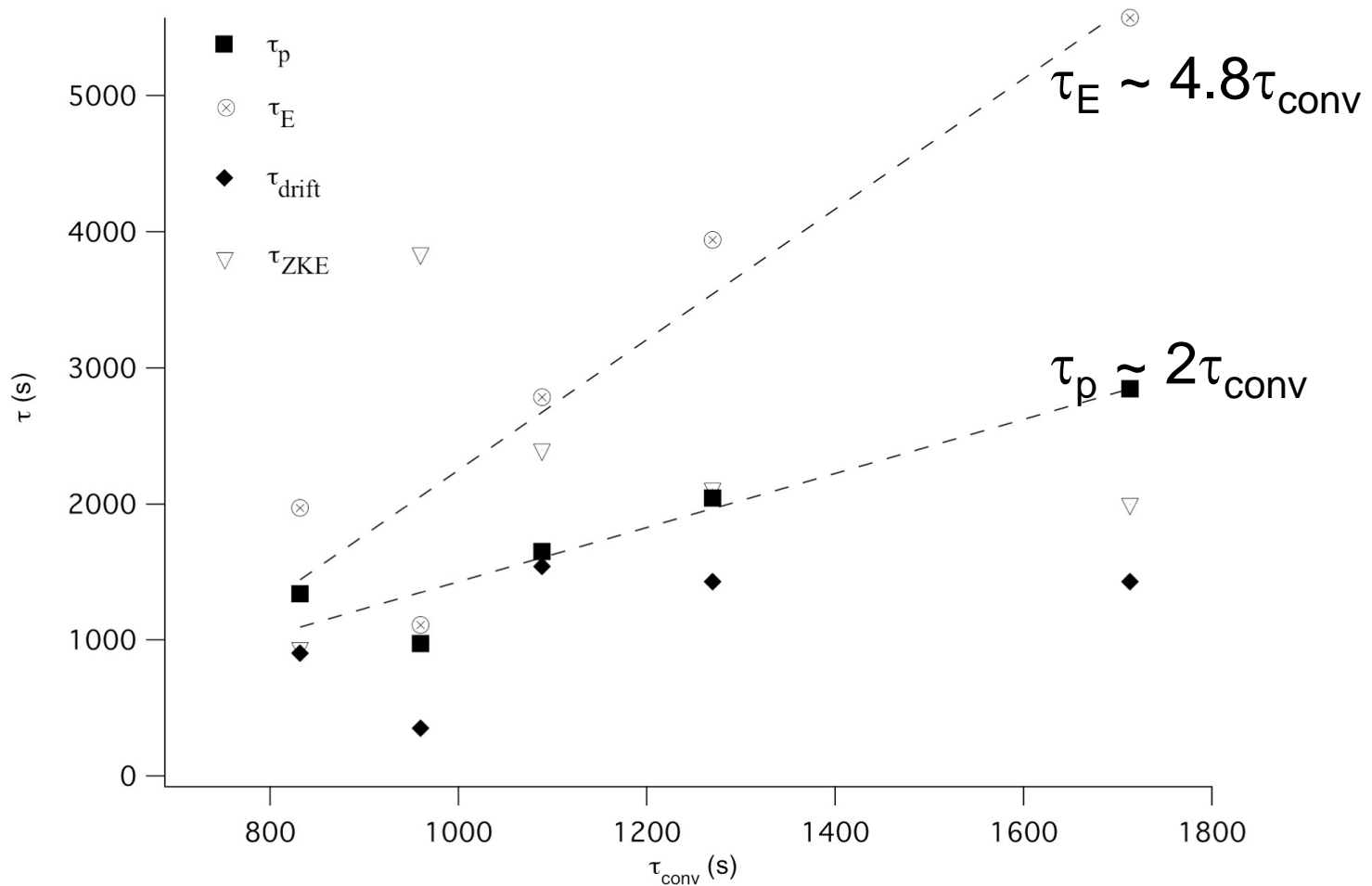


- Compute $C(K_E, K_Z)$ as a function of time -- Strongly **variable** [including its sign]
- Systematic KE conversion from eddies \rightarrow zonal mean flow [averaged in time]
 - Mean conversion rate $\sim 0.5\text{-}5 \times 10^{-10}$ W/kg ($\sim 0.1\text{-}1\%$ of F_B)
 - **Instantaneous conversion rate $\sim 10\text{-}50$ times bigger**
- Hint of a characteristic timescale/period $O(10^3 \text{ s})$
 - Cyclic decay/instability of zonal jets?

$$t_p = 1 / \sqrt{4u_{rms} b} \approx 1900 \text{ s; or}$$

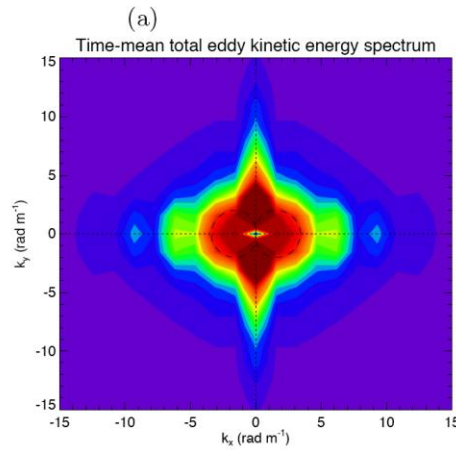
$$t_E = H / \sqrt{\overline{wN}} \approx 2000 \text{ s}$$

τ_{conv} vs timescales?

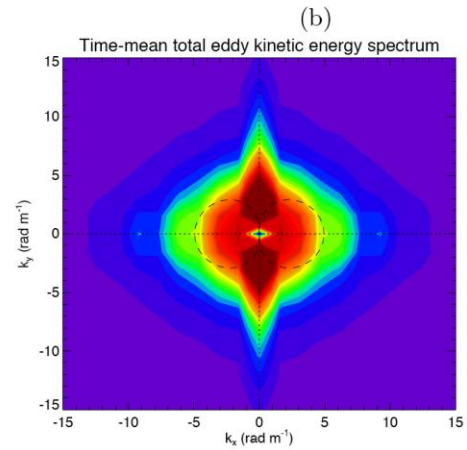


KE spectra

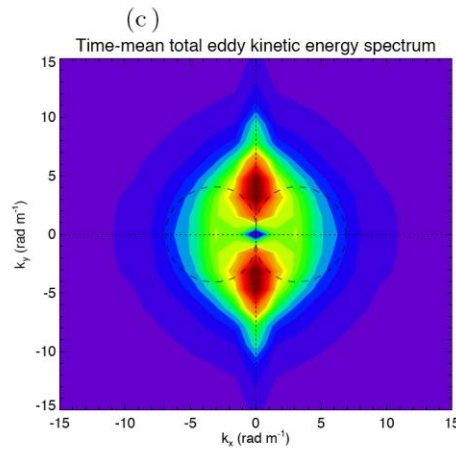
43



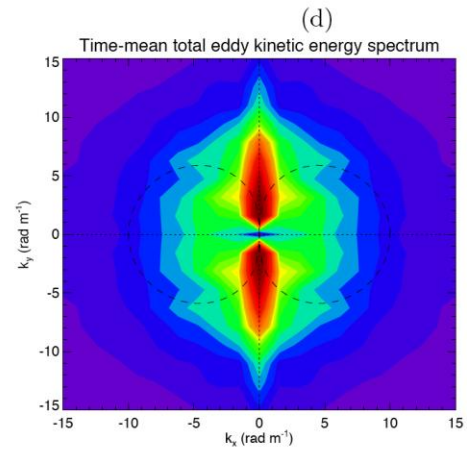
91



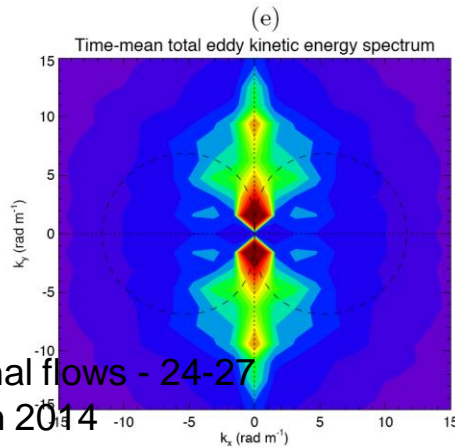
180



395



$$\frac{bL^2}{U_{rms}} = 581$$



$$E_Z = C_Z b^2 (n/a)^{-5} \quad (1)$$

$$E_R = C_R e^{2/3} (n/a)^{-5/3} \quad (2)$$

43

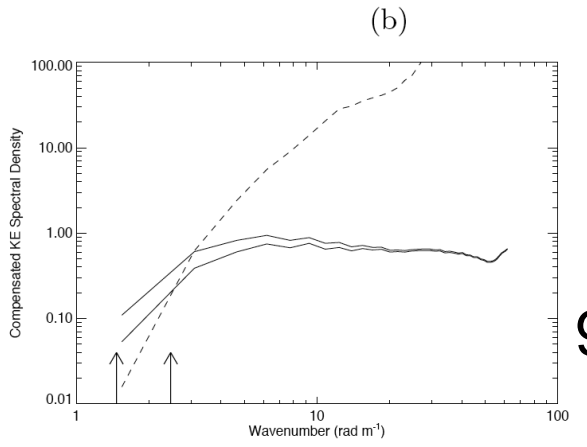
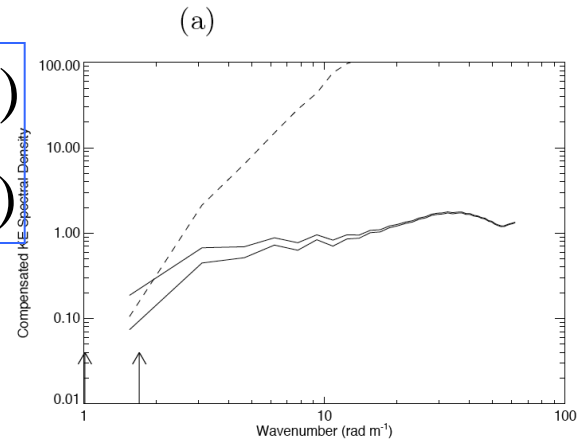
KE 180

spectra

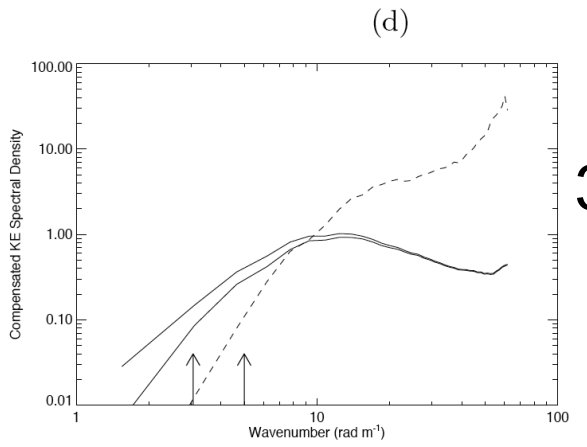
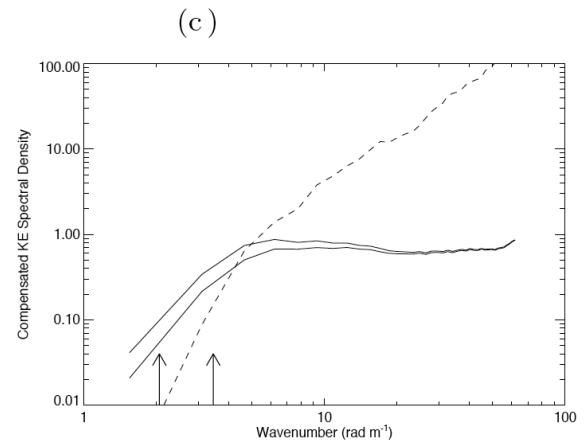
(compensated)

$$\begin{aligned} \varepsilon &\sim 5 \times 10^{-10} \text{ W kg}^{-1} \\ &= 0.2-1 \times C(K_E, K_Z) \\ &\approx 1\% \times F_B \end{aligned}$$

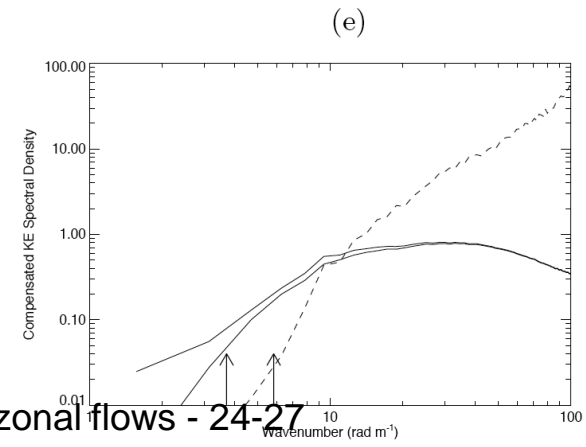
3/27/14



91



395

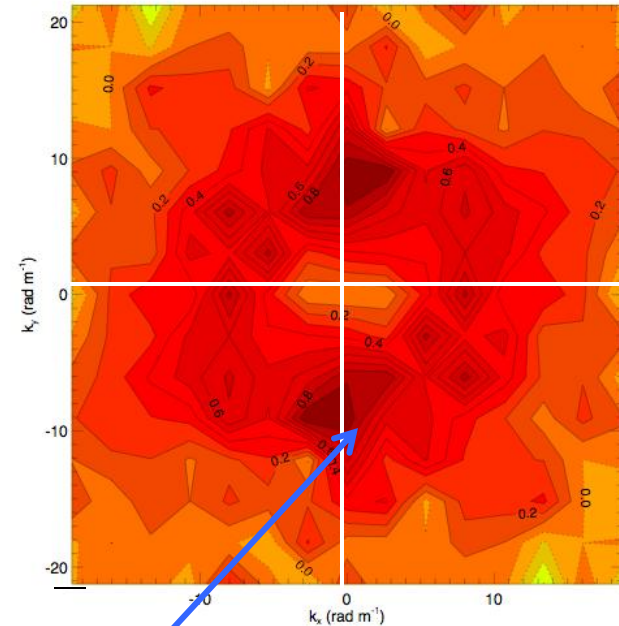
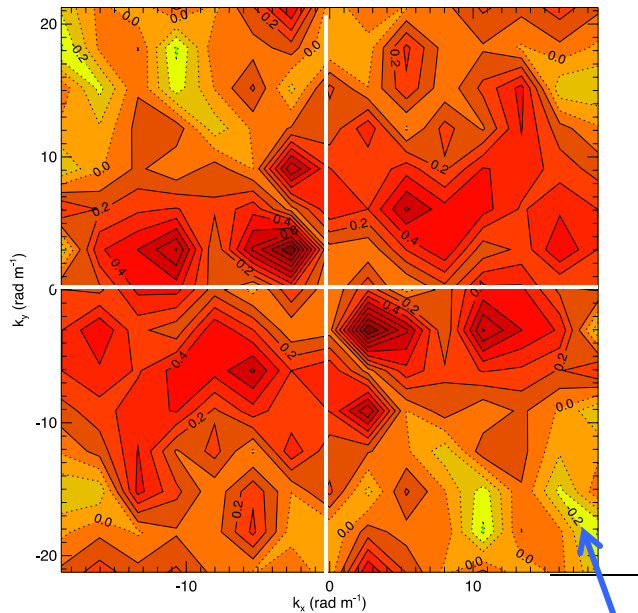


$$\frac{bL^2}{U_{rms}} = 581$$

KITP Wave-zonal flows - 24-27
March 2014

27

Eddy-zonal flow interactions: non-local spectral energy transfer?



- Spectral energy transfer function

- $T_{\Omega} = 2\pi k \operatorname{Re} \int_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \mathbf{p} \mathbf{p} \mathbf{d} \mathbf{p} \mathbf{d} \mathbf{q} / (2\pi)^2 \mathbf{p} \times \mathbf{q} / p^2 \langle \zeta(\mathbf{p}, t) \zeta(\mathbf{q}, t) \zeta(-\mathbf{k}, t) \rangle$; $(\mathbf{p}, \mathbf{q}) > |k_{\max}|$

- Peaks on k_y axis with β -effect – **NON-LOCAL transfer**

- **No such peak without** β -effect

Zonostrophy & tracer transport?

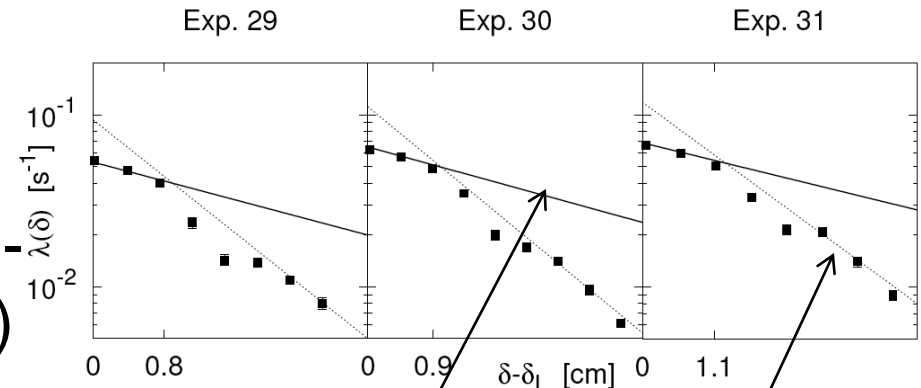
- Turbulence becomes anisotropic for $L > L_\beta$
- Tracer diffusivity (e.g. obtained from FSLEs) scale-dependent (Richardson law) for $L < L_\beta$ ($k > k_\beta$)

$$D_y = C_D e^{1/3} k^{-4/3}$$

- For $L > L_\beta$, however, D_y becomes scale-independent (Taylor law)

$$D_y = C_D^+ e^{1/3} k_b^{-4/3} = C_D^* e^{3/5} b^{-4/5}$$

FSLEs vs scale



Richardson law

Taylor law

(Laboratory experiments by Stefani Espa In Rome:

- Galperin et al. 2014 submitted)

- Break in gradient at $\sim 2L_\beta$

Zonostrophy & tracer transport?

- For $L > L_\beta$, however, D_y becomes scale-independent (Taylor law)

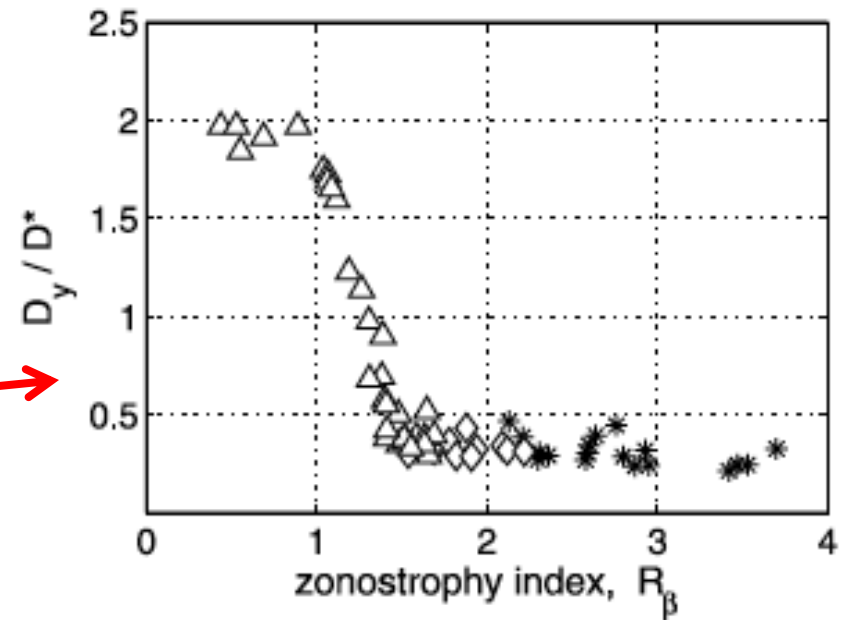
$$D_y = C_D^+ e^{1/3} k_b^{-4/3} = C_D^* e^{3/5} b^{-4/5}$$

- Large scale (Taylor) diffusivity a strong function of R_β

- Much weaker in transitional/zonostrophic regimes



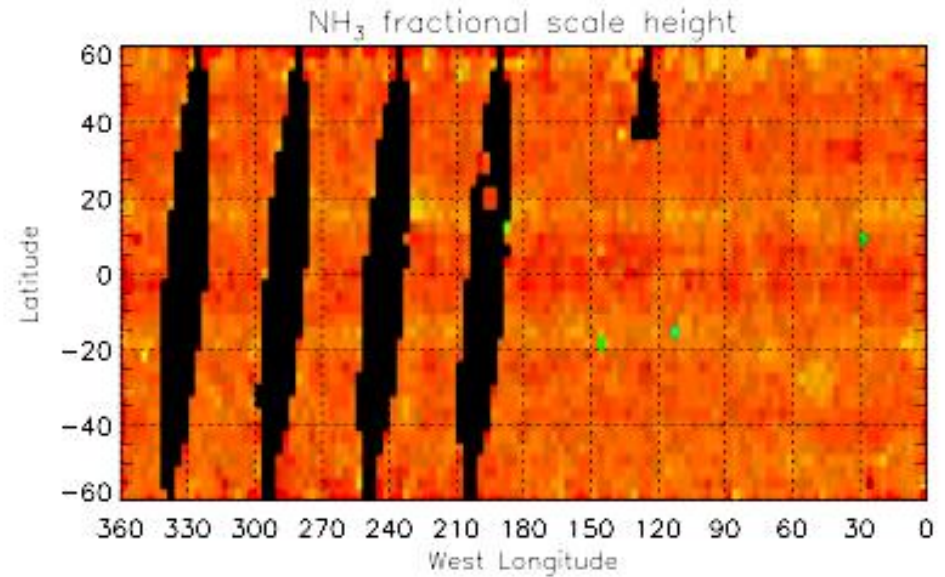
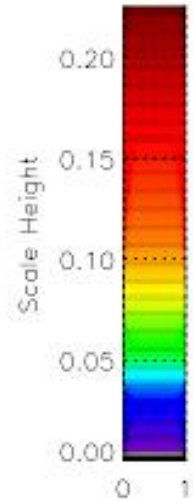
Friction-dominated | Transitional | Zonostrophic



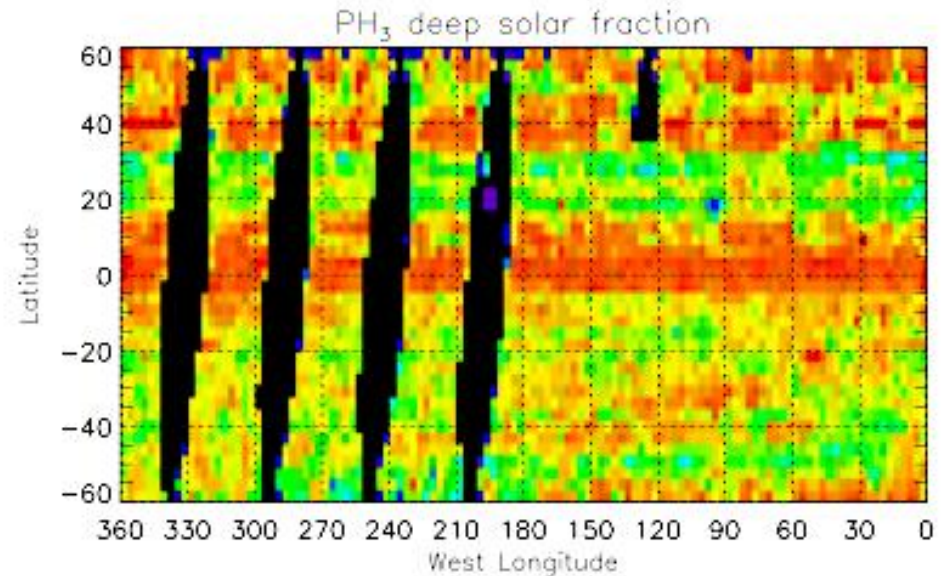
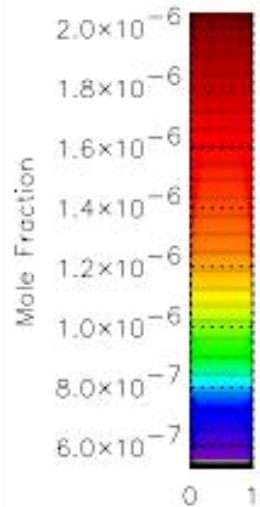
Sukoriansky et al. GRL (2009)
 - Uses barotropic vorticity equation on a sphere

Belts & Zones as transport barriers?

- Zones regions of enhanced NH_3 & PH_3 (from deep levels)
- Belts regions of weaker tracer conc.
 - (Irwin et al. 2004)

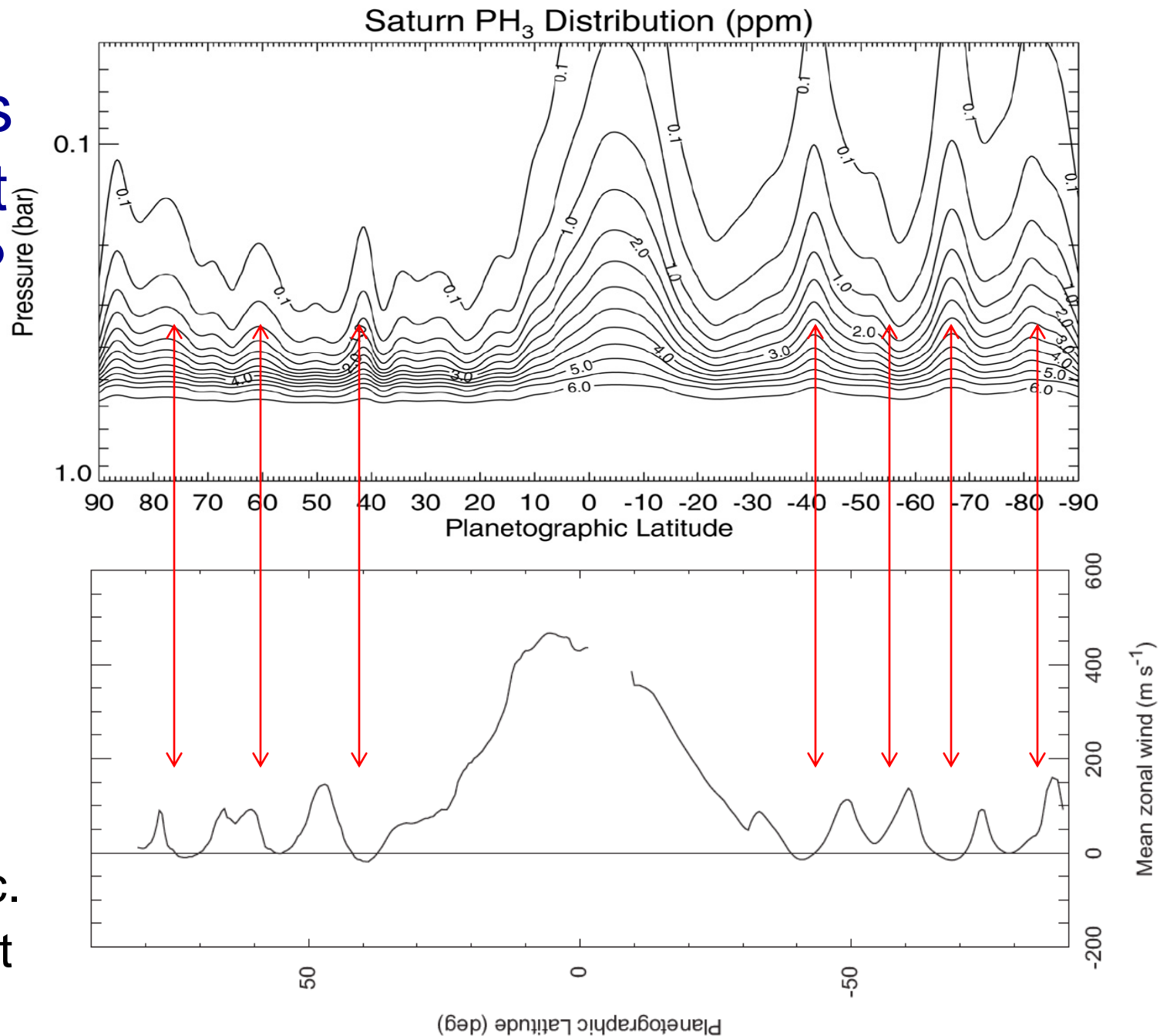


Jupiter (Cassini CIRS)



Belts & Zones as transport barriers?

- Zones regions of enhanced NH_3 & PH_3 (from deep levels)
- Belts regions of weaker tracer conc.
- (Fletcher et al. 2009)



Laboratory Experiments: the challenge of producing zonostrophic conditions?

- Experimental requirements

- Horizontal scale $L > L_{Rh} \sim \pi(2u_{rms}/\beta)^{1/2}$ i.e. $bL^2 / u_{rms} \gtrsim 100$
- AND Zonostrophy parameter $k_\beta (\sim [\beta^3/\varepsilon]^{1/5})/k_R$

$$R_b = \frac{k_b}{k_{Rh}} \gg \left(\frac{\varepsilon}{\rho} b u_{rms} t_E^2 \right)^{1/10} = [b^* \cdot Ro^2 \cdot E^{-1}]^{1/10} \gtrsim 2$$

- Translates to

$$\left(\frac{\varepsilon u_{rms} H \tan q}{\rho n} \right)^{1/3} \gtrsim 10^3 \quad \text{AND} \quad \frac{bL^2}{u_{rms}} \gtrsim 100$$

Approximating
 $\varepsilon \sim u_{rms}^2 / (2\tau_E)$

- Or $\frac{n}{WL^2 \tan^2 q} \ll 10^{-5}$ setting $\frac{bL^2}{u_{rms}} = 100$

Zonostrophic expts?

	Coriolis (Grenoble)	New Coriolis (Grenoble)	10Hz (Oxford)	100Hz (Grenoble)	Torino	Cryo He
$\nu \times 10^{-6}$ ($\text{m}^2 \text{s}^{-1}$)	1	1	1	1	1	0.02
Ω (rad/s)	0.18	0.5	60	600	2	3
L (m)	4.5	6	0.3	0.25	2.5	1.0
Θ ($^\circ$)	6	10	30	45	10	25
Ek_z	2.48E-05	1.65E-06	5.55E-07	2.66E-08	1.78E-06	3.07E-08
E	1.701	2.230	2.488	3.370	2.1	3.323

$$R_b \gg (bU_{rms} t_E^2)^{1/10}; \text{ but } U_{rms}^2 \propto \frac{F_B}{H} \text{ so } R_b \propto \frac{\epsilon [F_B H]^{1/2} \tan q \ddot{\theta}^{1/10}}{\epsilon n \theta}$$

Larger $R_\beta \Rightarrow$ deeper tank, steeper slope and/or stronger forcing (!!)

Conclusions

- Multiple-jet formation by nonlinear eddy-zonal flow processes in forced-dissipative geostrophic turbulence
 - Shows clear Rhines scaling in jet separation
 - Eddy->zonal flow energy exchanges dominate
 - Vorticity dynamics and jet stability?
 - Determines strength of jets?
 - Jets meander unless $R_\beta \square k_\beta/k_{Rhines} \square 2$
 - Mixing and transport barriers?
 - Reduced lateral dispersion in ~zonostrophic flow?
- We have [real experimental/observational] data!
 - Lab experiments [PIV velocities...]
 - Jupiter cloud winds, PV....

