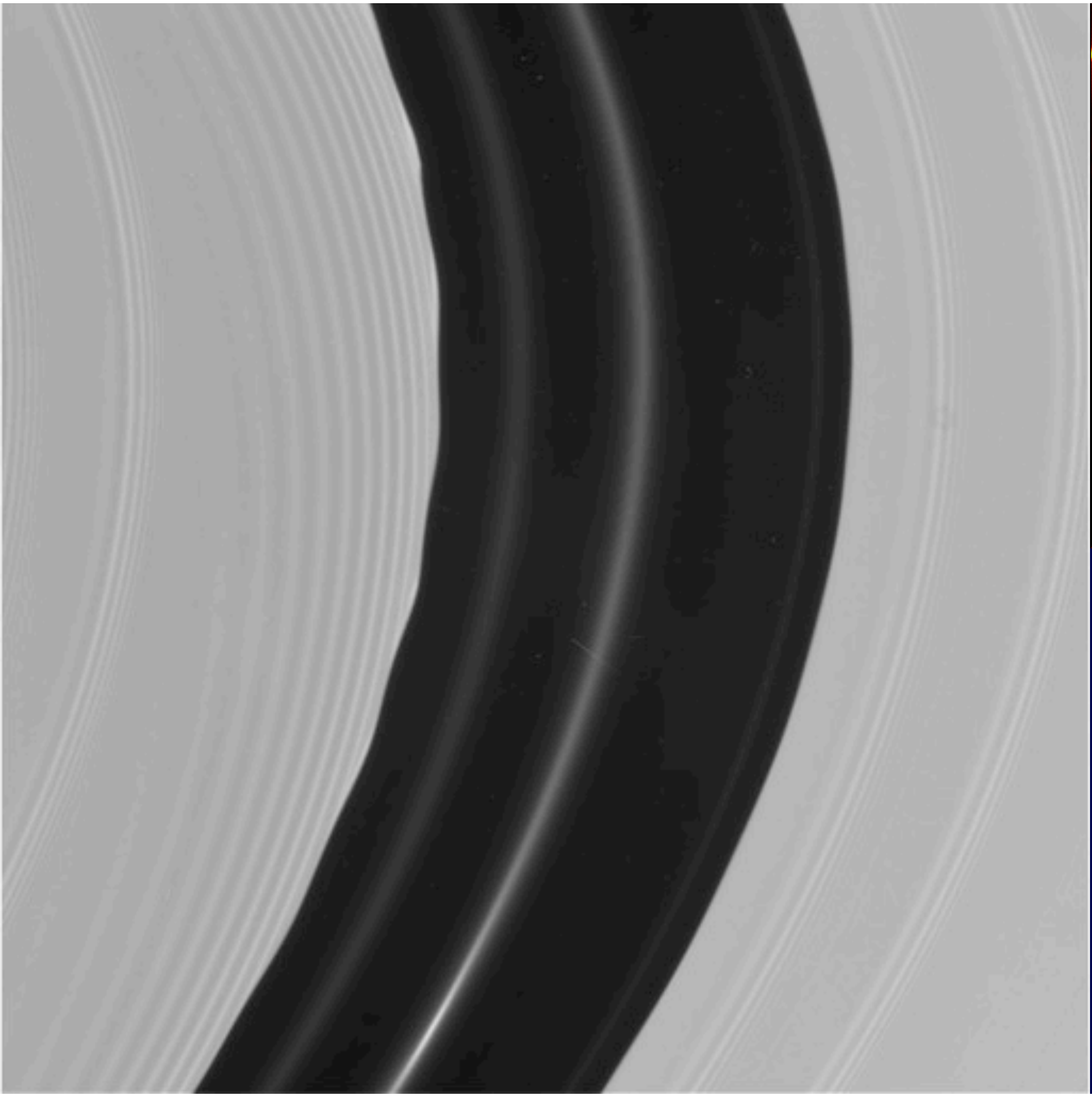
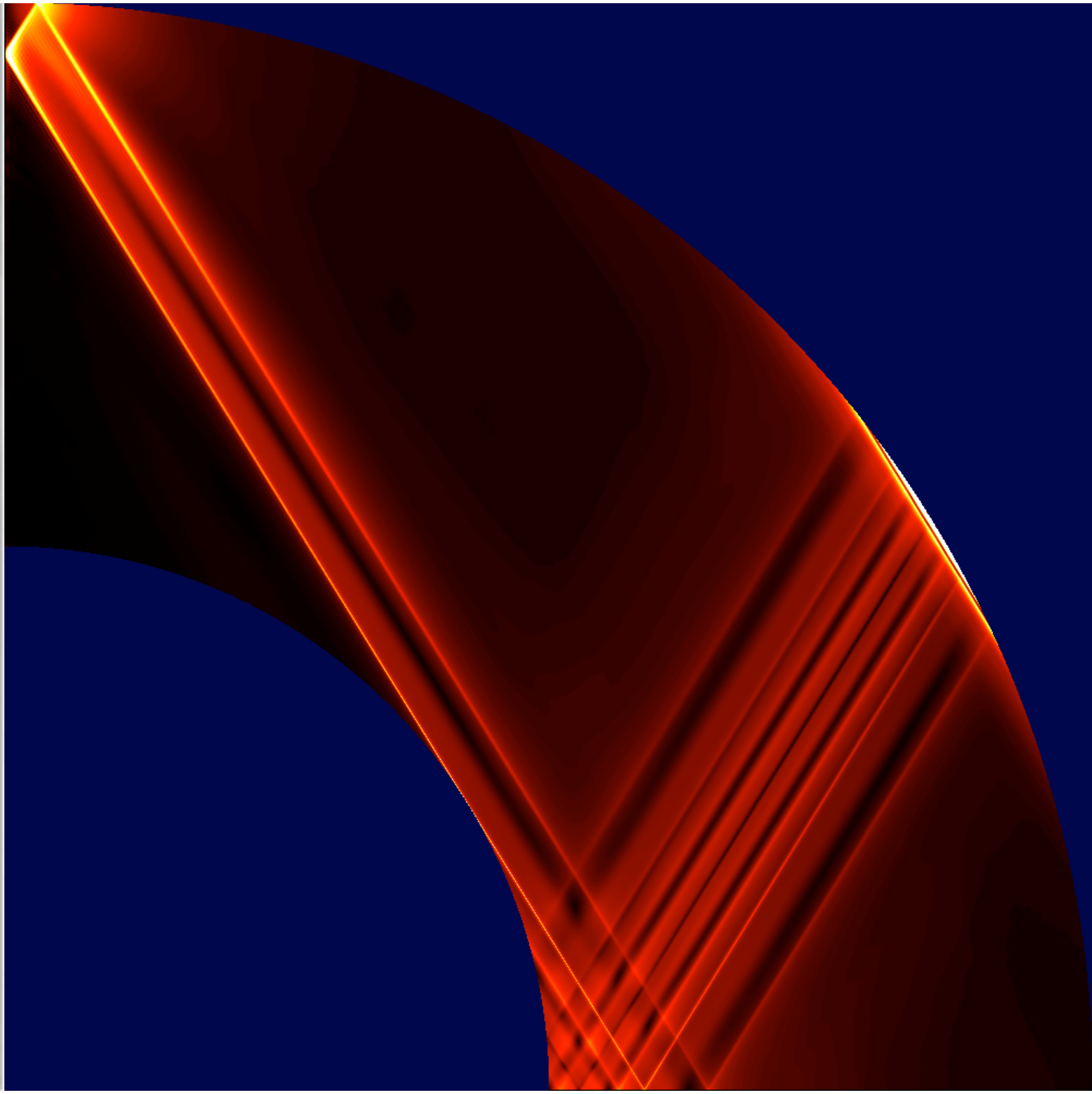
Wave–mean-flow interactions in astrophysical discs and stars

Gordon Ogilvie · DAMTP, University of Cambridge





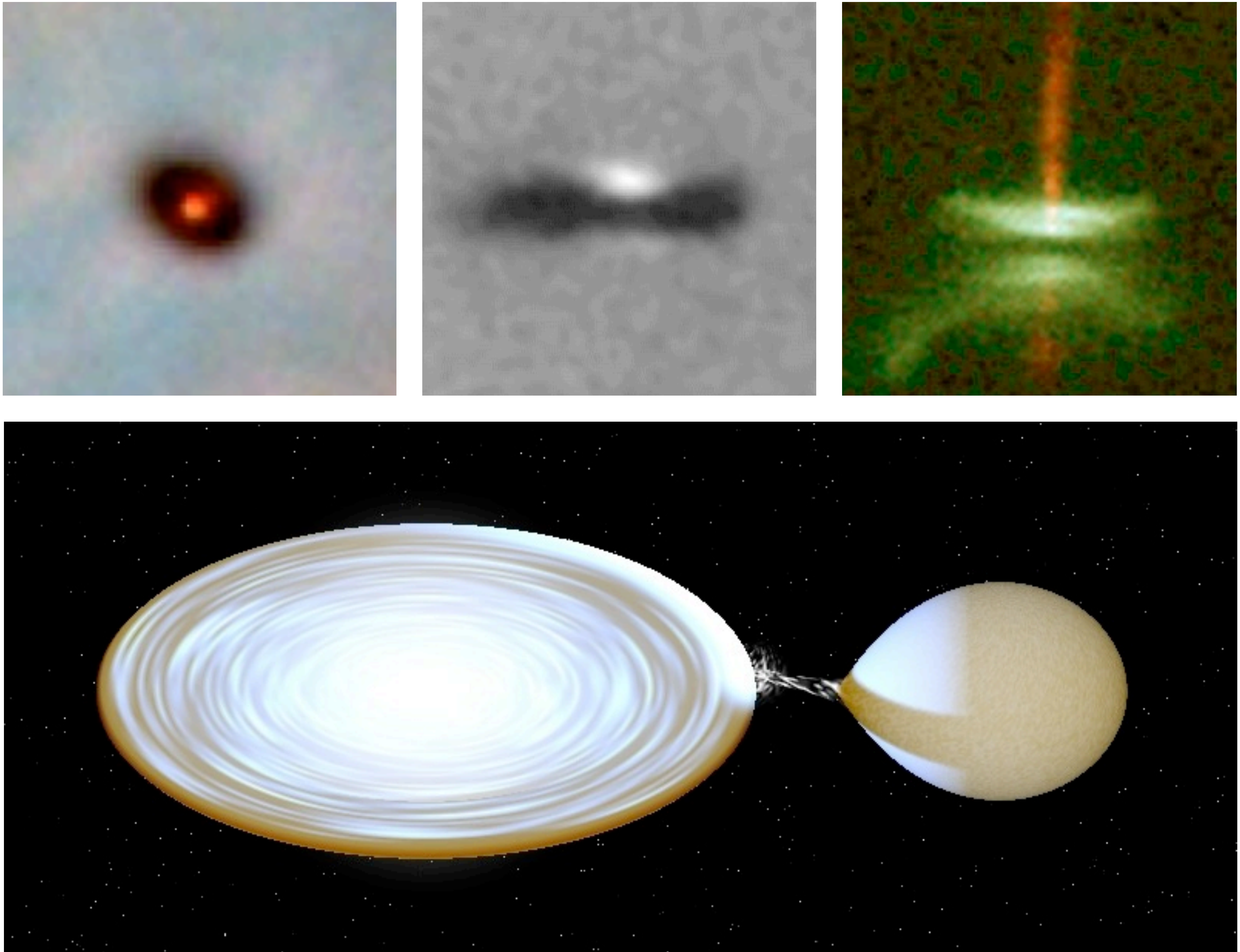
Eddy–mean-flow interactions in fluids

KITP, Santa Barbara · 26.03.14

Aims

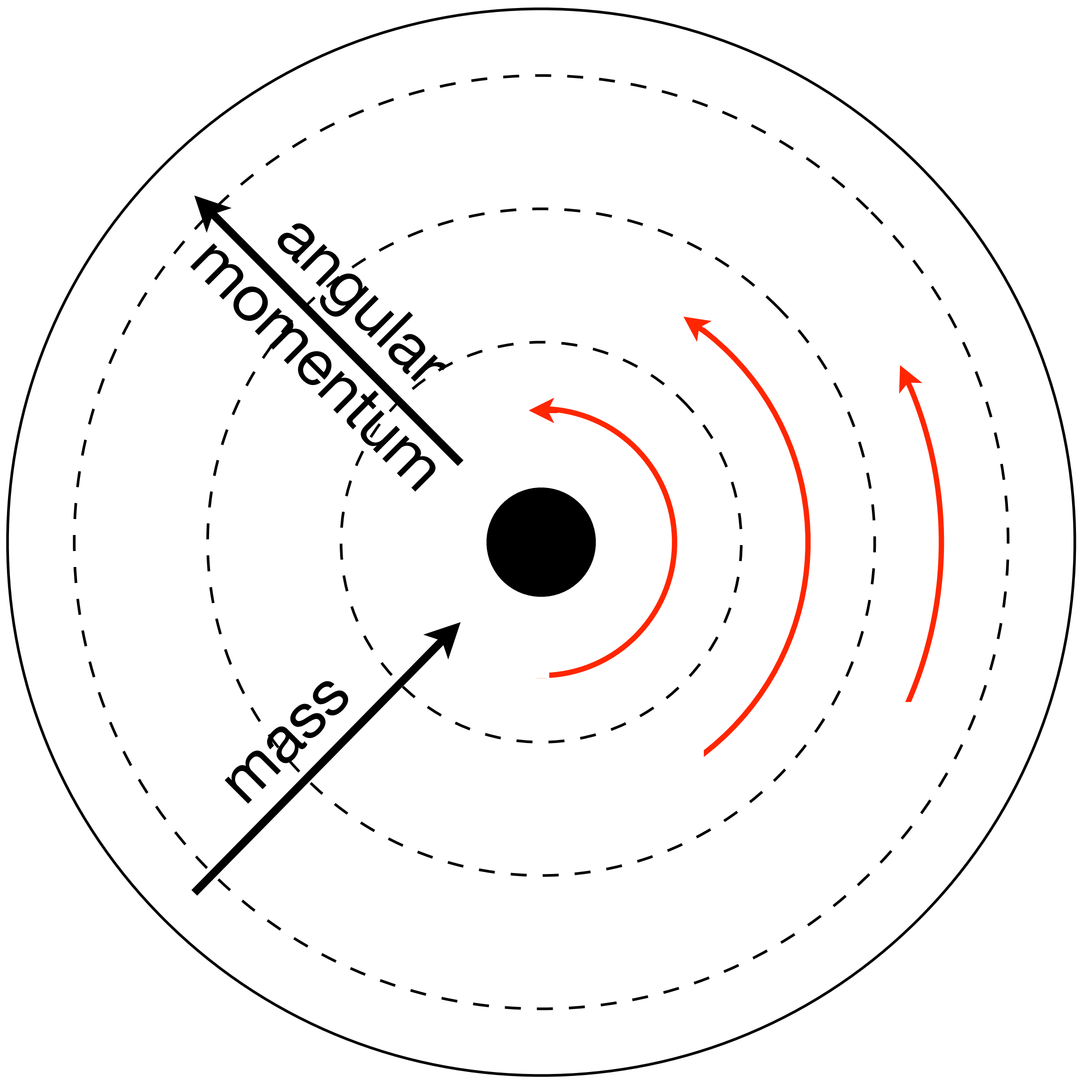
- Broad but selective overview
- Most technical details omitted
- Links between AFD and GFD

Waves and mean flows in astrophysical discs



Astrophysical discs

Continuous medium in orbital motion around a massive central body



- Usually circular, coplanar and thin
- Usually Keplerian (dominated by gravity of central mass)

$$\Omega = \left(\frac{GM}{r^3}\right)^{1/2}$$

- Hypersonic shear flow set by orbital dynamics
- Angular momentum transport ⇒ slow radial flow, not adjustment of azimuthal mean flow
- Asymptotics / scale separation:

$$\frac{H}{r} \ll 1$$

2D ideal compressible fluid model

- Basic equations (difficult to justify formally...)

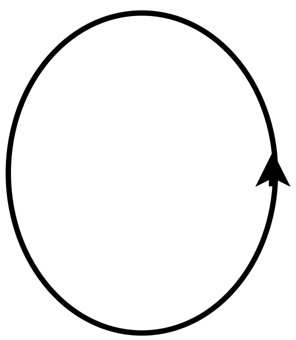
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} \Phi - \frac{1}{\Sigma} \boldsymbol{\nabla} P$$

$$\frac{\partial \Sigma}{\partial t} + \boldsymbol{\nabla} \cdot (\Sigma \boldsymbol{u}) = 0$$

- Potential vorticity / “vortensity” (Papaloizou & Lin 1989)

$$\zeta = \frac{(\boldsymbol{\nabla} \times \boldsymbol{u})_z}{\Sigma}$$

material invariant
(barotropic case)



$$\Gamma = \oint \boldsymbol{u} \cdot \mathrm{d}\boldsymbol{r} = \int (\boldsymbol{\nabla} \times \boldsymbol{u})_z \, \mathrm{d}A$$

$$M = \int \Sigma \, \mathrm{d}A$$

- Circular disc:
  specific angular momentum $h = r^2 \Omega$, vortensity $\zeta = \frac{1}{r\Sigma} \frac{\mathrm{d}h}{\mathrm{d}r}$

- Special case of MMSN model: $\Sigma \propto \Omega \propto r^{-3/2}$, $\zeta = \mathrm{const}$

2D ideal compressible fluid model

- Basic equations (difficult to justify formally...)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} \Phi - \frac{1}{\Sigma} \boldsymbol{\nabla} P$$

$$\frac{\partial \Sigma}{\partial t} + \boldsymbol{\nabla} \cdot (\Sigma \boldsymbol{u}) = 0$$

- Waves on a circular basic state with $\Sigma(r),\ P(r),\ \boldsymbol{u} = r\Omega(r)\,\boldsymbol{e}_\phi$:

$$\boldsymbol{u}' = \mathrm{Re}\left[\tilde{\boldsymbol{u}}'(r)\exp(\mathrm{i}m\phi - \mathrm{i}\omega t)\right] \qquad \text{etc.}$$

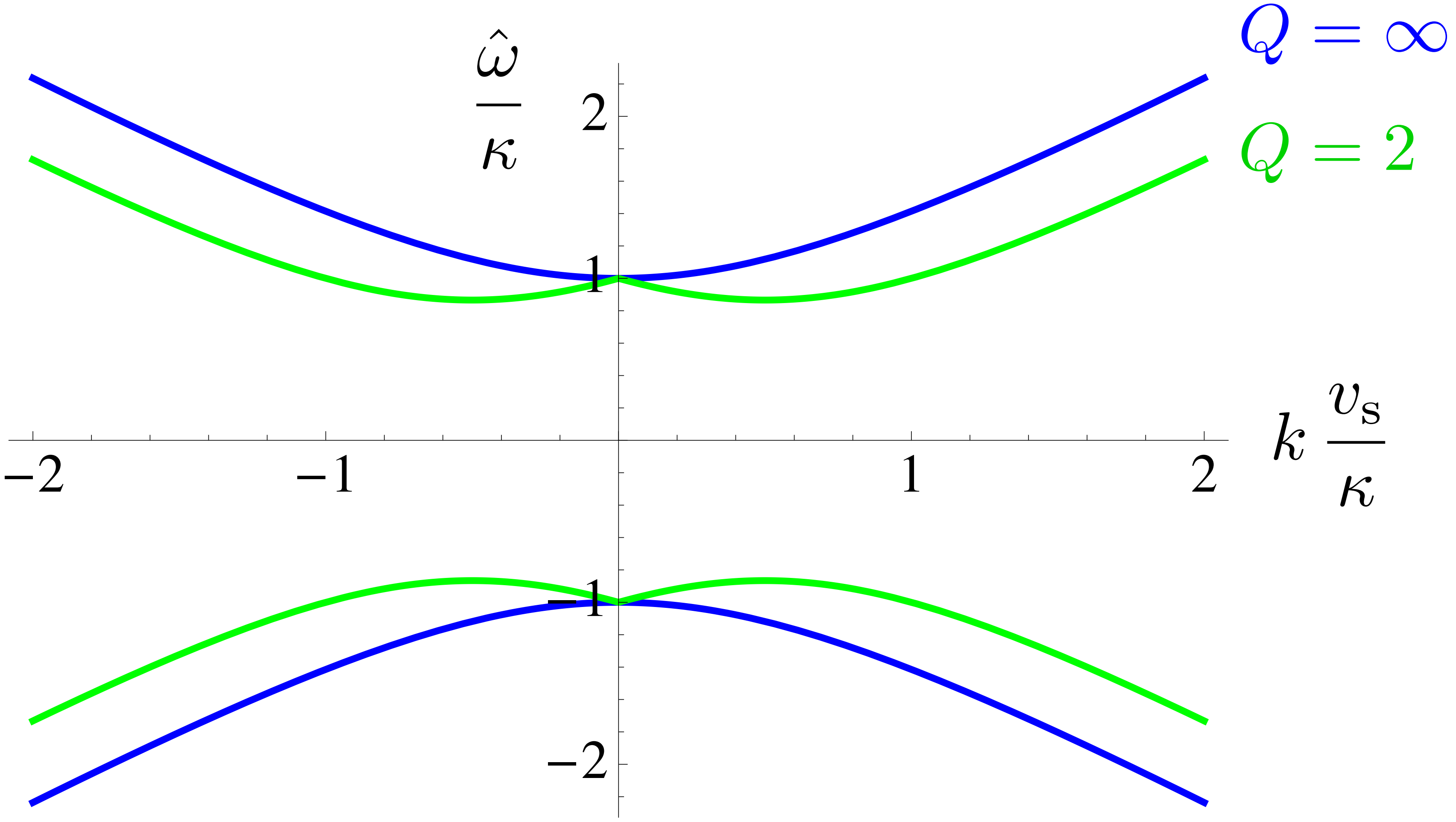
  - Fast acoustic-inertial “density wave”
  - Slow vortical / Rossby mode
  - Coupled near corotation where $\hat{\omega} = \omega - m\Omega = 0$

2D ideal compressible fluid model

- Local linear dispersion relation for density waves

$$\Sigma' = \mathrm{Re}\left\{\tilde{\Sigma}'(r)\exp\left[\mathrm{i}m\phi - \mathrm{i}\omega t + \mathrm{i}\int k(r)\,\mathrm{d}r\right]\right\}$$

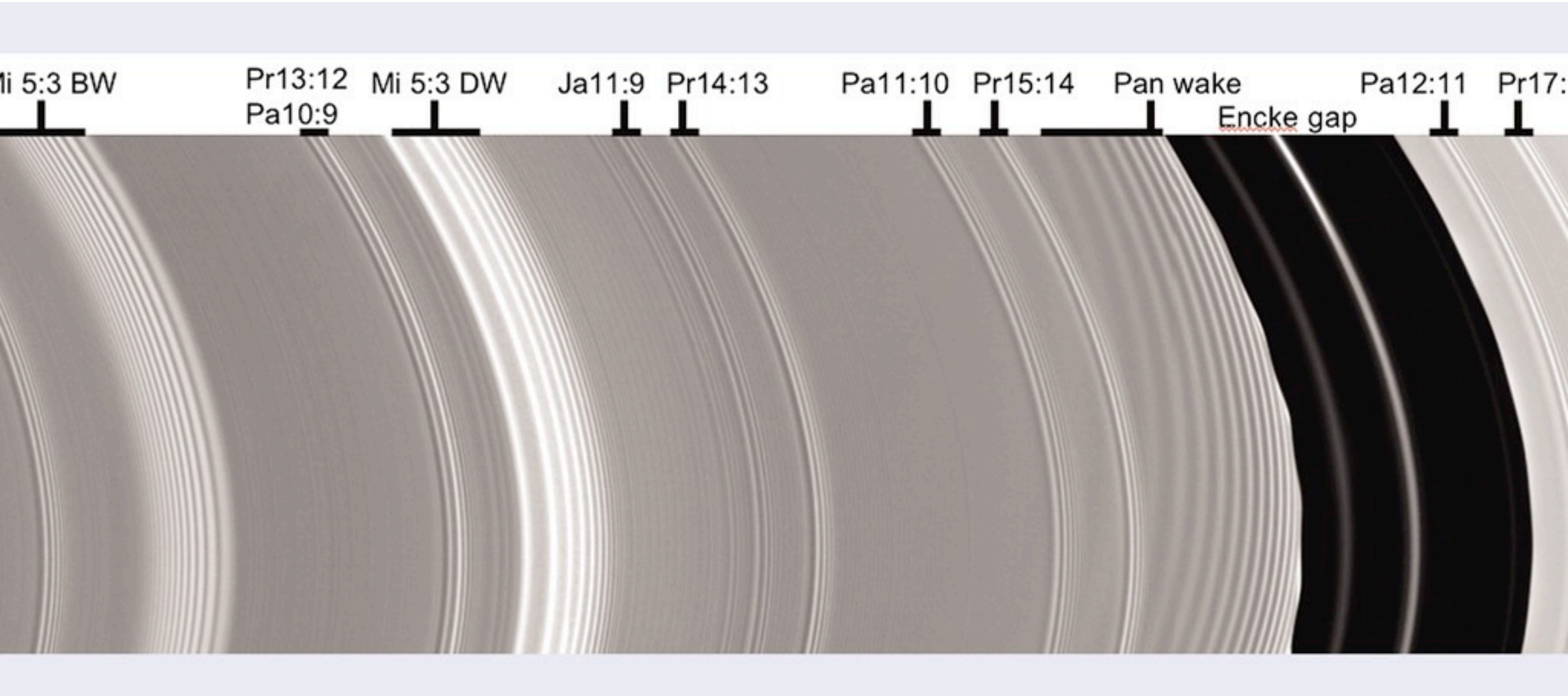
$$\hat{\omega}^2 = \kappa^2 - 2\pi G\Sigma|k| + v_\mathrm{s}^2 k^2 \qquad \hat{\omega} = \omega - m\Omega(r)$$



$$Q = \frac{v_\mathrm{s}\kappa}{\pi G\Sigma}$$

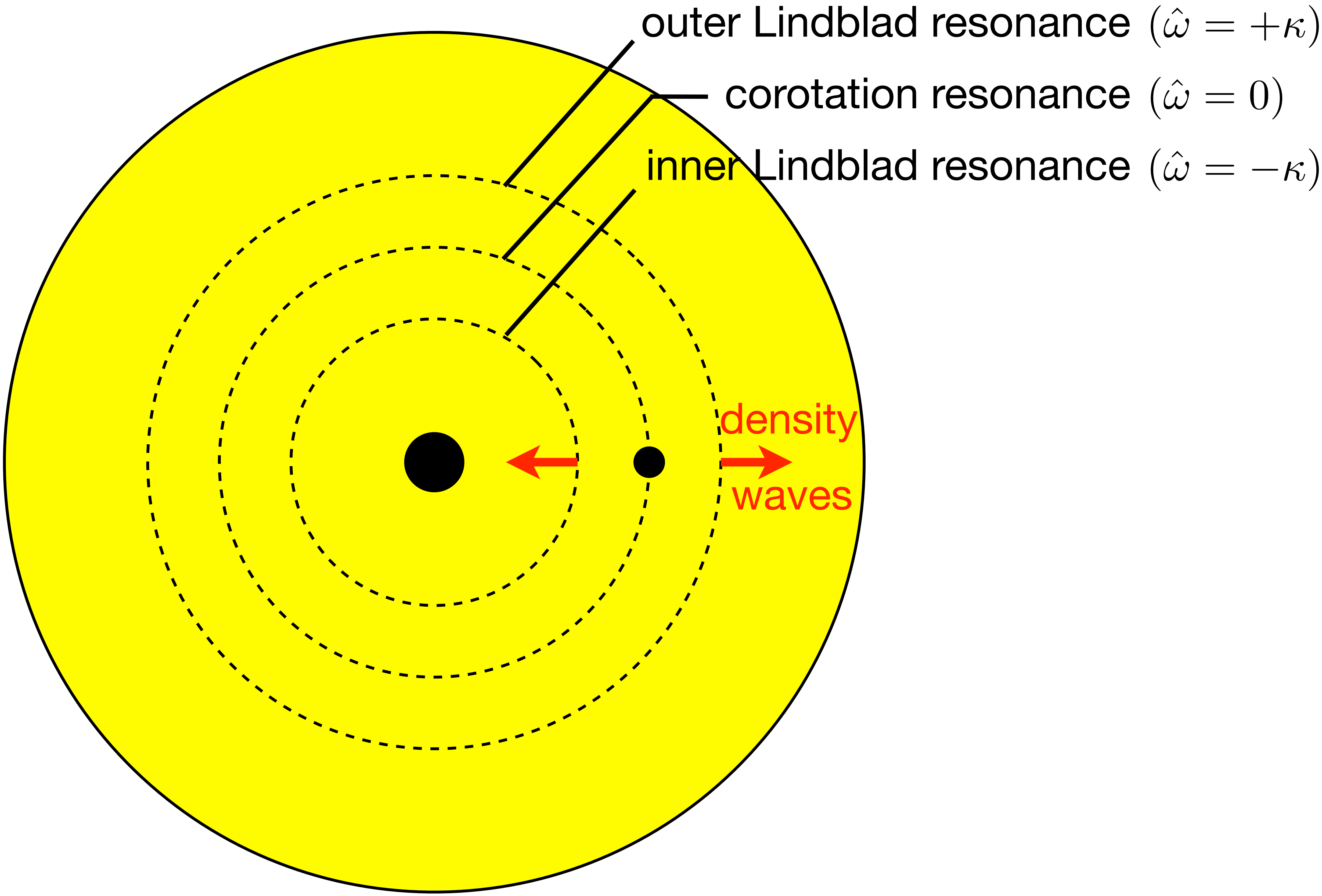
inverse measure of self-gravity

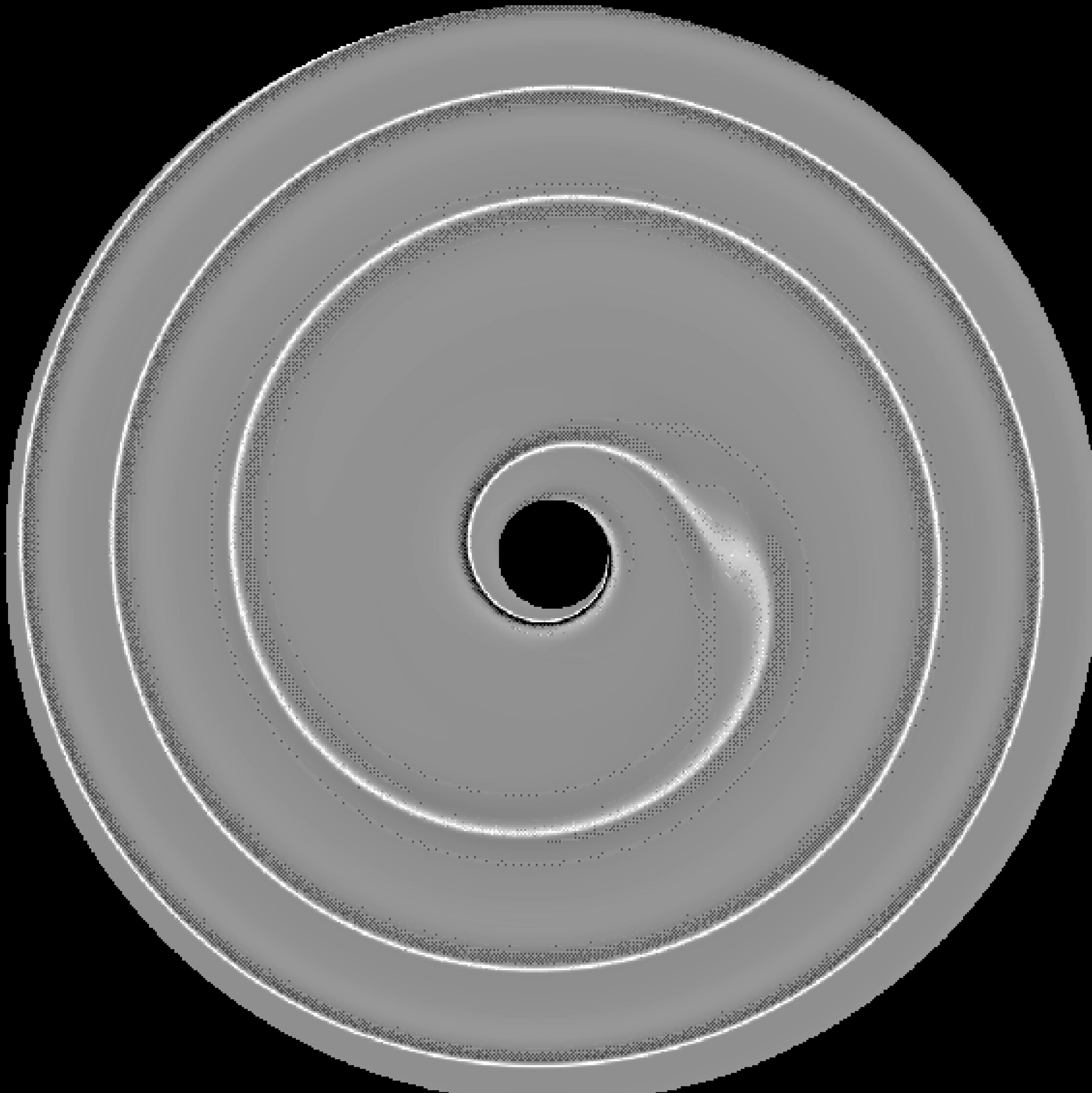
Nonlinear density waves and wakes in Saturn's rings



- Generally, rings are filled with nonlinear near-epicyclic oscillations

Density waves



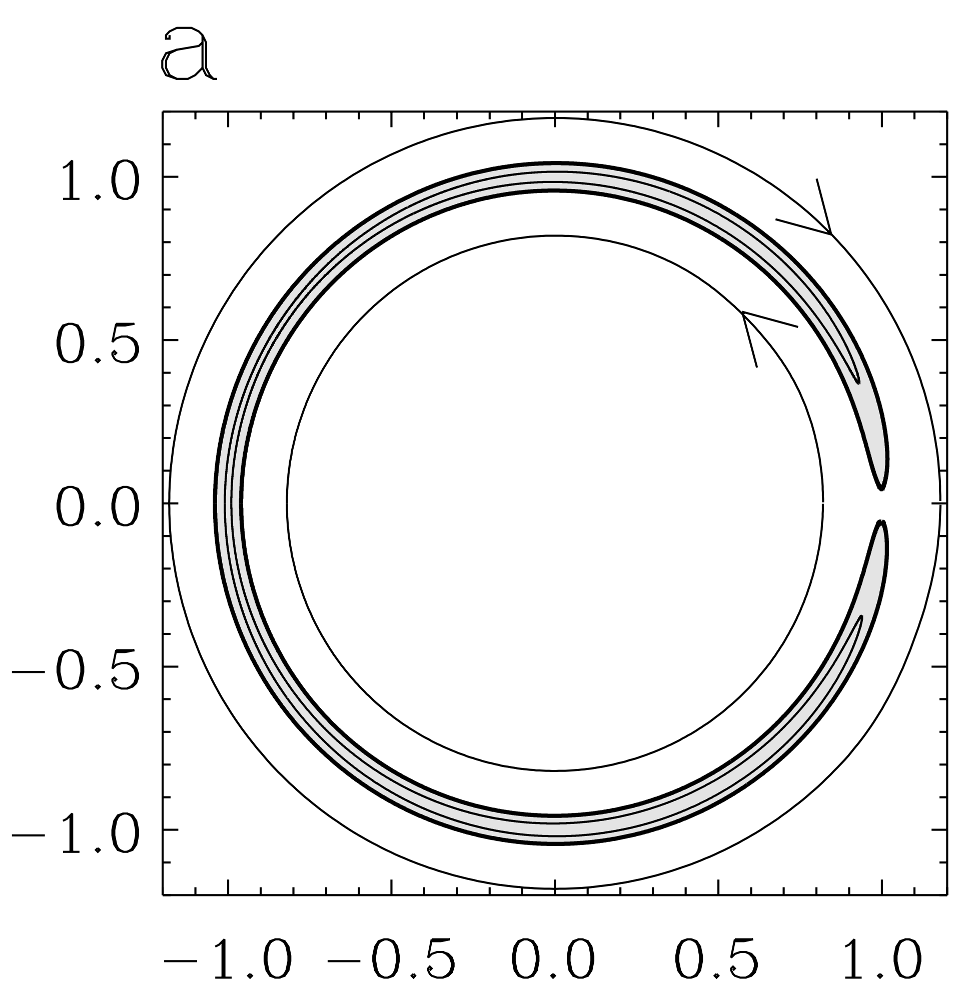


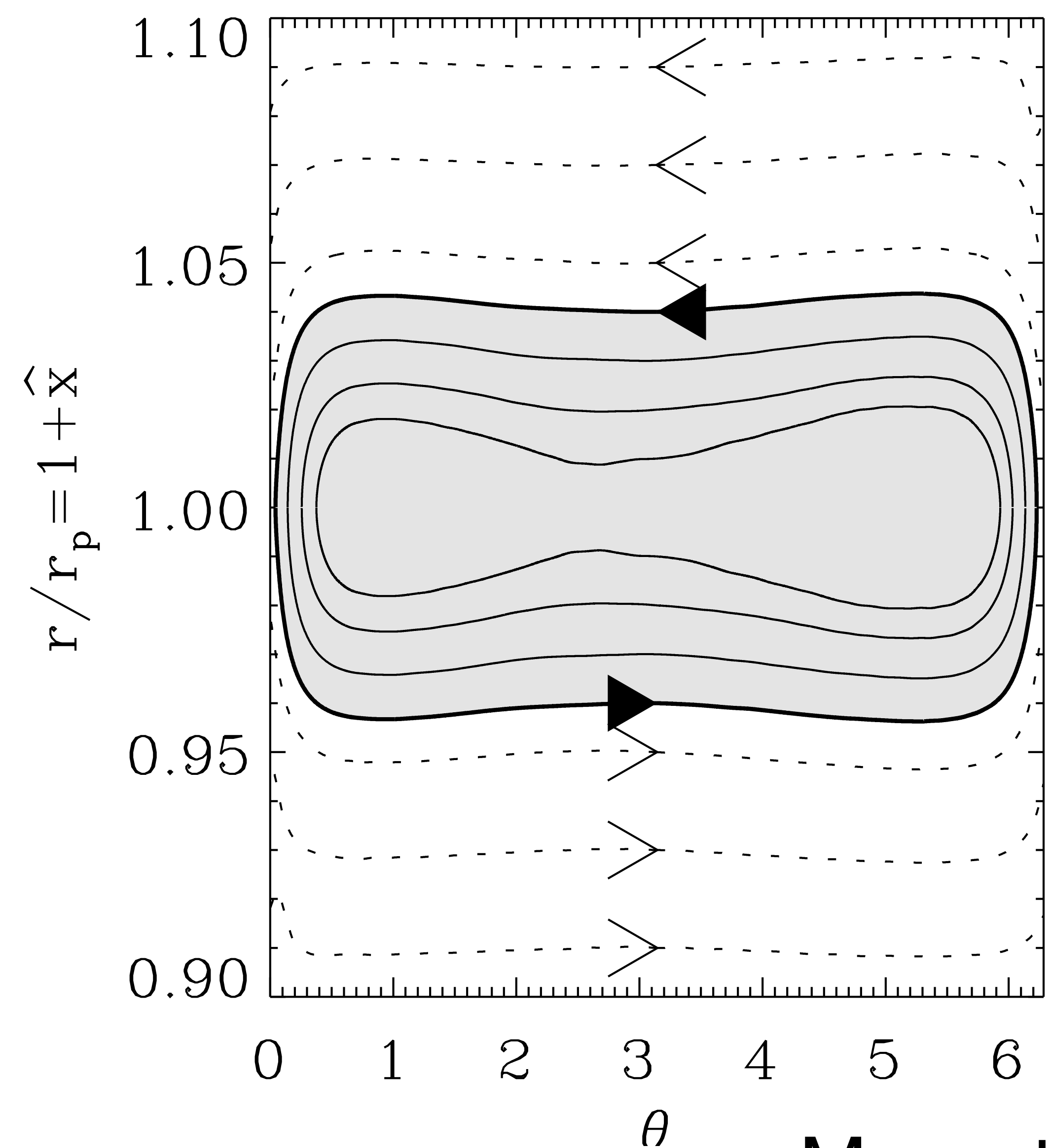
Corotational dynamics

- Linear corotation torque (Goldreich & Tremaine 1979)

$$T = \frac{m\pi^2\Psi^2}{\mathrm{d}\Omega/\mathrm{d}r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{\zeta}\right) \qquad \zeta = \frac{(\boldsymbol{\nabla}\times\boldsymbol{u})_z}{\Sigma}$$

- Streamline topology

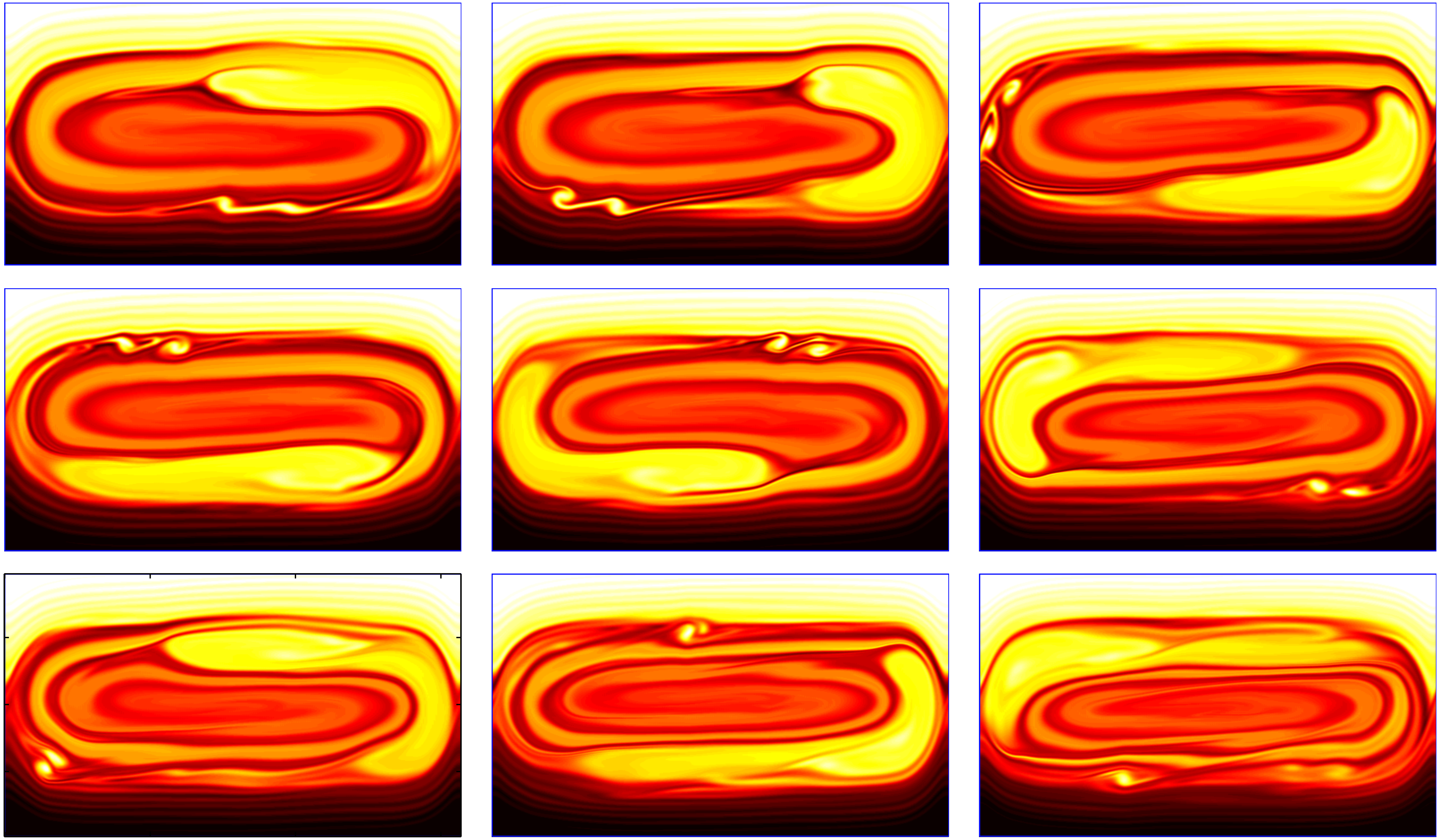




Masset 2001

Corotational dynamics

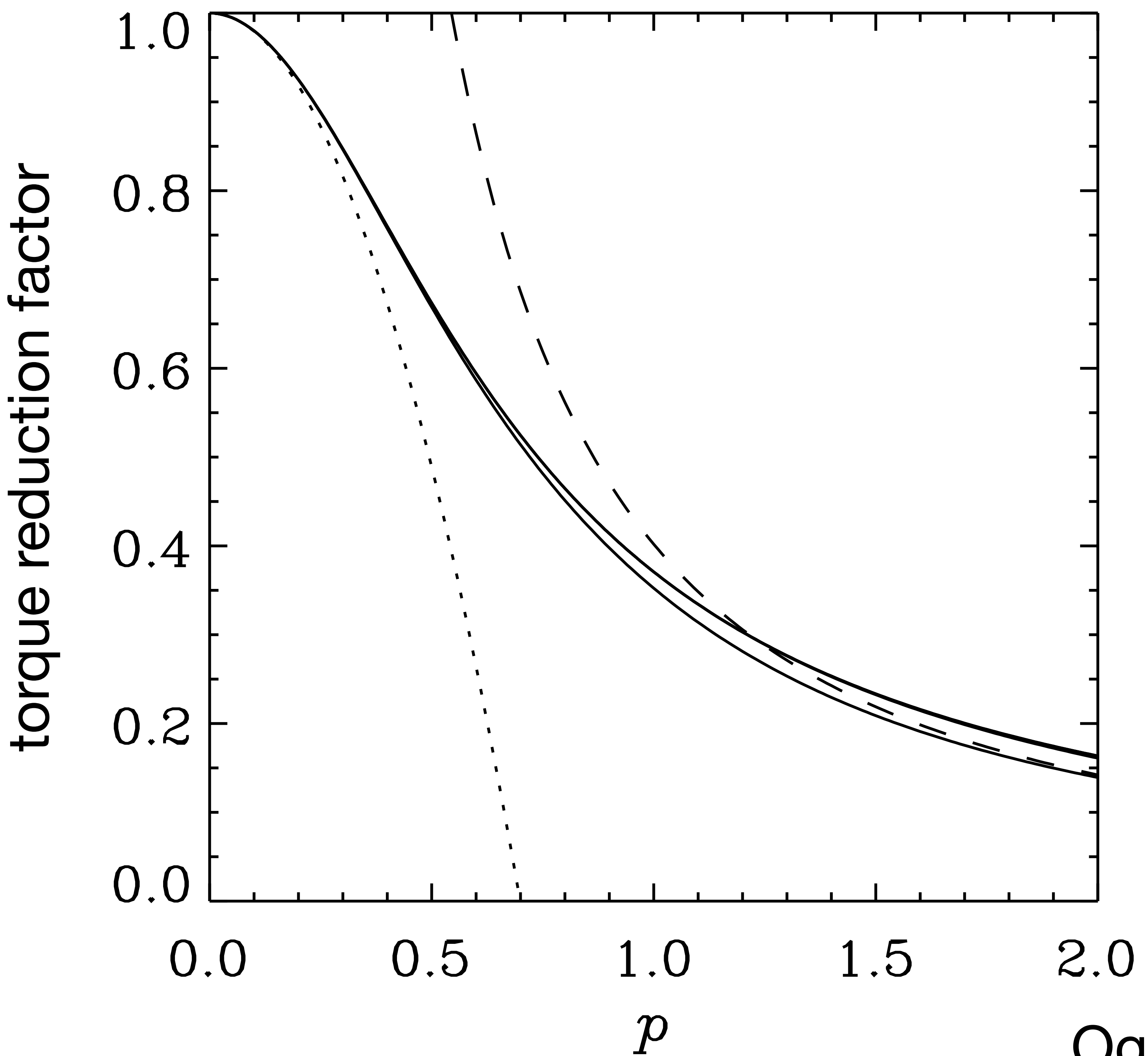
- Saturation of corotation resonance and torque through vortex formation, cf. critical layers in GFD



Balmforth & Korycansky 2001

Corotational dynamics

- Saturation of corotation resonance and torque



$$p \propto \frac{\Psi}{\nu^{2/3}}$$

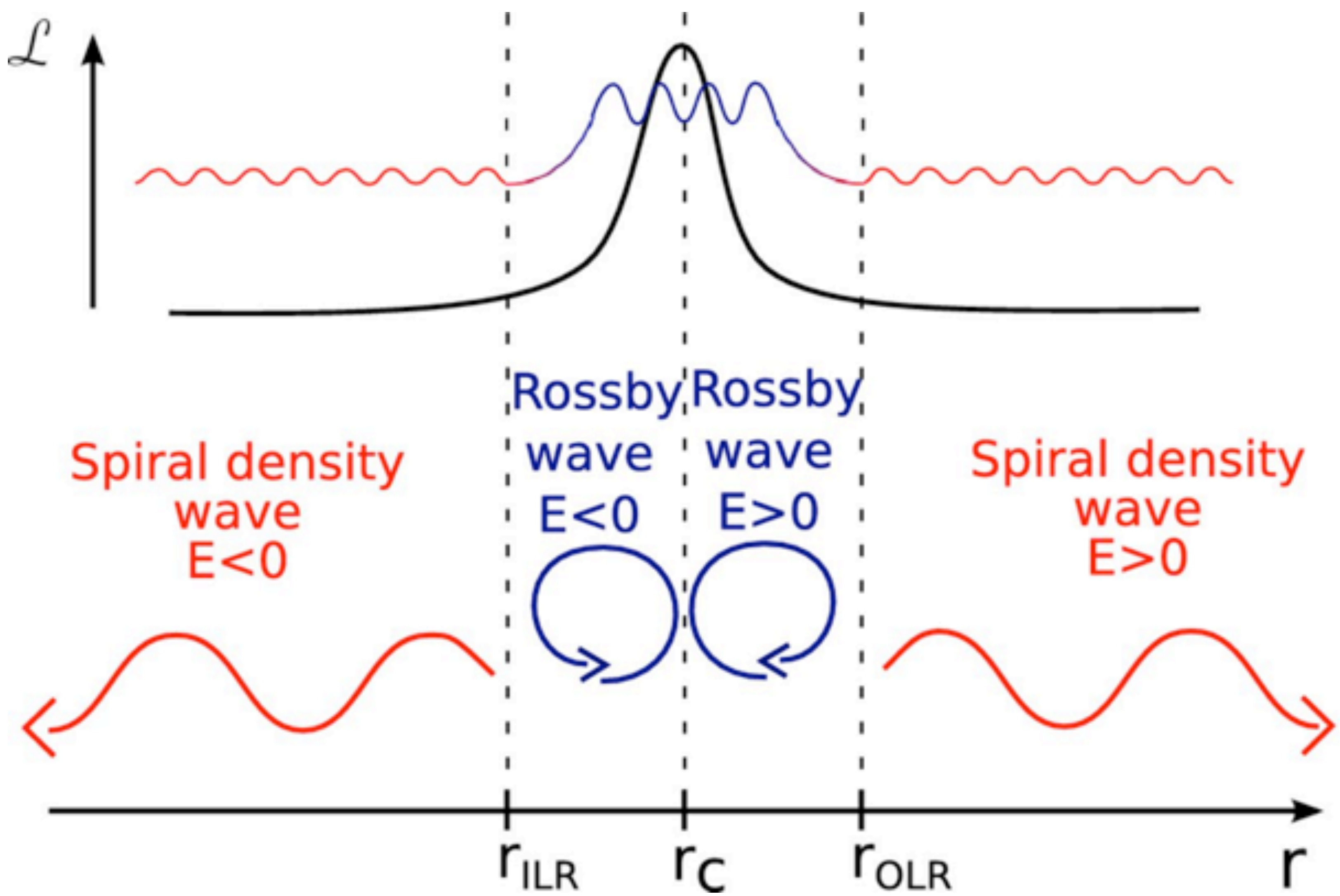
Ogilvie & Lubow 2003

Corotational dynamics

- Baroclinic, 3D, non-ideal and magnetic effects, e.g.:
  - Baruteau & Masset 2008
  - Paardekooper & Papaloizou 2009
  - Paardekooper+ 2011
  - Guilet+ 2013
- All cause modifications of PV / vortensity dynamics
- Importance, in competition with more robust Lindblad torques:
  - Rate and direction of planetary migration
  - Growth or decay of orbital eccentricity

Corotational dynamics

- Rossby vortex instability



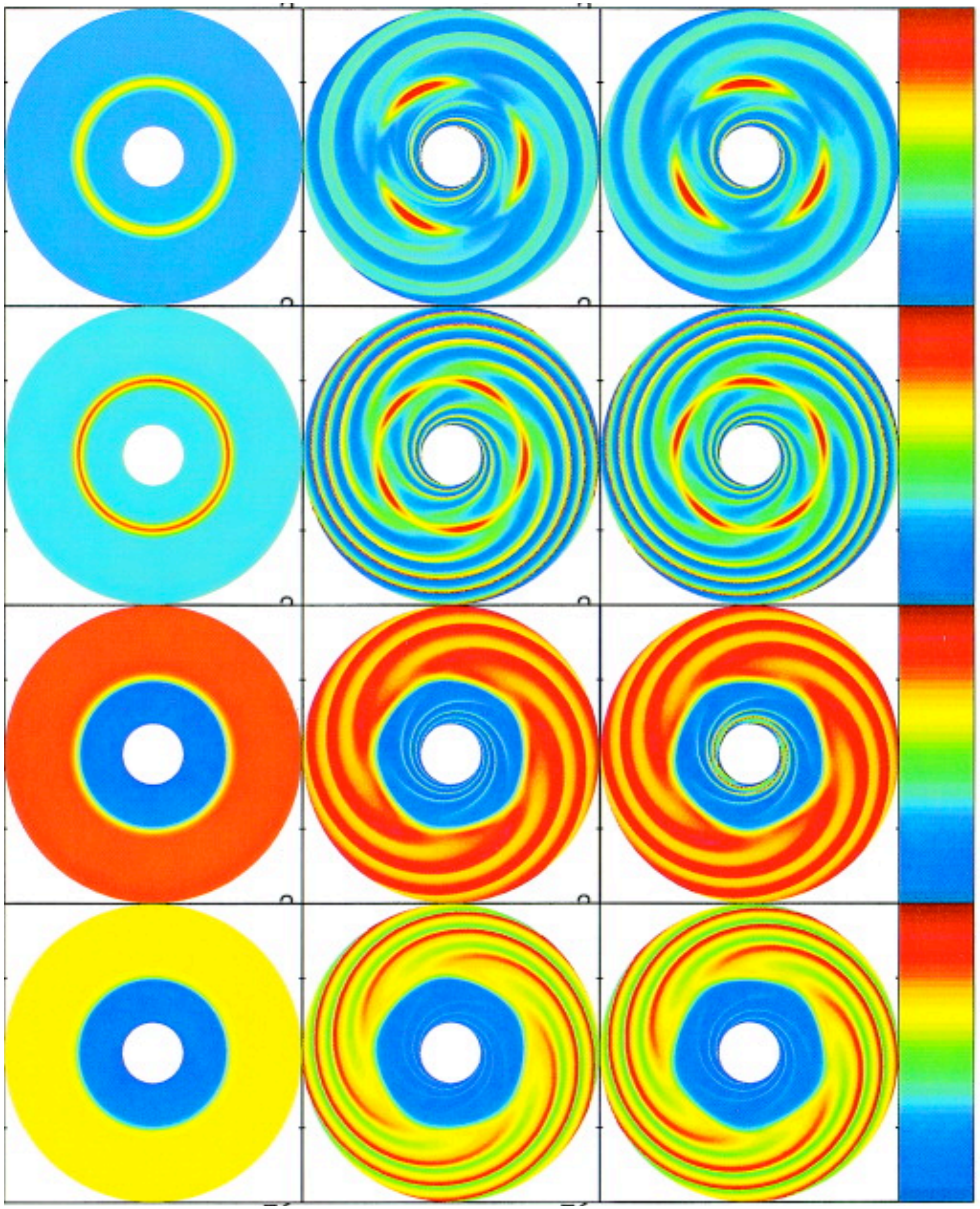
Meheut+ 2013

Lovelace & Hohlfeld 1978; Papaloizou & Lin 1989; Lovelace+ 1999

cf. Papaloizou–Pringle instability, which requires a reflecting edge

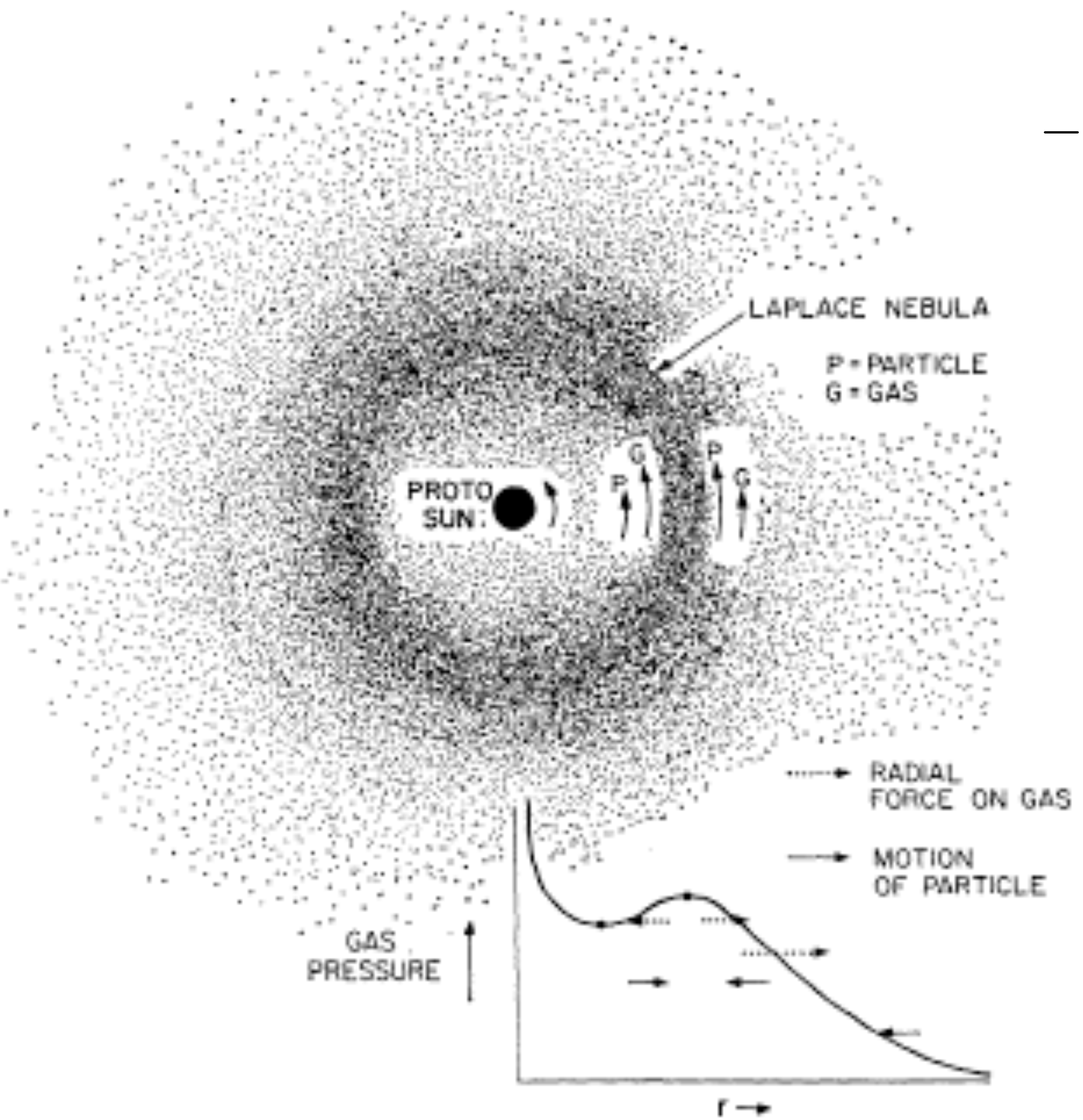
Corotational dynamics

- Rossby vortex instability
  Nonlinear outcome



Li+ 2001

Zonal flows in astrophysical discs

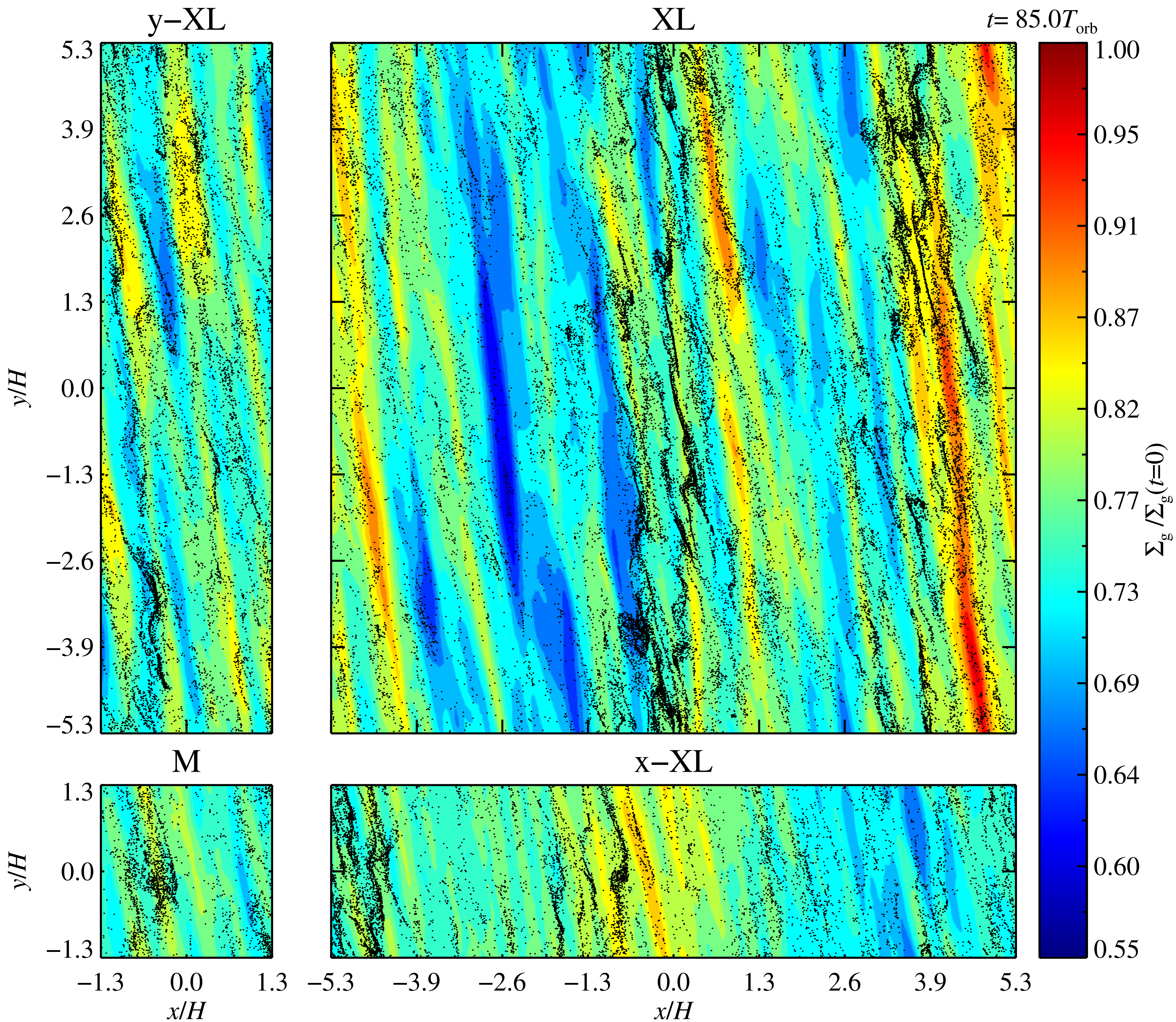


EFFECT OF GAS PRESSURE GRADIENT ON PARTICLE MOTION

$$-r\Omega^2 = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\partial p}{\partial r}$$

Whipple 1972

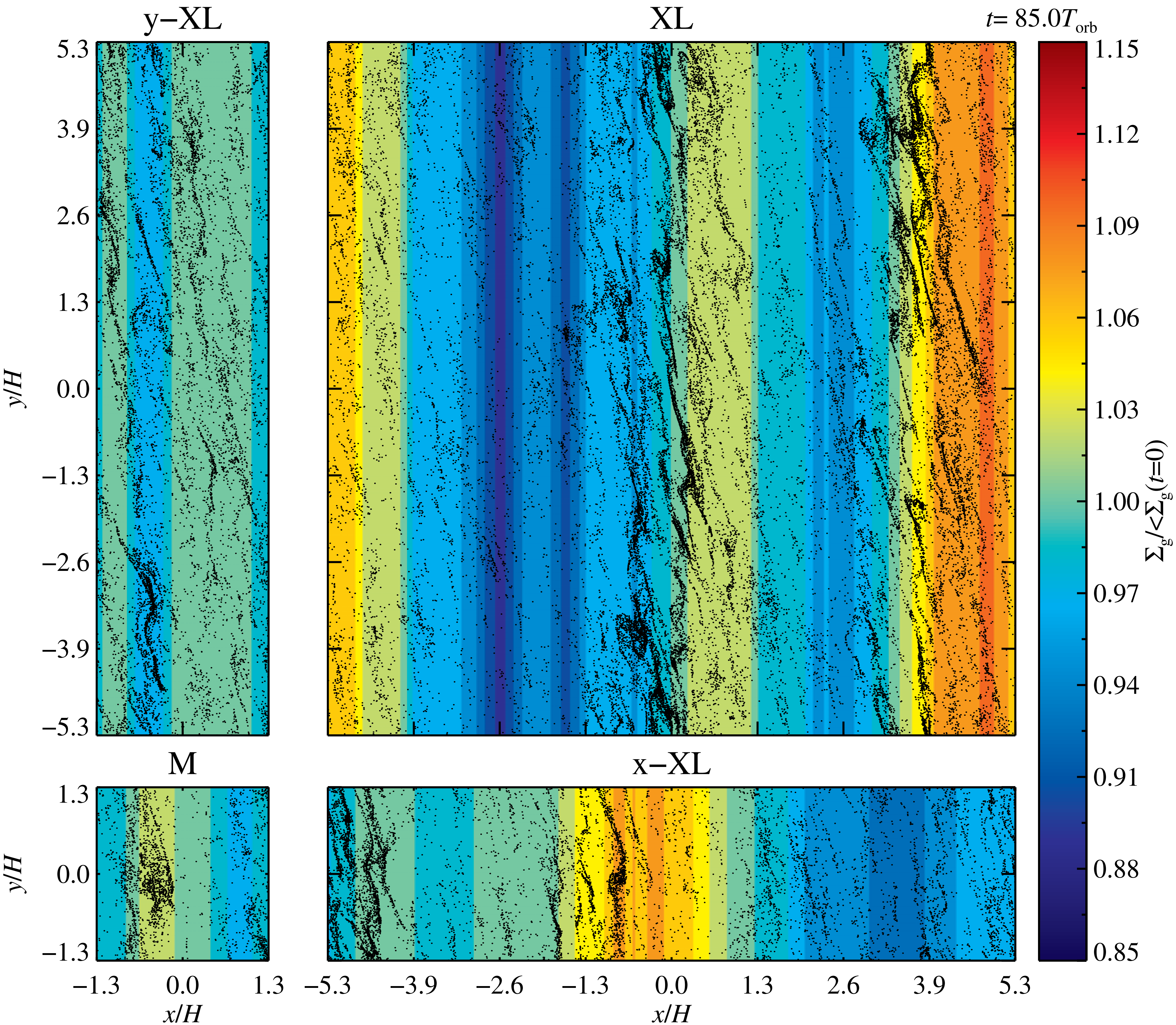
Zonal flows in astrophysical discs



surface density in xy plane

Dittrich+ 2013

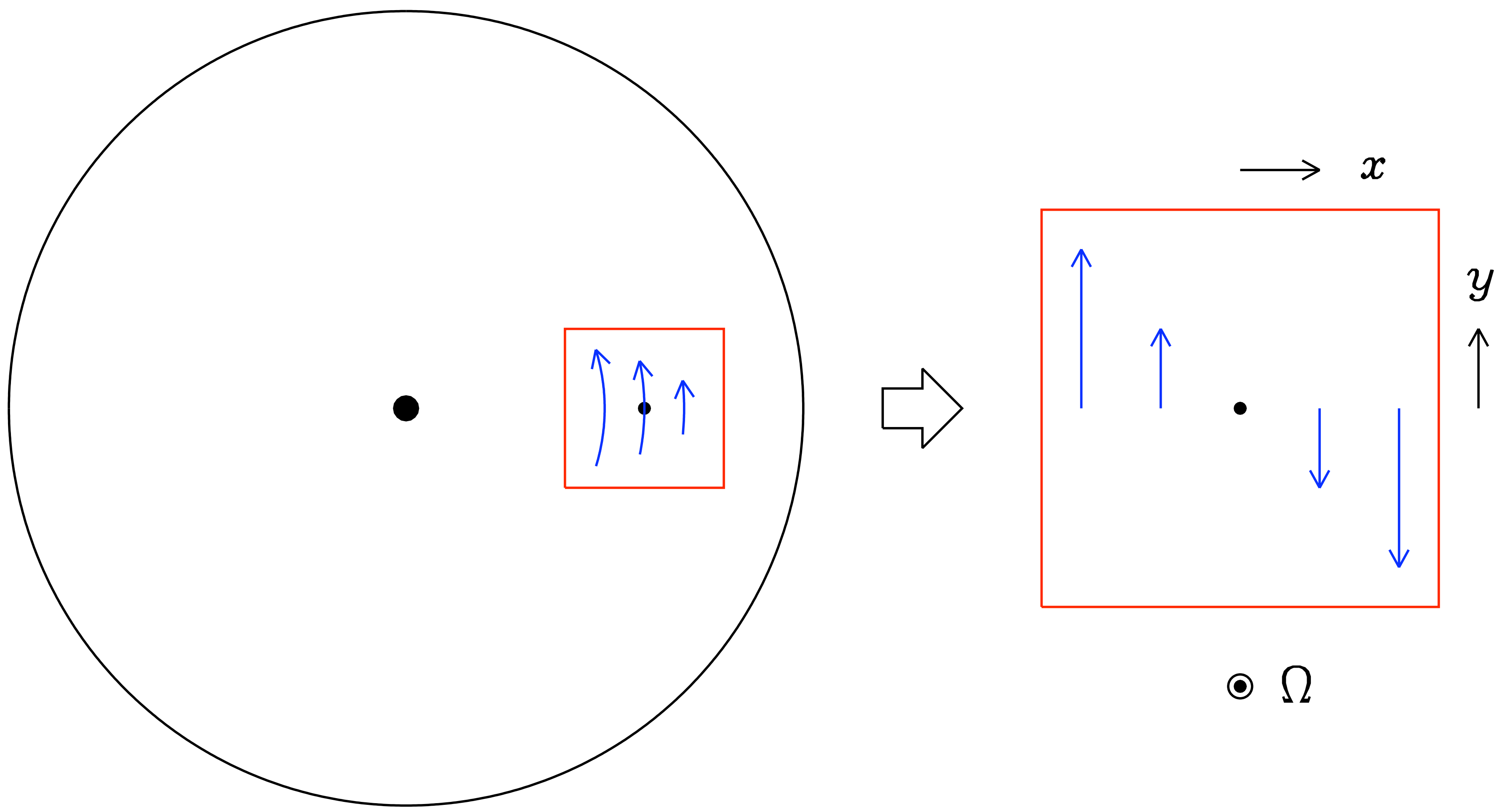
Zonal flows in astrophysical discs



zonally averaged surface density

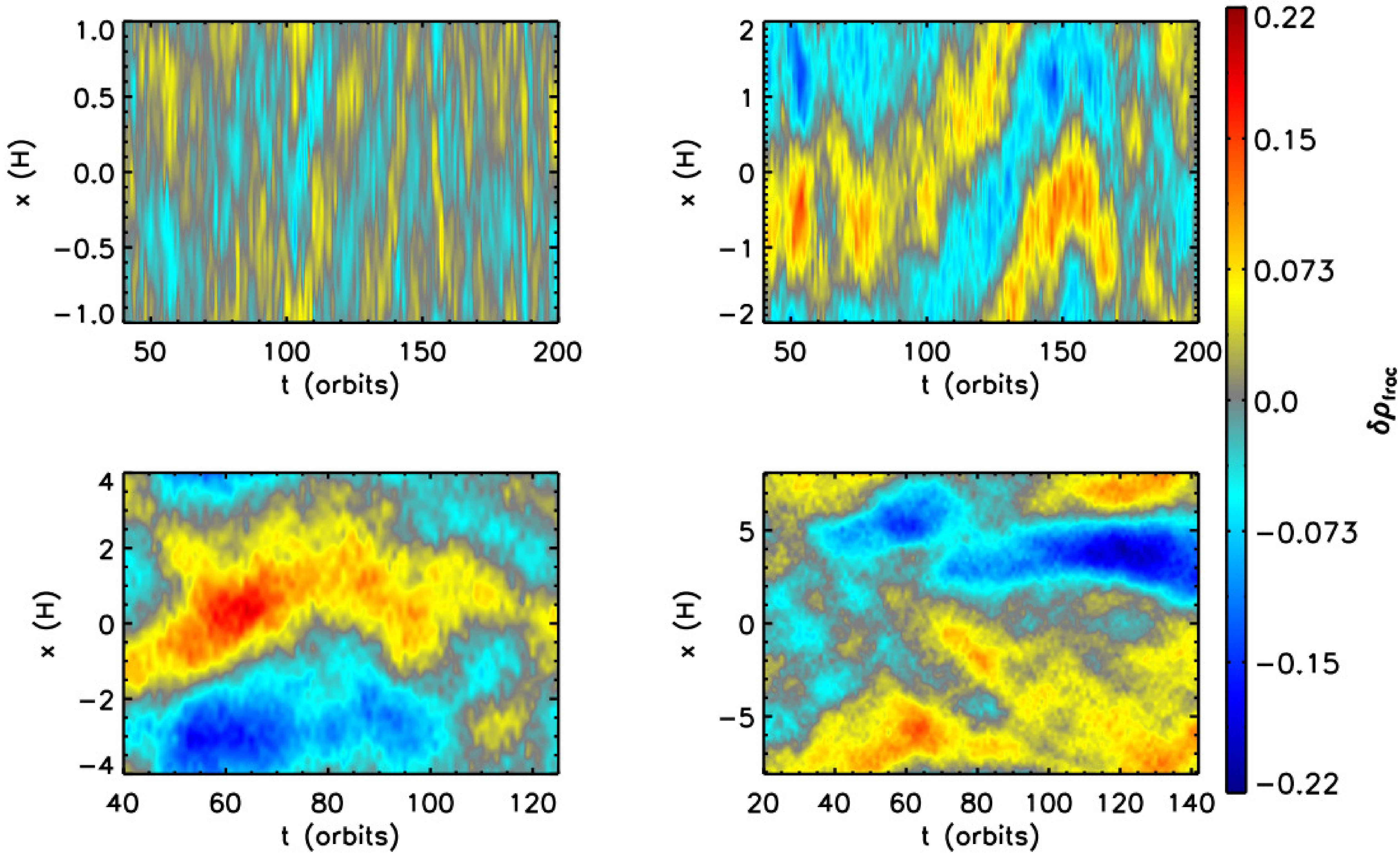
Dittrich+ 2013

Local approximation / shearing box



- Spatially homogeneous model (horizontally)
- Zonal-flow generation requires:
  - Inhomogeneous transport of angular momentum
  - Generation of non-uniform PV / vortensity
  - Modulational instability?

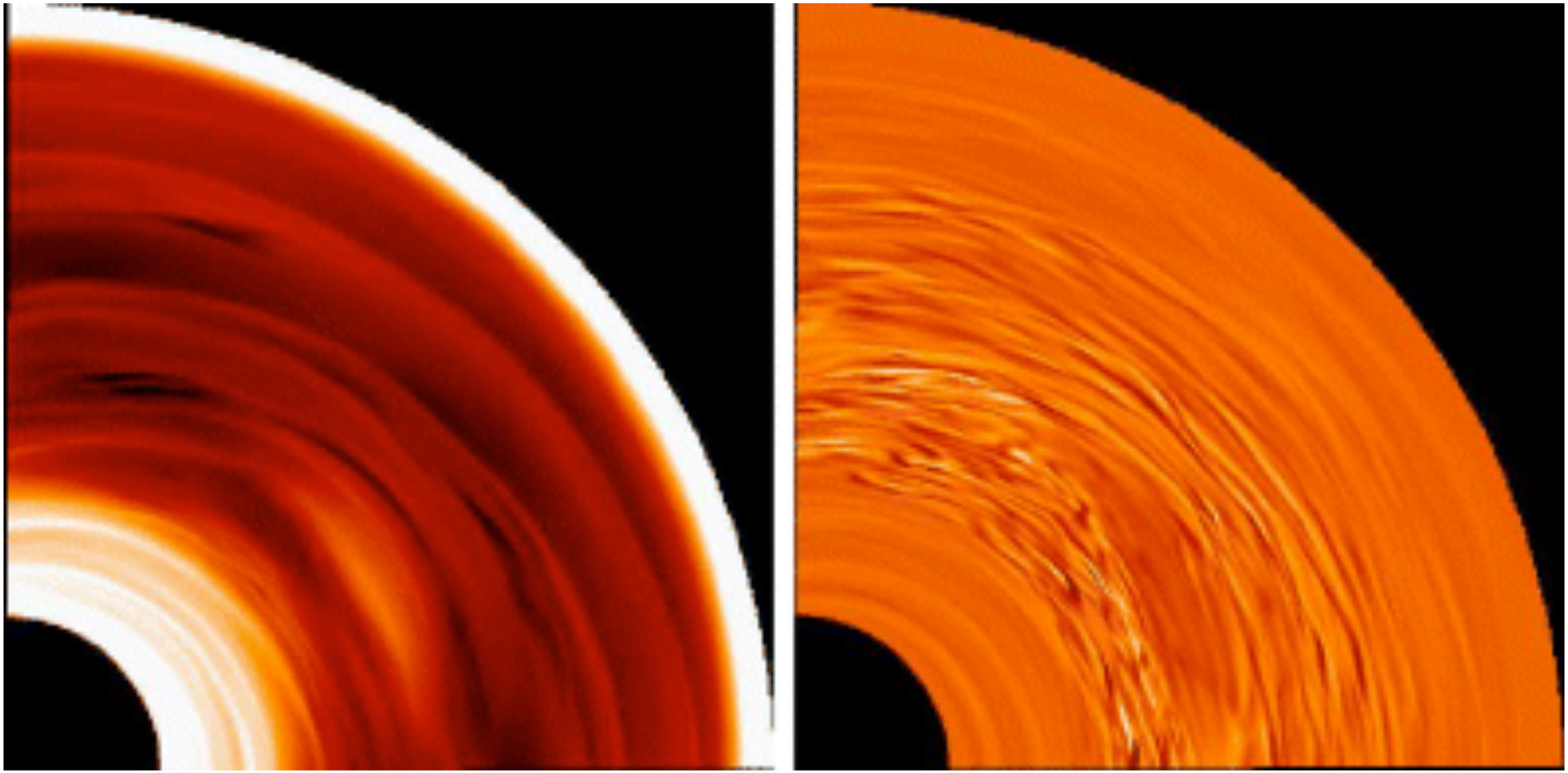
Zonal flows in astrophysical discs



Simon+ 2012

Vortices in astrophysical discs

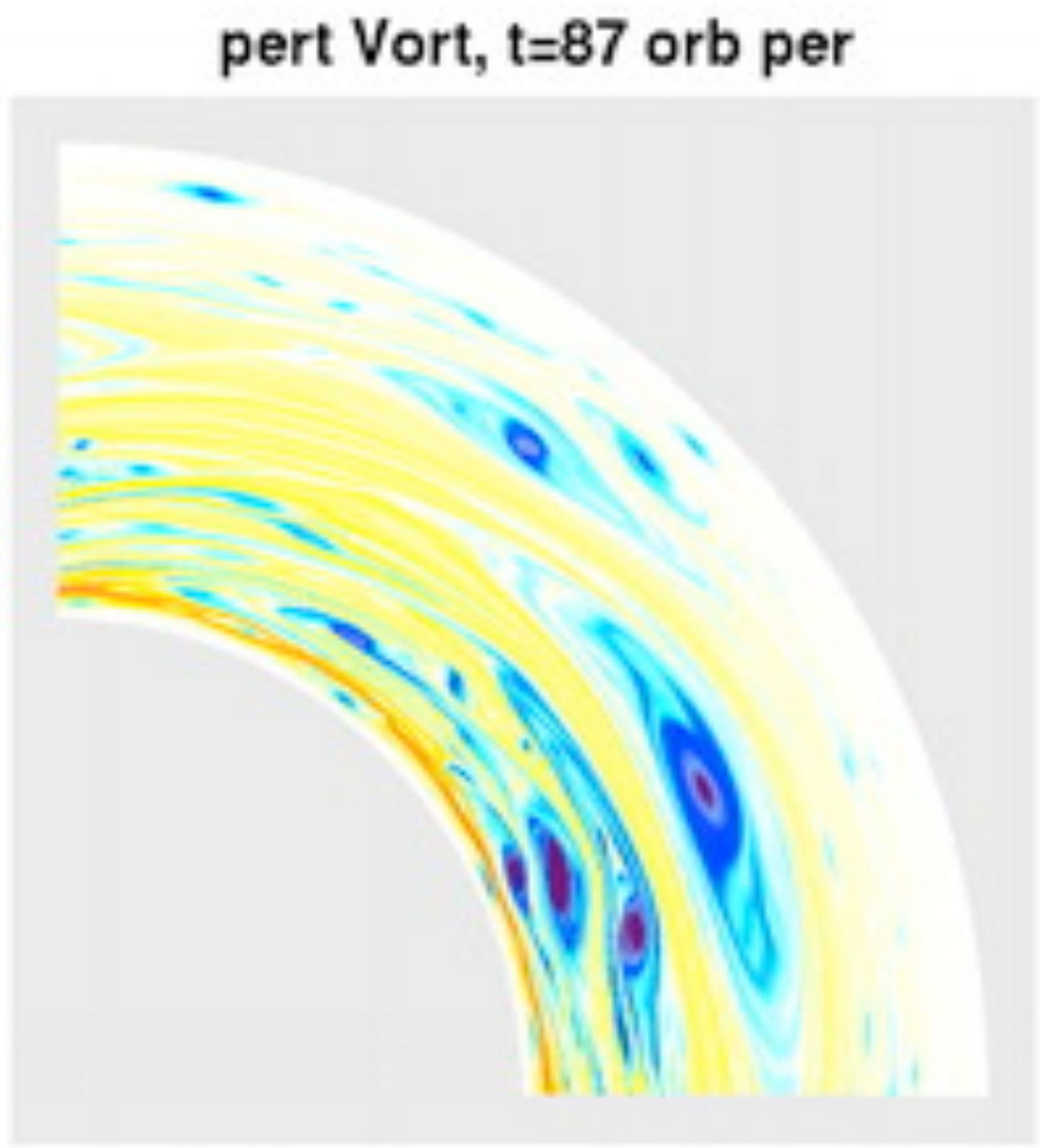
- Vortex formation in MHD turbulence



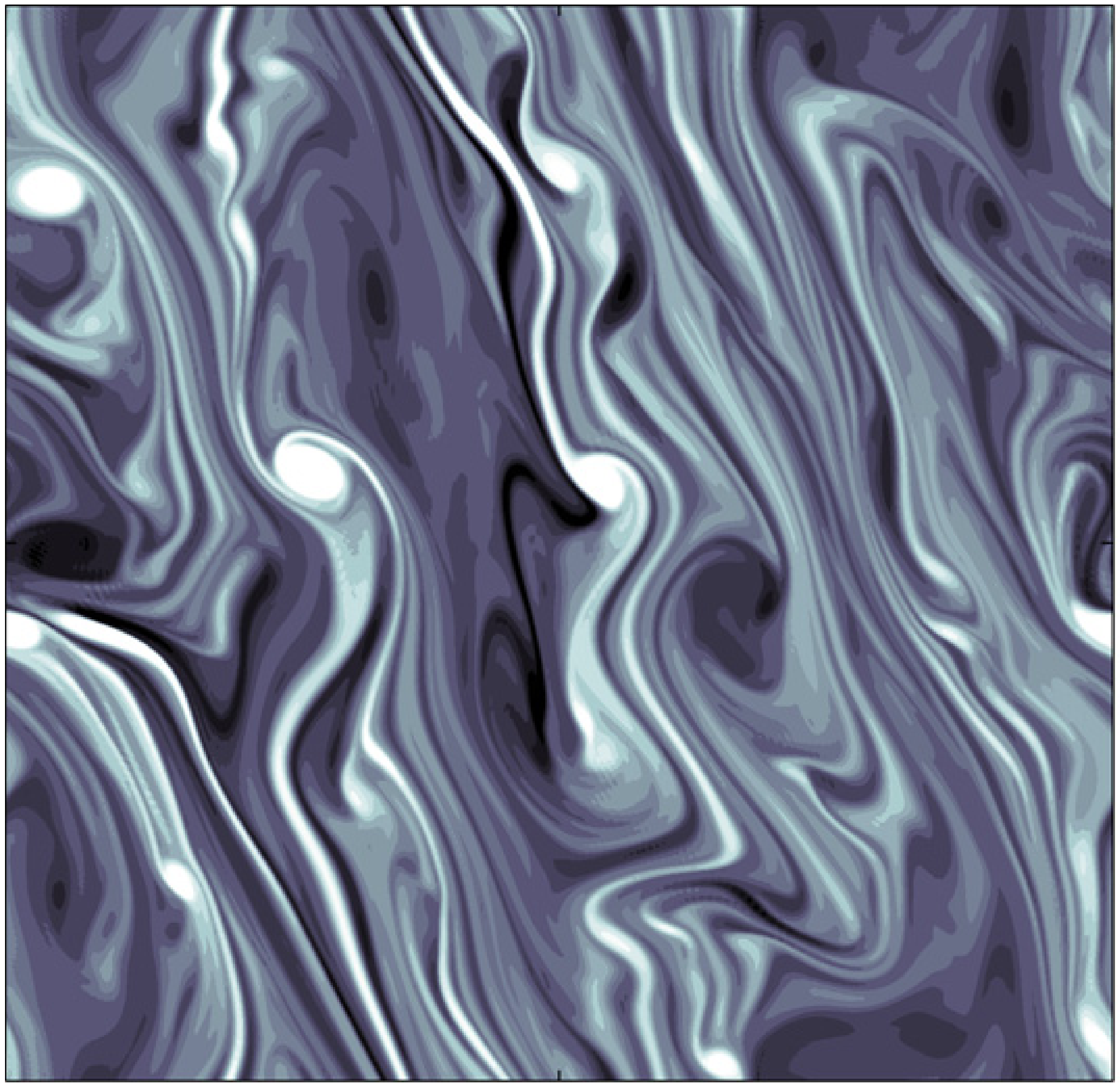
Fromang & Nelson 2005

Vortices in astrophysical discs

- Vortex formation through “subcritical baroclinic instability”



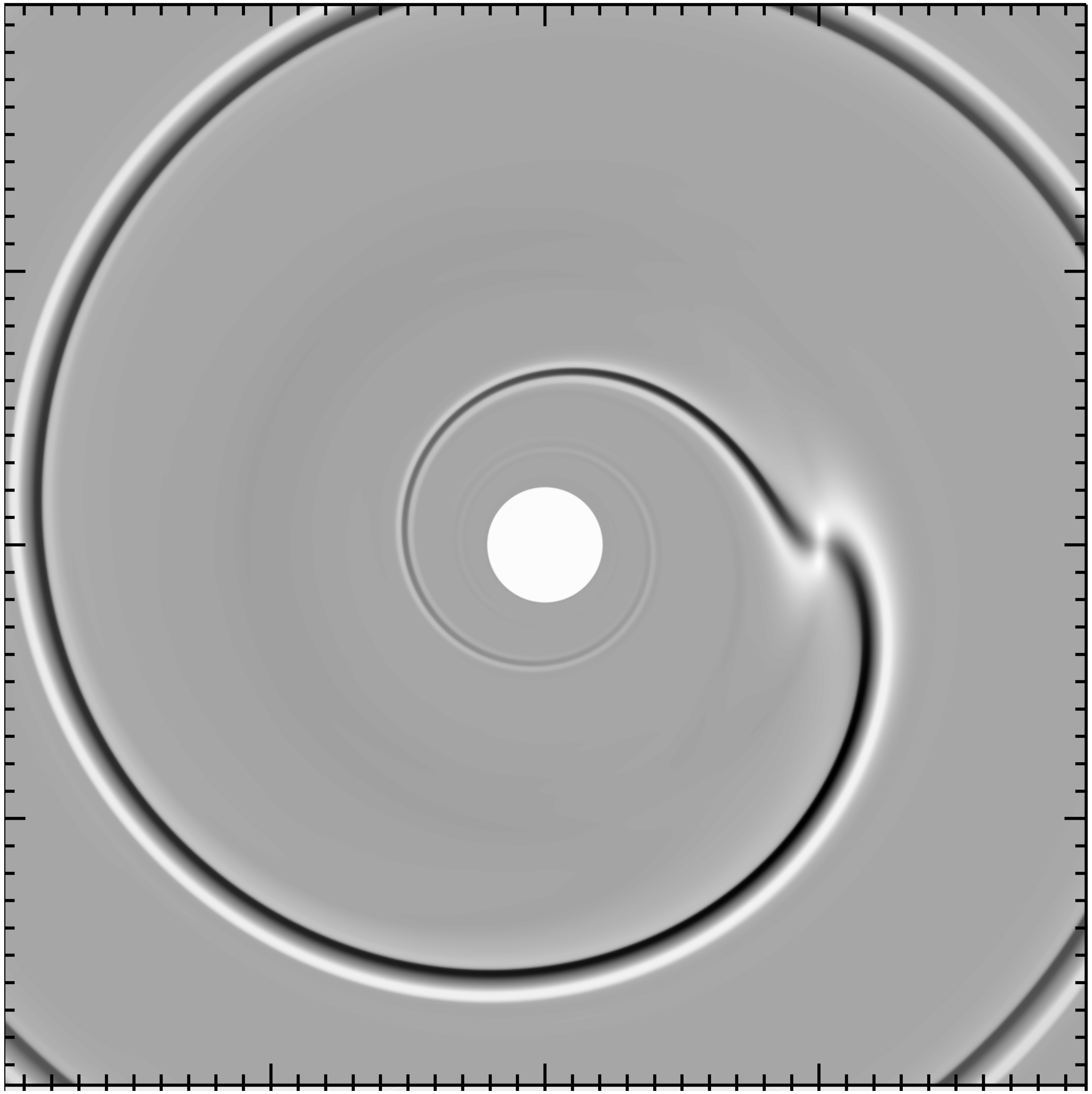
Petersen+ 2007



Lesur & Papaloizou 2010

Vortices in astrophysical discs

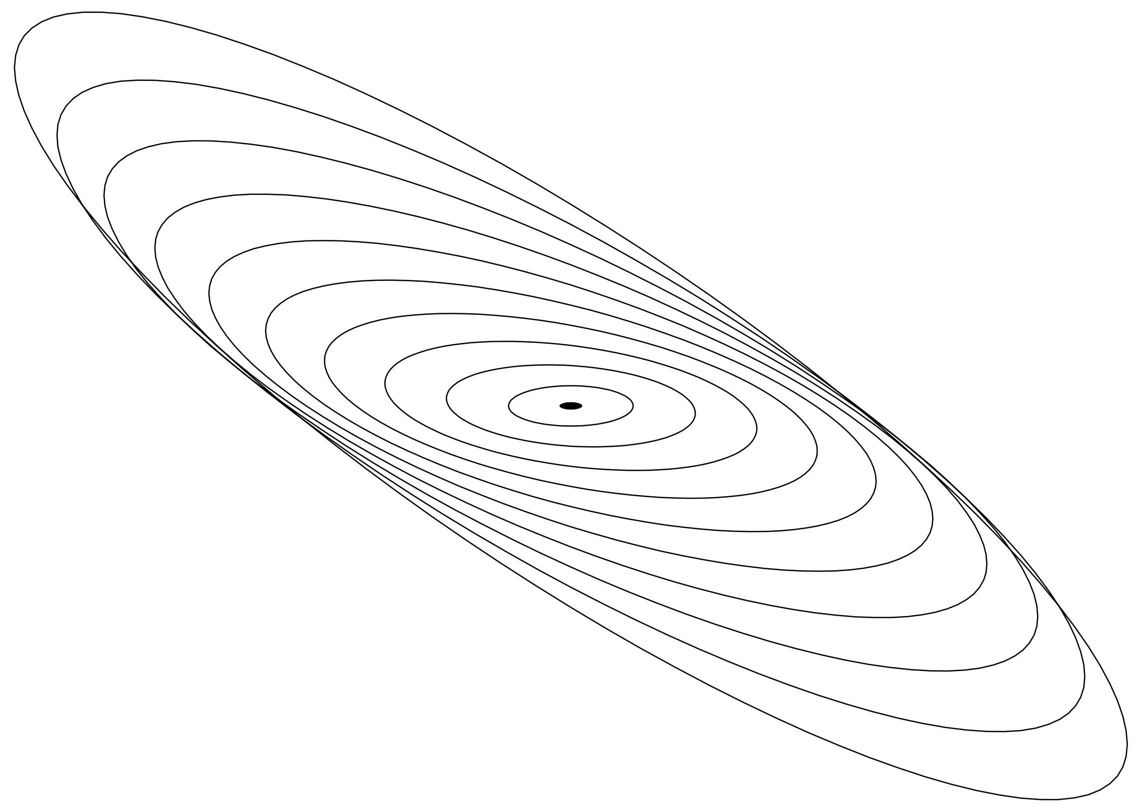
- Vortex migration through acoustic-inertial wave emission

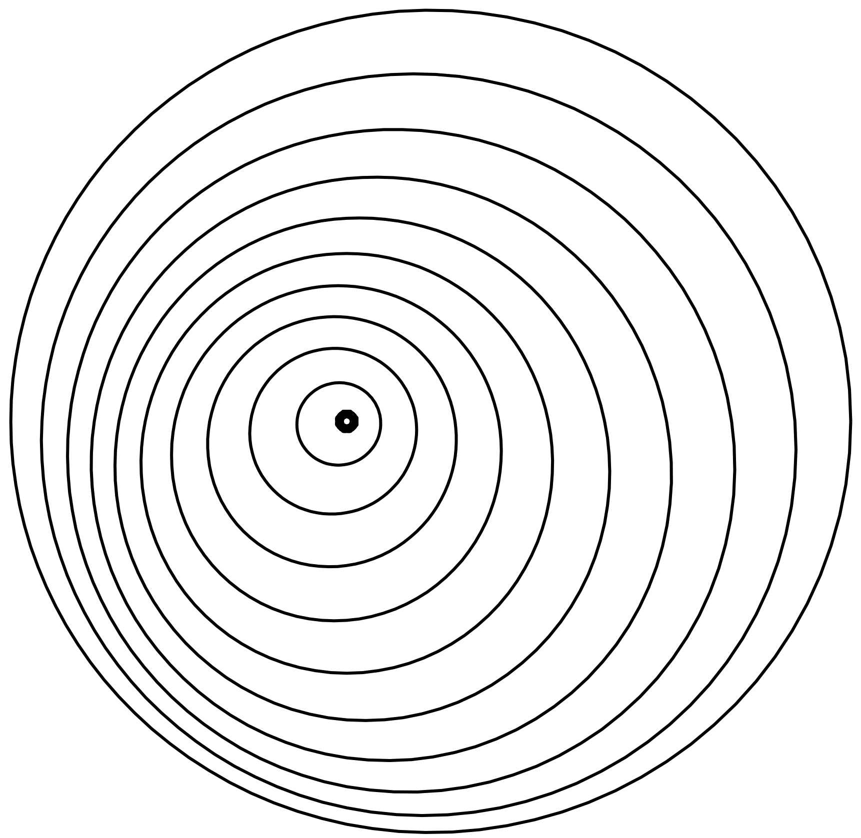


Paardekooper+ 2010

General Keplerian disc

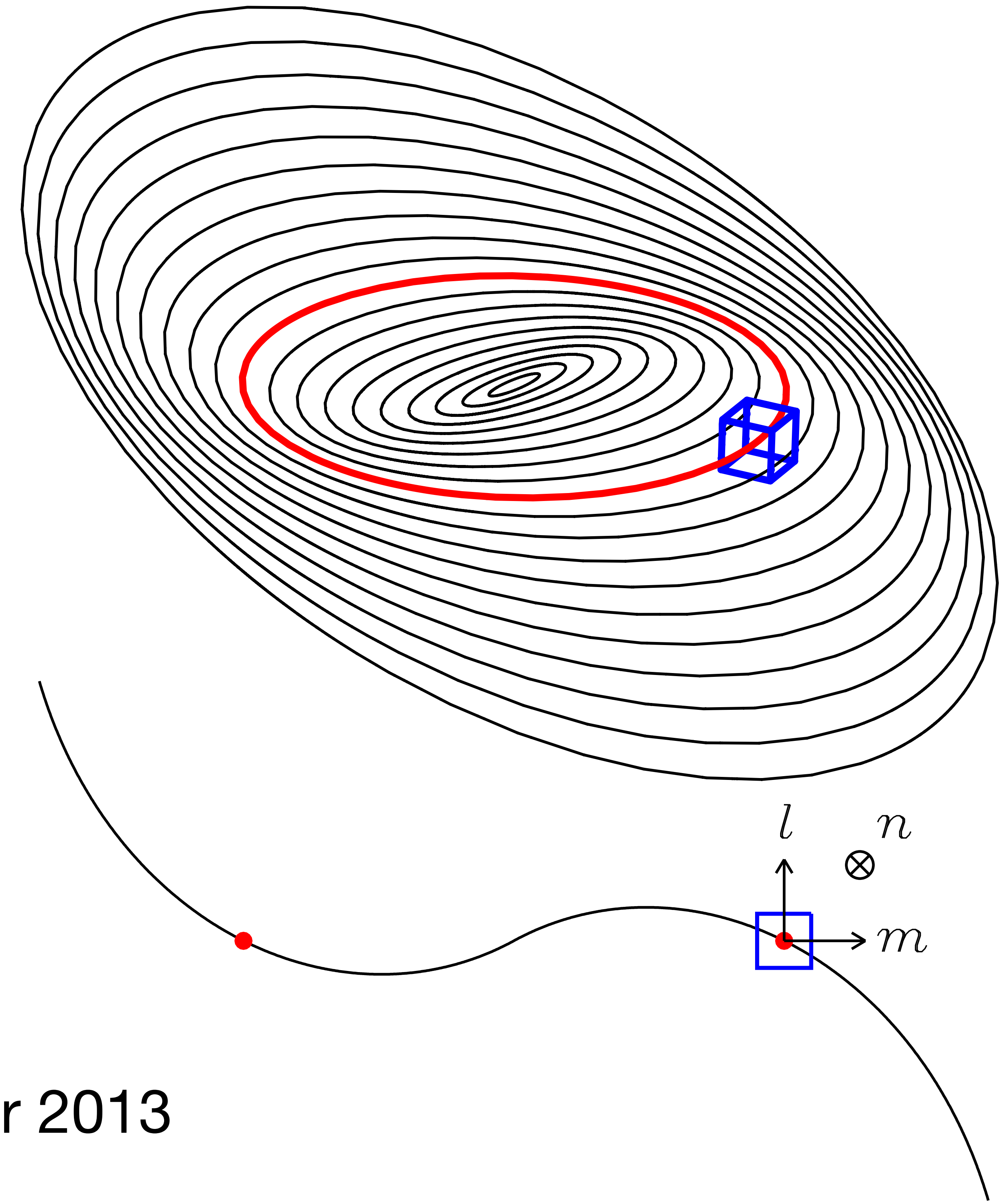
- Orbits can be variably elliptical and mutually inclined
- Smoothly nested streamlines
- Both shape and mass distribution evolve through collective effects
- Evolutionary equations (Ogilvie 1999, 2001)
- Need to determine how internal stresses depend on local geometry





Local model of a warped disc

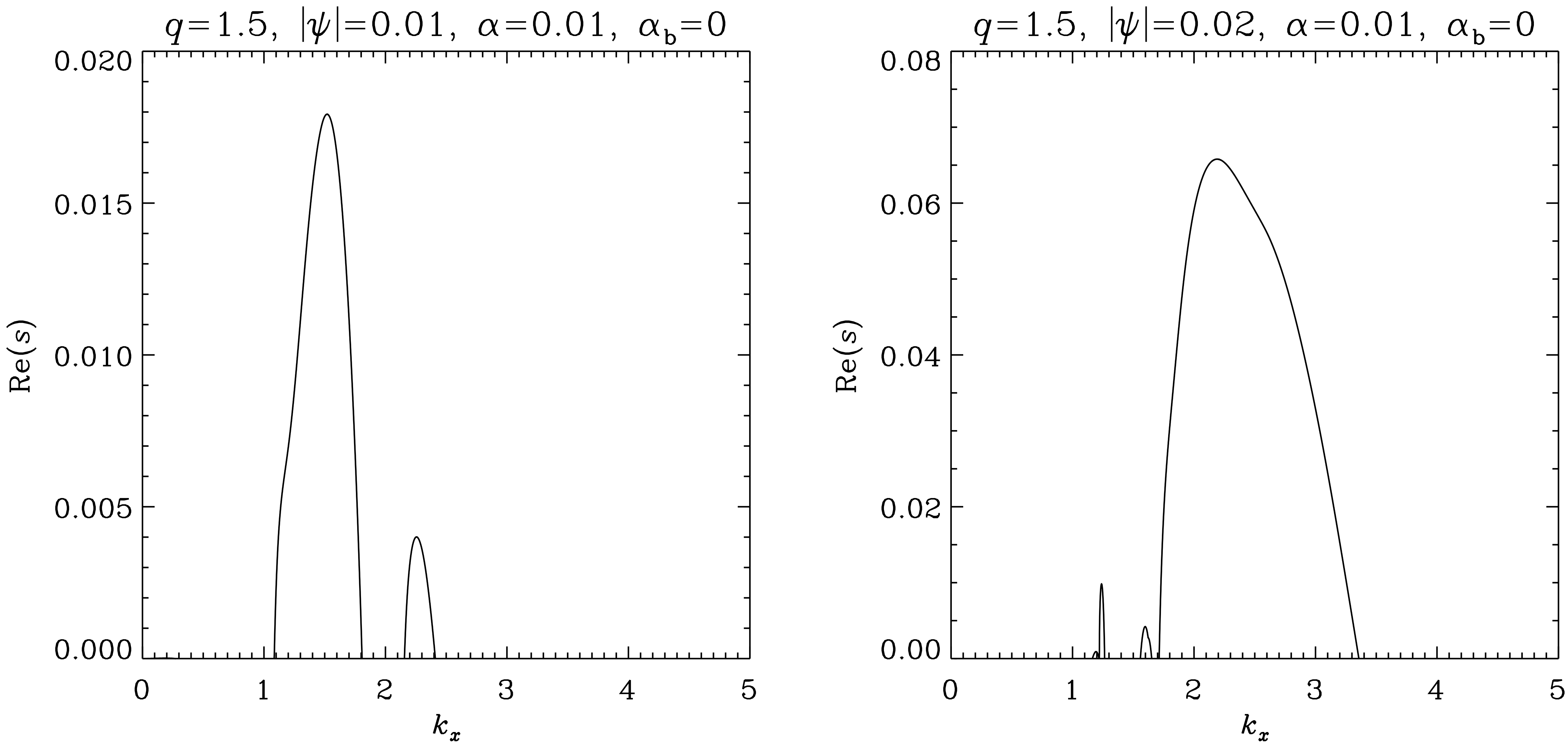
- Geometry oscillates at orbital frequency



Ogilvie & Latter 2013

Parametric instability of warped discs

- Floquet analysis of instability of oscillatory laminar flow
- Maximum growth rate versus radial wavenumber

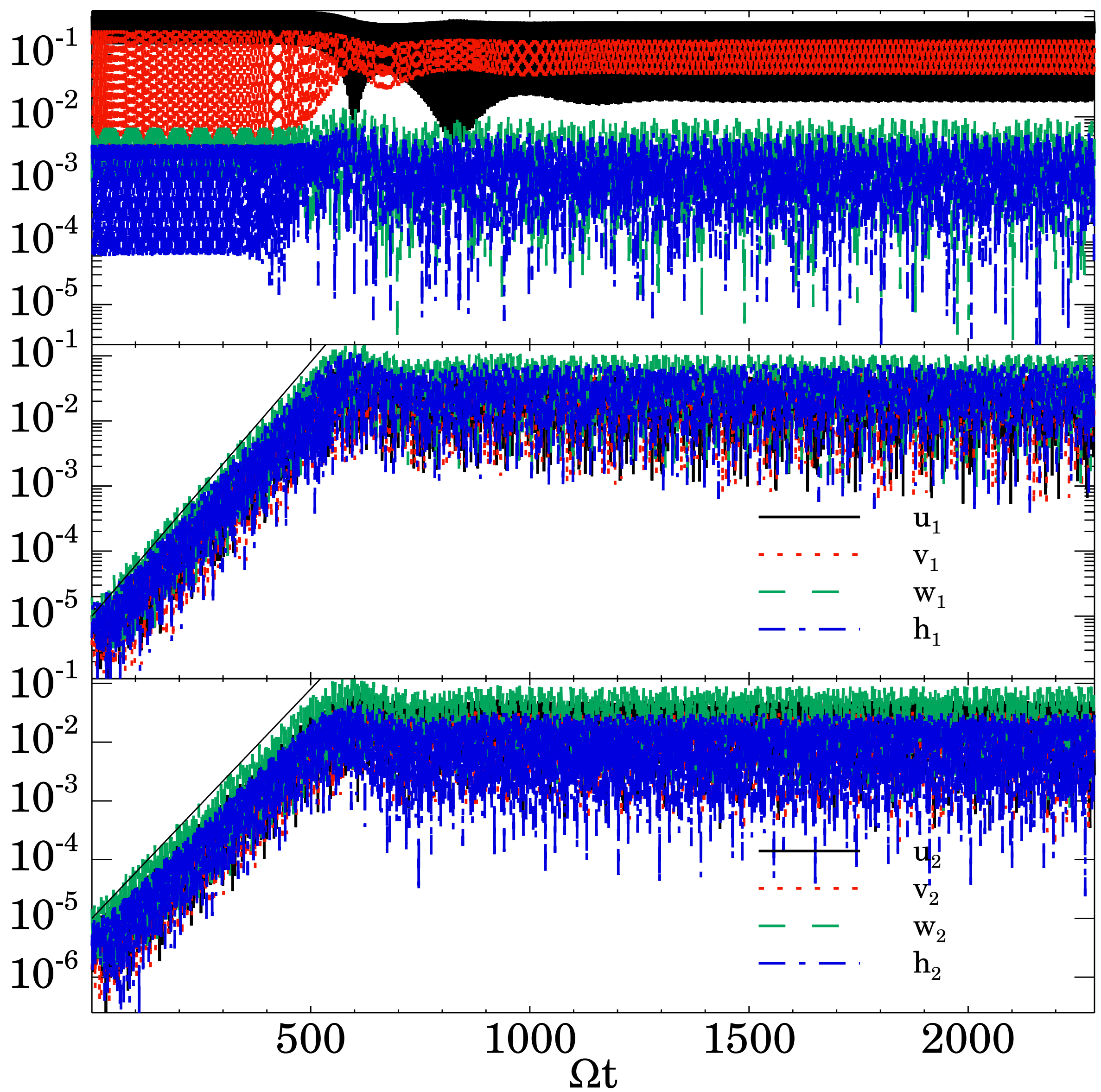


Ogilvie & Latter 2013

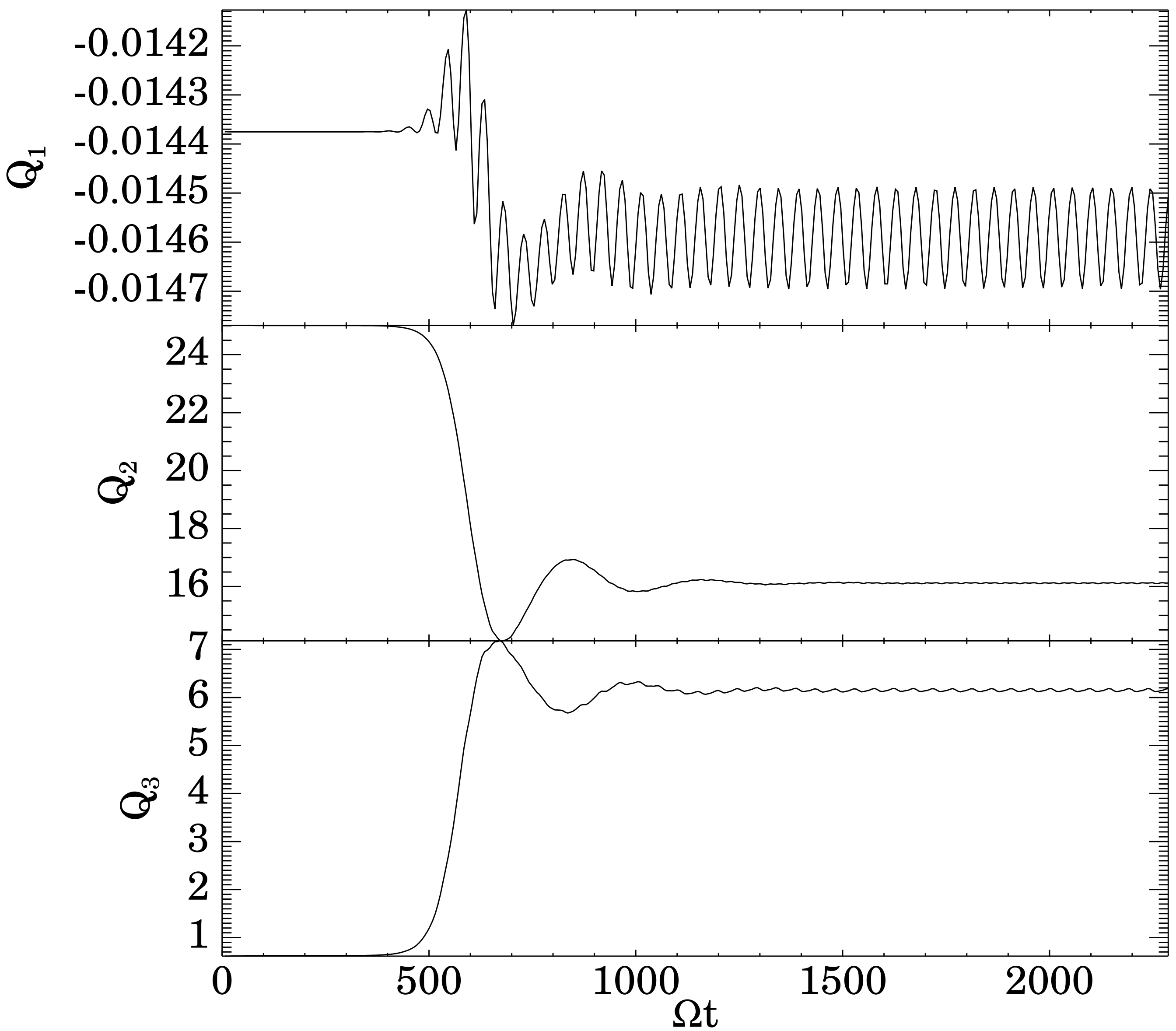
Nonlinear evolution in 2D (S.-J. Paardekooper)

- Keplerian $(q = 1.5,\ |\psi| = 0.01,\ \alpha = 0.01)$

amplitudes of internal waves



internal torque components



Waves and mean flows in stellar interiors

Internal gravity waves in solar-type stars

- Propagation:

$$\omega^2 \approx N^2 \frac{k_\mathrm{h}^2}{k_r^2 + k_\mathrm{h}^2} \qquad k_\mathrm{h}^2 = \frac{l(l+1)}{r^2}$$

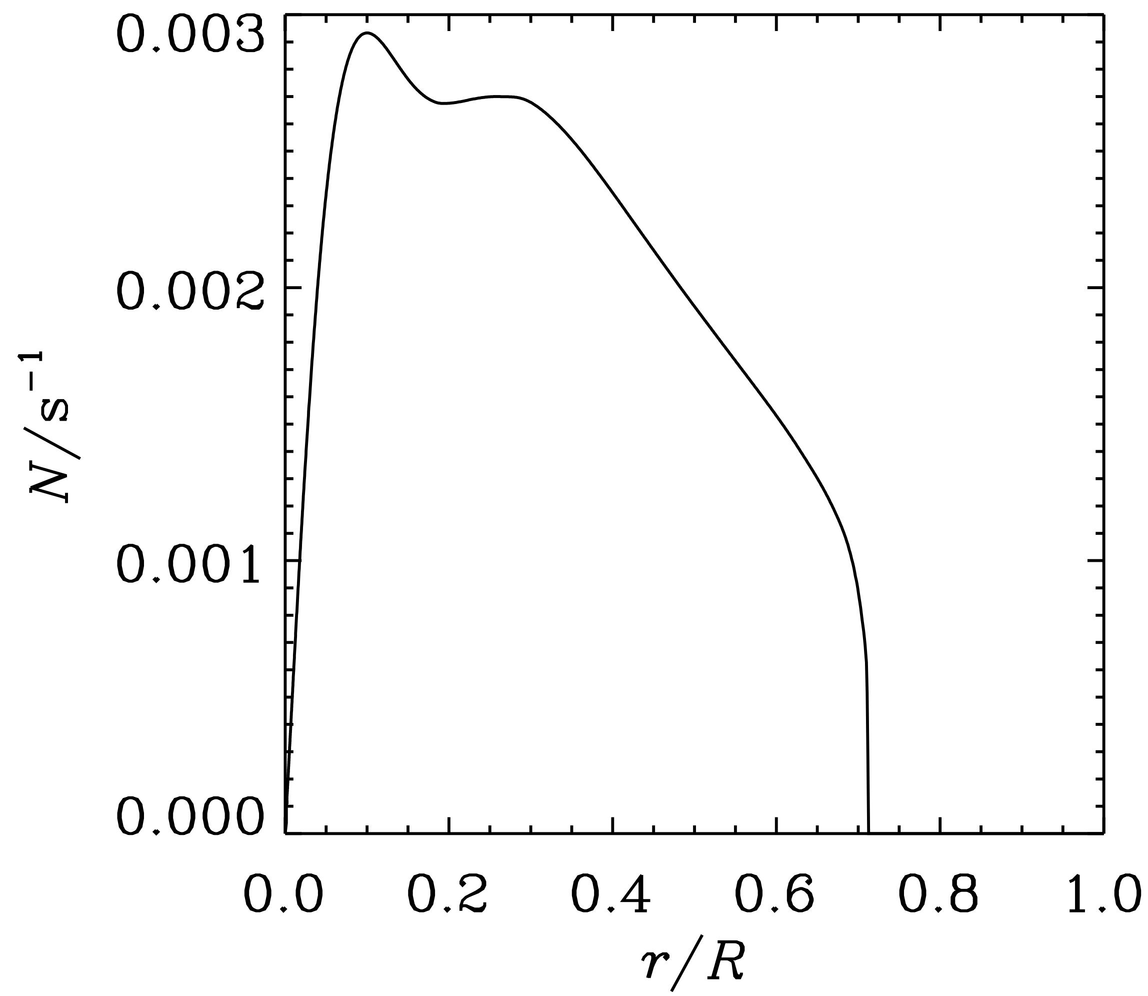
$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right)$$

- Excitation:
  - Convection
  - Instability
  - Tidal forcing
- Focusing towards stellar centre
- Dissipation:
  - Linear (radiative damping)
  - Nonlinear (wave breaking, parametric instability)

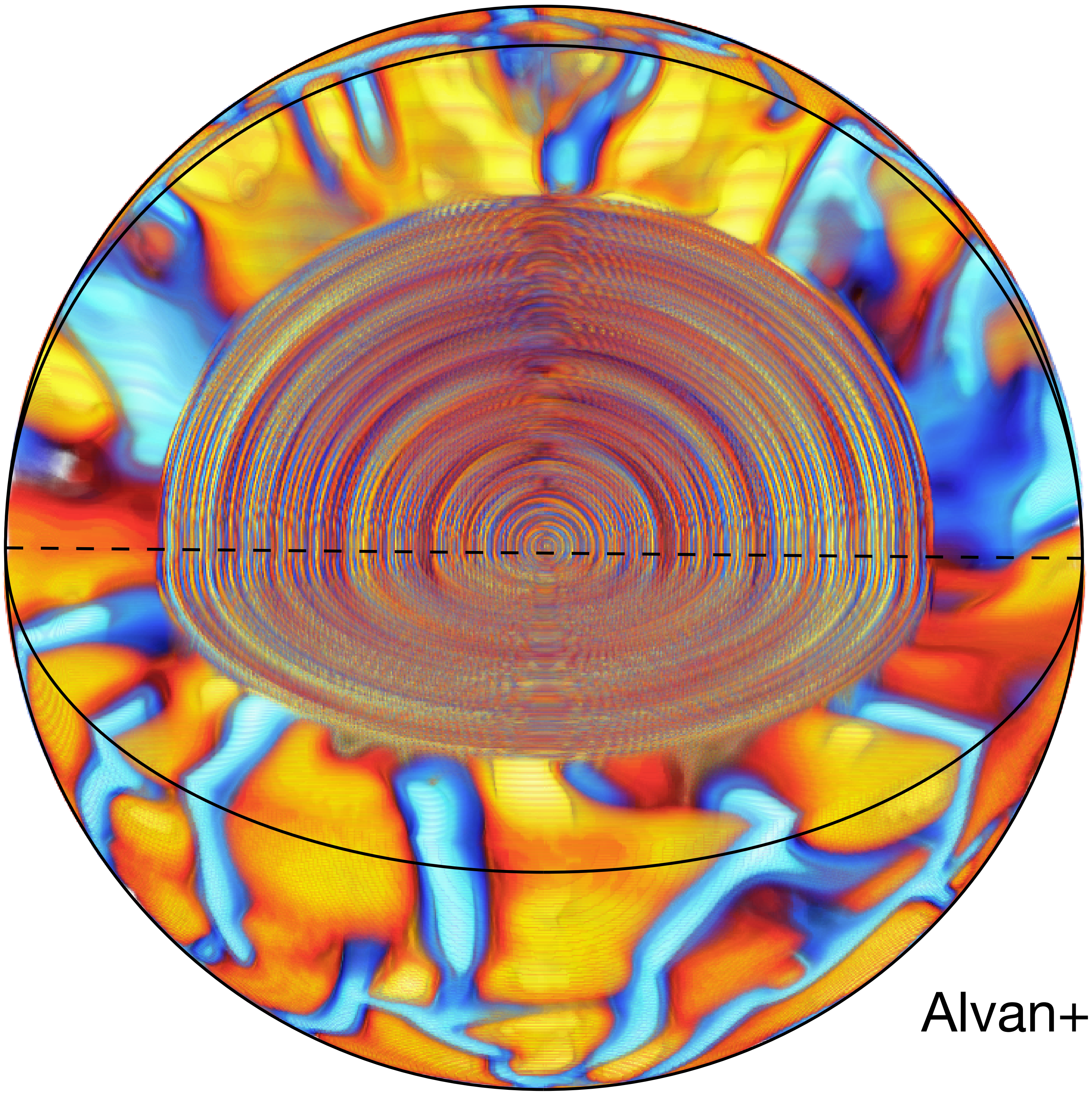
present Sun

slowly rotating

$(2\Omega \approx 5.4 \times 10^{-6}\,\mathrm{s}^{-1})$

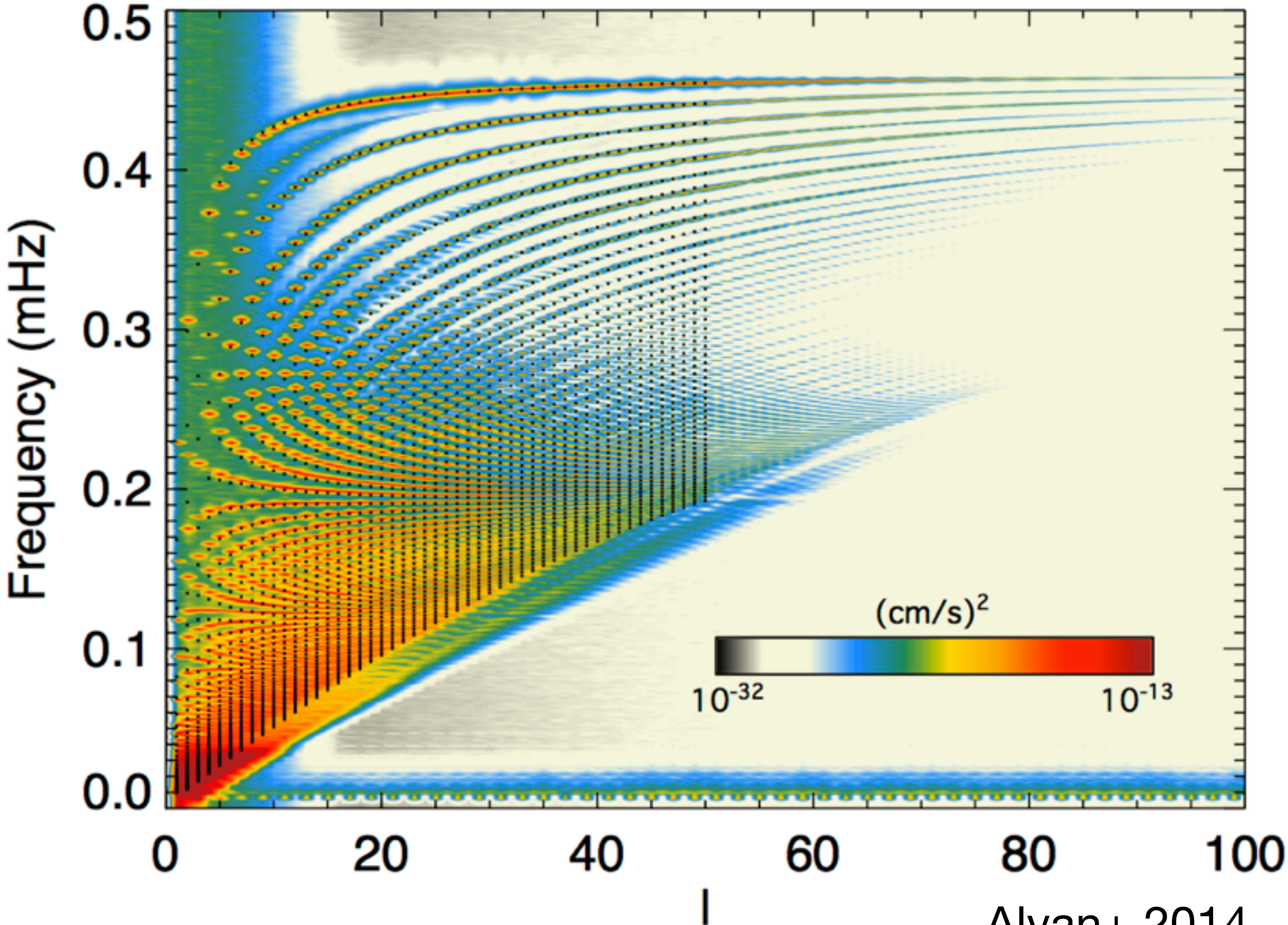


Excitation of internal gravity waves by convection



Alvan+ 2014

Excitation of internal gravity waves by convection



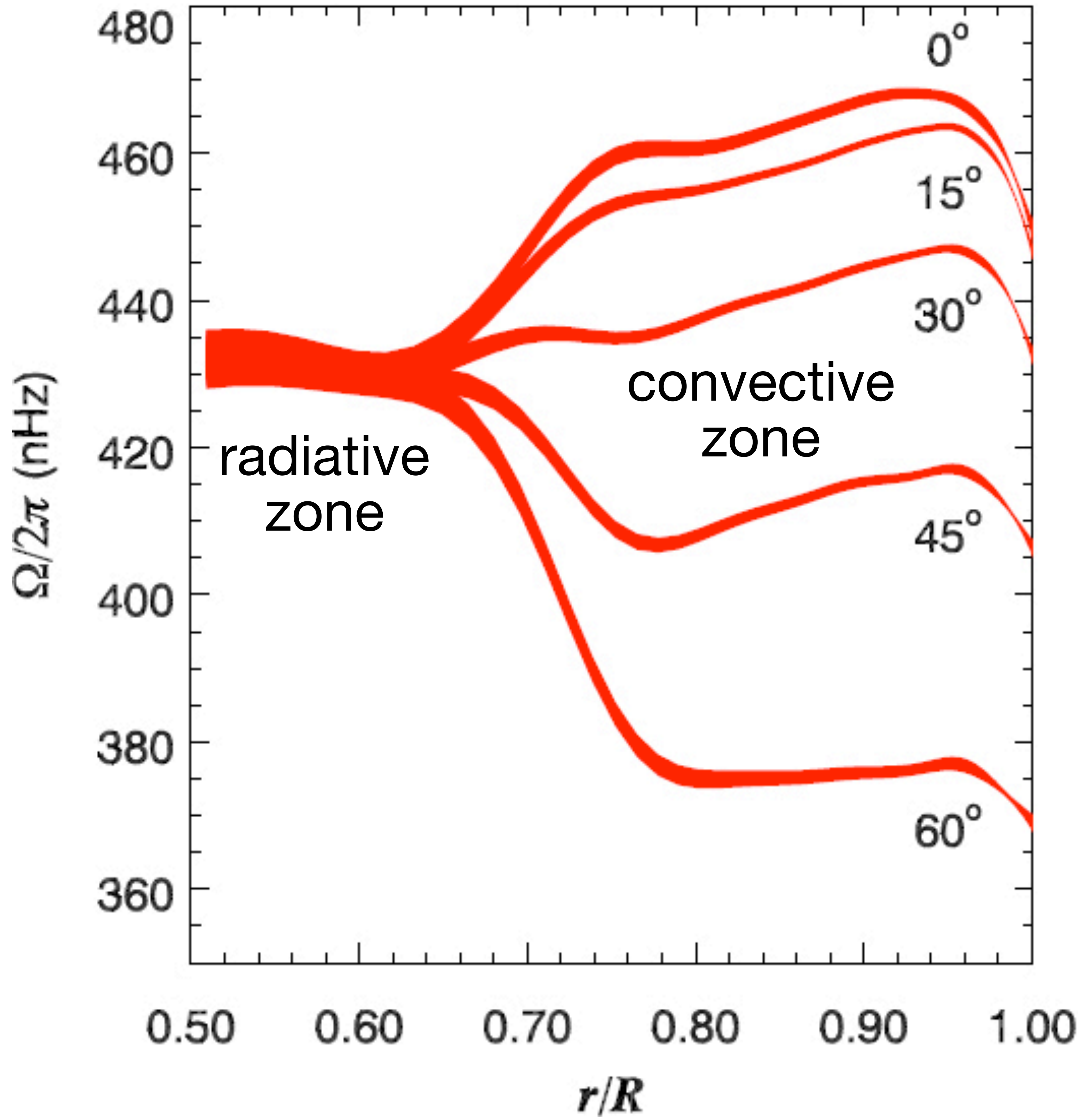
Alvan+ 2014

Excitation of internal gravity waves by convection

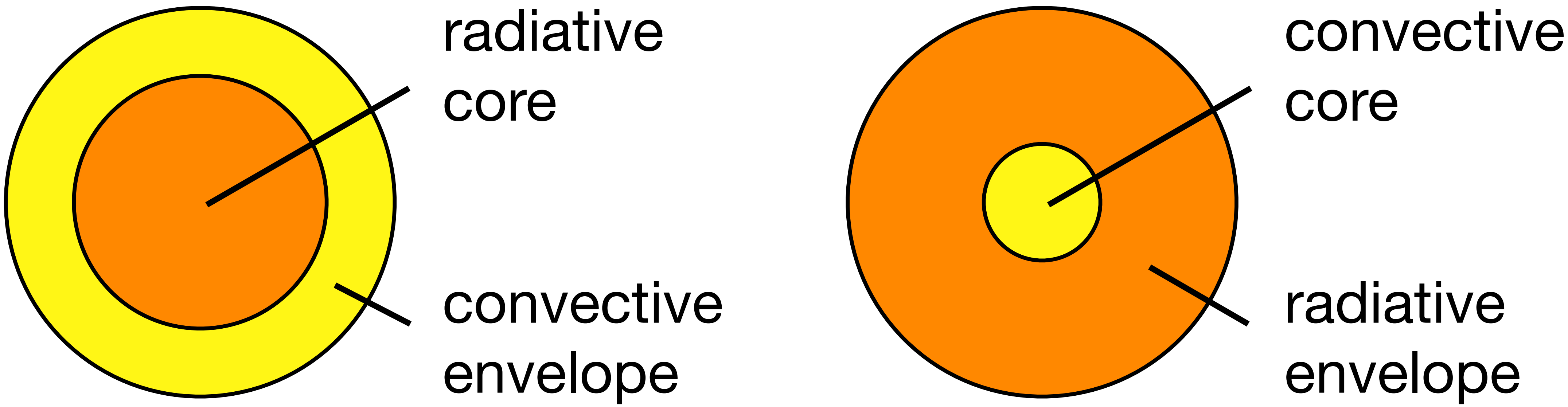
- Mixing of elements in solar core:
  - Solar neutrino problem (Press 1981)
  - Li abundance problem (García Lopez & Spruit 1991)
- Redistribution of angular momentum:
  - Mean flow of the form $\bar{\boldsymbol{u}} = \Omega(r, \theta)\, r \sin\theta\, \boldsymbol{e}_\phi$
  - Maintenance of uniform rotation?
    (Schatzman 1993; Kumar & Quataert 1997; Zahn+ 1997)
  - Sign error corrected! (Ringot 1998)
  - *Enhancement* of differential rotation (Kumar+ 1999)
  - Time-dependent behaviour, perhaps more complicated than QBO (Rogers & Glatzmaier 2005-6)
  - Magnetic field bound to be important

Excitation of internal gravity waves by convection

- Internal solar rotation determined from helioseismology



Stellar structure

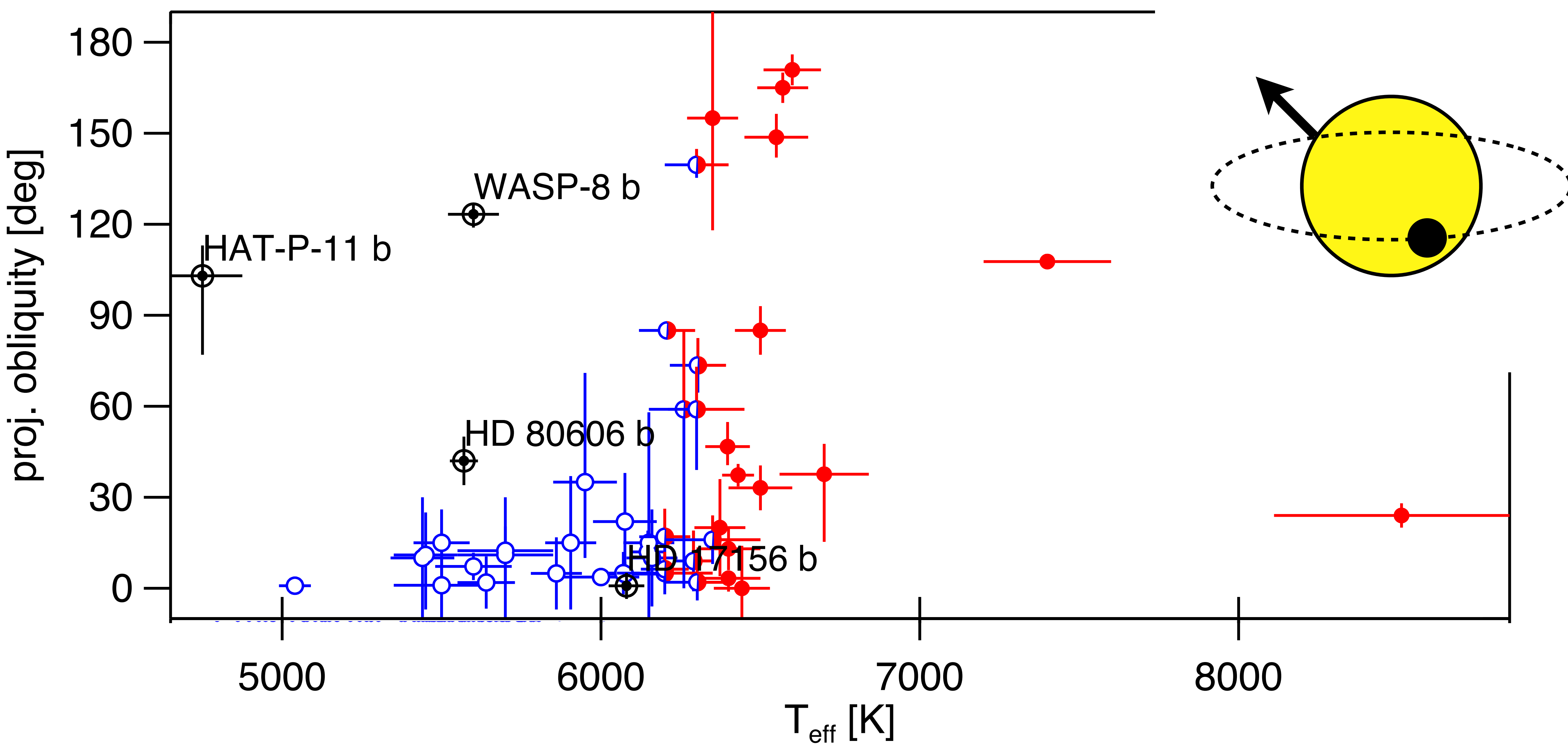


solar-type star

more massive star

More massive stars

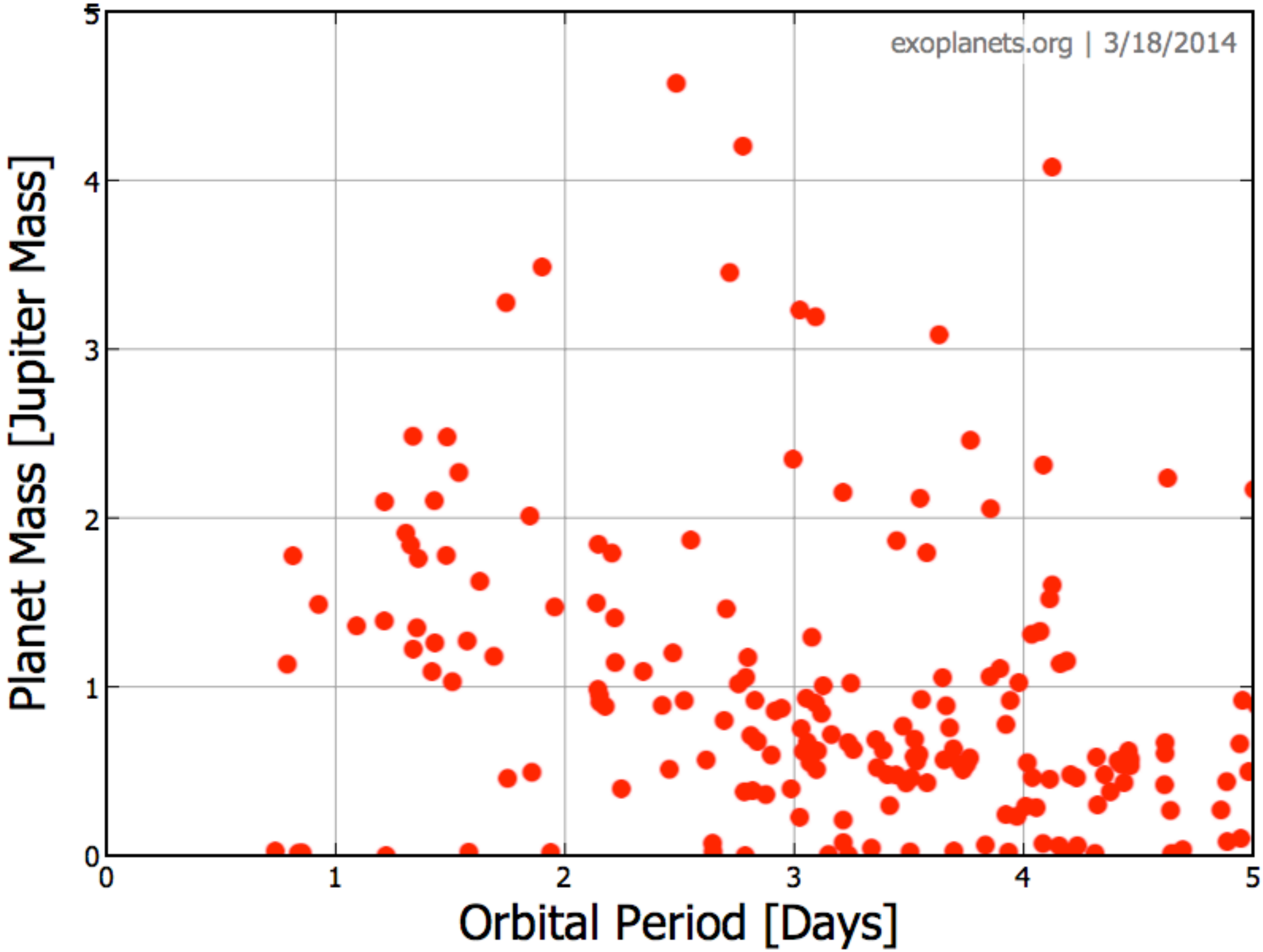
- Excitation by convection
  - Modulation of surface rotation (Rogers+ 2012-3)
  - Explanation of observed spin–orbit misalignments?



Albrecht+ 2012

Excitation of internal gravity waves by tidal forcing

Excitation of internal gravity waves by tidal forcing



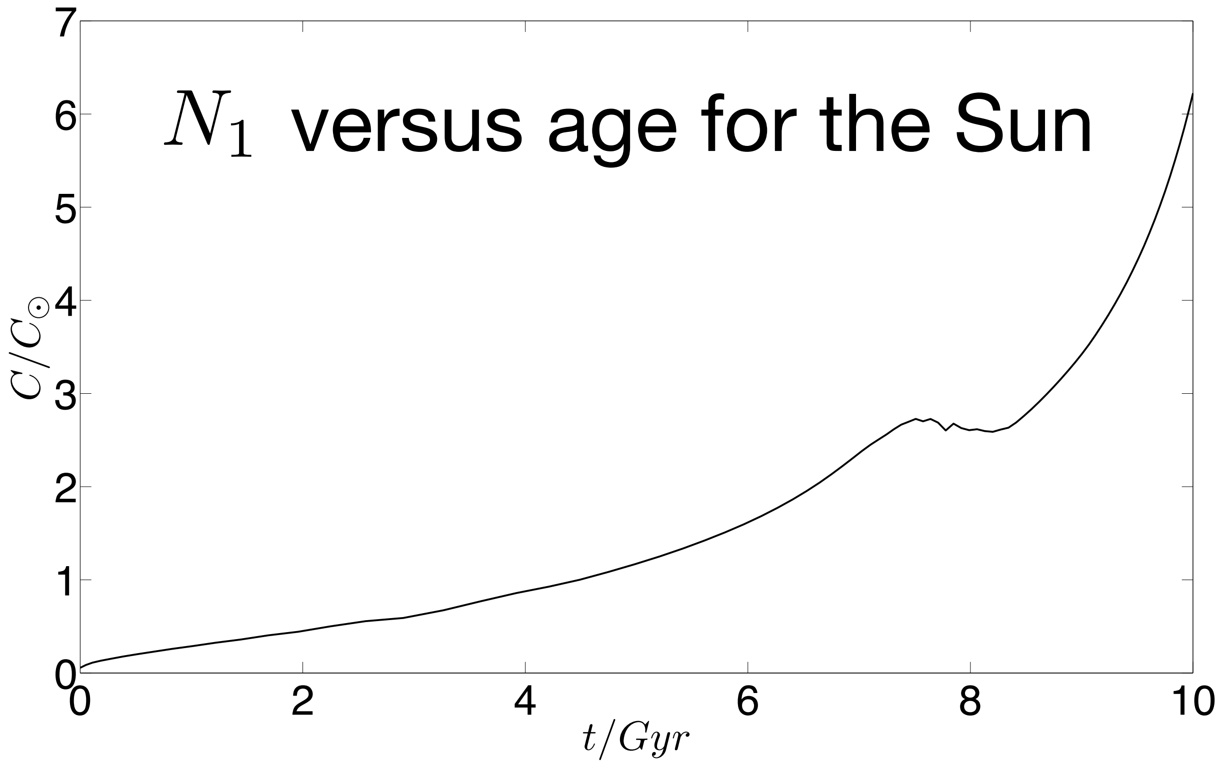
Breaking of internal gravity waves near stellar centre

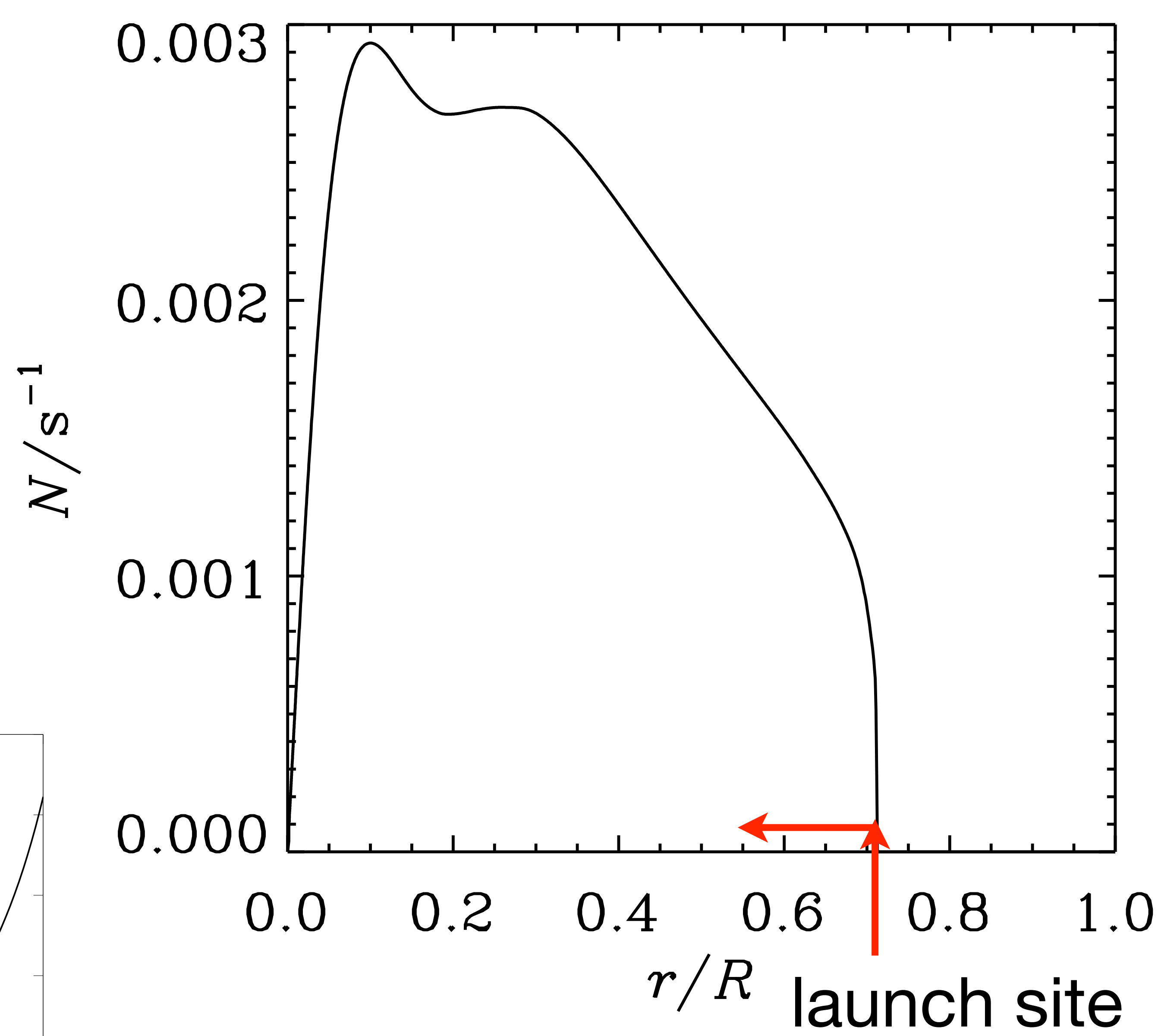
Near stellar centre:

$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right)$$

$$N = N_1 r + N_3 r^3 + \cdots$$

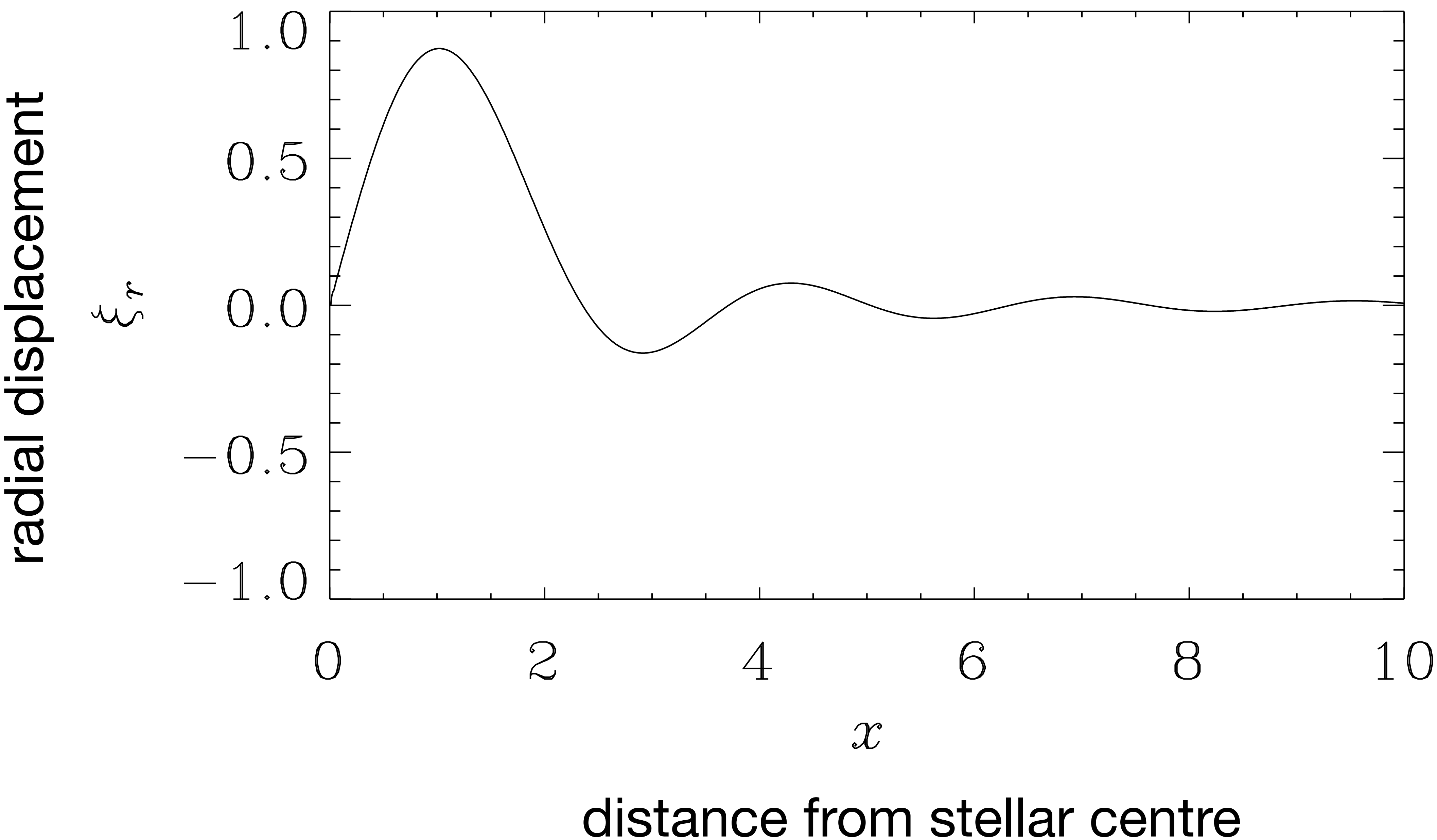
N_1 generally increases with stellar mass and age





monochromatic wave

Breaking of internal gravity waves near stellar centre



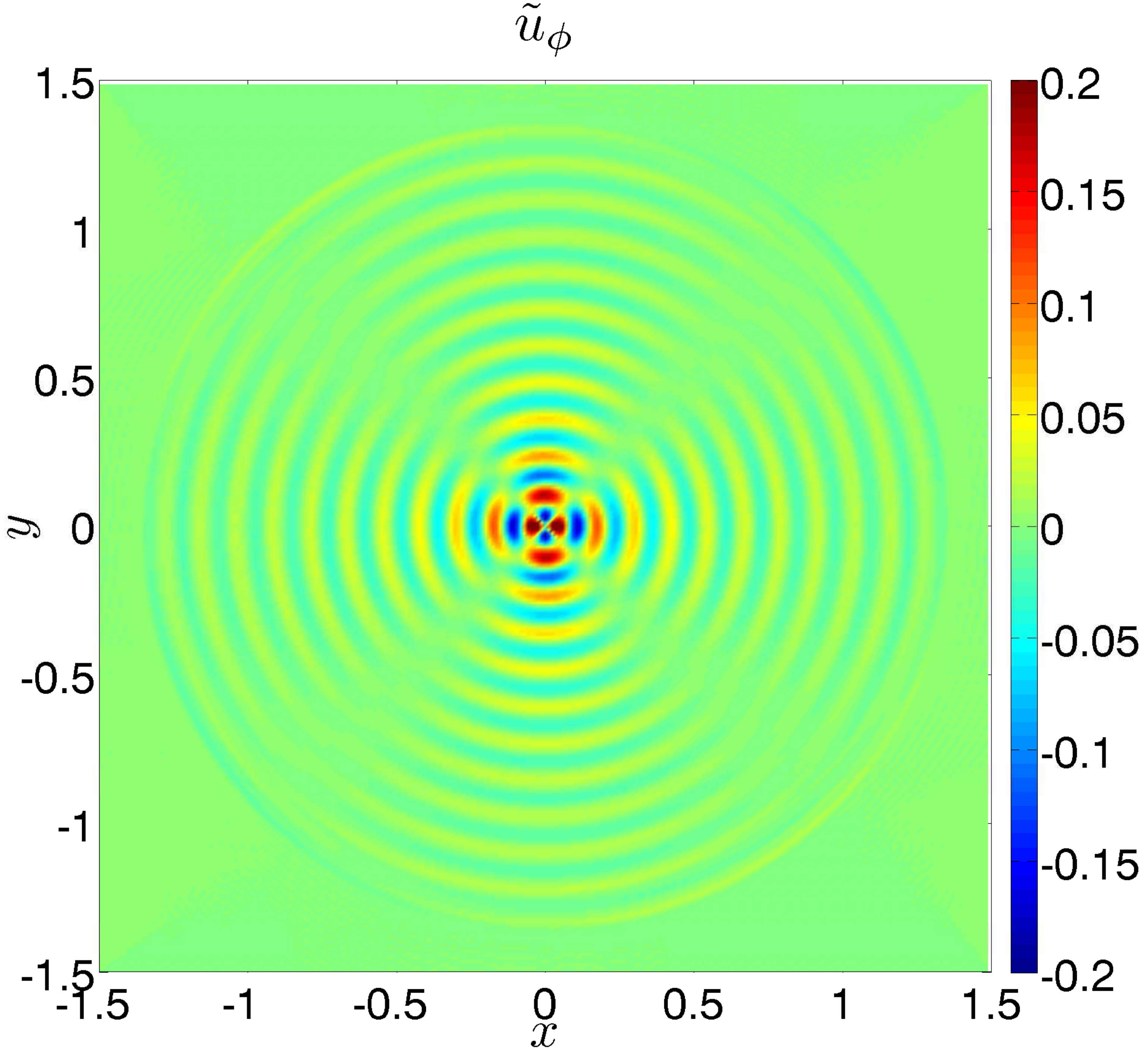
Barker & Ogilvie (2010), cf. Goodman & Dickson (1998)

Typical wavelength $0.001 - 0.01\,R_\odot$

3D numerical simulations

Barker & Ogilvie 2011

Lower amplitude: standing wave

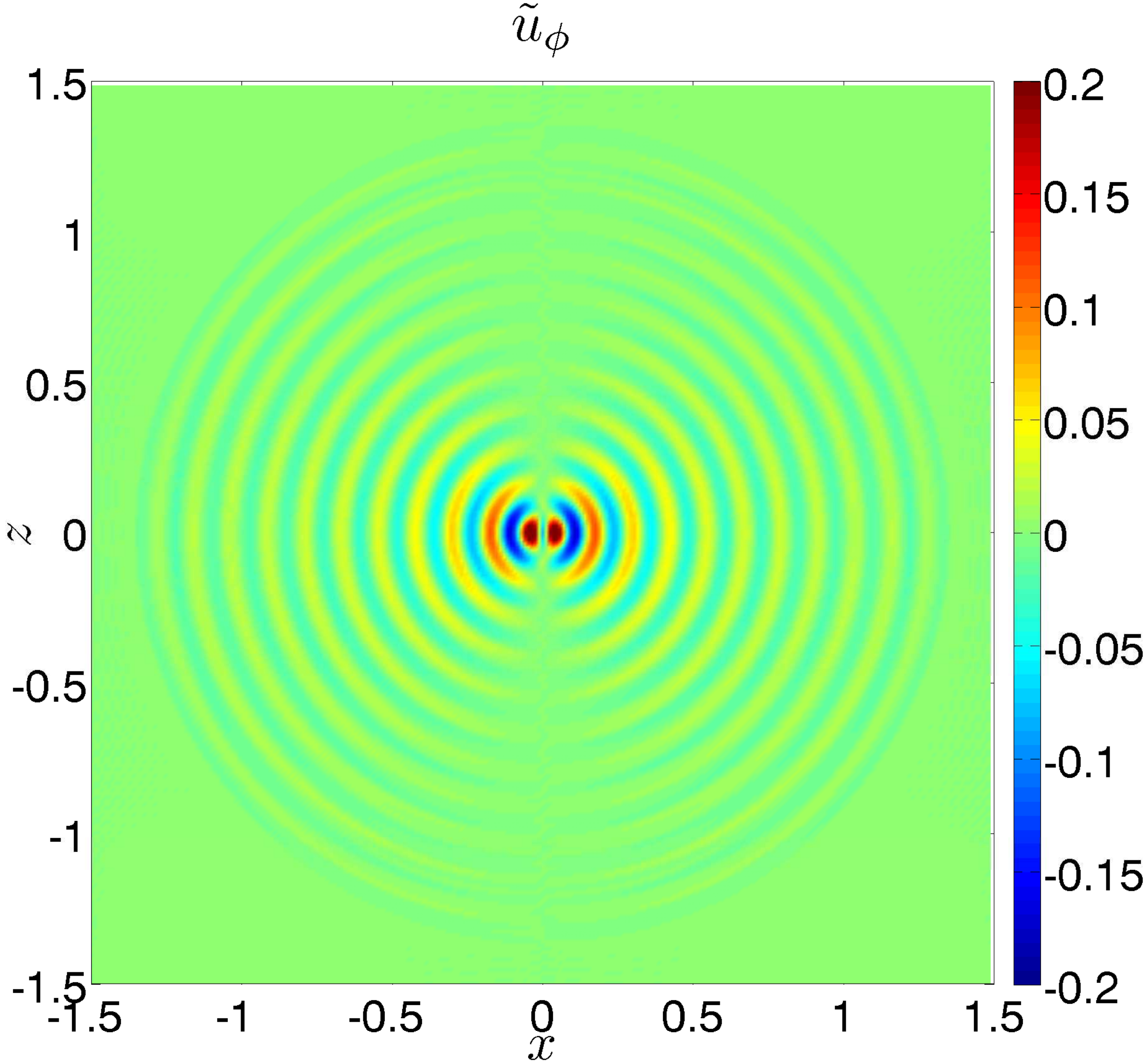


equatorial plane

3D numerical simulations

Barker & Ogilvie 2011

Lower amplitude: standing wave

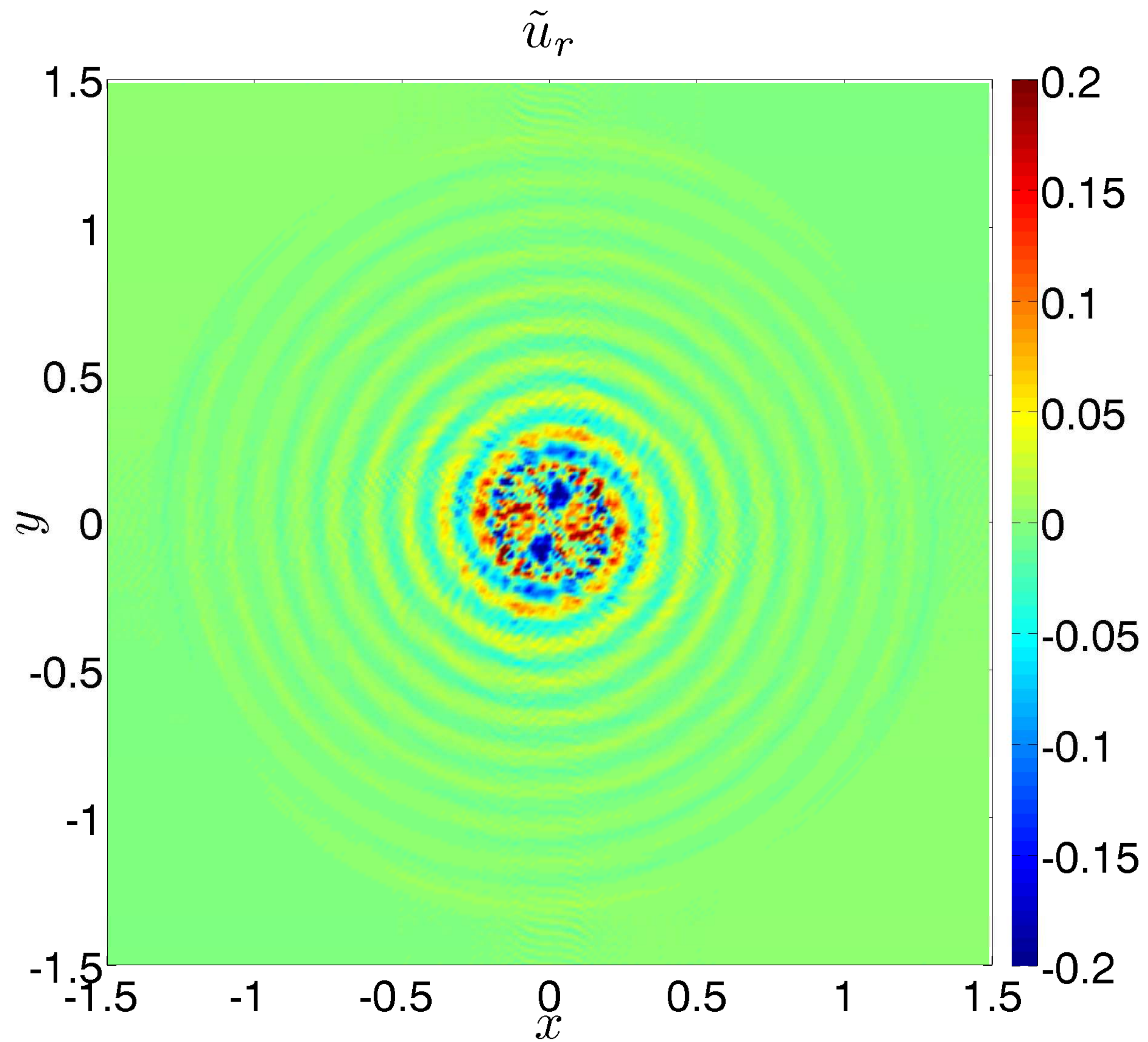


meridional plane

3D numerical simulations

Barker & Ogilvie 2011

Higher amplitude: breaking wave

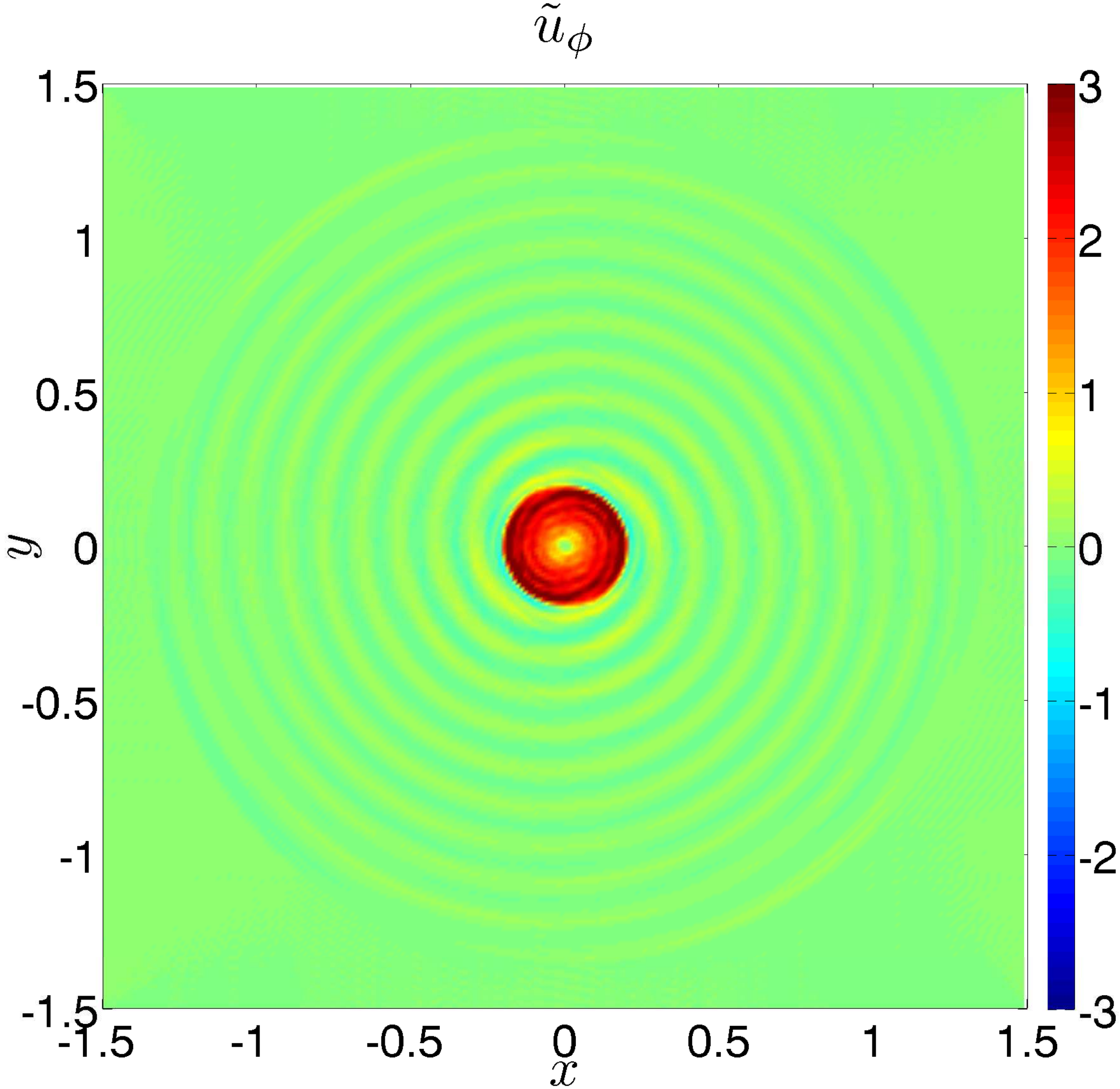


equatorial plane

3D numerical simulations

Barker & Ogilvie 2011

Higher amplitude: breaking wave



meridional
plane

3D numerical simulations

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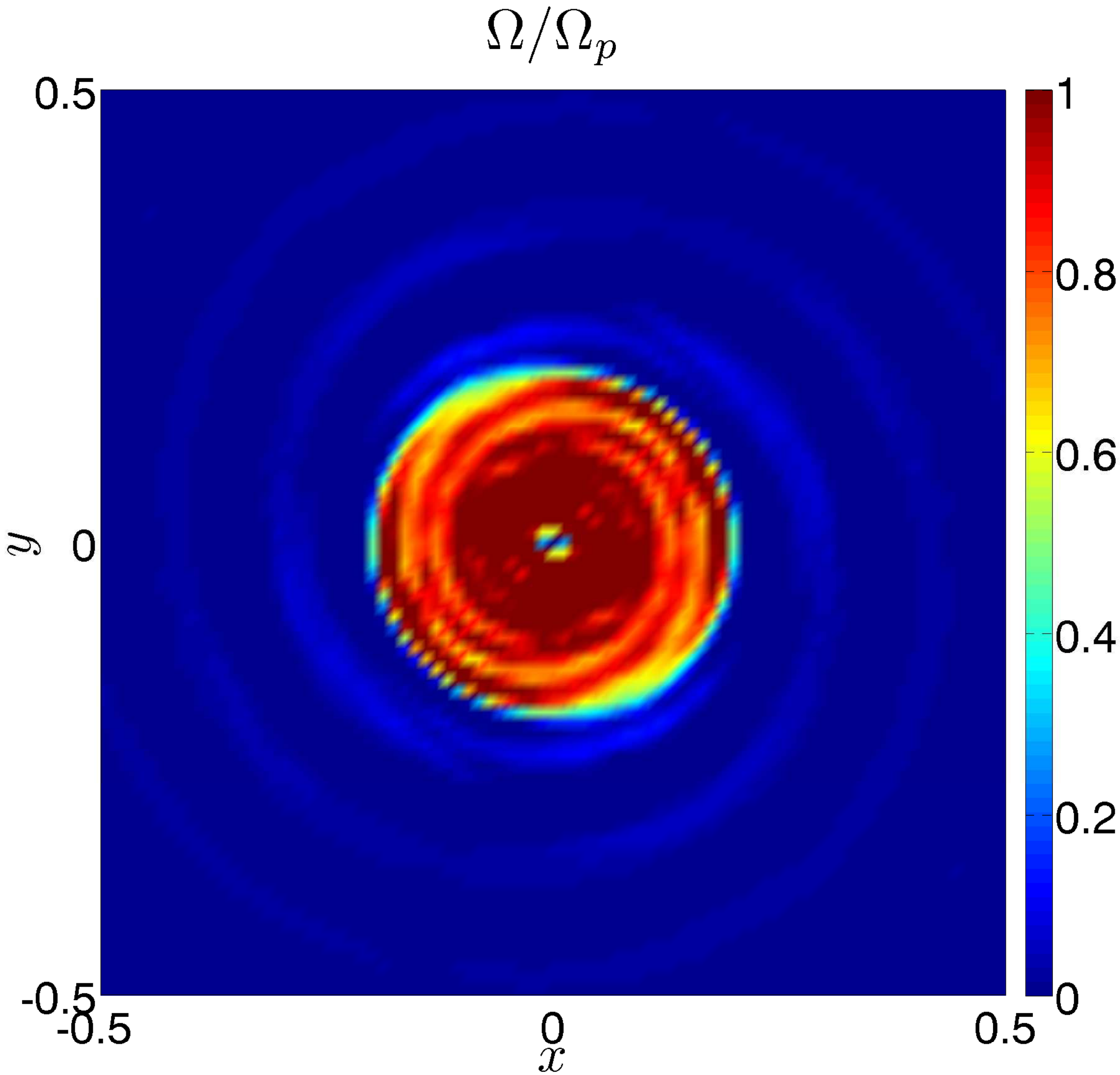
Breaking wave



equatorial plane

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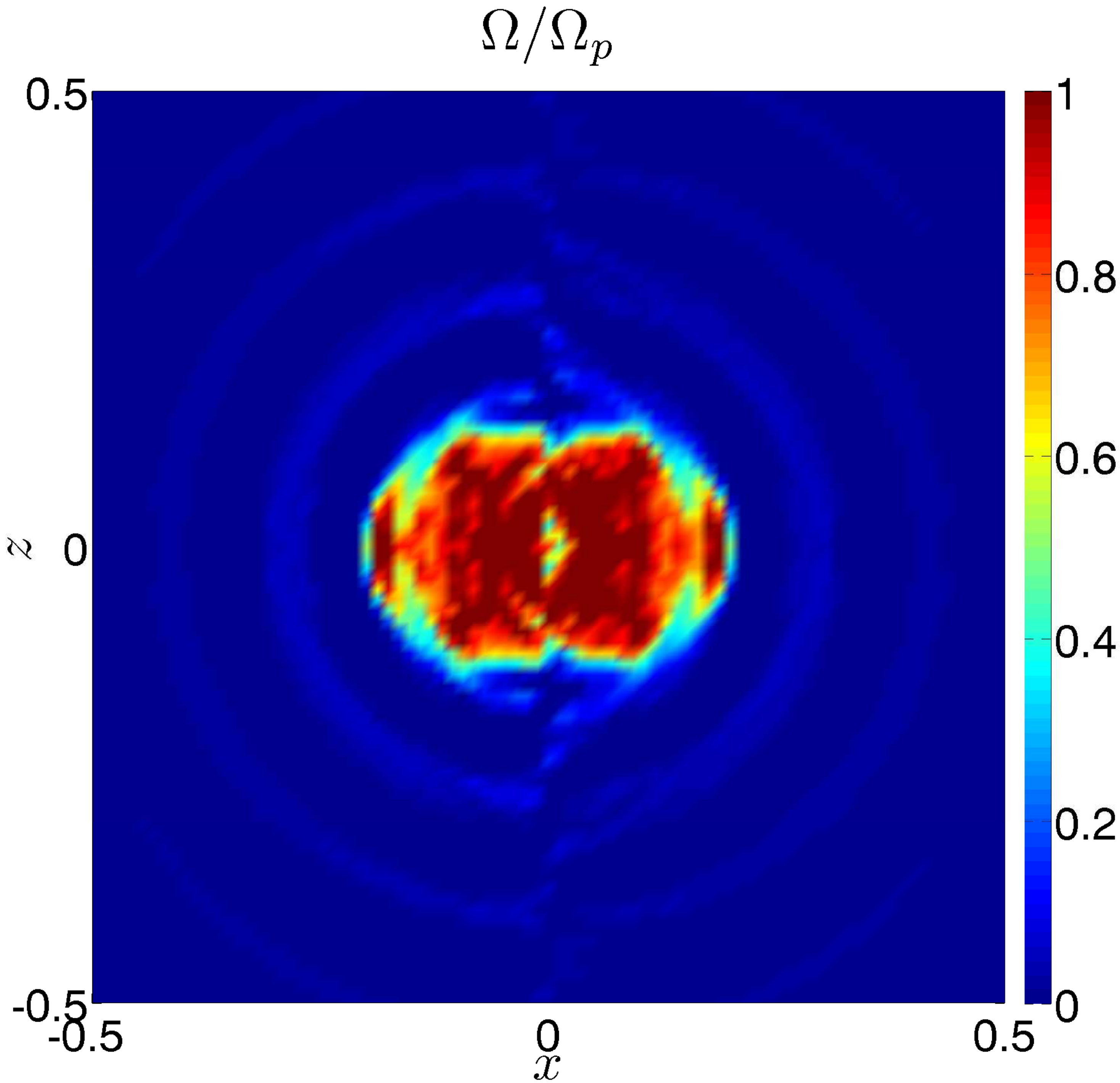
Breaking wave



meridional
plane

Implications

- Waves break at centre if

$$\frac{M_{\rm p}}{M_{\rm J}} > 3.6 \left(\frac{P_{\rm orb}}{\rm day} \right)^{-1/6}$$

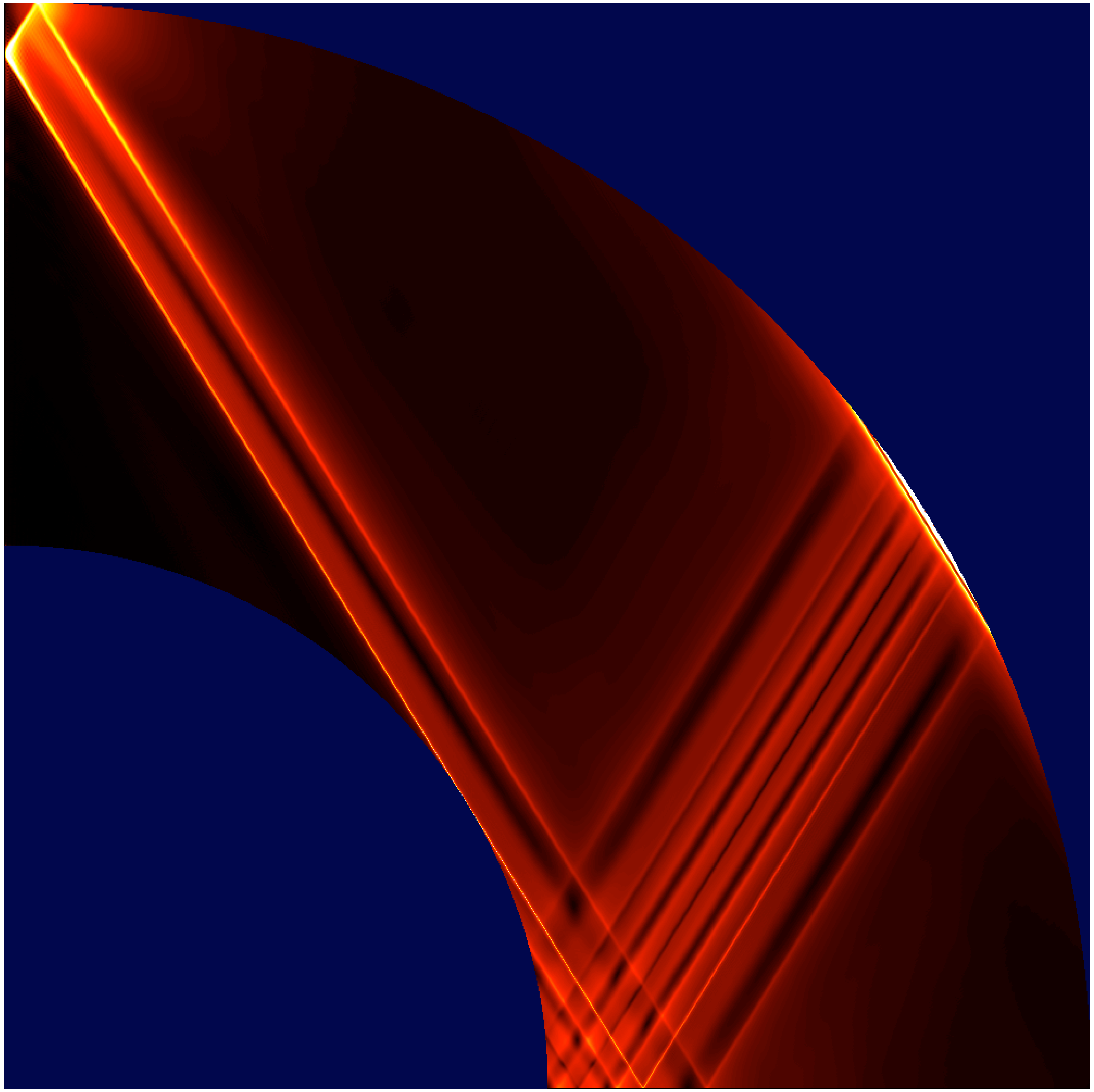
  or more easily in older or slightly more massive stars

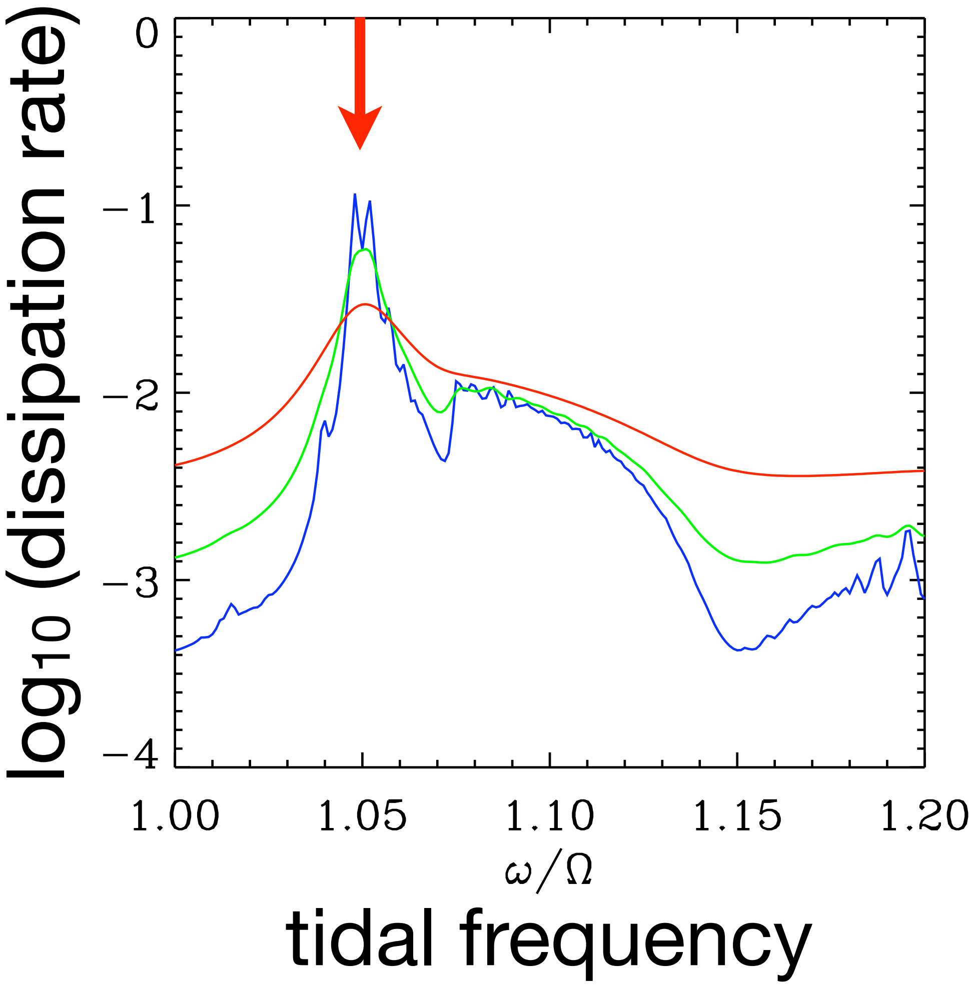
- If this occurs, planet is devoured within $1.4\,{\rm Myr} \left(\frac{M_{\rm p}}{M_{\rm J}} \right)^{-1} \left(\frac{P_{\rm orb}}{\rm day} \right)^{7.1}$

- Advancing critical layer could in principle be initiated by gradual radiative damping of waves of lower amplitude, but differential rotation may be erased by competing mechanisms

- More massive stars: Goldreich & Nicholson (1989)

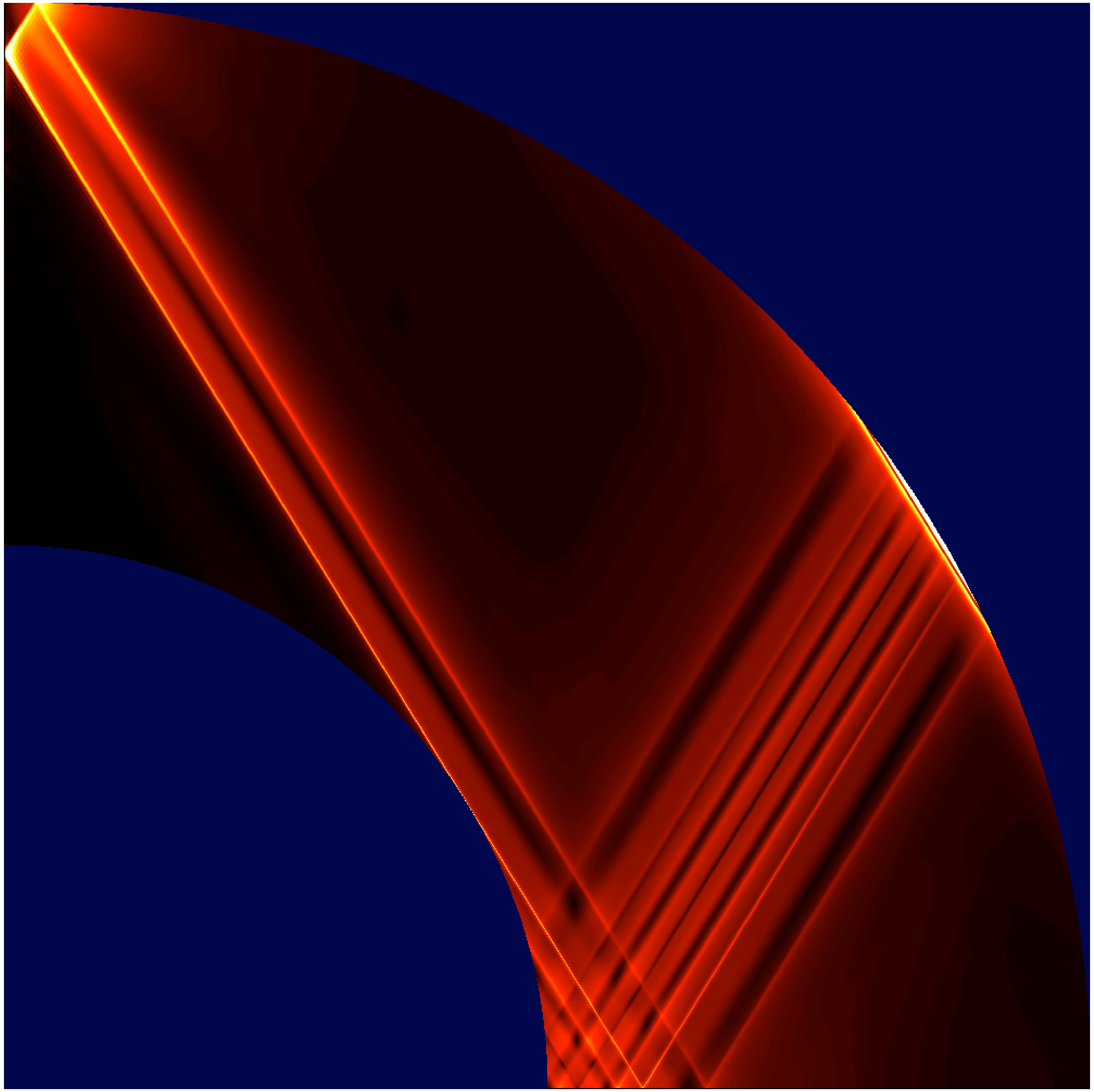
Tidally forced inertial waves and zonal flows

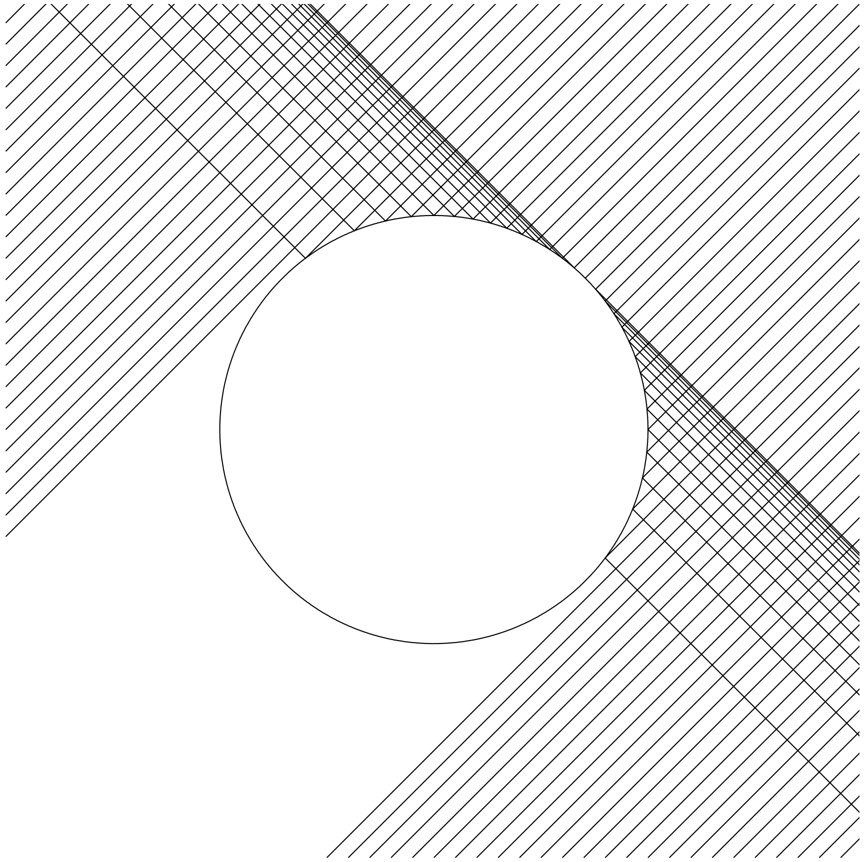




Ogilvie 2009

Tidally forced inertial waves and zonal flows

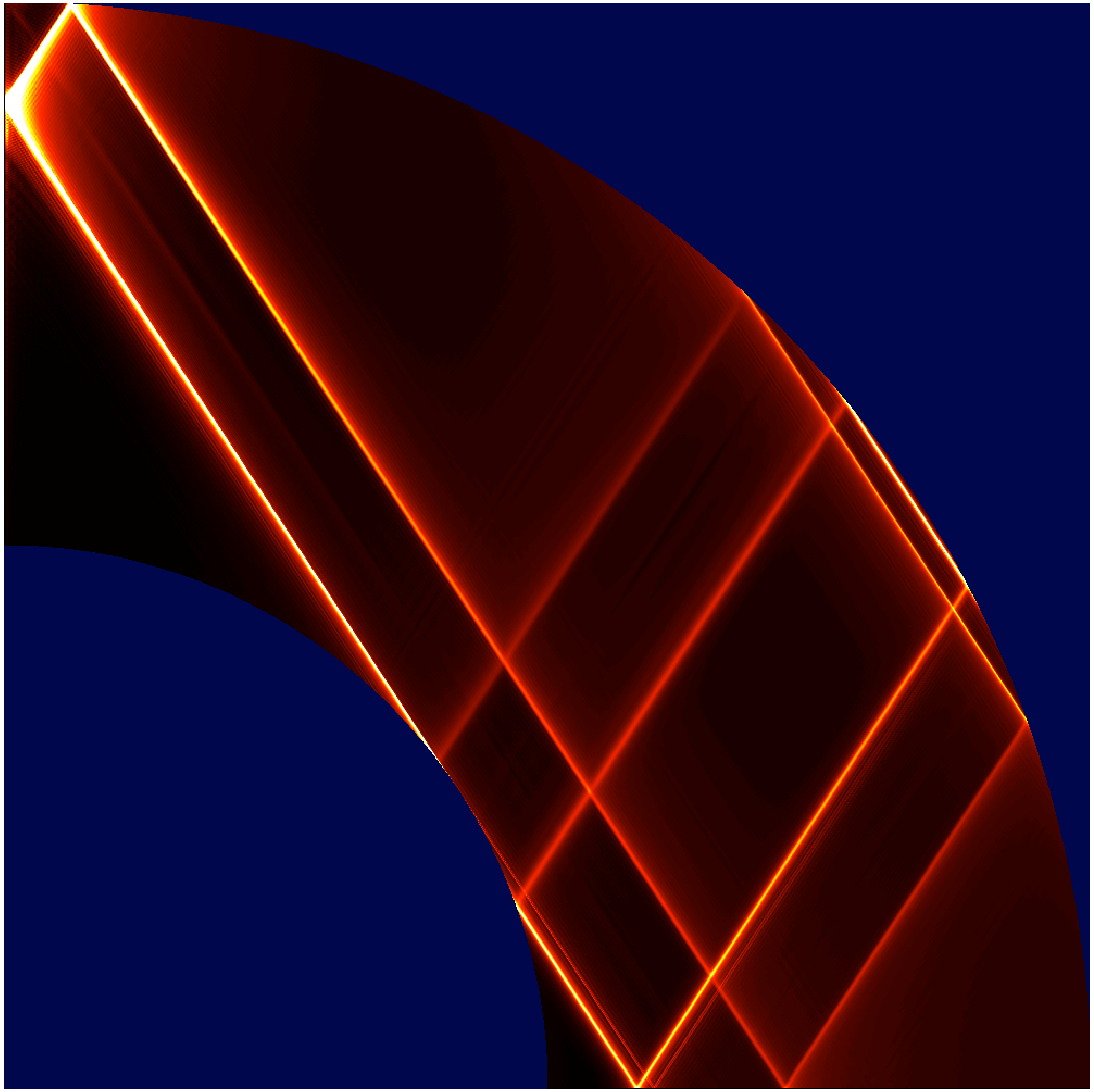


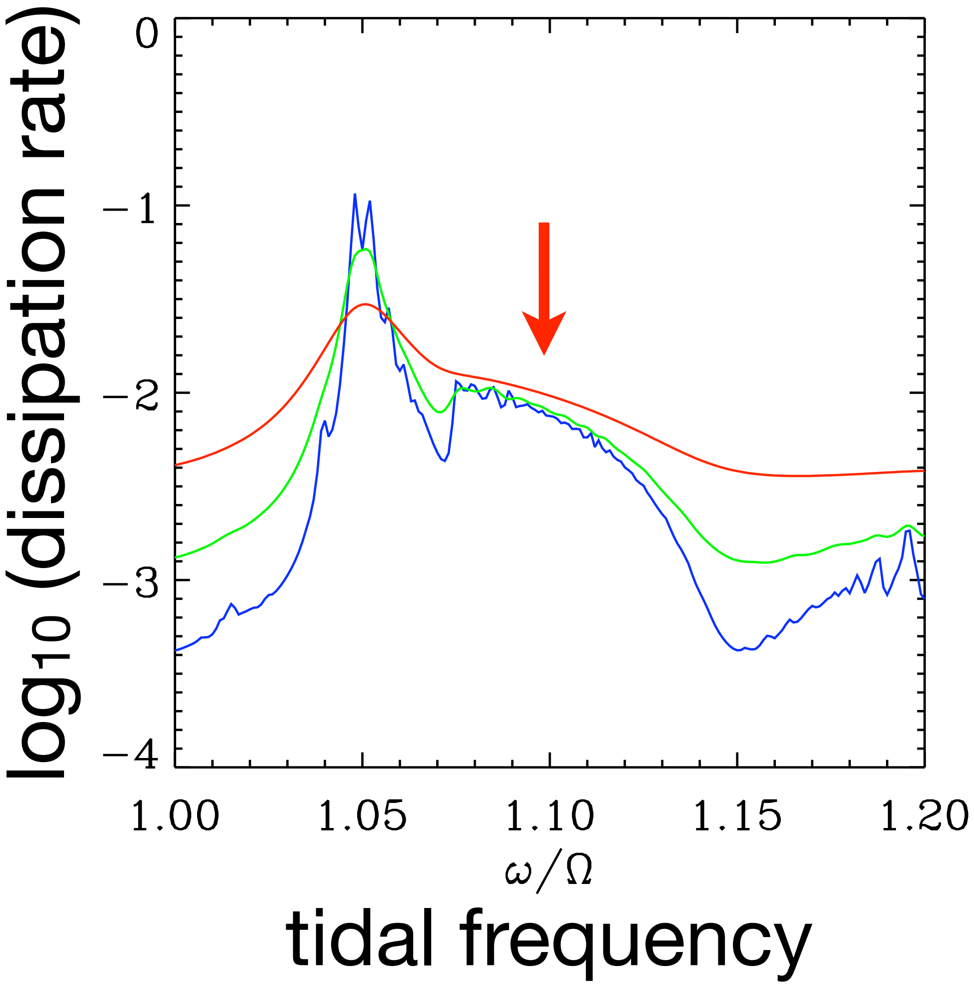


critical latitude singularity

Ogilvie 2009

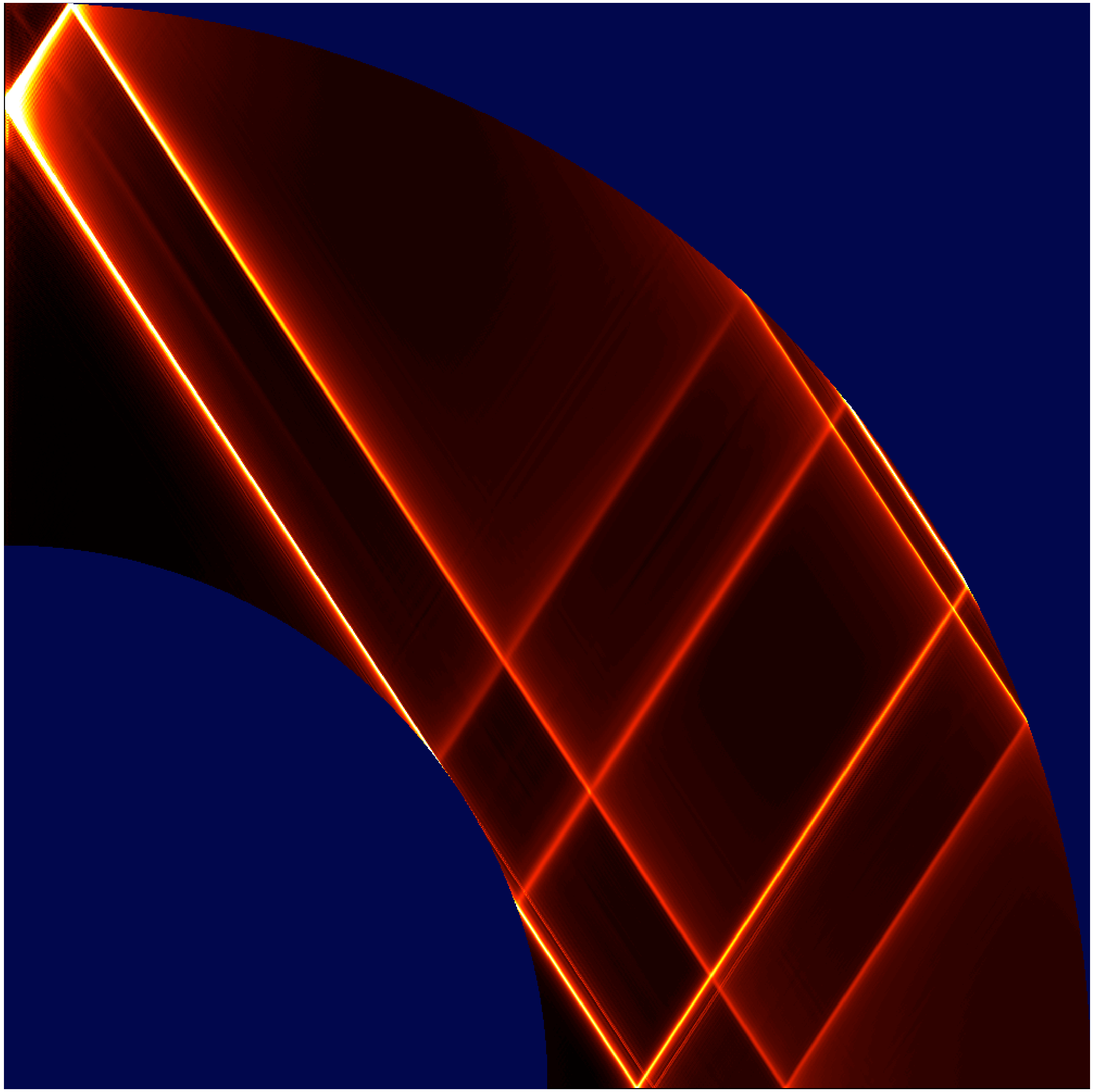
Tidally forced inertial waves and zonal flows

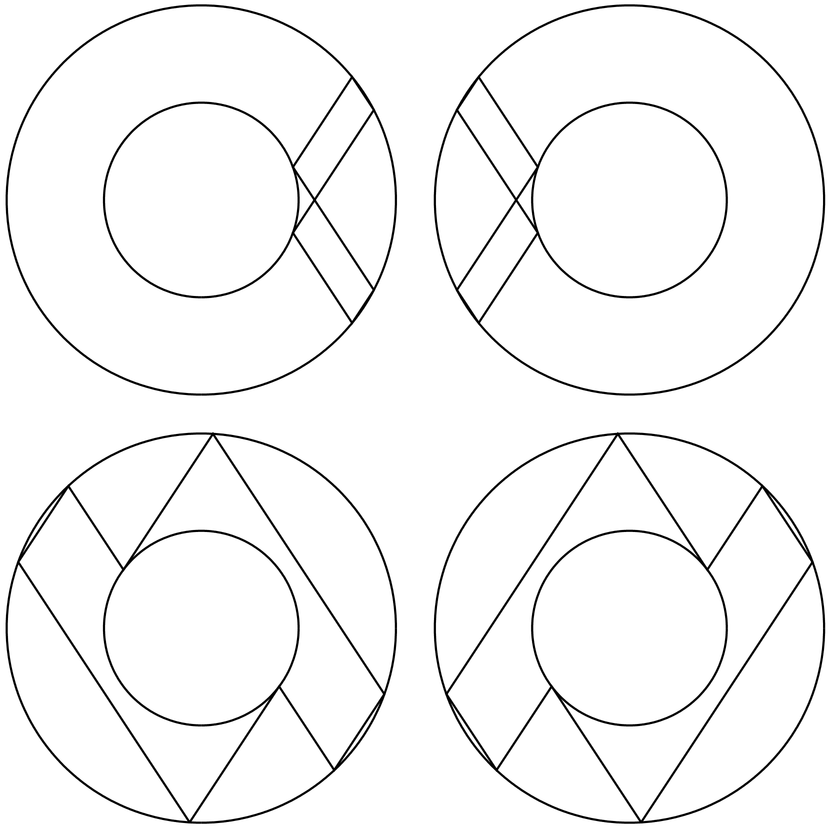




Ogilvie 2009

Tidally forced inertial waves and zonal flows

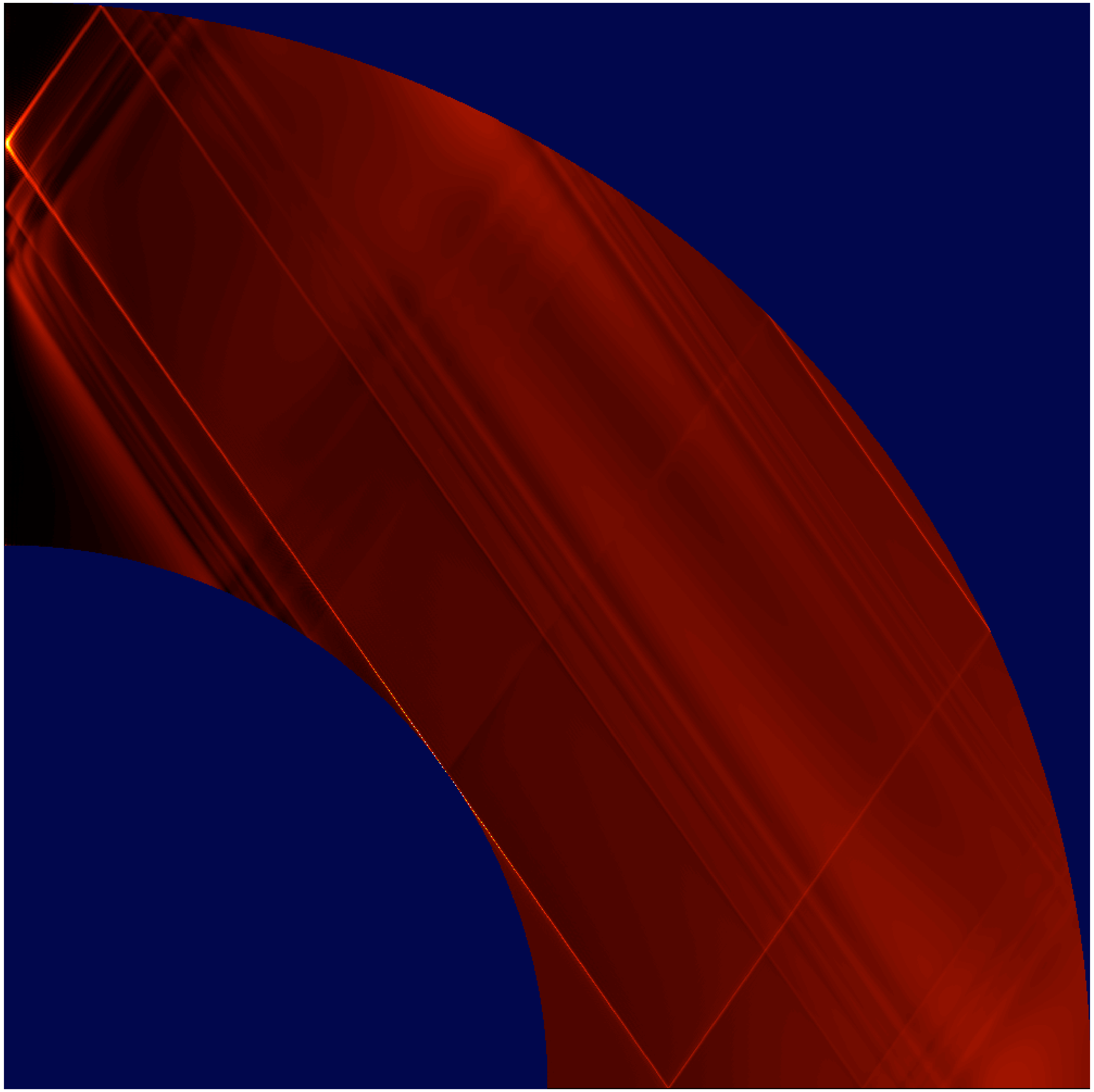


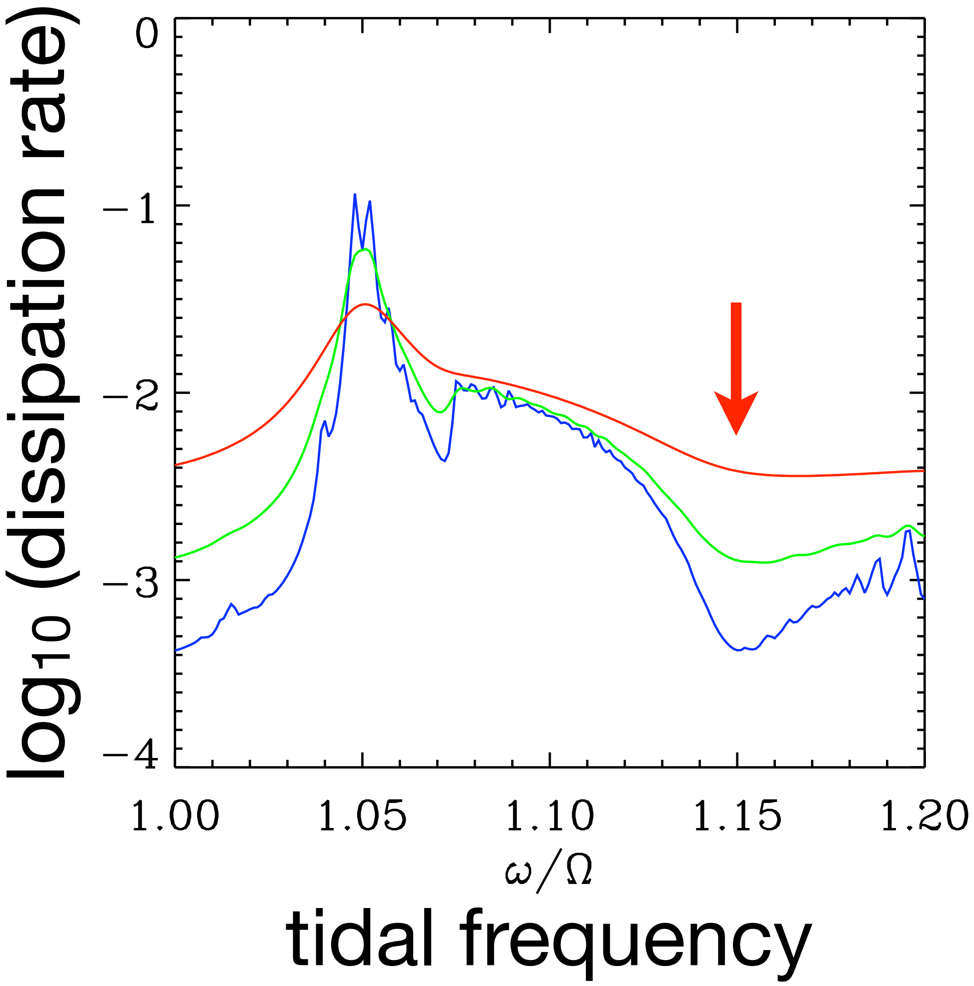


wave attractors

Ogilvie 2009

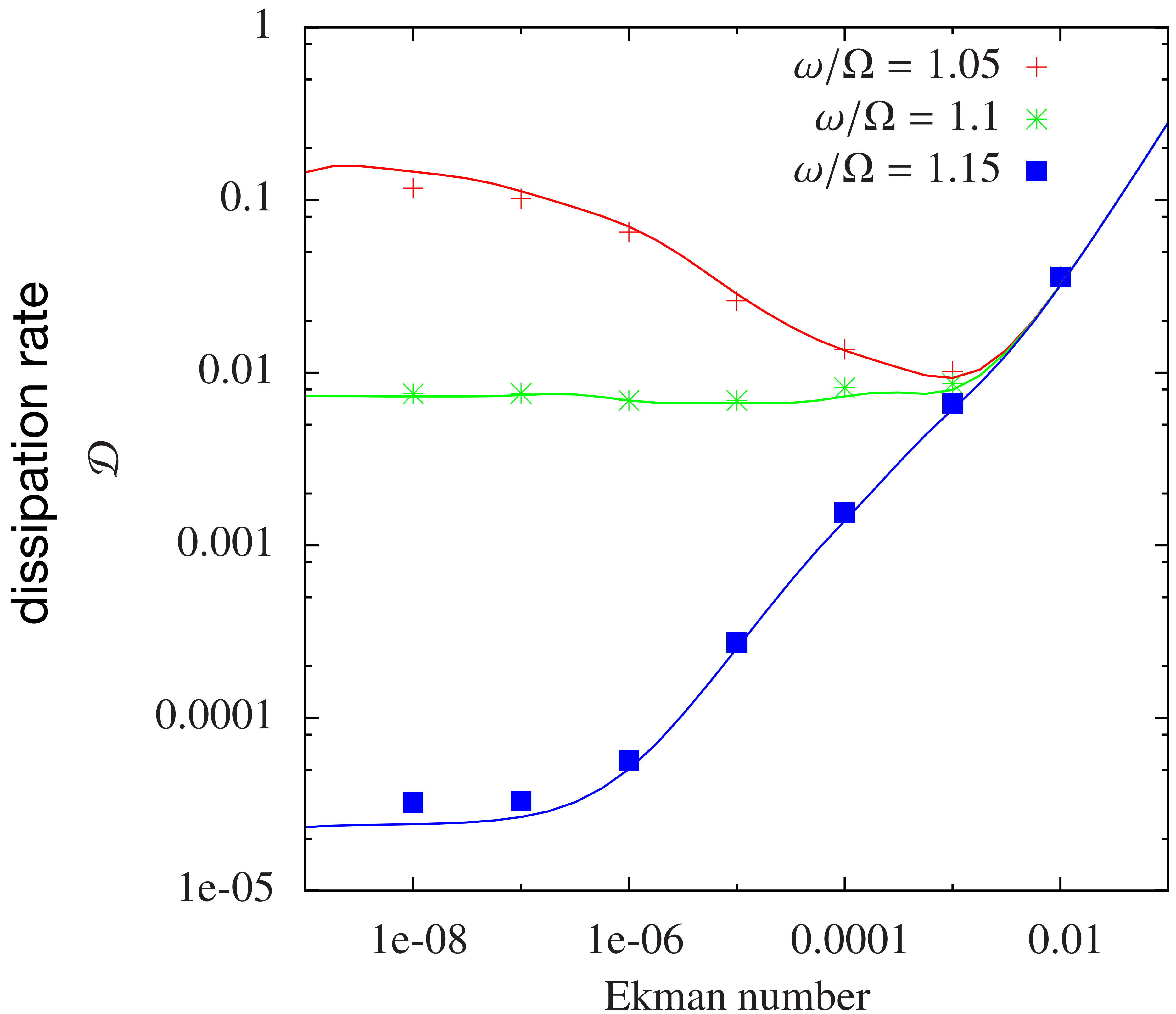
Tidally forced inertial waves and zonal flows





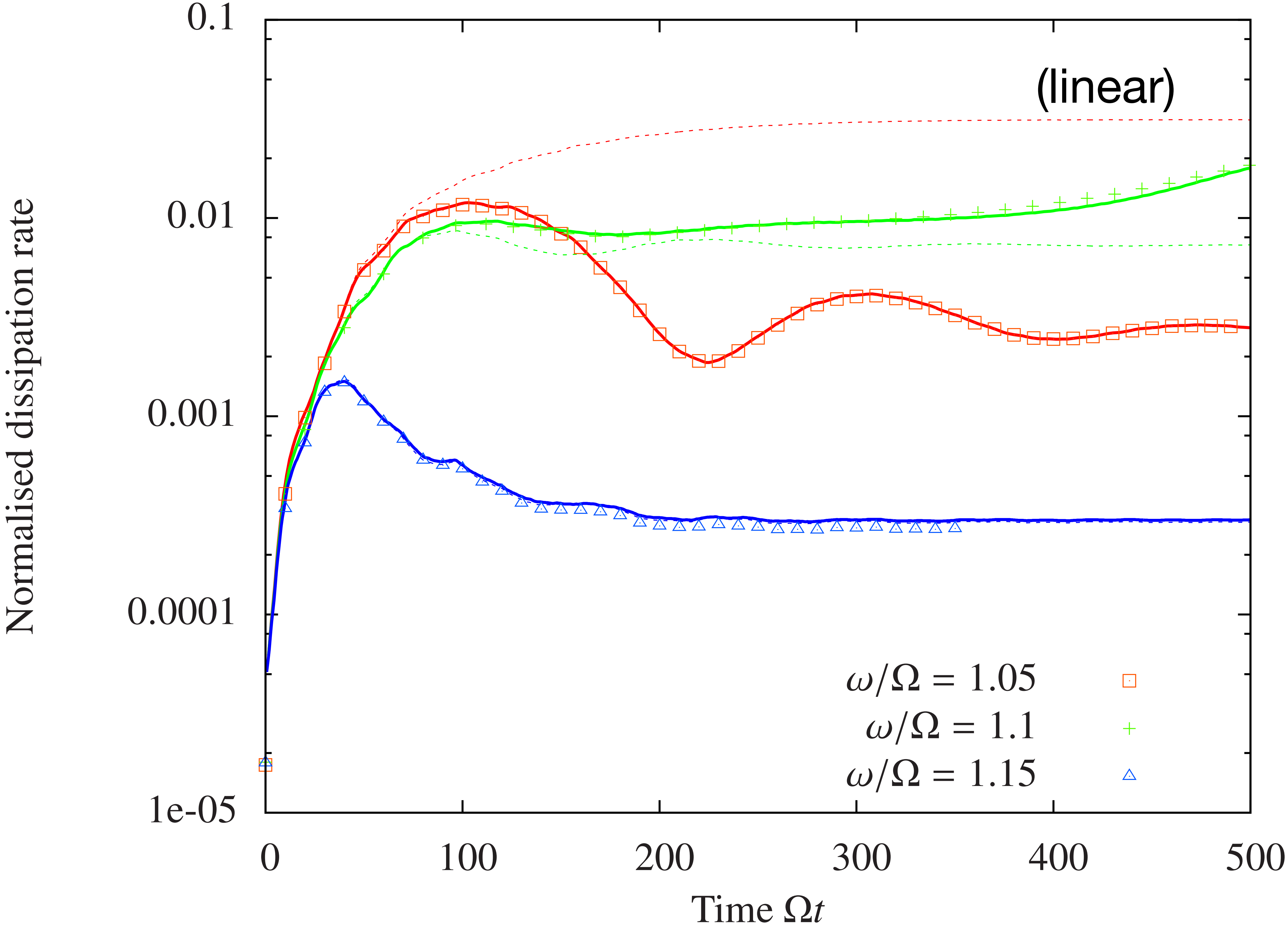
Ogilvie 2009

Tidally forced inertial waves and zonal flows



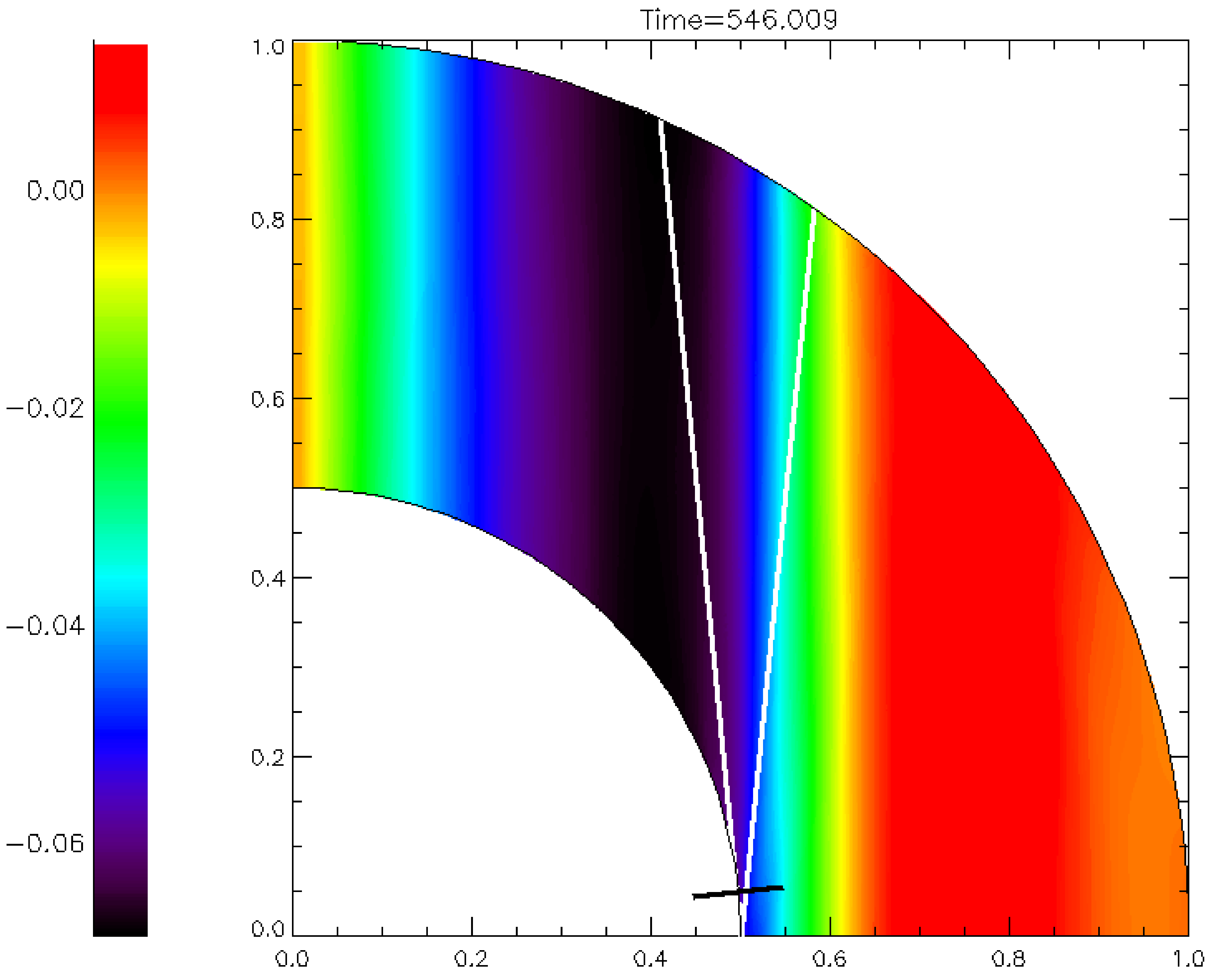
Favier+ 2014

Tidally forced inertial waves and zonal flows



Favier+ 2014

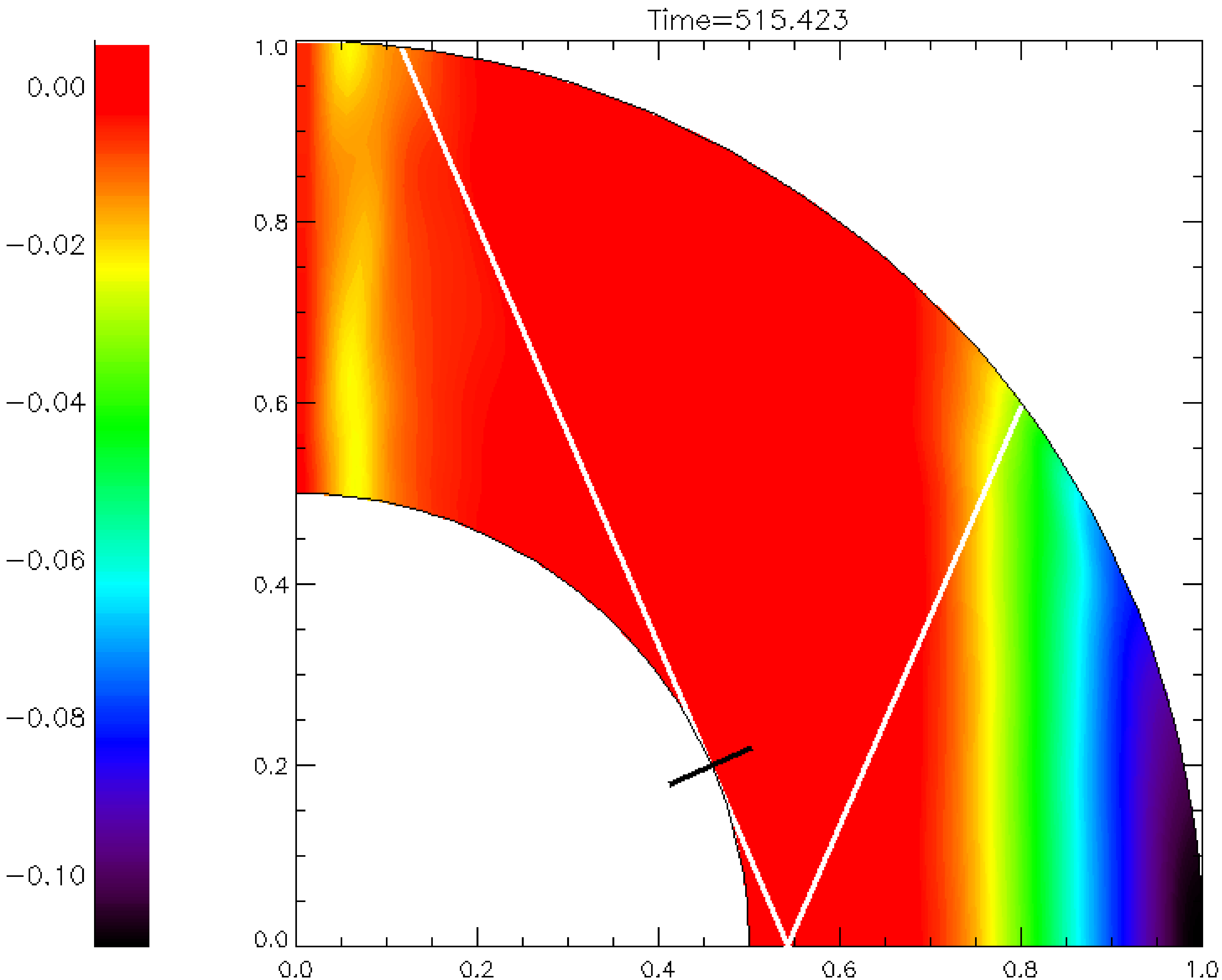
Tidally forced inertial waves and zonal flows



$\omega/\Omega = -0.2$

Favier+ 2014

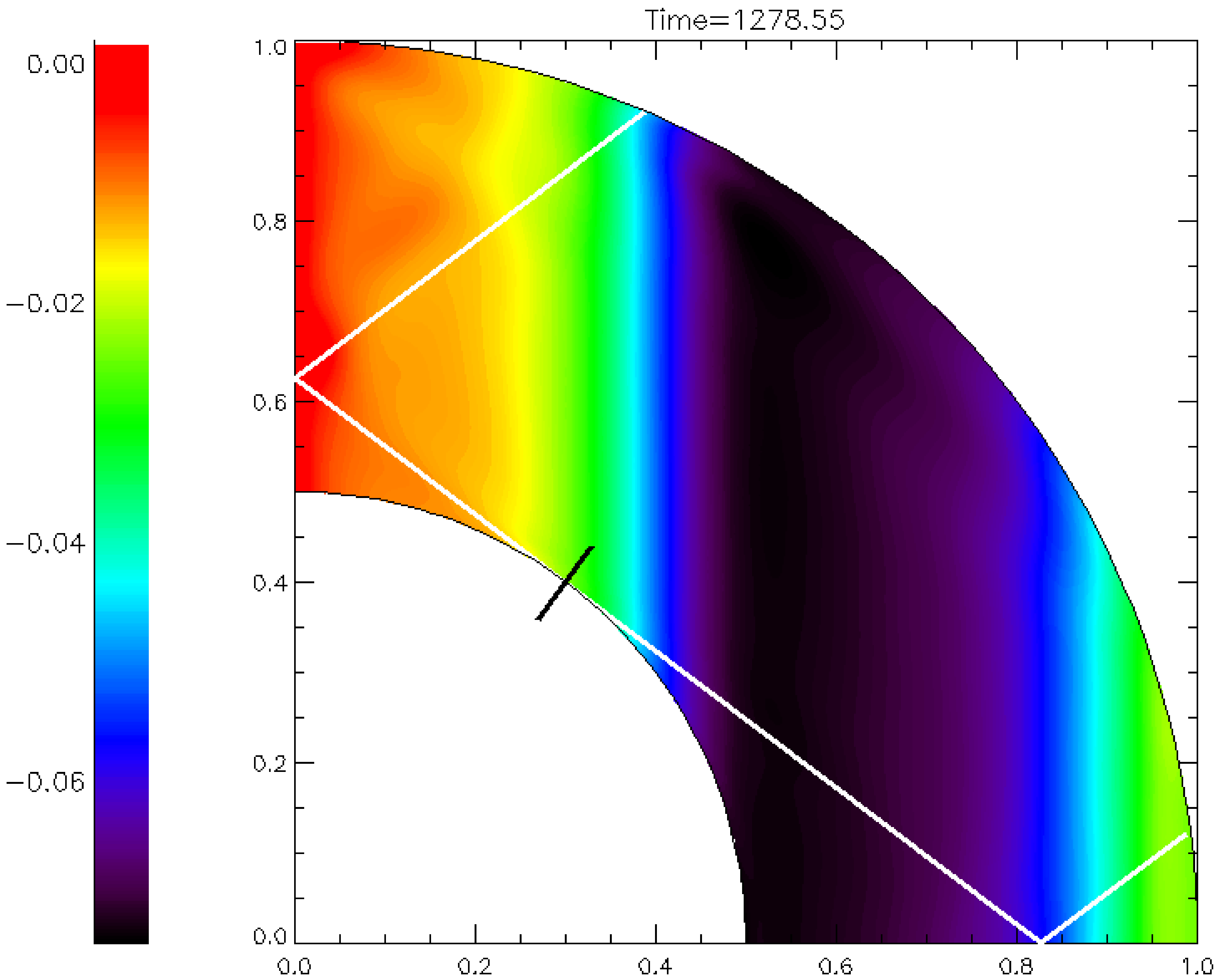
Tidally forced inertial waves and zonal flows



$\omega/\Omega = -0.8$

Favier+ 2014

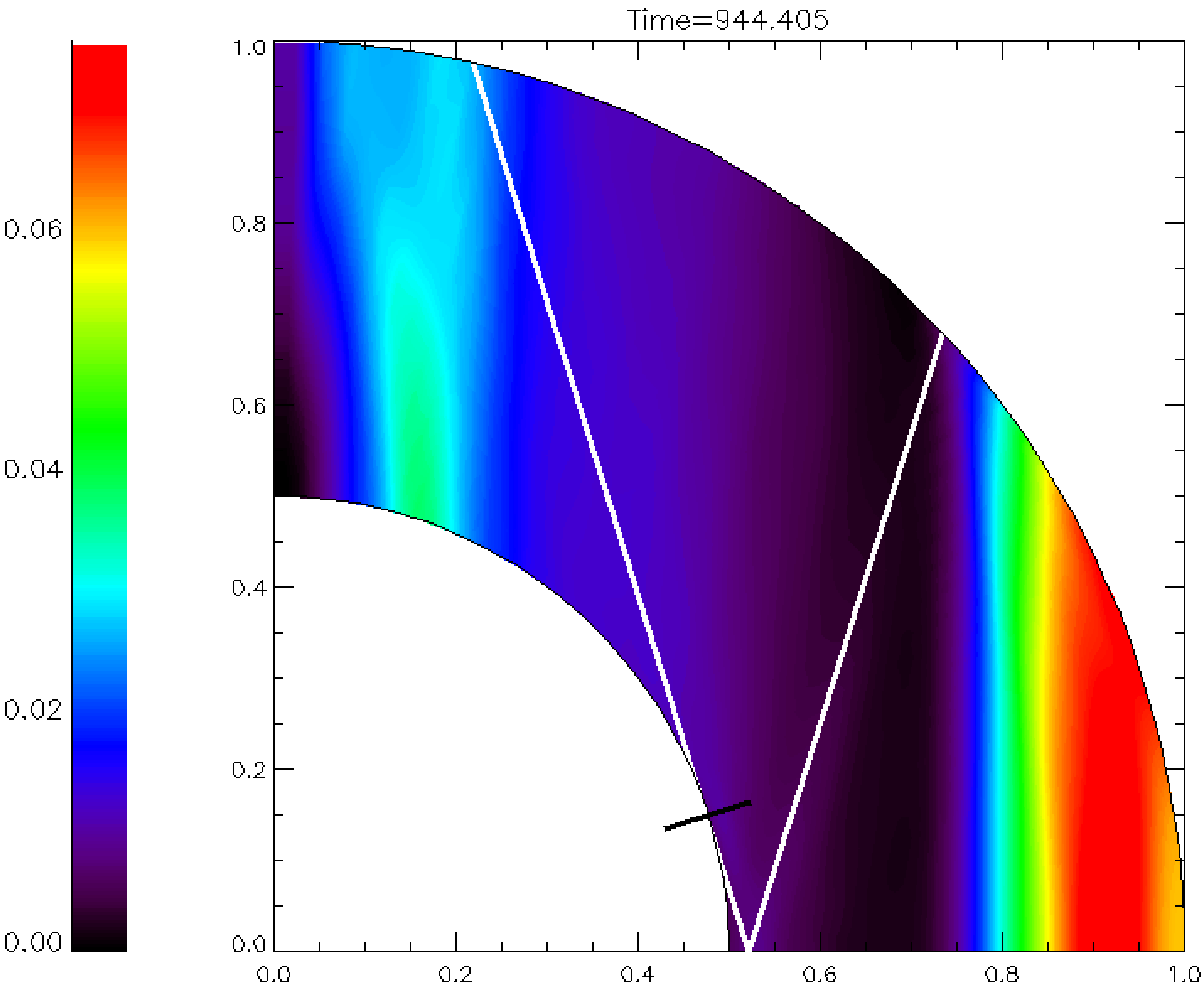
Tidally forced inertial waves and zonal flows



$\omega/\Omega = -1.6$

Favier+ 2014

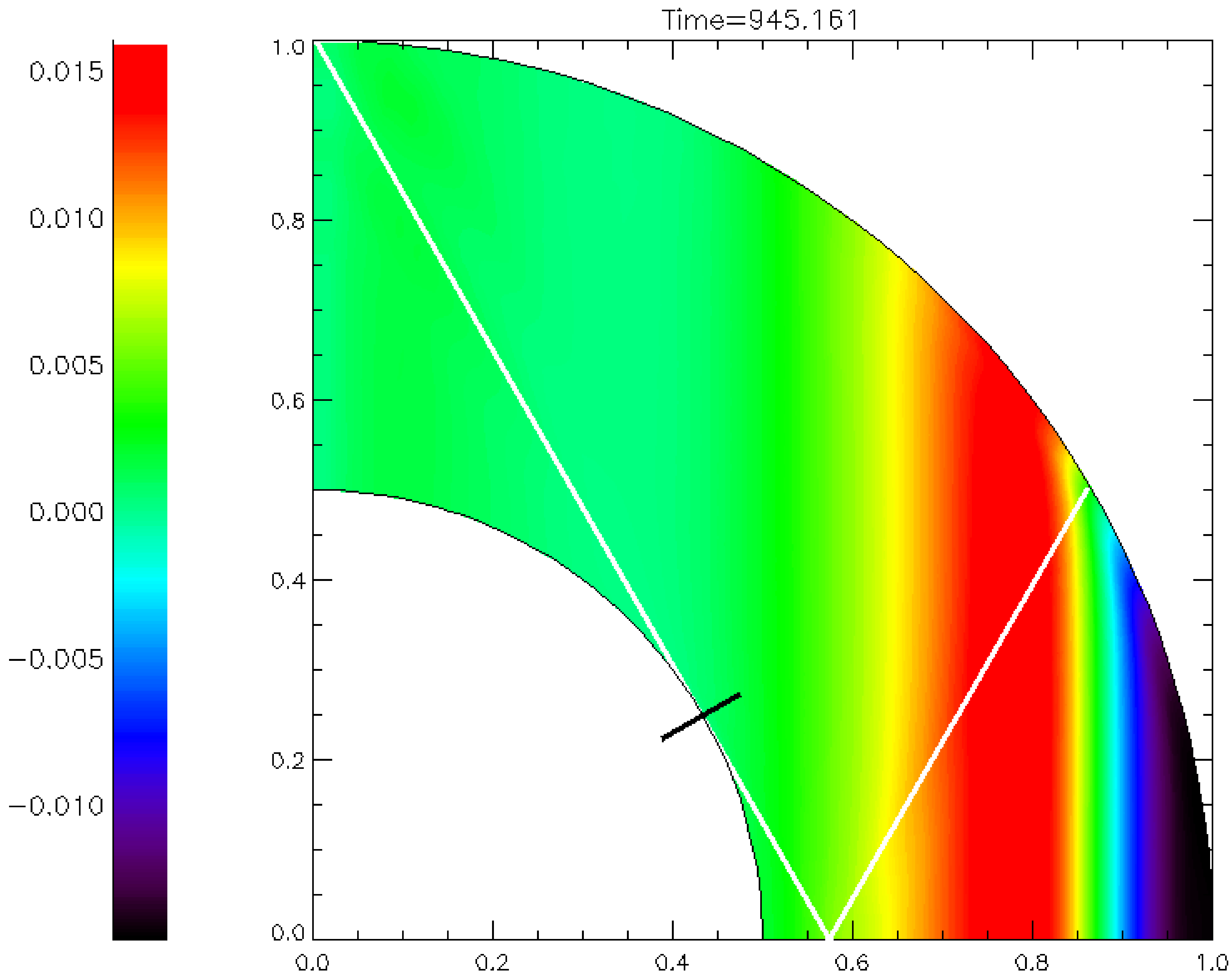
Tidally forced inertial waves and zonal flows



$\omega/\Omega = 0.6$

Favier+ 2014

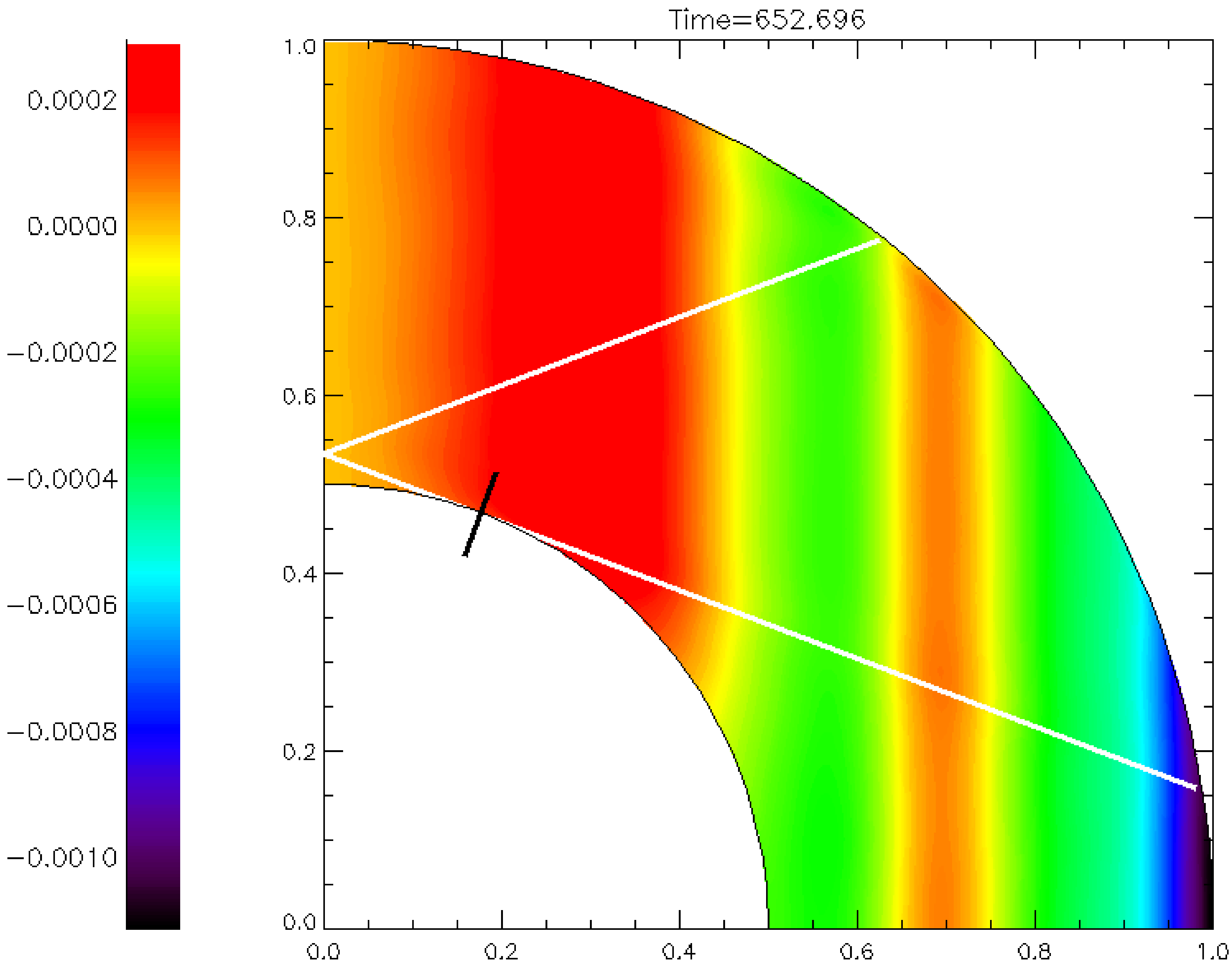
Tidally forced inertial waves and zonal flows



$\omega/\Omega = 1.0$

Favier+ 2014

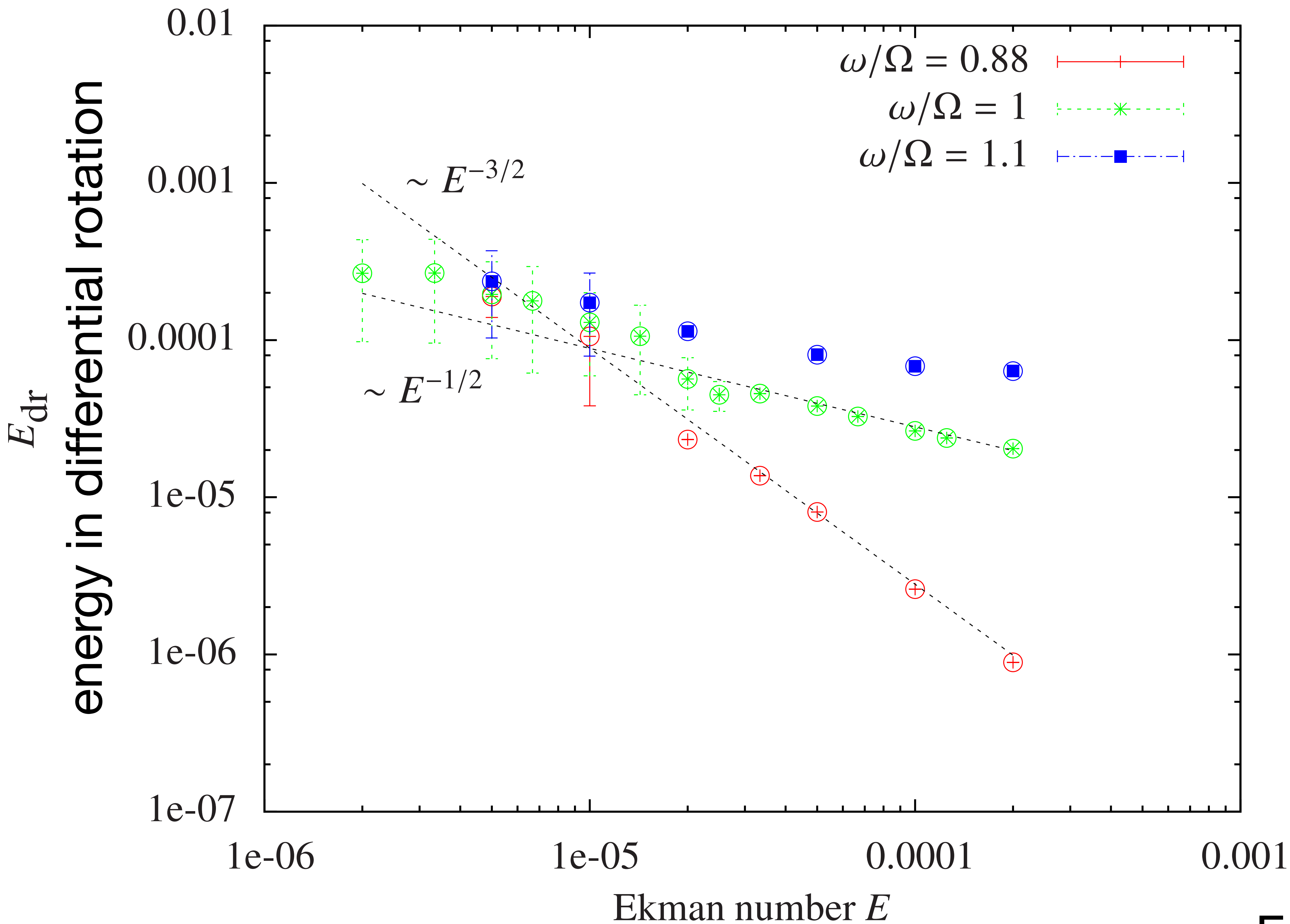
Tidally forced inertial waves and zonal flows



$$\omega/\Omega = 1.87$$

Favier+ 2014

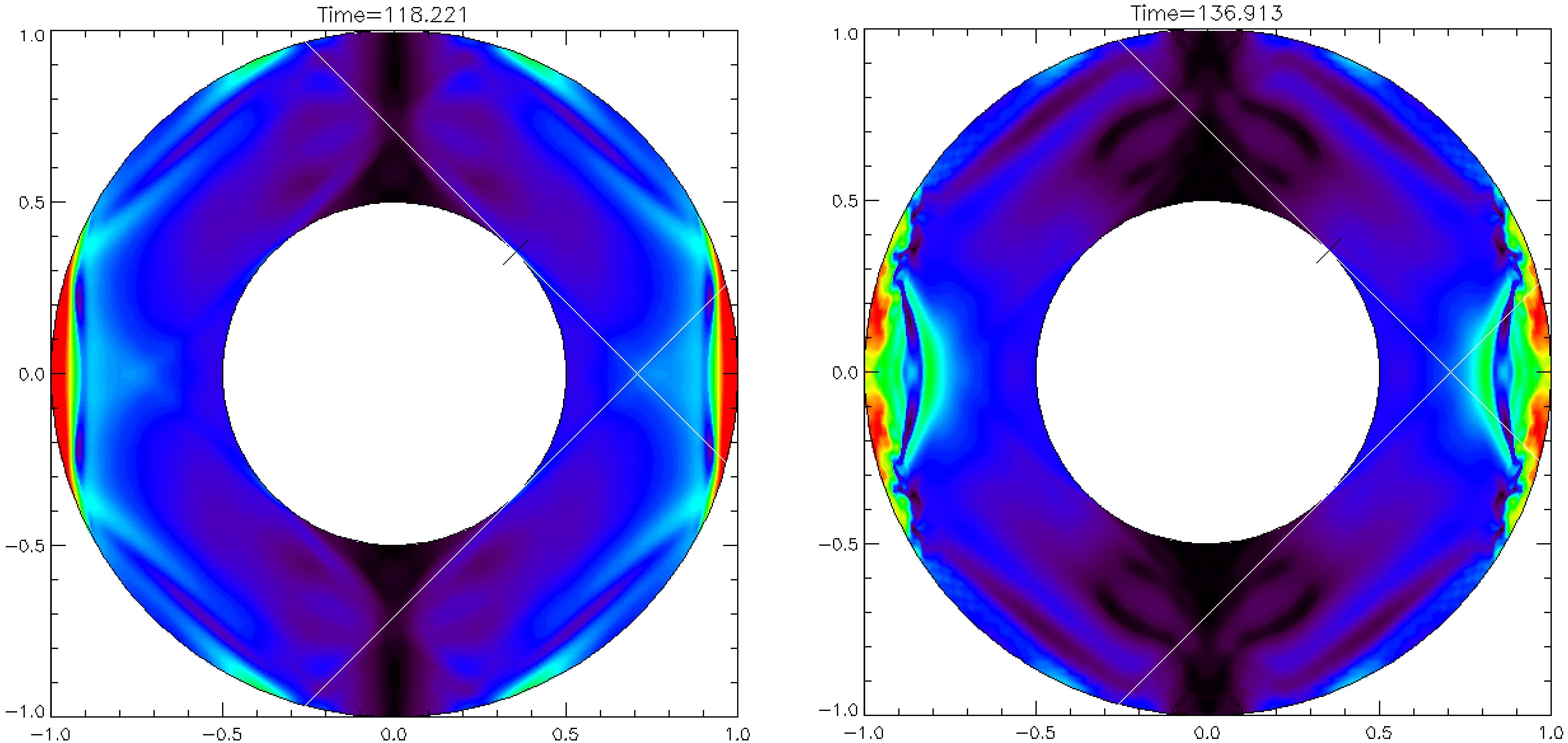
Tidally forced inertial waves and zonal flows



Favier+ 2014

Tidally forced inertial waves and zonal flows

- Instability of zonal flows

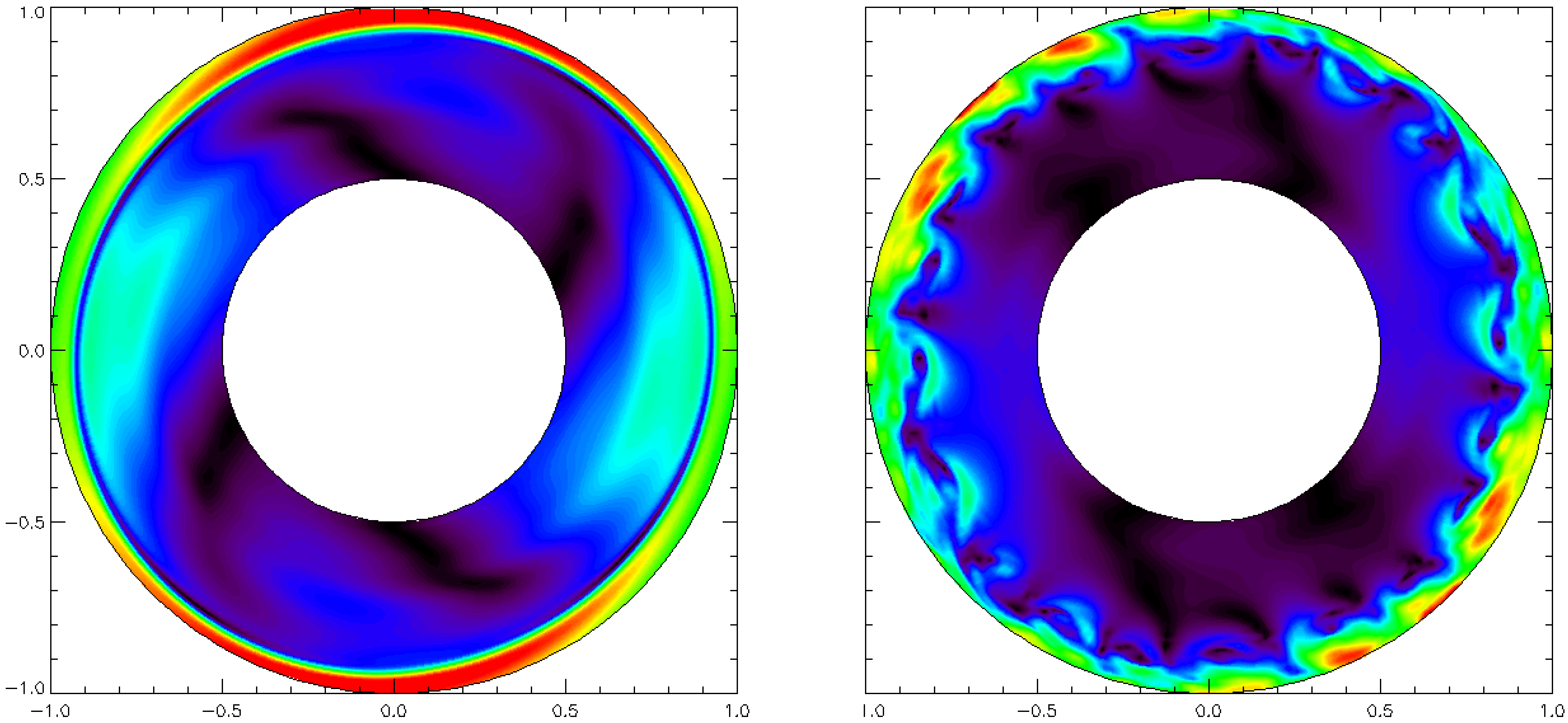


meridional plane

Favier+ 2014

Tidally forced inertial waves and zonal flows

- Instability of zonal flows



equatorial plane

Favier+ 2014

Summary

- Waves in discs: slow corotational dynamics involving mean flows determines torques and hence evolution of planetary orbits
- Localized zonal flows or vortices emerge from turbulence in a spatially homogeneous model
- In warped and eccentric discs internal waves are destabilized and their stresses may control the evolution of the disc
- Internal gravity waves are generated in stars by tidal forcing and convection
- Breaking of tidally forced gravity waves can lead to destruction of the planetary companion
- Interplay between tidally forced inertial waves and zonal flows is more complicated and merits further investigation