Surface Wave Effects on Oceanic Fronts, Filaments, and Turbulence

Baylor Fox-Kemper (Brown Geo.)
with Jim McWilliams (UCLA), Qing Li (Brown Geo), Nobu Suzuki (Brown Geo), and Sean Haney (CU-ATOC),

Expanding on past work with:
Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College),
Adrean Webb (TUMST), Keith Julien (CU-APPM), Greg Chini (UNH),
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Kavli Institute for Theoretical Physics
Sponsors: NSF 1258907, 1245944, 0934737, NASA NNX09AF38G
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The Earth’s Climate System is forced by the Sun on a global scale (24,000km).

Next-gen. ocean climate models simulate globe to 10km: Mesoscale Ocean Large Eddy Simulations (MOLES).

Turbulence cascades to scales about 10 billion times smaller $O(1\text{mm})$. All $<10\text{km}$ is parameterized.
Air-Sea Flux Errors vs. Data

Heat capacity & mode of transport is different in A vs. O
>90% of GW is oceanic, 10m O=whole A

With nearly incompressible (small density variations) approximation & approximated rotating Earth: A set of 5 vars

Summary of Boussinesq Equations

The simple Boussinesq equations are, for an inviscid fluid:

- momentum equations: \( \frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla \phi + b\mathbf{k}, \) (B.1)
- mass conservation: \( \nabla \cdot \mathbf{v} = 0, \) (B.2)
- buoyancy equation: \( \frac{Db}{Dt} = \dot{b}. \) (B.3)

If you want, it’s easy to distinguish buoyancy into contributions from Temperature and from Salinity

Don’t blame me that they are inviscid—they are the vallis equations...
Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

Inviscid (Re\(>>1\)), rapidly rotating (Ro\(<<1\)), and thin* (L\(>>H\))

**Full Momentum**

\[
\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla \phi + bk + \nu \nabla^2 \mathbf{v}
\]

\[
Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\partial b}{\partial z} \left(\frac{\partial U}{\partial z}\right)^2 \quad \alpha = \frac{H}{L}
\]

*closely related to strong statification & ocean dimensions*
In traditional oceanography & resolved flow in IPCC models, a special distinguished limit is inhabited:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Horizontal) Geostrophic Balance

\[
\frac{Dv}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla \phi + bk + \nu \nabla^2 \mathbf{v}
\]

\[
Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\partial b}{\partial z} \left( \frac{\partial U}{\partial z} \right)^2 \quad \alpha = \frac{H}{L}
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Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:
Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Vertical) Hydrostatic Balance

\[ \frac{Dv}{Dt} + f \times v = -\nabla \phi + bk + \nu \nabla^2 v \]

\[ Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{\partial b}{\partial z} \left( \frac{\partial U}{\partial z} \right)^2 \quad \alpha = \frac{H}{L} \]

*closerly related to strong stratification & ocean dimensions
Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit: Inviscid \((\text{Re} \gg 1)\), rapidly rotating \((\text{Ro} \ll 1)\), and thin* \((L \gg H)\)

(Combined) Thermal Wind Balance

\[
f \times \frac{\partial v}{\partial z} = -\nabla b
\]

Taken together with the forcing (air-sea) of buoyancy and the advection of buoyancy by this flow--you have the tools to study large-scale ocean physics!
Dimensionless Boussinesq
Spanning Mesoscale to Stratified Turbulence
McWilliams (85)

\[ \frac{Ro}{Ri} \left[ v_{i,t} + v_{j}v_{i,j} \right] + \frac{M_{Ro}}{Ri} wv_{i,z} + \epsilon_{ij}v_{j} = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,ij} \]

\[ \frac{\alpha^2}{Ri} \left[ w_{,t} + v_{j}w_{,j} + \frac{M_{Ro}}{Ri} w_{,z} \right] = -\pi_{,z} + b + \frac{\alpha^2}{ReRi} w_{,ij} \]

\[ b_{,t} + v_{j}b_{,j} + \frac{M_{Ro}}{Ri} wb_{,z} + w = 0 \]

\[ v_{j,j} + \frac{M_{Ro}}{Ri} w_{,z} = 0 \]

Plus boundary conditions

\[ Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U,z)^2} \quad \alpha = \frac{H}{L} \]

\[ M_{Ro} = \max(1, Ro) \quad \nu = \text{horiz. vel.} \quad w = \text{vert. vel.} \]
Let's see some examples of Bousinesq, Hydrostatic Models at work in the mesoscale (10–100km) & submesoscale (100m–10km)
Big, Deep (mesoscale) w/ Little, Shallow (submesoscale)

Note mixed layer heat capacity!

Big, Deep (mesoscale) w/ Little, Shallow (submeso)!

Note mixed layer heat capacity!

The Character of the Mesoscale

- Boundary Currents
- Eddies
- $Ro = O(0.1)$
- $Ri = O(1000)$
- Full Depth (4km)
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly baroclinic & barotropic instability. Parameterizations of baroclinic instability (GM, Visbeck...), will be routinely resolved in climate models in 2040.
The Character of the Submesoscale

(Capet et al., 2008)

- Fronts
- Eddies
- $\text{Ro} = O(1)$
- $\text{Ri} = O(1)$
- near-surface ($H=100\text{m}$)
- 1–10km, days

Eddy processes often baroclinic instability

Parameterizations = F-K et al (08-11).

Routinely resolved in 2100


The Character of the Submesoscale

(Capet et al., 2008)

- Fronts
- Eddies
- $R_o = O(1)$
- $R_i = O(1)$
- near-surface $(H=100m)$
- 1–10km, days

Eddy processes often baroclinic instability

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Routinely resolved in 2100


We can study a small-scale system, derive parameterizations, and then use them to improve climate models & assess impact globally.

This process often relies heavily on thermal wind scaling relationships.

But, what about the effects of things that aren't geostrophic & hydrostatic?

For example, waves and near-surface 3d turbulence.
Surface Waves are...

fast, small, irrotational solutions of the Boussinesq Equations
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fast, small, irrotational solutions of the Boussinesq Equations

NWW3 Polar Plot of Wave Energy Spectrum
at ILM01

Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)
Craik-Leibovich Boussinesq
Or Wave-Averaged Eqtns

Formally a multiscale asymptotic equation set:

- 3 classes: Small, Fast; Large, Fast; Large, Slow
- Solve first 2 types of motion in the case of limited slope (ka), irrotational \( \rightarrow \) Deep Water Waves!
- Average over deep water waves in space & time,
- Arrive at Large, Slow equation set.

All Wave-Mean coupling terms involve the Stokes Drift

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams et al. 2004
What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

\[ u^L(x_p(t_0), t) - u^E(x_p(t_0), t) \approx \left[ x_p(t) - x_p(t_0) \right]\cdot \nabla u^E(x_p(t_0), t) \]
\[ \approx \left[ \int_{t_0}^{t} u^E(x_p(t_0), s')ds' \right]\cdot \nabla u^E(x_p(t_0), t). \]

Examples:

Monochromatic:

\[ u^S = \hat{e}_w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} z} = \hat{e}_w a^2 \sqrt{gk^3} e^{2kz}. \]

Spectrum:

\[ u^S = \frac{16\pi^3}{g} \int_{0}^{\infty} \int_{-\pi}^{\pi} (\cos \theta, \sin \theta, 0) f^3 S_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df. \]

Wave-Averaged Equations following McWilliams & F-K (13) and Suzuki & F-K (14) (for horizontally uniform Stokes drift)

\[ \varepsilon = \frac{V^s H}{f LH_s} \]

\[ Ro \left[ v_{i,t} + v_j^L v_{i,j} \right] + \frac{M_{Ro}}{Ri} \omega v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi, i + \frac{Ro}{Re} v_{i,jj} \]

\[ \frac{\alpha^2}{Ri} \left[ w,y + v_j^L w,j + \frac{M_{Ro}}{RoRi} \right] = -\pi, z + b + \varepsilon v_j^L v_j^s, z + \frac{\alpha^2}{ReRi} w, jj \]

\[ b_t + v_j^L b,j + \frac{M_{Ro}}{RoRi} w b_z + w = 0 \]

\[ v_{j,j} + \frac{M_{Ro}}{RoRi} w_z = 0 \]

LAGRANGIAN (Eulerian+Stokes) advection of Eulerian momentum
Coriolis Effect is on LAGRANGIAN velocity
Stokes Shear Force is NEW in vertical momentum equation.

Plus boundary conditions
Near-surface


Ro\gg1

Ri<1: Nonhydro

1-100m (H=L)

10s to 1hr

w, u=O(10cm/s)

Stokes drift

Eqtns:Craik-Leibovich

Params: McWilliams & Sullivan, 2000, Van Roekel et al. 2011

Resolved routinely in 2170
Data + LES scaling, Southern Ocean mixing energy:

One way to estimate

So, waves can drive mixing via Stokes drift (combines with cooling & winds)

Data-driven offline parameterization:

Another way to estimate

Including Wave-driven Mixing (Harcourt 2013) Deepens the Mixed Layer!


Winter Waves in NCAR Community Earth System Model

Li et al., in prep.

Summer Waves in NCAR Community Earth System Model

Fig. 1. Summer mean MLD (m; Averaged over JAS for NH and JFM for SH) for cases (a) OBS, (b) CTRL, (c) MS2K and (d) VR12a.

Li et al., in prep.

Multi-Model Ensemble: Best Estimate

Results similar in pattern and magnitude to ours (in NCAR CESM) were found by Fan & Griffies (2014) using the NOAA Geophysical Fluid Dynamics Laboratory CM2M model.
Wave-Averaged Equations following McWilliams & F-K (13) and Suzuki & F-K (14) (for horizontally uniform Stokes drift)

\[
\begin{align*}
\frac{\text{Ro}}{\text{Ri}} \left[ v_{i,t} + v_j^L v_{i,j} \right] + \frac{M_{Ro}}{Ro} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi,i + \frac{\text{Ro}}{\text{Re}} v_{i,jj} \\
\frac{\alpha^2}{\text{Ri}} \left[ w_{,y} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} \right] = -\pi,z + b + \epsilon v_j^L v_j^s + \frac{\alpha^2}{\text{ReRi}} w_{,jj} \\
b_t + v_j^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_z + w = 0 \\
v_{j,j} + \frac{M_{Ro}}{RoRi} w_z = 0
\end{align*}
\]

Plus boundary conditions

LAGRANGIAN advection of Eulerian momentum

Coriolis Effect is on LAGRANGIAN velocity

Stokes Shear force is NEW in vertical momentum equation.

So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

(Combined) **Lagrangian Thermal Wind Balance**

\[ f \times \frac{\partial}{\partial z} (v + v_s) = f \times \frac{\partial v_L}{\partial z} = -\nabla b \]

Now the temperature gradients govern the **Lagrangian flow**, not the **Eulerian**!

Estimated importance of Stokes vortex on (sub)mesoscale: McWilliams & F-K (13)

\[ \varepsilon = \frac{V^s H}{f L H_s} \]

\[ Ro = \frac{U}{f L} \]

**Figure 1.** (Colour online) Estimated ratio \( \varepsilon/R \approx (u_s \cdot u)/(|u|^2 h_s) \) governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity \( (u) \) is taken as the AVISO weekly satellite geostrophic velocity or \( -u_s \) (for anti-Stokes flow) if \( |u_s| > |u| \). The front/filament depth \( (h) \) is estimated as the mixed layer depth from the de Boyer Montégut et al. (2004) climatology. An exponential fit to the Stokes drift of the upper 9 m projected onto the AVISO geostrophic velocity provides \( u_s \cdot u \) and \( h_s \). Stokes drift is taken from the Wave Watch 3 simulation described in Webb & Fox-Kemper (2011). \( u, u_s, \) and \( h_s \) are all for the year 2000, while \( h \) is from a climatology of observations over 1961–2008. The year 2000 average of \( \varepsilon/R \) is shown.

Wave-Averaged Equations following McWilliams & F-K (13) and Suzuki & F-K (14)
(for horizontally uniform Stokes drift)

\[ \text{LAGRANGIAN advection of Eulerian momentum} \]

\[ \frac{\alpha^2}{Ri} \left[ w_{,y} + v_{j,}^L w_{,j} + \frac{M_{Ro}}{RoRi} \right] = -\pi_{,z} + b + \varepsilon v_{j,}^L v_{j,}^s + \frac{\alpha^2}{ReRi} w_{,jj} \]

\[ b_t + v_{j,}^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_{,z} + w = 0 \]

\[ v_{j,}^j + \frac{M_{Ro}}{RoRi} w_{,z} = 0 \]

Coriolis Effect is on LAGRANGIAN velocity
Stokes Shear force is NEW in vertical momentum equation.

Plus boundary conditions

1) Consider a plane parallel balanced flow with \( b(y,z) \), \( q(y,z) \).

2) Perturb this flow by introducing Stokes forces (turn on waves).

3) Adiabatically rearrange the \( b(y,z) \) and \( q(y,z) \) until new forces are balanced.

4) This solution amounts to solving for \( b' \) and \( v' \) in:

\[
\partial_x^2 b' + \partial_z^2 \left( \frac{b'}{N^2} \right) = \mathcal{F}' + \partial_z \left( \frac{\epsilon \mathcal{Q}'}{N^2} \right) - \partial_x \epsilon \mathcal{P}'
\]

\[
v' = - \int_{-\infty}^{x} \left[ \partial_z \left( \frac{b'}{N^2} \right) + \frac{S_s}{N^2} \partial_z v_0 + \epsilon S_s \frac{\partial_x b'}{N^2} \right] \, dx'.
\]
Waves (Stokes Drift Vortex Force) -> Submeso, Meso, an example for $\varepsilon \ll 1$ near the “sweet spot”

Initial Submeso Front
Max: 1

Perturbation on that scale due to waves
Max: $7.1\varepsilon$

Waves (Stokes Drift Vortex Force) ->
Submeso, Meso, an example for finite waves

Initial Submeso Front
Max: 1

Perturbation on that scale
due to waves
Max: $6.8\varepsilon = 13.6$

LES of Langmuir-Submeso Interactions?

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front.

Use NCAR LES model to solve Craik-Leibovich equations (Moeng, 1984, McWilliams et al, 1997)

\[
\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \text{SGS} \\
\nabla \cdot \mathbf{u} = 0
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{\omega} + f \hat{z}) \times \mathbf{u}_L = -\nabla \pi - \frac{g \rho \hat{z}}{\rho_0} + \text{SGS}
\]

Computational parameters:
Domain size: 20km x 20km x -160m
Grid points: 4096 x 4096 x 128
Resolution: 5m x 5m x -1.25m

1000x more gridpoints than CESM

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\frac{\partial \rho}{\partial t} + u_L \cdot \nabla \rho &= \text{SGS} \\
\nabla \cdot \mathbf{u} &= 0 \\
\frac{\partial \mathbf{u}}{\partial t} + (\omega + f \hat{z}) \times \mathbf{u}_L &= -\nabla \pi - \frac{g \rho \hat{z}}{\rho_0} + \text{SGS}
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What’s plotted are surfaces of large vert. velocity, colored by temperature.

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Diverse types of interaction

Slide & Movies by Peter Hamlington

Diverse types of interaction

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Wave-influenced Sawyer-Eliassen eq.

\[
N^2 \frac{\partial^2 \psi}{\partial y^2} + F^* \frac{\partial^2 \psi}{\partial z^2} + 2M^* \frac{\partial^2 \psi}{\partial y \partial z} = 2Q_2 + S_{turb} + S_{SSF}
\]
Conclusions

Climate modeling is challenging partly due to the vast and diverse scales of fluid motions.

In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange and climate.

Interesting transition occurs on the Submeso to Langmuir scale boundary, as nonhydro. & ageostrophic effects begin to dominate.

The effects of the Stokes forces on mesoscale and submesoscale dynamics are under-appreciated.

All papers at: fox-kemper.com/pubs
FIG. 13. Fields of the mixed layer depth (in m) based on temperature, denoted $H_\theta$, (a,e) and on potential vorticity, denoted $H_q$, (b,f) for the LT (a,b) and ST (e,f) cases. The difference $H_\theta - H_q$ is shown in (c,g) and low-pass (submesoscale) vertical vorticity fields are shown in (d,h), where the filter cutoff for the vorticity fields is at 2km. Contour lines correspond to temperature contours taken from Figure 2.
So, no problems?
Just crunch away with CLB?

Let’s revisit our assumptions for scale separation:

- CLB wave equations require limited *wave steepness* and irrotational flow
- Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

Power Spectrum of wave height

\[
\langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_n}^{\infty} C_1 k^{-2} dk
\]

Power Spectrum of wave steepness: INFINITE!

\[
\langle k^2 \eta^2 \rangle = \int_0^{\infty} k^2 E(k) dk = D_0 + \int_{k_n}^{\infty} D_1 dk
\]

Steep waves break→vortex motion & small scale turbulence!
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Physical Sensitivity of Ocean Climate to MLE: (submeso) Mixed Layer Eddy Restratification Improves CFCs (water masses)

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Physical Sensitivity of Ocean Climate to MLE:
(submeso) Mixed Layer Eddy Restratiﬁcation
Improves CFCs (water masses)

$\mathbf{\text{CM2M } \text{H}_m \text{ Control-deBM (m) FEB}}$

$\mathbf{\text{CM2M } \text{H}_m \text{ Control-deBM (m) SEP}}$

$\mathbf{\text{CM2M } \text{H}_m \text{ Submeso-deBM (m) FEB}}$

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A consistently restratifying, $w'b' \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg,
M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels.

Parameterization of mixed layer eddies. III: Implementation and impact in
Physical Sensitivity of Ocean Climate to MLE: (submeso) Mixed Layer Eddy Restratification Improves CFCs (water masses)


\[
\begin{align*}
\overline{w'b'} & \propto \frac{H^2}{|f|} \left| \nabla_H \overline{b} \right|^2 \\
\overline{u'Hb'} & \propto -\frac{H^2}{|f|} \frac{\partial \overline{b}}{\partial z} \nabla_H \overline{b}
\end{align*}
\]

A consistently restratifying, and horizontally downgradient flux.
Mixed Layer Eddy Restratiﬁcation

Estimating eddy buoyancy/density ﬂuxes:

\[ \mathbf{u}'b' \equiv \Psi \times \nabla \bar{b} \]

A submeso eddy-induced overturning:

\[ \Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z} \]

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in ML only:

\[ \mu(z) = 0 \text{ if } z < -H \]

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For a consistently restratiﬁying,

$$\overline{w' b'} \propto \frac{H^2}{|f|} |\nabla H \bar{b}|^2$$

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For a consistently restratifying,

\[ \mathbf{w}' b' \propto \frac{H^2}{|f|} \left| \nabla_H \bar{b} \right|^2 \]

and horizontally downgradient flux.

\[ \mathbf{u}' H b' \propto -\frac{H^2}{|f|} \frac{\partial b}{\partial z} \nabla_H \bar{b} \]

Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

$$\overline{u'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced streamfunction:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \nabla \bar{b}$$

in ML only:

$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} \left| \nabla H \bar{b} \right|^2$$

and horizontally downgradient flux.

$$\overline{u'Hb'} \propto -\frac{H^2}{|f|} \frac{\partial b}{\partial z} \nabla H \bar{b}$$

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM+ minus CCSM−): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

May Stabilize AMOC

Affects sea ice

NO RETUNING NEEDED!!!

These are impacts: bias change unknown
CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves

Tricky: Misaligned Wind & Waves

Waves (Stokes Drift)

Vertical Velocity (m/s)

Distance (m)

Distance (m)

Tricky: Misaligned Wind & Waves

Tricky: Misaligned Wind & Waves

Vertical Velocity (m/s)

Waves (Stokes Drift)

Distance (m)

Distance (m)

Generalized Turbulent Langmuir No.,
Projection of $u^*$, $u_s$ into Langmuir Direction

\[
\langle \frac{\langle w^2 \rangle}{u_*^2} \rangle_{ML} = 0.6 \cos^2(\alpha_{LOW}) \left[ 1.0 + (3.1 L_{a_{proj}})^{-2} \right] + \left( 5.4 L_{a_{proj}} \right)^{-4},
\]
\[
L_{a_{proj}} = \frac{|u_s| \cos(\alpha_{LOW})}{|u_s| \cos(\theta_{ww} - \alpha_{LOW})},
\]
\[
\alpha_{LOW} \approx \tan^{-1} \left( \frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa} \ln \left( \frac{H_{ML}}{z_1} \right) + \cos(\theta_{ww})} \right).
\]

A scaling for LC strength & direction!

How well do we know Stokes Drift? <50% discrepancy

RMS error in measures of surface Stokes drift, 2 wave models (left), model vs. altimeter (right)

Year 2000 data & models

Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of \( \mathbf{u}_L = \mathbf{v} + \mathbf{v}_s \)

Misalignment enhances degree of wave-driven LT

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**Figure 1** Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).
Surface wave effects on oceanic fronts, filaments, and turbulence

Baylor Fox-Kemper
with contributions from Jim McWilliams (UCLA), Nobuhiro Suzuki (Brown), and Sean Haney (CU Boulder)

The upper ocean is home to many complex dynamical interactions—between the air and sea, between the waves, winds, and currents, between the gasses in the atmosphere and those dissolved in the ocean, and between spatiotemporal scales of variability. A standard approach to the upper ocean multiscale dynamical interaction was first proposed by Craik and Leibovich, and later improved by Holm, McWilliams, Lane, and Restrepo. The approach focuses on the Boussinesq equations averaged over the timescale of surface waves under the assumption that the fast scales are waves of limited steepness. Under this assumption, the leading order coupling between the fast and slow scales is through the Stokes drift of the surface waves, which affects the larger, slower scales through advection and the Coriolis and vortex forces. The approach is equivalent and complementary to the radiation stress theory of Longuet-Higgins. Analysis, theory, and Large Eddy Simulations of the wave-averaged equations will be presented that elucidate some effects of surface waves on upper ocean boundary layer turbulence and submesoscale fronts and filaments.
The \( u, v \), decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus, \( kH \) is a measure of depth.

\( ka \) is a measure of steepness.

Deep water waves don’t “feel” the bottom. Implies nonhydrostatic (\( Ro >> 1 \)) & fast timescale (\( Ro >> 1 \)).
So, no problems?

Just crunch away with CLB?

Let’s revisit our assumptions for scale separation:

CLB wave equations require limited *wave steepness* and irrotational flow

Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

\[
\langle \eta^2 \rangle = \int_0^\infty E(k) dk = C_0 + \int_{k_n}^\infty C_1 k^{-2} dk
\]

Power Spectrum of wave height

\[
\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k) dk = D_0 + \int_{k_n}^\infty D_1 dk
\]

Power Spectrum of wave steepness: INFINITE!

Steep waves break -> vortex motion & small scale turbulence!
So, no problems?
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Let’s revisit our assumptions for scale separation:

- CLB wave equations require limited *wave steepness* and irrotational flow
- Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

Power Spectrum of wave height

\[ \langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_n}^{\infty} C_1 k^{-2} dk \]

Power Spectrum of wave steepness: INFINITE!

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Steep waves break -> vortex motion & small scale turbulence!